How to choose the ADC resolution for short range low power communication?

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***Abstract*— We consider the maximization of the energy efficiency when communicating over a noisy *single-input single-output* channel, taking into account the transmit power as well as the power consumption of the analog-to-digital converter (ADC). This analysis is of interest in the context of energy constrained short range communication where low power, low cost and small size are key requirements (e.g. standard IEEE 802.15.4). In fact, the transmit power in such applications become smaller and reaches values in the order of the conversion or the processing power. Using an appropriate information-theoretic framework we show that the analog-to-digital converters (ADCs) for short range communication should be low-resolution, in order to reduce the overall power consumption. In addition we derive the optimal operating bit-resolution and signal-to-noise ratio (SNR) as function of the path-loss (i.e. the communication distance).**

I. Introduction

In his famous work [1], Shannon has shown that the maximal achievable rate of an AWGN channel with given transmit power P_T and bandwidth B is given by

$$R = B\log_2(1+\mathrm{SNR}) = B\log_2(1+\frac{\alpha P_T}{N_0\cdot B}) \tag{1}$$

where α is the radio path-gain [1] and N_0 is the one-sided noise spectral level. Since the rate function is monotonically increasing with bandwidth B, the minimum received signal energy per information bit is obtained when taking the bandwidth to infinity

$$\left[\frac{E_b}{N_0}\right]_{\min} = \lim_{B\to\infty}\frac{\alpha P_T}{N_0\cdot R} = \ln 2. \tag{2}$$

Obviously, since there is no penalty from taking the bandwidth to infinity, the maximum power-efficiency is obtained at infinite bandwidth. Besides it is assumed that the receiver has access to the channel data with infinite precision. This classical information theoretical approach was motivated by long range communication and thus neglects the conversion and processing power. However, when communicating over smaller distances using energy-constrained devices (e.g. sensor networks or on-chip communication), the transmit power may be comparable to the conversion or processing power. The relative simplicity of realizing and integrating functions digitally makes it desirable, to move the analog-to-digital interface further towards the antenna. Thus, the ADC may have a strong impact on the total energy consumption. Its complexity grows with the resolution b and the bandwidth B and it heavily affects the complexity of the following digital signal processing, e.g. the required memory size. In fact it has been observed that new ADC architectures like pipelined ADCs are thermal noise limited and thus their minimum possible power is proportional to $N_0\cdot 2^{2b}\cdot f_s$, where f_s is the sampling frequency [2]. In other words, under Nyquist rate sampling, the power needed for converting a complex signal with bandwidth B is [2]

$$P_{ADC} = 2\cdot c\cdot N_0\cdot 2^{2b}\cdot B, \tag{3}$$

where c is a proportionality constant depending on the ADC architecture. This results into a trade-off between power consumption and performance loss due to quantization. It is therefore of interest to design the system parameters like bandwidth and ADC resolution in order to minimize the total power consumption including the transmission energy as well as the conversion energy.

In [3], [4], we study the effects of quantization from an information theoretical point of view for MIMO systems, where the channel is perfectly known at the receiver. It turns out that the loss in channel capacity due to coarse quantization is surprisingly small at low to moderate SNR. In [5] a lower bound on the achievable rate under output quantization by means of an MMSE approach has been derived, as follows

$$R \geq B\log_2(\frac{1+\mathrm{SNR}}{1+\mathrm{SNR}/\lambda_q}), \tag{4}$$

where λ_q is the signal-to-distortion ratio related to the ADC resolution b. This bound is tight at low SNR and its derivation does not assume uncorrelated additive quantization noise. In order to maximize λ_q, an AGC-circuit is placed before the ADCs, which scales the noisy inphase and quadrature signals by a factor such that the ADCs are fed an optimal input power. In this case, $\lambda_q \propto 2^{2b}$ and we can approximate the rate under finite resolution b as

$$R \approx B\log_2(\frac{1+\mathrm{SNR}}{1+\mathrm{SNR}\cdot 2^{-2b}}), \tag{5}$$

which is consistent with the fact that the achievable rate with infinite P_T is $2bB$. ADC resolution has been commonly chosen, such that the ADC distortion noise is about 10dB below the overall noise level. However such an approach is inappropriate for designing low power systems.

Motivated by these results and using energy consumption models found in the literature, we aim to jointly minimize the transmission and conversion energy with respect to the different system parameters (resolution, bandwidth, operating SNR). We show that low resolution ADCs are optimal for short range communication in terms of overall energy efficiency.

