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MIMO Gaussian Bidirectional Broadcast Channels with Common Messages

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***Abstract*—In this work, the MIMO Gaussian *bidirectional broadcast channel (BBC) with common messages* is studied. The problem is motivated by the concept of bidirectional relaying in a three-node network, where a half-duplex relay node establishes a bidirectional communication between two other nodes using a decode-and-forward protocol and thereby adds an own multicast message to the communication. The capacity region of the broadcast phase is derived and the corresponding transmit covariance matrix optimization problem is analyzed in detail. Thereby, it is shown that the transmit covariance optimization problem is strongly connected to the corresponding one of the MIMO Gaussian BBC without common messages. In particular, this knowledge can be exploited to transfer results such as optimal transmit strategies from one scenario to the other one.**

***Index Terms*—Bidirectional relaying, MIMO, capacity region, optimization, wireless network.**

I. Introduction

THE recent research progress reveals that the use of relays has the potential to significantly increase the performance of wireless networks. Especially in cellular systems relays provide a promising approach to improve the throughput and coverage at the cell edges. Since a relay cannot transmit and receive at the same time and frequency, it needs orthogonal resources for transmission and reception which can be done more efficiently if bidirectional communication is considered [1–4]. In this work, we consider *bidirectional relaying* in a three-node network, where a relay node establishes a bidirectional communication between two other nodes and thereby adds an own multicast message to the communication. Since it is well-known that spatial MIMO techniques can improve the performance significantly [5], we assume a network where each node is equipped with multiple antennas.

In the initial multiple access (MAC) phase of a two-phase decode-and-forward protocol, the two nodes transmit their messages to the relay node. We assume the relay to decode both messages so that we end up with the classical MIMO Gaussian MAC in the first phase. Since the capacity region and optimal transmit strategies are well known, cf. for example [5, 6], we concentrate on the succeeding broadcast phase in this work. Here, the relay re-encodes and transmits both messages and an additional common message in such a way that each receiving node can decode the other's message and the common message using its own message from the previous phase as side information. Note that due to the available side information at the receiving nodes this channel differs from the classical broadcast channel with common messages and is therefore called *bidirectional broadcast channel (BBC) with common messages*.

Due to the decoding at the relay node, the processing of the two phases of the decode-and-forward protocol are decoupled. Hence, we have to distribute the available spectral resources between both phases and the achievable rate region for the bidirectional relay channel is given by the intersection of the corresponding scaled capacity regions of both phases. How to optimally distribute the resources between both phases is beyond the scope of this work. But it can be done in a similar way as, for example, in [7] or [8] respectively, where the optimal time-division for MISO and MIMO bidirectional relaying without additional common messages is discussed. In this work, we solely concentrate on the BBC phase, since it constitutes the indispensable basis for such a discussion.

The problem of jointly broadcasting bidirectional and multicast information arises for example in cellular systems. There, operators offer not only traditional services such as (bidirectional) voice communication, but also additional multicast services such as common signaling data or video broadcasts. Nowadays this is realized at the base station by a policy that allocates different services on different logical channels. But there is a trend to merge multiple coexisting services efficiently from an information theoretic point of view such that they work on the same wireless resources. In [9] it is presented how bidirectional relaying can efficiently be integrated in a cellular system, where the base station acts as the relay node. Accordingly, the convergence of wireless services is intensively discussed at the moment by the Third Generation Partnership Programs Long-Term Evolution Advanced (3GPP LTE-Advanced) group since this enables a joint resource allocation policy. Moreover, it is expected that this will result in a significantly reduced complexity and an improved energy efficiency.

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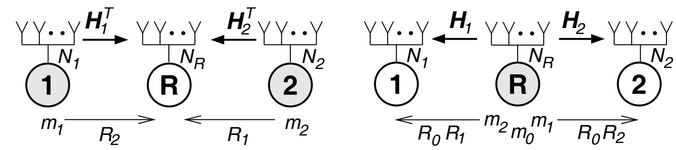
The BBC without common messages is widely studied. Capacity achieving strategies can be found, for instance, in [10–13] for discrete memoryless channels and in [14] for MIMO Gaussian channels. A deterministic approach that characterizes the capacity of the full-duplex bidirectional relay channel within a constant gap is given in [15]. There exist several strategies for bidirectional relaying classified by the processing at the relay node, e.g., refer [16] for a survey. In more detail, beside the decode-and-forward protocol [1, 10, 11, 17, 18], there are amplify-and-forward schemes that are analyzed, for example, in [19–23]. Other schemes are compress-and-forward [24, 25] or compute-and-forward [26–30] approaches, where the relay decodes a certain function of both individual messages. An extension that includes an additional common message of the relay node can be done in a similar way as for the decode-and-forward protocol using rate-splitting arguments.

The concept of bidirectional relaying and its extensions are subject of further research activities, e.g., in [31–34], extensions to the case where the relay supports the communication of multiple pairs of users are presented. Bidirectional relaying with an additional private message for the relay node in the MAC phase is addressed in [35]. A four-node network with bidirectional communication is discussed in [36], while [37] addresses the problem of joint network and channel coding for multi-way relaying. Some work on the SISO Gaussian broadcast channel with common messages and certain side information at the receivers can be found in [38] and [13] where the latter assumes degraded message sets. The throughput region of bidirectional multi-hop fading networks with common messages is analyzed in [39]. A general model for multi-user settings with correlated sources is given in [40]. The general broadcast channel with common messages is analyzed in [41] in terms of latent capacity, where the author shows that the achievability of a certain rate vector immediately implies the achievability of a whole non-trivial rate region. However, only the case of symmetric rates for all users is discussed.

The rest of this work is organized as follows. In Section II we introduce the MIMO Gaussian BBC with common messages and derive the corresponding capacity region by channel coding arguments only. Section III deals with the transmit covariance matrix optimization problem which leads to a characterization of the capacity achieving transmit strategies in Section IV. The exact knowledge of the optimal transmit strategies and in particular of the weighted rate sum optimal rate triples is of great interest, since it constitutes the basis for further cross-layer designs such as stability-optimal transmission policies or throughput-optimal scheduling strategies. For example, the stability region of the MIMO MAC is analyzed in [42] and throughput-optimal policies in relay networks are addressed in [43]. Further, in Section IV we establish a strong connection between the BBC with and without common messages which is used in Section V to demonstrate how results for the BBC without common messages immediately also provide solutions for the BBC with common messages. This emphasizes that the results are not only relevant in itself. Finally, we end up with a conclusion in Section VI.



(a) MIMO MAC phase (b) MIMO BBC phase

Fig. 1. Decode-and-forward bidirectional relaying with multiple antennas at all nodes. In the initial multiple access (MAC) phase, nodes 1 and 2 transmit their messages m_1 and m_2 with rates R_2 and R_1 to the relay node. In the succeeding bidirectional broadcast (BBC) phase, the relay forwards the messages m_1 and m_2 with rates R_2 and R_1 and adds its own common message m_0 with rate R_0 to the communication.

Notation

In this work, we denote random variables by non-italic capital letters and their corresponding realizations and ranges by lower case italic letters and script letters, respectively; scalars, vectors, and matrices are denoted by lower case letters, bold lower case letters, and bold capital letters; $H(\cdot)$ and $I(\cdot;\cdot)$ are the traditional entropy and mutual information; $\mathbb{N}$, $\mathbb{R}_+$, and $\mathbb{C}$ are the sets of non-negative integers, non-negative real, and complex numbers; $(\cdot)^{-1}$, $(\cdot)^T$, and $(\cdot)^H$ denote inverse, transpose, and Hermitian transpose, respectively; $\mathrm{tr}(\cdot)$ is the trace of a matrix; $\boldsymbol{Q} \succeq 0$ means the matrix $\boldsymbol{Q}$ is positive semidefinite; $\mathbb{E}\{\cdot\}$ is the expectation.

