BIDIRECTIONAL RELAYING IN WIRELESS NETWORKS– IMPACT OF DEGREE OF COORDINATION

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ABSTRACT

The concept of bidirectional relaying is a key technique to improve the performance in wireless networks such as sensor, ad-hoc, and even cellular systems. It applies to three-node networks, where a relay node establishes a bidirectional communication between two other nodes using a decode-and-forward protocol. We assume that the communication is disturbed by unknown varying interference and analyze the impact of the degree of coordination. We show that the unknown variation of the interference has a dramatic impact on the communication. For traditional interference coordination it can lead to channels which completely prohibit any reliable communication. Anyhow, by allowing a relay-to-receivers coordination, communication can also be established in such situations where the traditional approach fails.

***Index Terms*—** Bidirectional Relaying, Wireless Network, Interference Coordination, Capacity

1. INTRODUCTION

To meet the performance targets of future wireless communication systems, there is the need to significantly improve the throughput and coverage. The use of relays is a promising approach, which is intensively discussed at the moment by the Third Generation Partnership Program's Long-Term Evolution Advanced (3GPP LTE-Advanced) group.

Due to practical constraints a relay cannot transmit and receive at the same time and frequency. Consequently, the relay has to allocate orthogonal resources which can be realized more efficiently, if bidirectional communication is considered [1], [2], [3]. In this work, we consider the scenario where a relay node establishes a bidirectional communication between two other nodes. This is known as the *bidirectional relay channel*.

The processing at the relay node usually classifies the relaying strategy. Here, we consider a two-phase decode-and-forward protocol. In the initial phase both nodes transmit their messages to the relay node. Since the relay decodes the messages, we end up with the classical multiple access channel. In the succeeding phase the relay broadcasts a re-encoded composition of them so that both nodes are able to decode the other's message using their own message as side information. This is the *bidirectional broadcast channel* (BBC) for which the optimal coding and transmit strategies are known [4]. Bidirectional relaying under channel uncertainty is analyzed in [5].

So far bidirectional relaying is studied as an isolated three-node network. Since it shows to have the potential to significantly enhance the performance, the next step is to consider bidirectional relaying within a wireless network. Inevitably, the communication is now disturbed by interference from other transmitting nodes as illustrated in Figure 1. For example in a cellular system the receiving nodes, especially at the cell edges, are confronted with interference from other transmitting nodes that is added onto the intended signal in a destructive way. The interference model used in this work is introduced in Section 2.

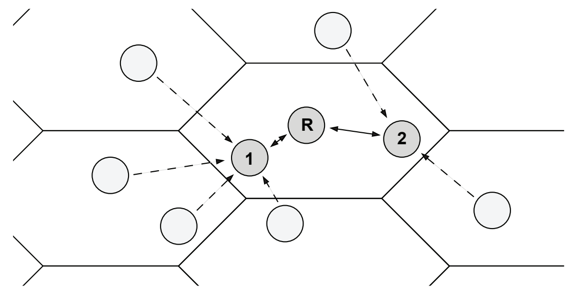

Fig. 1. Bidirectional relaying in a cellular network with inter-cell interference.

Since the unknown interference disturbs the communication, interference coordination is indispensable for reliable communication. In Section 3 we analyze different types of coordination. Thereby, we show that the traditional approach can lead to situations which prohibit reliable communication, whereas a relay-to-receivers coordination can enable a bidirectional communication even in such situations. Finally, a discussion is given in Section 4.[1]

2. INTERFERENCE MODELING

In this work, we concentrate on the bidirectional broadcast phase, where the transmission is corrupted by unknown varying additive interference. We call this a *BBC with unknown varying interference*. Clearly, the interference at both receivers may differ so that we introduce two artificial interferers, one for each receiver, to model this behavior. Then the flat fading input-output relation between the relay node and node k is given by

$$y_k = x + i_k + n_k, \quad k = 1, 2,$$

[1]*Notation:* Random variables are denoted by capital letters and sets by calligraphic letters; $\mathbb{N}$ is the set of natural numbers and $\mathbb{R}_+$ the set of non-negative real numbers; $\mathbb{P}\{\cdot\}$ denotes the probability; $(\cdot)^c$ is the complement of a set; the Euclidean norm is denoted by $\|\cdot\|^2$; lhs := rhs means the value of the right hand side (rhs) is assigned to the left hand side (lhs).

where $y_k \in \mathbb{R}$ denotes the output, $x \in \mathbb{R}$ the input, $i_k \in \mathbb{R}$ the additive interference, and $n_k \in \mathbb{R}$ the Gaussian noise of the channel distributed according to $\mathcal{N}(0, \sigma^2)$.