[1] Although $\alpha \leq 1$, it is more convenient to refer to it as path-gain, i.e. the inverse path-loss ratio.

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The optimization is performed in two steps. First, we derive the optimal resolution and operating SNR under separate ADC and transmission power constraints, i.e. P_{ADC} and P_T, in Section II. Then, in Section III, we consider a combined power minimization problem to find out the fraction of power that have to be optimally allocated to the ADC as function of the communication distance. Finally, in Section IV, some numerical results are presented.

II. Optimizing the Resolution for fixed Powers

Let us first consider fixed ADC and transmission powers. We aim to maximize the rate expression (5) with respect to b. Using the substitution $\mathrm{SNR} = \alpha P_T/(N_0 B)$ and (3) we rewrite (5) as

$$\begin{aligned} R &\approx \frac{P_{ADC}}{2cN_0 2^{2b}} \log_2\left(\frac{\frac{P_{ADC}}{2c\alpha P_T} + 2^{2b}}{\frac{P_{ADC}}{2c\alpha P_T} + 1}\right) \\ &= \frac{\alpha P_T}{N_0}\frac{a}{x}\log_2\left(\frac{a+x}{a+1}\right), \end{aligned} \tag{6}$$

with the substitutions

$$x = 2^{2b} \quad \text{and} \quad a = \frac{P_{ADC}}{2c\alpha P_T}. \tag{7}$$

We note that the variable a is related to the ADC-to-receive power ratio, and it will play a crucial role throughout this work. To obtain the optimal resolution we compute the first derivative of R with respect to x

$$\frac{\mathrm{d}R}{\mathrm{d}x} = \frac{\alpha P_T}{N_0}\frac{a}{\ln 2}\left(\frac{1}{x(a+x)} - \frac{\ln(\frac{a+x}{a+1})}{x^2}\right). \tag{8}$$

This admits one zero that is greater than 1, given by

$$x_{\mathrm{opt}} = -\frac{a}{W(-\frac{a}{\mathrm{e}(a+1)})} - a, \tag{9}$$

where $W(\cdot)$ denotes the Lambert function verifying the identity $W(z)\mathrm{e}^{W(z)} = z$. Using the following expansion of the Lambert function $W(z)$ around $z = -\mathrm{e}^{-1}$ [6]

$$W(z) = -1 + \sqrt{2\mathrm{e}z + 2} - \frac{2}{3}(\mathrm{e}z + 1) + o(\mathrm{e}z + 1), \tag{10}$$

we get a good approximation to x_{opt}, for all $a \geq 0$ (see also Fig. 1)

$$x_{\mathrm{opt}} \approx \sqrt{2a + (\mathrm{e} - 4/3)^2} + 4/3. \tag{11}$$

Next, a back-substitution yields the optimal ADC resolution

$$\begin{aligned} b_{\mathrm{opt}} = \frac{1}{2}\log_2(x_{\mathrm{opt}}) &= \frac{1}{2}\log_2\left(-\frac{a}{W(-\frac{a}{\mathrm{e}(a+1)})} - a\right) \\ &\approx \frac{1}{2}\log_2(\sqrt{2a + (\mathrm{e} - 4/3)^2} + 4/3). \end{aligned} \tag{12}$$

In addition, the optimal operating SNR turns to be

$$\mathrm{SNR}_{\mathrm{opt}} = \frac{\alpha P_T}{N_0 B} = \frac{x_{\mathrm{opt}}}{a} = -\frac{1}{W(-\frac{a}{\mathrm{e}(a+1)})} - 1. \tag{13}$$

Consequently the rate attains a maximal value of

$$R_{\max} = -\frac{\alpha P_T}{N_0}\frac{W(-\frac{a}{\mathrm{e}(a+1)})}{\ln 2}, \tag{14}$$