II. MIMO Bidirectional Broadcast Channel

We assume N_R antennas at the relay node and N_i antennas at node i, $i = 1, 2$, as shown in Figure 1. In the bidirectional broadcast phase, the discrete-time complex-valued input-output relation between the relay node and node i, $i = 1, 2$, is given by

$$\boldsymbol{y}_i = \boldsymbol{H}_i \boldsymbol{x} + \boldsymbol{n}_i, \tag{1}$$

where $\boldsymbol{y}_i \in \mathbb{C}^{N_i \times 1}$ denotes the output at node i, $\boldsymbol{H}_i \in \mathbb{C}^{N_i \times N_R}$ the multiplicative channel matrix, $\boldsymbol{x} \in \mathbb{C}^{N_R \times 1}$ the input of the relay node, and $\boldsymbol{n}_i \in \mathbb{C}^{N_i \times 1}$ the independent additive noise according to a circular symmetric complex Gaussian distribution $\mathcal{CN}(\boldsymbol{0}, \sigma^2 \boldsymbol{I}_{N_i})$. We assume perfect channel state information at all nodes and an average transmit power constraint $\mathrm{tr}(\boldsymbol{Q}) \leq P$ with $\boldsymbol{Q} = \mathbb{E}\{\boldsymbol{x}\boldsymbol{x}^H\}$.

We consider the standard model with a block code of arbitrary but fixed length n. Let $\mathcal{M}_i := \{1, ..., M_i^{(n)}\}$ be the individual message set of node i, $i = 1, 2$, which is also known at the relay node. Further, $\mathcal{M}_0 := \{1, ..., M_0^{(n)}\}$ is the common message set of the relay node.

For the bidirectional broadcast phase we assume that the relay has successfully decoded the individual messages $m_1 \in \mathcal{M}_1$ from node 1 and $m_2 \in \mathcal{M}_2$ from node 2 that it received in the previous MAC phase. If there is no additional common message for the relay to transmit, it remains for the relay to broadcast a re-encoded message based on the network coding idea that allows both nodes to decode the other's message using their own message from the previous phase as side information. In this case, we know from [14] that it is optimal to use Gaussian distributed input that carries all the re-encoded information. A direct consequence is the following theorem.

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Theorem 1 ([14]): The capacity region of the MIMO Gaussian BBC without common messages and with average power constraint P is given by

$$\bigcup_{\boldsymbol{Q}:\,\mathrm{tr}(\boldsymbol{Q})\leq P,\,\boldsymbol{Q}\succeq 0} \mathcal{R}'(\boldsymbol{Q})$$

with

$$\mathcal{R}'(\boldsymbol{Q}) := \big\{(R_1',R_2')\in\mathbb{R}_+^2 : R_1'\leq C_1(\boldsymbol{Q}), \quad R_2'\leq C_2(\boldsymbol{Q})\big\} \tag{2}$$

and $C_i(\boldsymbol{Q}) = \log\det(\boldsymbol{I}_{N_i} + \frac{1}{\sigma^2}\boldsymbol{H}_i\boldsymbol{Q}\boldsymbol{H}_i^H)$, $i=1,2$.

Thereby, R_1' denotes the individual rate for message $m_2\in\mathcal{M}_2$ from the relay to node 1 and, similarly, R_2' denote the rate for $m_1\in\mathcal{M}_1$ from the relay to node 2.

Now, we turn to our broadcast scenario, where the relay transmits the individual messages and an additional common message $m_0\in\mathcal{M}_0$. This is the bidirectional broadcast channel (BBC) with common messages.

Definition 1: An $(n,M_0^{(n)},M_1^{(n)},M_2^{(n)})$*-code* for the MIMO Gaussian BBC with common messages and average power constraint P consists of one encoder at the relay node

$$f:\mathcal{M}_0\times\mathcal{M}_1\times\mathcal{M}_2\to\mathcal{X}^n$$

with $\mathcal{X}^n := \{(\boldsymbol{x}_1,\boldsymbol{x}_2,...,\boldsymbol{x}_n)\in\mathbb{C}^{N_R\times n} : \frac{1}{n}\sum_{k=1}^n \boldsymbol{x}_k^H\boldsymbol{x}_k \leq P\}$ satisfying the average power constraint and corresponding decoders at nodes 1 and 2

$$g_1:\mathbb{C}^{N_R\times n}\times\mathcal{M}_1\to\mathcal{M}_0\times\mathcal{M}_2\cup\{0\},$$
$$g_2:\mathbb{C}^{N_R\times n}\times\mathcal{M}_2\to\mathcal{M}_0\times\mathcal{M}_1\cup\{0\}.$$

The element 0 in the definition of the decoder plays the role of an erasure symbol and is included for convenience only.

When the relay has sent the message $m=(m_0,m_1,m_2)$, and nodes 1 and 2 have received $\boldsymbol{y}_1^n$ and $\boldsymbol{y}_2^n$, the decoder at node 1 is in error if $g_1(\boldsymbol{y}_1^n,m_1)\neq(m_0,m_2)$. Accordingly, the decoder at node 2 is in error if $g_2(\boldsymbol{y}_2^n,m_2)\neq(m_0,m_1)$. Then, the average probability of error at node i is given by

$$\mu_i^{(n)} := \frac{1}{M_0^{(n)}M_1^{(n)}M_2^{(n)}}\sum_{m_0=1}^{M_0^{(n)}}\sum_{m_1=1}^{M_1^{(n)}}\sum_{m_2=1}^{M_2^{(n)}}\lambda_i(m_0,m_1,m_2)$$

where $\lambda_i(m_0,m_1,m_2)$ denotes the probability that the decoder at node i decodes the sent message $m=(m_0,m_1,m_2)$ incorrectly, $i=1,2$.

Definition 2: A rate triple $(R_0,R_1,R_2)\in\mathbb{R}_+^3$ is said to be *achievable* for the MIMO Gaussian BBC with common messages and average power constraint P if for any $\delta>0$ there is an $n(\delta)\in\mathbb{N}$ and a sequence of $(n,M_0^{(n)},M_1^{(n)},M_2^{(n)})$-codes satisfying the power constraint P such that for all $n\geq n(\delta)$ we have $\frac{\log M_0^{(n)}}{n}\geq R_0-\delta$, $\frac{\log M_2^{(n)}}{n}\geq R_1-\delta$, and $\frac{\log M_1^{(n)}}{n}\geq R_2-\delta$ while $\mu_1^{(n)},\mu_2^{(n)}\to 0$ as $n\to\infty$. The set of all achievable rate triples is the *capacity region of the MIMO Gaussian BBC with common messages* and is denoted by $\mathcal{C}_{\mathrm{BBC}}^{\mathrm{MIMO}}$.

Now, we can state the capacity region of the MIMO Gaussian BBC with common messages. For this purpose we define the region

$$\mathcal{R}(\boldsymbol{Q}) := \big\{(R_0,R_1,R_2)\in\mathbb{R}_+^3 : R_0+R_1\leq C_1(\boldsymbol{Q}), \quad R_0+R_2\leq C_2(\boldsymbol{Q})\big\}. \tag{3}$$

Remark 1: Clearly, the sum constraints in the definition of $\mathcal{R}(\boldsymbol{Q})$ immediately implies that the rate of the common message has to fulfill $R_0\leq\min\{C_1(\boldsymbol{Q}),C_2(\boldsymbol{Q})\}$.