The transmit powers of the relay and the artificial interferers are restricted by average power constraints Γ and Λ_k, $k = 1, 2$, respectively. This means, all permissible input sequences $x_1, x_2, ..., x_n$ of length n must satisfy

$$\frac{1}{n}\sum_{j=1}^{n} x_j^2 \leq \Gamma \tag{1}$$

and all permissible interfering sequences $i_{k,1}, i_{k,2}, ..., i_{k,n}$ of length n must satisfy

$$\frac{1}{n}\sum_{j=1}^{n} i_{k,j}^2 \leq \Lambda_k, \quad k = 1, 2. \tag{2}$$

From the conditions (1) and (2) follow that all permissible codewords and interfering sequences lie on or within an n-dimensional sphere of radius $\sqrt{n\Gamma}$ or $\sqrt{n\Lambda_k}$, $k = 1, 2$, respectively.

Of course, interference in the initial multiple access phase can be modeled and analyzed in a similar way.

3. TYPES OF COORDINATION

The aim is to analyze different approaches of coordination and to specify their impact on the transmission in the bidirectional broadcast phase. Therefore we characterize all achievable rate pairs at which reliable communication is possible for two different types of coordination, namely the traditional interference and the relay-to-receivers coordination.

3.1. Preliminaries

We consider the standard model with a block code of arbitrary but fixed length n. Let $\mathcal{M}_k := \{1, ..., M_k^{(n)}\}$ be the message set of node k, $k = 1, 2$, which is also known at the relay node. Further, we use the abbreviation $\mathcal{M} := \mathcal{M}_1 \times \mathcal{M}_2$. We start with the deterministic strategy for the traditional interference coordination, where the relay and the receivers use prespecified encoder and decoders.

Definition 1 *A deterministic $(M_1^{(n)}, M_2^{(n)}, n)$-code $\mathcal{C}_{trad}$ for the BBC with unknown varying interference is a family*

$$\mathcal{C}_{trad} := \big\{(x_m^n, D^{(1)}_{m_2|m_1}, D^{(2)}_{m_1|m_2}) : m_1 \in \mathcal{M}_1, m_2 \in \mathcal{M}_2\big\}$$

with codewords x_m^n, one for each message $m = (m_1, m_2)$, and decoding sets at nodes 1 and 2, i.e., $D^{(1)}_{m_2|m_1}$ and $D^{(2)}_{m_1|m_2}$ for all $m_1 \in \mathcal{M}_1$ and $m_2 \in \mathcal{M}_2$. For given m_1 at node 1 the decoding sets have to be disjoint and similarly for given m_2 at node 2 the decoding sets have to be disjoint.

When x_m^n with $m = (m_1, m_2)$ has been sent, and y_1^n and y_2^n have been received at nodes 1 and 2, the decoder at node 1 is in error if y_1^n is not in $D^{(1)}_{m_2|m_1}$. Accordingly, the decoder at node 2 is in error if y_2^n is not in $D^{(2)}_{m_1|m_2}$. This allows us to define the probabilities of error for given message $m = (m_1, m_2)$ and interfering sequence i_k^n, $k = 1, 2$, at nodes 1 and 2 as

$$\lambda_1(m, i_1^n) := \mathbb{P}\big\{x_m^n + i_1^n + n_1^n \notin D^{(1)}_{m_2|m_1}\big\}$$
$$\lambda_2(m, i_2^n) := \mathbb{P}\big\{x_m^n + i_2^n + n_2^n \notin D^{(2)}_{m_1|m_2}\big\}$$

and the *average probability of error* for interfering sequence i_k^n at node k, $k = 1, 2$, as

$$\bar{\lambda}_k^{(n)}(i_k^n) := \frac{1}{|\mathcal{M}|}\sum_{m\in\mathcal{M}} \lambda_k(m, i_k^n). \tag{3}$$