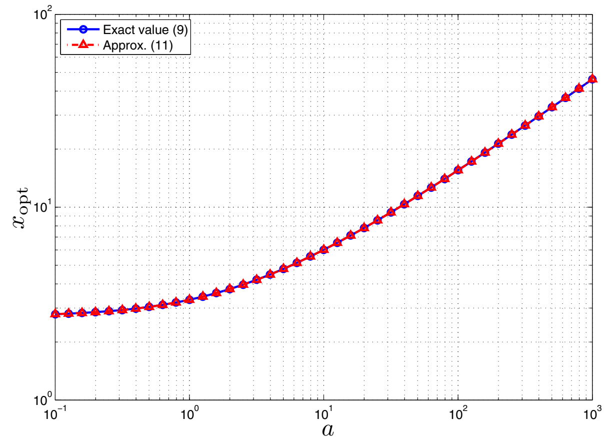

Fig. 1. x_{opt} as function of a and its approximation.

and we get the bit energy E_b over N_0 as

$$\begin{aligned} \left[\frac{E_b}{N_0}\right]_{\mathrm{opt}} &= \frac{\alpha P_T}{N_0 R_{\mathrm{opt}}} = -\frac{\ln 2}{W(-\frac{a}{\mathrm{e}(a+1)})} \\ &\approx \ln 2\left(1 + \sqrt{\frac{2}{a} + \left(\frac{\mathrm{e} - 4/3}{a}\right)^2} + \frac{4}{3a}\right), \end{aligned} \tag{15}$$

which indicates that the minimum $[E_b/N_0]_{\min} = \ln 2$ is only achievable with infinite a.

III. Total Power Minimization

Now we consider the total power spent in the ADC and in the transmission,

$$P_{total} = P_T + P_{ADC}. \tag{16}$$

We can reuse substitution (7) to have the fraction of power dedicated for transmission [2]

$$P_T = \frac{P_{total}}{1 + 2c\alpha \cdot a}. \tag{17}$$

We aim to minimize the total power consumed for given target rate, a problem that might be interesting for low power applications. For that we substitute (17) into (14)

$$R_{\max} = -\frac{\alpha P_{total}}{N_0 \ln 2}\frac{W(-\frac{a}{\mathrm{e}(a+1)})}{1 + 2c\alpha \cdot a}. \tag{18}$$

Evidently, to minimize the total power for given target rate, we should minimize the following objective function with respect to a

$$f(a) = -\frac{1 + 2c\alpha \cdot a}{W(-\frac{a}{\mathrm{e}(a+1)})}. \tag{19}$$

Fortunately, this optimization does not depend on the special target rate. However, we were not able to find a closed form solution for a. Thus we make use of the following approximation as done in (15)

$$f(a) \approx (1 + 2c\alpha \cdot a)\left(1 + \sqrt{\frac{2}{a}} + \frac{4}{3a}\right). \tag{20}$$

[2] It is also possible to include the power amplifier losses into P_T, by just substituting α by $\eta\alpha$ in all equations, where η is the power amplifier efficiency.

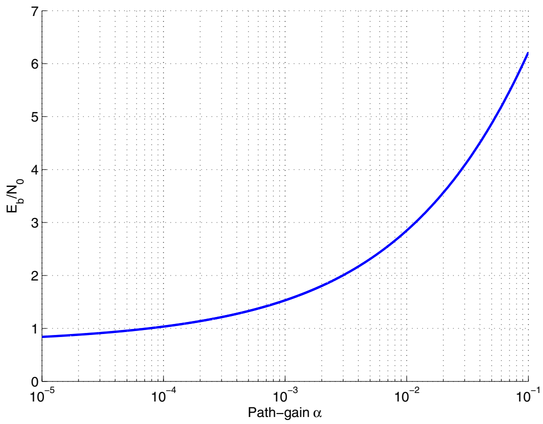

Fig. 2. Optimal transmit energy per bit vs. the path-gain for $c = 96$.

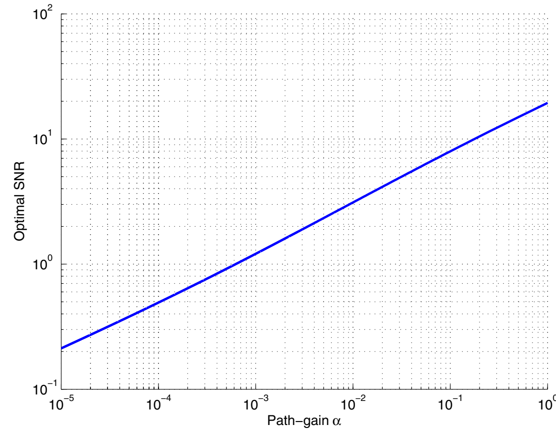

Fig. 3. Optimal operating SNR vs. the path-gain for $c = 96$.