Remark 2: At a first glance, (3) suggests a rate splitting approach between the common rate and the individual rates, but one has to be careful since the coding strategy has to be designed in such a way that the common message can be decoded at both receivers. This observation reveals some interesting connections to compound channels [44–46] where the coding strategy has to ensure that the message to transmit is decodable for a whole set of possible channels.

Theorem 2: The capacity region $\mathcal{C}_{\mathrm{BBC}}^{\mathrm{MIMO}}$ of the MIMO Gaussian BBC with common messages and average power constraint P is given by

$$\mathcal{C}_{\mathrm{BBC}}^{\mathrm{MIMO}} = \bigcup_{\boldsymbol{Q}:\,\mathrm{tr}(\boldsymbol{Q})\leq P,\,\boldsymbol{Q}\succeq 0}\mathcal{R}(\boldsymbol{Q}). \tag{4}$$

Since the $\log\det$ function is concave in $\boldsymbol{Q}$ [47, Theorem 7.6.7], the region in (4) is already convex. Hence, an auxiliary random variable that realizes an additional time-sharing operation is not necessary since such an operation will not enlarge the region.

A. Proof of Achievability

In this section, we present a construction of a coding strategy that achieves all rate triples $(R_0,R_1,R_2)\in\mathcal{R}(\boldsymbol{Q})$, cf. (3), for a given covariance matrix $\boldsymbol{Q}$. Then the desired region (4) is immediately obtained by taking the union over all covariance matrices that satisfy the input power constraint. The construction is mainly based on the idea of [48] for the classical broadcast channel with common messages, where the whole information sent to each receiver is split into an individual part and a common part. We use this idea to extend the proof of achievability for the BBC without common messages, cf. Theorem 1 and [14, Sec. III-A], to our scenario.

Recall the broadcast situation considered here. The relay node wants to transmit a common message $m_0\in\mathcal{M}_0$ with rate R_0 and individual messages $m_1\in\mathcal{M}_1$ and $m_2\in\mathcal{M}_2$ with rates R_2 and R_1, respectively. Node 1 knows its own message m_1 that it transmitted in the previous MAC phase and wants to recover the common message m_0 and the individual message m_2. Similarly, node 2 knows m_2 and wants to recover m_0 and m_1. Having [48] in mind the broadcast situation can also be interpreted in a slightly different way by combining the desired individual messages and the common message. In more detail, node 1 knows its own message m_1 and is interested in the (combined) individual message $m_2'=(m_0,m_2)\in\mathcal{M}_0\times\mathcal{M}_2=:\mathcal{M}_2'$ with rate $R_1'=R_0+R_1$ and, similarly, node 2 knows m_2 and is interested in $m_1'=(m_0,m_1)\in\mathcal{M}_0\times\mathcal{M}_1=:\mathcal{M}_1'$ with rate $R_2'=R_0+R_2$.

Basically, we see that due to this reinterpretation the problem of coding for the BBC with common messages reduces to the problem of coding for the BBC without common messages. The only difference is that while in the classical BBC without common messages each receiving node has complementary side information, i.e., it knows exactly the message the other one is interested in, in our scenario each

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receiving node knows only a part of the information the other one is interested in, i.e., it has only the other individual message and not the common message as available side information. We see that our scenario is not precisely included in [14], but it is a straightforward extension. Therefore, we go through the proof of achievability in the following and sketch only the differences to [14, Sec. III-A], cf. also Theorem 1.

Similarly as in [14, Sec. III-A] we show by random coding arguments that for a given covariance matrix $\boldsymbol{Q}$ there exists a coding strategy such that all rate pairs $(R_1', R_2') \in \mathcal{R}'(\boldsymbol{Q})$, i.e., satisfying $R_i' = R_0 + R_i \leq C_i(\boldsymbol{Q})$, $i = 1, 2$, cf. also (2), are achievable. Therefore, we generate $M_0^{(n)} M_1^{(n)} M_2^{(n)}$ codewords of length n with $M_0^{(n)} := 2^{nR_0}$, $M_1^{(n)} := 2^{nR_2}$, and $M_2^{(n)} := 2^{nR_1}$, where for each $m = (m_0, m_1, m_2) \in \mathcal{M}_0 \times \mathcal{M}_1 \times \mathcal{M}_2$ each entry of the corresponding codeword is independently chosen according to $\mathcal{CN}(\mathbf{0}, \boldsymbol{Q})$. Each receiving node uses typical set decoding in a similar way as in [14, Sec. III-A]. Now, it is straightforward to show that the probability of a decoding error, averaged over all codewords and all codebooks, at receiving node 1 gets arbitrarily small if the rate of the intended (combined) message $m_2' = (m_0, m_2) \in \mathcal{M}_0 \times \mathcal{M}_2 = \mathcal{M}_2'$ fulfills $R_1' = R_0 + R_1 \leq C_1(\boldsymbol{Q})$. Clearly, the same is also true for receiving node 2 which is able to determine $m_1' = (m_0, m_1) \in \mathcal{M}_0 \times \mathcal{M}_1 = \mathcal{M}_1'$ if $R_2' = R_0 + R_2 \leq C_2(\boldsymbol{Q})$. With the (combined) individual messages $m_1' \in \mathcal{M}_1'$ and $m_2' \in \mathcal{M}_2'$ with rates R_2' and R_1' the receiving nodes immediately obtain the common message $m_0 \in \mathcal{M}_0$ with rate R_0 and the individual messages $m_1 \in \mathcal{M}_1$ and $m_2 \in \mathcal{M}_2$ with rate $R_2' - R_0$ and $R_1' - R_0$, respectively. Thus, similar to [41,48], all rate triples $(R_0, R_1' - R_0, R_2' - R_0) = (R_0, R_1, R_2) \in \mathcal{R}(\boldsymbol{Q})$ with $R_0 \leq \min\{C_1(\boldsymbol{Q}), C_2(\boldsymbol{Q})\}$, cf. Remark 1, are achievable for the MIMO Gaussian BBC with common messages which already proves the achievability. ■

B. Proof of Weak Converse

We have to show that for any given sequence of $(n, M_0^{(n)}, M_1^{(n)}, M_2^{(n)})$-codes with $\mu_1^{(n)}, \mu_2^{(n)} \to 0$ there exists a covariance matrix $\boldsymbol{Q}$ satisfying the average power constraint $\mathrm{tr}(\boldsymbol{Q}) \leq P$ such that

$$R_0 + R_1 := \frac{1}{n}\big(\log M_0^{(n)} + \log M_2^{(n)}\big) \leq C_1(\boldsymbol{Q}) + o(n^0)$$
$$R_0 + R_2 := \frac{1}{n}\big(\log M_0^{(n)} + \log M_1^{(n)}\big) \leq C_2(\boldsymbol{Q}) + o(n^0)$$

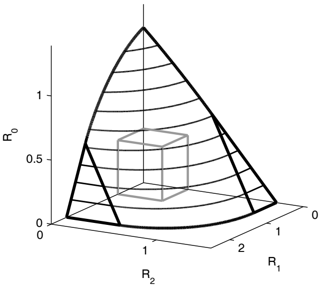
are satisfied. For this purpose we need a version of Fano's lemma suitable for the MIMO Gaussian BBC with common messages.