Definition 2 *A rate pair $(R_{R1}, R_{R2}) \in \mathbb{R}_+^2$ is said to be deterministically achievable for the BBC with unknown varying interference if for any $\delta > 0$ there exists an $n(\delta) \in \mathbb{N}$ and a sequence of deterministic $(M_1^{(n)}, M_2^{(n)}, n)$-codes such that for all $n \geq n(\delta)$ we have*

$$\frac{\log M_1^{(n)}}{n} \geq R_{R2} - \delta \quad \textit{and} \quad \frac{\log M_2^{(n)}}{n} \geq R_{R1} - \delta$$

while $\max_{i_k^n} \bar{\lambda}_k^{(n)}(i_k^n) \to 0$ as $n \to \infty$, $k = 1, 2$. The set of all achievable rate pairs is the capacity region of the BBC with unknown varying interference and is denoted by $\mathcal{R}_{trad}$.

The definitions above require that we have to find a universal strategy which works for all possible interfering sequences simultaneously. Next, we introduce the random strategy for the relay-to-receivers coordination, where the relay and the receivers have the additional flexibility to coordinate their choice of the encoder and decoders.

Definition 3 *A random $(M_1^{(n)}, M_2^{(n)}, n, \mathcal{Z}, \mu)$-code $\mathcal{C}_{coord}$ for the BBC with unknown varying interference is a collection of deterministic $(M_1^{(n)}, M_2^{(n)}, n)$-codes $\mathcal{C}(Z)$, $Z \in \mathcal{Z}$, $Z \sim \mu$,*

$$\mathcal{C}(Z) := \big\{(x_m^n(Z), D^{(1)}_{m_2|m_1}(Z), D^{(2)}_{m_1|m_2}(Z)) : m_1 \in \mathcal{M}_1, m_2 \in \mathcal{M}_2\big\}$$

where μ is the uniform distribution on $\mathcal{Z}$.

This means that the codewords and the decoding sets are chosen according to a common random experiment. The definitions of probability of error, a *randomly achievable* rate pair, and the *random code capacity region* $\mathcal{R}_{coord}$ follow accordingly.

Remark 1 *From the definitions it is clear that the random strategy is more general and includes the deterministic one as a special case. Consequently, the deterministic code capacity region $\mathcal{R}_{trad}$ is contained in the random code capacity region $\mathcal{R}_{coord}$. This means we have $\mathcal{R}_{trad} \subseteq \mathcal{R}_{coord}$.*

3.2. Traditional Interference Coordination

The traditional interference coordination is in general based on a system design which ensures that the interference at the receivers do not exceed a certain threshold. For example in current cellular networks this is realized by separating cells in space which operate at the same frequency.

Theorem 1 *The deterministic code capacity region $\mathcal{R}_{trad}$ of the BBC with unknown varying interference with input constraint Γ and interferer constraints Λ_1 and Λ_2 is the set of all rate pairs $(R_{R1}, R_{R2}) \in \mathbb{R}_+^2$ satisfying*

$$R_{Rk} \leq \begin{cases} \frac{1}{2}\log\left(1 + \frac{\Gamma}{\Lambda_k + \sigma^2}\right) & \textit{if } \Gamma > \Lambda_k \\ 0 & \textit{if } \Gamma \leq \Lambda_k \end{cases} \tag{4}$$

$k = 1, 2$. This means interior$(\mathcal{R}_{trad}) \neq \emptyset$ if and only if $\Gamma > \Lambda_1$ and $\Gamma > \Lambda_2$.

First, we consider the case when $\Gamma \leq \Lambda_1$ or $\Gamma \leq \Lambda_2$. Let $x^n_{m_1,m_2} \in \mathbb{R}^n$, $m_1 = 1, ..., M_1^{(n)}$, $m_2 = 1, ..., M_2^{(n)}$ with $M_1^{(n)} \geq 2$ and $M_2^{(n)} \geq 2$ be arbitrary codewords satisfying the input constraint (1). For $\Gamma \leq \Lambda_1$ we can consider the interfering sequences $i^n_{1,m_1,m_2} = x^n_{m_1,m_2}$, $m_1 = 1, ..., M_1^{(n)}$, $m_2 = 1, ..., M_2^{(n)}$. Then for each $m_1 \in \mathcal{M}_1$ at node 1 the following holds. For each pair $(l,j) \in \mathcal{M}_2 \times \mathcal{M}_2$ with $l \neq j$ we have for the probability of error at node 1