Omitting computational details for ease of exposition, we deliver directly an approximate global optimizer of this function, which holds true for small α

$$a_{\text{opt}} \approx \frac{1}{2}(c\alpha)^{-\frac{2}{3}} + \frac{5}{9}(c\alpha)^{-\frac{1}{3}}. \tag{21}$$

Thus, the optimum ADC to transmit power ratio is given by

$$\left[\frac{P_{ADC}}{P_T}\right]_{\text{opt}} = 2c\alpha a_{\text{opt}} \approx (c\alpha)^{\frac{1}{3}} + \frac{10}{9}(c\alpha)^{\frac{2}{3}}. \tag{22}$$

As expected, the fraction of power that should be dedicated to the ADC becomes significant, as α increases, i.e. the path-loss decreases.

Finally the optimal value of the resolution b and the resulting SNR and E_b/N_0 can be obtained from (12), (13) and (15), respectively.

IV. Numerical Results

As example, let us take $c = 96$, which is yields the absolute Minimum Power of comparator-based ADCs [2].[3]. Fig. 2 and 3 illustrate the behavior of the optimal operating E_b/N_0 and SNR as function of the path-gain α, respectively. Observe that, only for very small α (long range communication), it is optimal to operate at an SNR close to 0 and E_b/N_0 attains its minimal value of $\ln 2$, as obtained from the classical approach.

Fig. 4 shows the behavior of the optimized conversion-to-transmission power ratio versus the path-gain α. As already mentioned, for large α, ADC power becomes significant compared to the transmit power.

Afterwards, the normalized combined energy per bit required to communicate across the noisy channel is depicted in Fig. 5. It is obtained from (18) as

$$\frac{P_{total}}{N_0 R_{\max}} = -\frac{\ln 2}{\alpha}\frac{1+2c\alpha a}{W(-\frac{a}{\mathrm{e}(a+1)})}, \tag{23}$$

and it is minimized when a takes the value from (21). Remarkably, even there is no path-loss ($\alpha = 1$), the required total energy per bit is quite large due to the ADC power consumption.

[3]Note that state-of-the art ADCs still exhibit much larger c values than this theoretical limit, which means that the following numerical results are rather optimistic regarding the ADC power consumption.

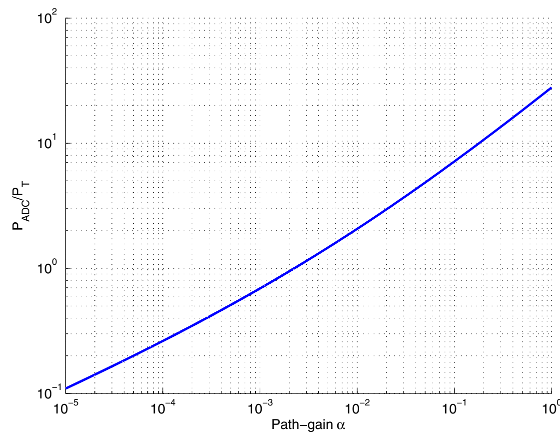

Fig. 4. Optimal ADC-to-transmit power ratio vs. the path-gain for $c = 96$.

Fig. 6 shows that the optimal resolution for short rage communication is indeed quite low and converges to the value

$$\lim_{c\alpha\to\infty} b_{\text{opt}} = \frac{1}{2}\log_2 \mathrm{e} \cong 0.7213. \tag{24}$$

On the other hand, for smaller α, it increases quite slowly, almost as

$$b_{\text{opt}} \approx -\frac{1}{6}\log_2(c\alpha). \tag{25}$$

This motivates us to consider the achievable rate with a fixed resolution $b = 1$, i.e. just a 1-bit ADC for each the real part and the imaginary part of the signal. From (6) we get the capacity for this case

$$R_{1-\text{bit}} \approx \frac{\alpha P_{total}}{4N_0}\frac{a}{1+2c\alpha\cdot a}\log_2(\frac{a+4}{a+1}). \tag{26}$$