Lemma 1: For the BBC with common messages we have the following versions of Fano's inequality

$$H(\mathrm{M}_0, \mathrm{M}_2 | \mathbf{Y}_1^n, \mathrm{M}_1) \leq \mu_1^{(n)} \log(M_0^{(n)} M_2^{(n)}) + 1 = n\epsilon_1^{(n)} \quad \text{(5a)}$$
$$H(\mathrm{M}_0, \mathrm{M}_1 | \mathbf{Y}_2^n, \mathrm{M}_2) \leq \mu_2^{(n)} \log(M_0^{(n)} M_1^{(n)}) + 1 = n\epsilon_2^{(n)} \quad \text{(5b)}$$

with $\epsilon_1^{(n)} = \frac{\log(M_0^{(n)} M_2^{(n)})}{n} \mu_1^{(n)} + \frac{1}{n} \to 0$ and $\epsilon_2^{(n)} = \frac{\log(M_0^{(n)} M_1^{(n)})}{n} \mu_2^{(n)} + \frac{1}{n} \to 0$ for $n \to \infty$ as $\mu_1^{(n)}, \mu_2^{(n)} \to 0$.

Proof: The main difference to the classical version of Fano's inequality is that the entropy terms given in (5) are conditioned on the messages known at the corresponding receivers. This is due to the fact that the receiving nodes can use their own message from the previous phase as side information for decoding. For completeness, the proof can be found in Appendix A. ■



$$\boldsymbol{H}_1 = \begin{bmatrix} 0.2 - 0.5i & 0.5i \\ -1 & 1.3 + 0.3i \end{bmatrix}$$
$$\boldsymbol{H}_2 = \begin{bmatrix} 0.7 - 0.2i & 0.7 \\ 0 & -0.8i \end{bmatrix}$$
$$P = \sigma^2 = 1$$

Fig. 2. Capacity region of the MIMO Gaussian BBC with common messages (black) and a comparable TDMA approach (gray) with $N_R = N_1 = N_2 = 2$.

With this, we can bound $H(\mathrm{M}_0) + H(\mathrm{M}_2)$ as follows

$$\begin{aligned} H(\mathrm{M}_0) + H(\mathrm{M}_2) &= H(\mathrm{M}_0|\mathrm{M}_1, \mathrm{M}_2) + H(\mathrm{M}_2|\mathrm{M}_1) \\ &= H(\mathrm{M}_0, \mathrm{M}_2|\mathrm{M}_1) \\ &\leq I(\mathrm{M}_0, \mathrm{M}_2; \mathbf{Y}_1^n|\mathrm{M}_1) + n\epsilon_1^{(n)} \\ &\leq I(\mathrm{M}_0, \mathrm{M}_1, \mathrm{M}_2; \mathbf{Y}_1^n) + n\epsilon_1^{(n)} \\ &\leq I(\mathbf{X}^n; \mathbf{Y}_1^n) + n\epsilon_1^{(n)} \end{aligned} \quad (6)$$

where the equalities and inequalities follow from the independence of M_0, M_1, and M_2, the chain rule for entropy, Lemma 1, the chain rule for mutual information, the positivity of mutual information, and the data processing inequality. Accordingly, using the same arguments we also obtain

$$H(\mathrm{M}_0) + H(\mathrm{M}_1) \leq I(\mathbf{X}^n; \mathbf{Y}_2^n) + n\epsilon_2^{(n)}. \quad (7)$$

Note that (6) and (7) immediately imply that $H(\mathrm{M}_0) \leq \min\{I(\mathbf{X}^n; \mathbf{Y}_1^n) + n\epsilon_1^{(n)}, I(\mathbf{X}^n; \mathbf{Y}_2^n) + n\epsilon_2^{(n)}\}$, cf. also Remark 1.

The rest of the proof is almost identical to [14, Sec. III-B] and follows from standard arguments. It only remains to bound the term $I(\mathbf{X}^n; \mathbf{Y}_i^n)$, $i = 1, 2$, in such a way that we end up with the well known and expected $\log\det$ expression. Exactly as in [14, Lemma 3] it can be shown that $\frac{1}{n} I(\mathbf{X}^n; \mathbf{Y}_i^n) \leq \log\det\big(\boldsymbol{I}_{N_i} + \frac{1}{\sigma^2}\boldsymbol{H}_i\big(\frac{1}{n}\sum_{k=1}^n \boldsymbol{Q}_k\big)\boldsymbol{H}_i^H\big)$. Following [14, Sec. III-B] this immediately leads to

$$R_0 + R_i \leq \log\det(\boldsymbol{I}_{N_i} + \tfrac{1}{\sigma^2}\boldsymbol{H}_i \boldsymbol{Q} \boldsymbol{H}_i^H), \quad i = 1, 2$$

which proves the weak converse. ■

C. Example

As an example Figure 2 depicts the capacity region of a MIMO Gaussian BBC with common messages and illustrates how the optimal strategy outperforms a simple TDMA approach that realizes the same routing task with three orthogonal time slots.

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III. Covariance Optimization Problem

Since the capacity region $\mathcal{C}_{\text{BBC}}^{\text{MIMO}}$ is convex, the rate triples on the dominant surface characterize the capacity region completely. Therefore, one is interested in finding the optimal transmit covariance matrices that achieve the rate triples on the dominant surface since they constitute the basis for further cross-layer designs such as stability-optimal scheduling policies.

A rate triple on the dominant surface of the capacity region is a solution of a weighted rate sum problem so that we consider the corresponding convex optimization problem

$$
\begin{aligned}
R_\Sigma(\boldsymbol{w}) = \max_{R_0,R_1,R_2} \quad & \sum_{i=0}^{2} w_i R_i && \text{(8a)}\\
\text{s.t.} \quad & R_0 + R_i \leq C_i(\boldsymbol{Q}), \quad i=1,2 && \text{(8b)}\\
& R_i \geq 0, \quad i = 0,1,2 && \text{(8c)}\\
& \operatorname{tr}(\boldsymbol{Q}) \leq P,\ \boldsymbol{Q} \succeq 0 && \text{(8d)}
\end{aligned}
$$

with $\boldsymbol{w} = (w_0, w_1, w_2) \in \mathbb{R}_+^3$ the weight vector and $C_i(\boldsymbol{Q}) = \log\det(\boldsymbol{I}_{N_i} + \frac{1}{\sigma^2}\boldsymbol{H}_i\boldsymbol{Q}\boldsymbol{H}_i^H)$, $i = 1,2$, in the following.