$$\begin{aligned}
&\lambda_1((m_1,l), i^n_{1,m_1,j}) + \lambda_1((m_1,j), i^n_{1,m_1,l}) \\
&= \mathbb{P}\{x^n_{m_1,l} + i^n_{1,m_1,j} + n^n_1 \notin D^{(1)}_{l|m_1}\} \\
&\qquad + \mathbb{P}\{x^n_{m_1,j} + i^n_{1,m_1,l} + n^n_1 \notin D^{(1)}_{j|m_1}\} \\
&= \mathbb{P}\{x^n_{m_1,l} + i^n_{1,m_1,j} + n^n_1 \in (D^{(1)}_{l|m_1})^c\} \\
&\qquad + \mathbb{P}\{x^n_{m_1,l} + i^n_{1,m_1,j} + n^n_1 \notin D^{(1)}_{j|m_1}\} \\
&\geq \mathbb{P}\{x^n_{m_1,l} + i^n_{1,m_1,j} + n^n_1 \in (D^{(1)}_{l|m_1})^c\} \\
&\qquad + \mathbb{P}\{x^n_{m_1,l} + i^n_{1,m_1,j} + n^n_1 \in D^{(1)}_{l|m_1}\} \\
&= \mathbb{P}\{x^n_{m_1,l} + i^n_{1,m_1,j} + n^n_1 \in (D^{(1)}_{l|m_1})^c \cup D^{(1)}_{l|m_1}\} = 1.
\end{aligned}$$

Hence, for a fixed $m_1 \in \mathcal{M}_1$ this leads for the average probability of error to

$$\begin{aligned}
&\frac{1}{M_2^{(n)}} \sum_{j=1}^{M_2^{(n)}} \bar{\lambda}_1^{(n)}(i^n_{1,m_1,j}) \\
&= \frac{1}{M_2^{(n)}} \frac{1}{M_1^{(n)} M_2^{(n)}} \sum_{j=1}^{M_2^{(n)}} \sum_{m_1'=1}^{M_1^{(n)}} \sum_{m_2'=1}^{M_2^{(n)}} \lambda_1((m_1', m_2'), i^n_{1,m_1,j}) \\
&\geq \frac{1}{M_1^{(n)} (M_2^{(n)})^2} \sum_{m_1'}^{M_1^{(n)}} \frac{M_2^{(n)}(M_2^{(n)} - 1)}{2} \\
&= \frac{M_1^{(n)} M_2^{(n)} (M_2^{(n)} - 1)}{2 M_1^{(n)} (M_2^{(n)})^2} = \frac{M_2^{(n)} - 1}{2 M_2^{(n)}} \geq \frac{1}{4}.
\end{aligned}$$

This implies that $\bar{\lambda}_1^{(n)}(i^n_{1,m_1,m_2}) \geq \frac{1}{4}$ for at least one $(m_1, m_2) \in \mathcal{M}_1 \times \mathcal{M}_2$. Since the average probability of error is bounded from below by a positive constant, a reliable transmission from the relay to node 1 is not possible so that we have $R_{\text{R}1} = 0$. The case $\Gamma \leq \Lambda_2$ leads similarly to $R_{\text{R}2} = 0$.

This is intuitively clear, if one realizes the following. Since we have $\Gamma \leq \Lambda_k$, it can happen that the interfering sequence looks like another valid codeword. Node k receives now a superimposed version of two codewords and cannot distinguish which of the codewords was transmitted by the relay so that reliable communication can no longer be guaranteed.

Remark 2 *Interestingly, the theorem shows that the existence of positive rates only depends on the interference and is completely independent of the noise. Consequently, the goal of the traditional interference coordination is to ensure that the received interference will be small enough. Otherwise, there is no communication possible, not even at very low rates.*

Now, we turn to the case when $\Gamma > \Lambda_1$ and $\Gamma > \Lambda_2$. To show that the rates given in (4) are actually achievable, we follow [6] where a similar result is proved for the corresponding single-user scenario. The strategy is outlined in the following.