Again, an approximate global optimizer $a_{\text{opt},1-\text{bit}}$ of this function can be found as follows

$$a_{\text{opt},1-\text{bit}} \approx \sqrt{\frac{5}{4c\alpha}}. \tag{27}$$

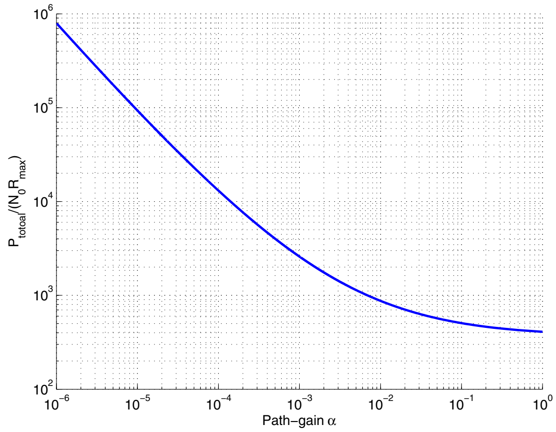

Fig. 5. Normalized combined energy per bit vs. the path-gain for $c = 96$.

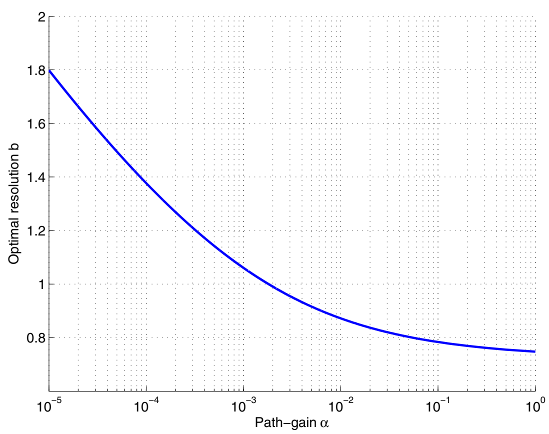

Fig. 6. Optimal ADC resolution vs. the path-gain for $c = 96$.

Thus, the optimum conversion-to-transmission power ratio is given by

$$\left[\frac{P_{ADC}}{P_T}\right]_{\text{opt},1-\text{bit}} = 2c\alpha a_{\text{opt},1-\text{bit}} \approx \sqrt{5c\alpha}, \tag{28}$$

and similarly, the optimal operating SNR reads as

$$\text{SNR}_{\text{opt},1-\text{bit}} = \frac{2^2}{a_{\text{opt},1-\text{bit}}} \approx 8\sqrt{\frac{c\alpha}{5}}. \tag{29}$$

The ratio of $R_{1-\text{bit}}$ to $R_{\max}$ computed with the respective optimized value of a is shown in Fig. 7. Clearly, the the 1-bit system nearly achieve the maximal rate for $\alpha > 10^{-5}$, which suggests that 1-bit ADCs may be a good choice for low power short range communication. Note that if a single bit hard-decision is used, the implementation of the all digital receiver is considerably simplified [7], [8], [9]. In particular, automatic gain control (AGC), linearity requirements for RF components and multipliers for signal correlation are no more necessary.

V. Conclusion

We presented a new channel capacity optimization framework that takes into account the resolution and the power

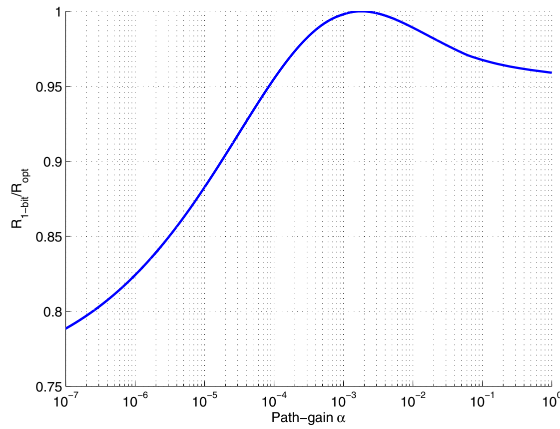

Fig. 7. Ratio of 1-bit rate (26) and maximal value (14) vs. the path-gain for $c = 96$ and fixed total power.

consumption of ADCs. Based on this, we showed that, in the context of low power short range communication, low-resolution (or maybe 1-bit) sampling performs adequately, while reducing total power consumption. Besides, even if the system is free of a bandwidth constraint, there is no advantage from taking the bandwidth to infinity, contrary to the result stated by the Shannon theory. We also believe that these results also hold for more general channel settings, e.g. MIMO channels. Additionally, taking into account the decoding energy would be an interesting extension of this work and could be a research topic for the future.

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