For the optimal rate triples the constraints (8b) will be satisfied with equality for positive weights. Since otherwise, if $R_0 + R_i < C_i(\boldsymbol{Q})$, we can increase the rate R_i up to the point where we have equality, i.e., $R_0 + R_i = C_i(\boldsymbol{Q})$, without affecting the other rates and therewith increasing the weighted rate sum $R_\Sigma(\boldsymbol{w})$. On the other hand if some weights are zero, there exists also an optimal solution where (8b) will be satisfied with equality. Therefore, we concentrate on those rate triples that satisfy (8b) with equality and rewrite the optimization problem as follows

$$
\begin{aligned}
\max_{\boldsymbol{Q},R_0} \quad & (w_0 - w_1 - w_2)R_0 + w_1 C_1(\boldsymbol{Q}) + w_2 C_2(\boldsymbol{Q}) && \text{(9)}\\
\text{s.t.} \quad & 0 \leq R_0 \leq C_i(\boldsymbol{Q}), \quad i = 1,2\\
& \operatorname{tr}(\boldsymbol{Q}) \leq P,\ \boldsymbol{Q} \succeq 0.
\end{aligned}
$$

Then the Lagrangian for the corresponding minimization problem is given by

$$
\begin{aligned}
L(\boldsymbol{Q}, R_0, \boldsymbol{\nu}, \xi, \mu, \boldsymbol{\Psi}) = &-(w_0 - w_1 - w_2)R_0 - \sum_{i=1}^{2} w_i C_i(\boldsymbol{Q})\\
&+ \nu_1\big(R_0 - C_1(\boldsymbol{Q})\big) + \nu_2\big(R_0 - C_2(\boldsymbol{Q})\big)\\
&- \xi R_0 + \mu\big(\operatorname{tr}(\boldsymbol{Q}) - P\big) - \operatorname{tr}(\boldsymbol{Q}\boldsymbol{\Psi})
\end{aligned}
$$

with Lagrange multipliers $\xi, \mu \in \mathbb{R}$, $\boldsymbol{\nu} = (\nu_1, \nu_2) \in \mathbb{R}^2$, and $\boldsymbol{\Psi} \in \mathbb{C}^{N_R \times N_R}$, from which we get the Karush-Kuhn-Tucker (KKT) conditions with $C_i'(\boldsymbol{Q}) = \boldsymbol{H}_i^H(\sigma^2\boldsymbol{I}_{N_i} + \boldsymbol{H}_i\boldsymbol{Q}\boldsymbol{H}_i^H)^{-1}\boldsymbol{H}_i$, $i = 1,2$, as

$$
\begin{aligned}
\mu\boldsymbol{I}_{N_R} - \boldsymbol{\Psi} = (w_1 + \nu_1)C_1'(\boldsymbol{Q}) + (w_2 + \nu_2)C_2'(\boldsymbol{Q}) && \text{(10a)}\\
w_0 = w_1 + w_2 + \nu_1 + \nu_2 - \xi && \text{(10b)}\\
0 \leq R_0 \leq C_i(\boldsymbol{Q}),\ i = 1,2 && \text{(10c)}\\
\boldsymbol{Q} \succeq 0, \qquad \operatorname{tr}(\boldsymbol{Q}) \leq P && \text{(10d)}\\
\boldsymbol{\Psi} \succeq 0, \qquad \nu_1, \nu_2, \xi, \mu \geq 0 && \text{(10e)}\\
\operatorname{tr}(\boldsymbol{Q}\boldsymbol{\Psi}) = 0, \qquad \mu\big(\operatorname{tr}(\boldsymbol{Q}) - P\big) = 0 && \text{(10f)}\\
\xi R_0 = 0, \qquad \nu_i\big(R_0 - C_i(\boldsymbol{Q})\big) = 0,\ i = 1,2 && \text{(10g)}
\end{aligned}
$$

with primal, dual, and complementary slackness conditions (10c)-(10d), (10e), and (10f)-(10g) respectively. Since the constraint functions satisfy a generalized version of Slater's condition [49, Sec. 5.9], the KKT conditions (10a)-(10g) are necessary and sufficient and therefore characterize the optimal transmit covariance matrix for a certain weight vector $\boldsymbol{w} = (w_0, w_1, w_2)$.

Although the optimization problem (8) is a convex optimization problem and can therefore be efficiently solved using interior point method, further insights can be obtained by studying its structure in more detail as done in the following section.

IV. Capacity Achieving Transmit Strategies

Already the proof of achievability, cf. Section II-A, indicates that the BBC with common messages is closely related to the BBC without common messages. Motivated by this observation, we analyze the optimization problem from Section III in detail and establish a strong connection between these two cases in the following so that results such as transmit strategies obtained for one case will also be applicable for the other one.

It is reasonable to distinguish three different kinds of weight vectors based on the relation between the weight of the common message and the weights of the individual messages. For notational convenience we collect the corresponding weight vectors in three sets

$$
\begin{aligned}
\mathcal{W}^{(<)} &:= \{(w_0, w_1, w_2) \in \mathbb{R}_+^3 : w_0 < w_1 + w_2\},\\
\mathcal{W}^{(>)} &:= \{(w_0, w_1, w_2) \in \mathbb{R}_+^3 : w_0 > w_1 + w_2\},\\
\mathcal{W}^{(=)} &:= \{(w_0, w_1, w_2) \in \mathbb{R}_+^3 : w_0 = w_1 + w_2\}.
\end{aligned}
$$

In the following three subsections we analyze the optimization problem for each set of weight vectors separately. This will be a reasonable division since they characterize the cases with no common message rate, full common message rate, and the case with a trade-off between the common rate and the individual rates.

A. Zero Common Message Rate

If $w_0 < w_1 + w_2$, the formulation (9) of the optimization problem already shows that the weighted rate sum is maximized by setting $R_0 = 0$. Since otherwise, an increasing common rate R_0 would result in a decreasing weighted rate sum. Our intuition is confirmed by the following results.

Proposition 1: Let $\boldsymbol{w} \in \mathcal{W}^{(<)}$ be a weight vector for the BBC with common messages. Then for the weighted rate sum optimal rate triple we have $R_0 = 0$.

Proof: Since $\nu_1, \nu_2 \geq 0$, cf. (10e), condition (10b) shows that for $w_0 < w_1 + w_2$ we must have $\xi > 0$ which indeed implies $R_0 = 0$ by (10g). ■

Clearly, if the rate of the common message is zero, the BBC with common messages reduces to the BBC without common messages. Therefore, for weight vector $\boldsymbol{w}' = (0, w_1', w_2') \in \mathbb{R}_+^3$ let $\boldsymbol{Q}_{\text{opt}}'(\boldsymbol{w}')$ be the optimal transmit covariance matrix for the BBC without common messages. Thereby, we know from [50, 51] that it suffices to consider normalized weight vectors only, i.e., $w_1' + w_2' = 1$, since the optimal transmit covariance matrix depends only the relation between the two individual

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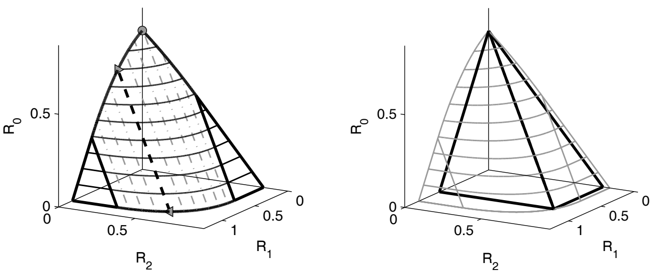

(a) Capacity region of the MISO Gaussian BBC with common messages

(b) Achievable rate region for the XOR coding approach (black)

Fig. 3. MISO Gaussian BBC with common messages with $N_R = 2$, $N_1 = N_2 = 1$ for $\boldsymbol{h}_1 = [1.3 \;\; 1.3i]^T$, $\boldsymbol{h}_2 = [1 \;\; -ie^{i\frac{\pi}{3}}]^T$, $P = 1$, and $\sigma^2 = 1$. For fixed individual weights w_1 and w_2, in Fig. 3(a) the point $\triangleleft$ characterizes $\boldsymbol{R}_{\text{opt}}(\boldsymbol{w})$ for all $\boldsymbol{w} \in \mathcal{W}^{(<)}$ with the fixed individual weights. The solid dashed line between $\triangleleft$ and $\triangleright$ corresponds to all $\boldsymbol{R}_{\text{opt}}(\boldsymbol{w})$ for $\boldsymbol{w} \in \mathcal{W}^{(=)}$. For all $\boldsymbol{w} \in \mathcal{W}^{(>)}$ the optimal rate triples moves along the curved section and tends to the XOR solution in point $\circ$.

weights and not on the exact values. Then the optimum for the BBC with common messages is achieved by the same transmit covariance matrix, i.e., $\boldsymbol{Q}_{\text{opt}}(\boldsymbol{w}) = \boldsymbol{Q}'_{\text{opt}}(\boldsymbol{w}')$, for all weight vectors $\boldsymbol{w} \in \mathcal{W}^{(<)}$ with $w_i = w'_i$, $i = 1, 2$, as long as $w_0 < w'_1 + w'_2$ is satisfied. This means the BBC without common messages immediately determines the capacity achieving transmit strategies for the BBC with common messages.