Without loss of generality we assume that $\Gamma = 1$ and further $0 < \Lambda_k < 1$, $k = 1, 2$. Then it suffices to show that for every small $\delta > 0$ and sufficiently large n there exist $M_1^{(n)} M_2^{(n)}$ codewords $x^n_{m_1,m_2}$ (on the unit sphere) with $M_1^{(n)} = \exp(nR_{\text{R}2})$ and $M_2^{(n)} = \exp(nR_{\text{R}1})$ and $C_k - 2\delta < R_{\text{R}k} < C_k - \delta$ with $C_k := \frac{1}{2}\log(1 + \frac{1}{\Lambda_k + \sigma^2})$, $k = 1, 2$, cf. (4), such that the average probability (3) is arbitrarily small for all i^n_k satisfying (2). To ensure that the probability of error gets arbitrarily small, the codewords must possess certain properties which are guaranteed by the following lemma. This is a straightforward extension of [6, Lemma 1] to the BBC with unknown varying interference.

Lemma 1 *For every $\epsilon > 0$, $8\sqrt{\epsilon} < \eta < 1$, $K > 2\epsilon$, and $M_1^{(n)} = \exp(nR_{R2})$, $M_2^{(n)} = \exp(nR_{R1})$ with $2\epsilon \leq R_{Rk} \leq K$, $k = 1, 2$, for $n \geq n_0(\epsilon, \eta, K)$ there exist unit vectors $x^n_{m_1,m_2}$, $m_1 = 1, ..., M_1^{(n)}$, $m_2 = 1, ..., M_2^{(n)}$ such that for every unit vector u^n and constants α, β in $[0,1]$, we have for each $m_1 \in \mathcal{M}_1$*

$$\begin{aligned}
&\big|\{m_2 : (x^n_{m_1,m_2}, u^n) \geq \alpha\}\big| \\
&\qquad \leq \exp(n(|R_{R1} + \tfrac{1}{2}\log(1 - \alpha^2)|^+ + \epsilon))
\end{aligned}$$

and, if $\alpha \geq \eta$, $\alpha^2 + \beta^2 > 1 + \eta - \exp(-2R_{R1})$

$$\begin{aligned}
&\frac{1}{M_2^{(n)}} \big|\{m_2' : |(x^n_{m_1,m_2}, x^n_{m_1,m_2'})| \geq \alpha, |(x^n_{m_1,m_2}, u^n)| \geq \beta, \\
&\qquad \text{for some } m_2 \neq m_2'\}\big| \leq \exp(-n\epsilon)
\end{aligned}$$

and similarly for each $m_2 \in \mathcal{M}_2$

$$\begin{aligned}
&\big|\{m_1 : (x^n_{m_1,m_2}, u^n) \geq \alpha\}\big| \\
&\qquad \leq \exp(n(|R_{R2} + \tfrac{1}{2}\log(1 - \alpha^2)|^+ + \epsilon))
\end{aligned}$$

and, if $\alpha \geq \eta$, $\alpha^2 + \beta^2 > 1 + \eta - \exp(-2R_{R2})$

$$\begin{aligned}
&\frac{1}{M_1^{(n)}} \big|\{m_1' : |(x^n_{m_1,m_2}, x^n_{m_1',m_2})| \geq \alpha, |(x^n_{m_1,m_2}, u^n)| \geq \beta, \\
&\qquad \text{for some } m_1 \neq m_1'\}\big| \leq \exp(-n\epsilon).
\end{aligned}$$

At the receiving nodes it suffices to use a minimum-distance decoder. Then for each $m_1 \in \mathcal{M}_1$ the decoding sets at node 1 are given by

$$\begin{aligned}
D^{(1)}_{m_2|m_1} := \{y^n_1 : \|y^n_1 - x^n_{m_1,m_2}\|^2 < \|y^n_1 - x^n_{m_1,m_2'}\|^2 \\
\text{for } m_2 \neq m_2'\}.
\end{aligned} \tag{5}$$

The decoding sets at node 2 are defined accordingly. With the presented coding and decoding rule, the probability of error gets arbitrarily small for increasing block length, which can be shown analogously to [6]. The details are very technical and omitted for brevity.

It remains to show that the described strategy is optimal, which means that no other rate pairs are achievable. From Remark 1 we already observed that the capacity region of the traditional interference coordination is included in the capacity region of the relay-to-receivers coordination. From the following Theorem 2 we see that for $\Gamma > \Lambda_k$, $k = 1, 2$, the maximal achievable rates for both strategies are equal. Since the described strategy already achieves these rates, the optimality is proved.