Another important issue is to characterize the optimal rate triples $\boldsymbol{R}_{\text{opt}}(\boldsymbol{w}) = (R_0(\boldsymbol{w}), R_1(\boldsymbol{w}), R_2(\boldsymbol{w}))$ for given weight vector $\boldsymbol{w} = (w_0, w_1, w_2)$.

Proposition 2: For weight vector $\boldsymbol{w} \in \mathcal{W}^{(<)}$ let $\boldsymbol{Q}_{\text{opt}}(\boldsymbol{w})$ be the optimal transmit covariance matrix for the BBC with common messages. Then the weighted rate sum optimal rate triple $\boldsymbol{R}_{\text{opt}}(\boldsymbol{w})$ is

$$R_0(\boldsymbol{w}) = 0, \tag{11a}$$
$$R_i(\boldsymbol{w}) = C_i\big(\boldsymbol{Q}_{\text{opt}}(\boldsymbol{w})\big), \; i = 1, 2. \tag{11b}$$

Proof: The weighted rate sum optimal rate triple $\boldsymbol{R}_{\text{opt}}(\boldsymbol{w})$ follows immediately from Proposition 1. ■

Remark 3: It shows that the weights for the common and individual messages have a strong impact on further cross-layer designs. We see that for all weight vectors $\boldsymbol{w} \in \mathcal{W}^{(<)}$ it is optimal to allocate no resources to the common message and solely transmit the individual messages which indeed influences the scheduling policy at the relay node.

Figure 3(a) depicts the capacity region of a MISO Gaussian BBC with common messages, where the relay node has multiple antennas, while each receiving node is equipped with a single antenna only. For all weight vectors $\boldsymbol{w} \in \mathcal{W}^{(<)}$ we see from (11) that $R_0(\boldsymbol{w}) = 0$ and the weighted rate sum optimal rate triples $\boldsymbol{R}_{\text{opt}}(\boldsymbol{w})$ describe the boundary of the capacity region on the R_1/R_2-plane as further studied in [50, 51].

B. Full Common Message Rate

Next, we turn to the more interesting case where the weight of the common message exceeds the weights of the individual messages, i.e., $w_0 > w_1 + w_2$.

Proposition 3: Let $\boldsymbol{w} \in \mathcal{W}^{(>)}$ be a weight vector for the BBC with common messages. Then for the weighted rate sum optimal rate triple we have $R_0 > 0$.

Proof: We prove the proposition by contradiction. Let us assume $R_0 = 0$ so that $\nu_1 = 0$ and $\nu_2 = 0$ by (10g). Since $w_0 > w_1 + w_2$, condition (10b) can only be satisfied if $\xi < 0$ which is a contradiction to (10e). Therefore, if $w_0 > w_1 + w_2$, we must have $R_0 > 0$ which proves the proposition. ■

Intuitively one would expect that it is optimal to allocate all available resources to the common message. But the rate of the common message is limited by a min operation, cf. Remark 1, so that this might not maximize the weighted rate sum in general. Therefore, similar to the previous case we want to know when a given transmit covariance matrix that is optimal for the BBC without common messages is also optimal for the BBC with common messages.

Theorem 3: For weight vector $\boldsymbol{w}' = (0, w'_1, w'_2) \in \mathbb{R}^3_+$ let $\boldsymbol{Q}'_{\text{opt}}(\boldsymbol{w}')$ be the optimal transmit covariance matrix for the BBC without common messages. For all weight vectors $\boldsymbol{w} \in \mathcal{W}^{(>)}$ that further satisfy, if $C_1(\boldsymbol{Q}'_{\text{opt}}(\boldsymbol{w}')) < C_2(\boldsymbol{Q}'_{\text{opt}}(\boldsymbol{w}'))$, the condition

$$w_0 = w'_1 + w'_2, \; w_1 < w'_1, \; w_2 = w'_2, \tag{12a}$$

or, if $C_1(\boldsymbol{Q}'_{\text{opt}}(\boldsymbol{w}')) > C_2(\boldsymbol{Q}'_{\text{opt}}(\boldsymbol{w}'))$, the condition

$$w_0 = w'_1 + w'_2, \; w_1 = w'_1, \; w_2 < w'_2, \tag{12b}$$

or, if $C_1(\boldsymbol{Q}'_{\text{opt}}(\boldsymbol{w}')) = C_2(\boldsymbol{Q}'_{\text{opt}}(\boldsymbol{w}'))$, the condition

$$w_0 = w'_1 + w'_2, \; w_1 \leq w'_1, \; w_2 \leq w'_2, \tag{12c}$$

the optimum for the BBC with common messages is achieved by the same transmit covariance matrix, i.e., $\boldsymbol{Q}_{\text{opt}}(\boldsymbol{w}) = \boldsymbol{Q}'_{\text{opt}}(\boldsymbol{w}')$.

Proof: We start with case (12a) and note that we have $\xi = 0$ by (10g) since $R_0 > 0$ which follows from Proposition 3. If $C_1(\boldsymbol{Q}'_{\text{opt}}(\boldsymbol{w}')) < C_2(\boldsymbol{Q}'_{\text{opt}}(\boldsymbol{w}'))$, then from (10c) follows that $R_0 < C_2(\boldsymbol{Q}'_{\text{opt}}(\boldsymbol{w}'))$ which immediately implies together with (10g) that $\nu_2 = 0$. With this, (10a) reads as

$$\mu \boldsymbol{I}_{N_R} - \boldsymbol{\Psi} = (w_1 + \nu_1) C'_1\big(\boldsymbol{Q}'_{\text{opt}}(\boldsymbol{w}')\big) + w_2 C'_2\big(\boldsymbol{Q}_{\text{opt}}(\boldsymbol{w}')\big)$$

which is exactly the same structure as the MIMO Gaussian BBC without common messages has, cf. for example [51, Eq. (2a)]. Consequently, the optimization problem of the BBC with common messages reduces to the BBC without common messages but with modified individual weights $w'_2 = w_2$ and $w'_1 = w_1 + \nu_1 = w_0 - w_2$ where the last equality follows from (10b). Hence, the optimal transmit covariance matrix $\boldsymbol{Q}'_{\text{opt}}(\boldsymbol{w}')$ for the BBC without common messages and weight vector $\boldsymbol{w}' = (0, w'_1, w'_2)$ is also a solution of the corresponding problem of the BBC with common messages for all weight vectors $\boldsymbol{w} \in \mathcal{W}^{(>)}$ that further satisfy $w_0 = w'_1 + w'_2$, $w_1 < w'_1$, and $w_2 = w'_2$ which proves the first assertion (12a).

Now, the case (12b) follows accordingly using the same arguments. Furthermore, the third assertion (12c) follows immediately from (10a) and (10b) and $\nu_i \geq 0$, $i = 1, 2$. ■

Remark 4: A given weight vector $\boldsymbol{w}'$ uniquely characterizes the optimal transmit covariance matrix $\boldsymbol{Q}'_{\text{opt}}(\boldsymbol{w}')$ for the BBC without common messages, cf. [50, 51] and immediately determines the maximal unidirectional rates $C_1(\boldsymbol{Q}'_{\text{opt}}(\boldsymbol{w}'))$ and

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$C_2(\boldsymbol{Q}'_{\text{opt}}(\boldsymbol{w}'))$ for given $\boldsymbol{Q}'_{\text{opt}}(\boldsymbol{w}')$. But more important, it directly affects the common rate, since it is restricted by the minimum of the two maximal unidirectional rates, cf. also Remark 1. This substantiates the result that an optimal transmit covariance matrix $\boldsymbol{Q}'_{\text{opt}}(\boldsymbol{w}')$ for the BBC without common messages is also optimal for the BBC with common messages for three different sets of weight vectors based on the relation between the maximal unidirectional rates $C_1(\boldsymbol{Q}'_{\text{opt}}(\boldsymbol{w}'))$ and $C_2(\boldsymbol{Q}'_{\text{opt}}(\boldsymbol{w}'))$, respectively.

Furthermore, the results so far allow to characterize the weighted rate sum optimal rate triples $\boldsymbol{R}_{\text{opt}}(\boldsymbol{w})$ for weight vectors $\boldsymbol{w} \in \mathcal{W}^{(>)}$ in detail. Similarly as in Theorem 3 we have to distinguish between three cases.

Proposition 4: For weight vector $\boldsymbol{w} \in \mathcal{W}^{(>)}$ let $\boldsymbol{Q}_{\text{opt}}(\boldsymbol{w})$ be the optimal transmit covariance matrix for the BBC with common messages. If $C_1(\boldsymbol{Q}_{\text{opt}}(\boldsymbol{w})) < C_2(\boldsymbol{Q}_{\text{opt}}(\boldsymbol{w}))$, then the weighted rate sum optimal rate triple $\boldsymbol{R}_{\text{opt}}(\boldsymbol{w})$ is

$$R_0(\boldsymbol{w}) = C_1\big(\boldsymbol{Q}_{\text{opt}}(\boldsymbol{w})\big), \tag{13a}$$

$$R_1(\boldsymbol{w}) = 0, \tag{13b}$$

$$R_2(\boldsymbol{w}) = C_2\big(\boldsymbol{Q}_{\text{opt}}(\boldsymbol{w})\big) - C_1\big(\boldsymbol{Q}_{\text{opt}}(\boldsymbol{w})\big). \tag{13c}$$

If $C_1(\boldsymbol{Q}_{\text{opt}}(\boldsymbol{w})) > C_2(\boldsymbol{Q}_{\text{opt}}(\boldsymbol{w}))$, then the weighted rate sum optimal rate triple is $\boldsymbol{R}_{\text{opt}}(\boldsymbol{w}) = (C_2(\boldsymbol{Q}_{\text{opt}}(\boldsymbol{w})), C_1(\boldsymbol{Q}_{\text{opt}}(\boldsymbol{w})) - C_2(\boldsymbol{Q}_{\text{opt}}(\boldsymbol{w})), 0)$.

If $C_1(\boldsymbol{Q}_{\text{opt}}(\boldsymbol{w})) = C_2(\boldsymbol{Q}_{\text{opt}}(\boldsymbol{w}))$, then $\boldsymbol{R}_{\text{opt}}(\boldsymbol{w})$ is

$$R_0(\boldsymbol{w}) = C_1\big(\boldsymbol{Q}_{\text{opt}}(\boldsymbol{w})\big), \tag{14a}$$

$$R_i(\boldsymbol{w}) = 0, \quad i = 1, 2. \tag{14b}$$

Proof: We start with the proof of the first case, then the second one follows accordingly using the same arguments. If $C_1(\boldsymbol{Q}_{\text{opt}}(\boldsymbol{w})) < C_2(\boldsymbol{Q}_{\text{opt}}(\boldsymbol{w}))$, then $R_0(\boldsymbol{w}) < C_2(\boldsymbol{Q}_{\text{opt}}(\boldsymbol{w}))$ so that $\nu_2 = 0$ by (10g). Further, from Proposition 3 we know that in the optimal rate triple we have $R_0(\boldsymbol{w}) > 0$ so that $\xi = 0$ by (10g). With this, (10b) reads as $w_0 = w_1 + w_2 + \nu_1$ which implies that $\nu_1 > 0$ since $w_0 > w_1 + w_2$ by assumption. From this follows $R_0(\boldsymbol{w}) = C_1(\boldsymbol{Q}_{\text{opt}}(\boldsymbol{w}))$ by (10g) so that the weighted rate sum optimal rate triple $\boldsymbol{R}_{\text{opt}}(\boldsymbol{w})$ is given by (13).

It remains to prove the third case $C_1(\boldsymbol{Q}_{\text{opt}}(\boldsymbol{w})) = C_2(\boldsymbol{Q}_{\text{opt}}(\boldsymbol{w}))$. Since $R_0(\boldsymbol{w}) > 0$, we have $\xi = 0$ so that (10b) becomes $w_0 = w_1 + w_2 + \nu_1 + \nu_2$. Since $w_0 > w_1 + w_2$ by assumption, this immediately implies that $\nu_1 > 0$ or $\nu_2 > 0$. If $\nu_1 > 0$ then $R_0(\boldsymbol{w}) = C_1(\boldsymbol{Q}_{\text{opt}}(\boldsymbol{w})) = C_2(\boldsymbol{Q}_{\text{opt}}(\boldsymbol{w}))$ by (10g). Similarly, $\nu_2 > 0$ leads to $R_0(\boldsymbol{w}) = C_2(\boldsymbol{Q}_{\text{opt}}(\boldsymbol{w})) = C_1(\boldsymbol{Q}_{\text{opt}}(\boldsymbol{w}))$ so that the weighted rate sum optimal rate triple $\boldsymbol{R}_{\text{opt}}(\boldsymbol{w})$ is given by (14). ■

Going back to our example in Figure 3(a) we see that for all weight vectors $\boldsymbol{w} \in \mathcal{W}^{(>)}$ the weighted rate sum optimal rate triples $\boldsymbol{R}_{\text{opt}}(\boldsymbol{w})$ describe the boundaries of the capacity region on the R_0/R_1- and R_0/R_2-plane respectively. In more detail, all rate triples $\boldsymbol{R}_{\text{opt}}(\boldsymbol{w})$ with $C_1(\boldsymbol{Q}_{\text{opt}}(\boldsymbol{w})) < C_2(\boldsymbol{Q}_{\text{opt}}(\boldsymbol{w}))$ lie on the R_0/R_2-plane and with $C_1(\boldsymbol{Q}_{\text{opt}}(\boldsymbol{w})) > C_2(\boldsymbol{Q}_{\text{opt}}(\boldsymbol{w}))$ on the R_0/R_1-plane. For equality, i.e., $C_1(\boldsymbol{Q}_{\text{opt}}(\boldsymbol{w})) = C_2(\boldsymbol{Q}_{\text{opt}}(\boldsymbol{w}))$, the rate triple $\boldsymbol{R}_{\text{opt}}(\boldsymbol{w})$ characterizes the XOR solution on the R_0-axis (denoted by point $\circ$ in Figure 3(a)).