3.3. Relay-to-Receivers Coordination

Next, we study a strategy with a different degree of coordination. We assume that the relay and the receivers are synchronized in such a manner that they can coordinate their choice of the encoder and decoders based on an access to a common resource which is independent of the current message.

This can be realized by using a random code as given in Definition 3. If we transmit at rates R_{R1} and R_{R2} with exponentially many messages, i.e., $\exp(nR_{\mathrm{R1}})$ and $\exp(nR_{\mathrm{R2}})$, we know from [7] that it suffices to use a random code which consists of n^2 pairs of encoder and decoders and a uniformly distributed random variable whose value indicates which of the pair all nodes have to use. The access to the common random variable can be realized by an external source, e.g., a satellite signal, or a preamble prior to the transmission. Clearly, for sufficiently large block length the (polynomial) costs for the coordination are negligible. We call this *relay-to-receivers coordination*. Due to the more involved coordination we expect an improvement in the performance compared to the traditional coordination approach, especially for high interference.

Theorem 2 *The random code capacity region $\mathcal{R}_{coord}$ of the BBC with unknown varying interference with input constraint Γ and interferer constraints Λ_1 and Λ_2 is the set of all rate pairs $(R_{R1}, R_{R2}) \in \mathbb{R}_+^2$ satisfying*

$$R_{Rk} \leq \frac{1}{2}\log\left(1 + \frac{\Gamma}{\Lambda_k + \sigma^2}\right), \quad k = 1, 2. \tag{6}$$

The theorem can be proved analogously to [8] where a similar result is proved for the single-user case. The random strategy which achieves the rates given in (6) is outlined in the following.

The codewords $x^n_{m_1,m_2}$ are uniformly distributed on the n-sphere of radius $\sqrt{n\Gamma}$. Similar to the traditional approach, a minimum-distance decoder as given in (5) at the receiving nodes is sufficient. It remains to show that for all rate pairs satisfying (6) the probability of error gets arbitrarily small for increasing block length. This can be done similarly to [8].

The optimality of the presented random strategy, which means that no other rate pairs are achievable, follows immediately from [8] and can be shown by standard arguments.

Remark 3 *The capacity region $\mathcal{R}_{coord}$ is identical to the one if the interfering sequences would consist of iid Gaussian symbols distributed according to $\mathcal{N}(0, \Lambda_k)$, $k = 1, 2$. This means, the arbitrary, possibly non-Gaussian, unknown interference do not more affect the achievable rates than Gaussian noise of the same power.*

4. DISCUSSION

The analysis shows that unknown varying interference has a dramatic impact on the bidirectional communication. It can completely prohibit any reliable communication if traditional interference coordination is applied. The difficulty with the traditional approach is that it considers the interference as some kind of additional noise. Unfortunately, in general this is to imprecise and leads to a performance loss since the interference is caused by other transmitting nodes which maybe use the same or a similar codebook. The consequence is that the interference can look like other valid codewords so that the receivers cannot reliably distinguish between the intended signal and the interference anymore. As we see from Theorem 1 this occurs if the power of the interference is greater than the transmit power of the relay. Consequently, a traditional interference coordination is only reasonable if the interference can be made small enough.

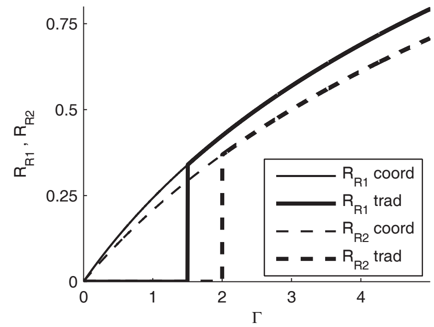

Fig. 2. Achievable rates for the BBC with unknown varying interference for the traditional interference (trad) and relay-to-receivers coordination (coord) with $\Lambda_1 = 1.5$, $\Lambda_2 = 2$, and $\sigma^2 = 1$.

We see that, especially for the case of high interference, we need a more sophisticated coordination to establish a bidirectional communication. Theorem 2 shows that a coordination of the encoder and decoders based on a common resource is sufficient to handle the interference even if it is stronger than the desired signal.

Figure 2 depicts the maximal achievable rates for the traditional interference and the relay-to-receivers coordination for increasing transmit power Γ and illustrates how the transmission collapses for the traditional interference coordination if the interference exceeds the transmit power.

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