This substantiates the fact that an optimal transmit strategy for the BBC without common messages for one specific weight vector is optimal for the BBC with common messages for a whole set of weight vectors as specified in (12). For example consider the following. For all $\boldsymbol{w} \in \mathcal{W}^{(>)}$ it is optimal to allocate as much as rate to the common message as possible. If $C_1(\boldsymbol{Q}_{\text{opt}}(\boldsymbol{w})) < C_2(\boldsymbol{Q}_{\text{opt}}(\boldsymbol{w}))$, then $R_1(\boldsymbol{w}) = 0$ which implies that $\boldsymbol{R}_{\text{opt}}(\boldsymbol{w})$ is the same for weight vectors $\boldsymbol{w} \in \mathcal{W}^{(>)}$ with fixed w_0 and w_2 as long as $w_1 < w_0 - w_2$. Moreover, it follows that the boundary of the capacity region on the R_0/R_2-plane is solely characterized by the relation between the weights w_0 and w_2.

C. Dominant Surface

Already in (9) we see that if $w_0 = w_1 + w_2$, the weighted rate sum is independent of the common rate. This indicates that we can interchange the rate of the common message and the rates of the individual messages.

Theorem 4: For weight vector $\boldsymbol{w}' = (0, w'_1, w'_2) \in \mathbb{R}^3_+$ let $\boldsymbol{Q}'_{\text{opt}}(\boldsymbol{w}')$ be the optimal transmit covariance matrix for the BBC without common messages. Then for all weight vectors $\boldsymbol{w} \in \mathcal{W}^{(=)}$ with $w_0 = w'_1 + w'_2$ and $w_i = w'_i$, $i = 1, 2$, the optimum for the BBC with common messages is achieved by the same transmit covariance matrix, i.e., $\boldsymbol{Q}_{\text{opt}}(\boldsymbol{w}) = \boldsymbol{Q}'_{\text{opt}}(\boldsymbol{w}')$.

Proof: If $R_0 = 0$, then $\nu_1 = \nu_2 = 0$ by (10g) which implies that (10a) becomes $\mu \boldsymbol{I}_{N_R} - \boldsymbol{\Psi} = w_1 C'_1(\boldsymbol{Q}) + w_2 C'_2(\boldsymbol{Q})$. Again, this is the BBC without common messages and individual weights $w'_i = w_i$, $i = 1, 2$, so that the optimal transmit covariance matrix for the BBC without common messages immediately determines the one for the BBC with common messages.

If $R_0 > 0$, then $\xi = 0$ by (10g) so that (10b) reads as $w_0 = w_1 + w_2 + \nu_1 + \nu_2$. Since $\nu_i \geq 0$, $i = 1, 2$, by (10e), (10b) is only valid if $\nu_1 = \nu_2 = 0$. Consequently, (10a) becomes $\mu \boldsymbol{I}_{N_R} - \boldsymbol{\Psi} = w_1 C'_1(\boldsymbol{Q}) + w_2 C'_2(\boldsymbol{Q})$. The same arguments as in the first case finish the proof. ■

Proposition 5: For weight vector $\boldsymbol{w} \in \mathcal{W}^{(=)}$ let $\boldsymbol{Q}_{\text{opt}}(\boldsymbol{w})$ be the optimal transmit covariance matrix for the BBC with common messages. Then the weighted rate sum optimal rate triples $\boldsymbol{R}_{\text{opt}}(\boldsymbol{w})$ are

$$R_0(\boldsymbol{w}) \leq \min\big\{C_1\big(\boldsymbol{Q}_{\text{opt}}(\boldsymbol{w})\big), C_2\big(\boldsymbol{Q}_{\text{opt}}(\boldsymbol{w})\big)\big\},$$

$$R_i(\boldsymbol{w}) = C_i\big(\boldsymbol{Q}_{\text{opt}}(\boldsymbol{w})\big) - R_0(\boldsymbol{w}), \quad i = 1, 2.$$

Proof: $\boldsymbol{R}_{\text{opt}}(\boldsymbol{w})$ follows immediately from Theorem 4. ■

We see that for the weighted rate sum optimal rate triples there is a trade-off between the common rate and the individual rates as illustrated in Figure 3(a). For a given weight vector $\boldsymbol{w} \in \mathcal{W}^{(=)}$ the optimal rate triples $\boldsymbol{R}_{\text{opt}}(\boldsymbol{w})$ correspond to a line that begins on the boundary on the R_1/R_2-plane and ends on the R_0/R_1- or R_0/R_2-plane (denoted by gray dashed-dotted lines). Consequently, the weight vectors $\boldsymbol{w} \in \mathcal{W}^{(=)}$ characterizes the dominant surface of the capacity region completely.

D. Interpretation

There is another interpretation. If one fixes some individual weights w_1 and w_2, the optimal rate triple is always on the R_1/R_2-plane (for example given by the point $\triangleleft$ in Figure 3(a)), as long as the common weight fulfills $w_0 < w_1 + w_2$. This immediately implies that the sum rate performance is the same as for the corresponding BBC without common messages.

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In the case of equality, i.e., $w_0 = w_1 + w_2$, all rate triples on the connecting line between the points $\lhd$ and $\rhd$ are optimal. If $w_0 > w_1 + w_2$ the optimal rate triple is on the R_0/R_1- or R_0/R_2-plane and with increasing common weight w_0 the optimal rate triple moves along the corresponding boundary and tends to the XOR solution in point $\circ$ as $w_0 \to \infty$. Figure 3(b) depicts the achievable rate region that is characterized by the XOR solution. Since the common message is transmitted to both receiving nodes, a positive common rate reduces the maximal achievable rates for both individual messages, cf. also (3). Moreover, for the XOR solution in point $\circ$ the common rate uses all available resources so that both individual rates are zero.

V. Discussion

In the previous section we established a strong connection between the transmit covariance matrix optimization problems for the BBC with and without common messages. This was done by showing that an optimal transmit covariance matrix for the BBC without common messages is also a solution for certain optimization problems for the BBC with common messages.

Here we indicate that this connection can be exploited to easily transfer results from one case to the other one which shows that the results obtained in the previous section are not only relevant in itself. In the following we briefly review results from [50, 51] where we assume that the reader is familiar with these references. But we want to stress that these are only examples and that there are much more results which can be transfered in a similar way. The aim of this section is to demonstrate the usefulness of the established connection between the BBC with and without common messages.

First, we consider a MISO scenario, where the relay node is equipped with multiple antennas, while the two other nodes each have a single antenna. Then we know from [50] that beamforming into the subspace spanned by the channels is always optimal for the BBC without common messages. This means that for all weight vectors $\boldsymbol{w}'$ the optimal transmit covariance matrix $\boldsymbol{Q}'_{\text{opt}}(\boldsymbol{w}')$ is of rank one. From the previous section we know that for certain weights the transmit strategy $\boldsymbol{Q}'_{\text{opt}}(\boldsymbol{w}')$ is also optimal for the BBC with common messages. More precisely, this means that for weight vectors $\boldsymbol{w}$ as specified by the results from the previous section, the optimal transmit covariance matrix $\boldsymbol{Q}_{\text{opt}}(\boldsymbol{w})$ for the BBC with common messages is immediately determined by $\boldsymbol{Q}_{\text{opt}}(\boldsymbol{w}) = \boldsymbol{Q}'_{\text{opt}}(\boldsymbol{w}')$. Consequently, $\boldsymbol{Q}_{\text{opt}}(\boldsymbol{w})$ is also of rank one and beamforming into the subspace spanned by the channels is optimal for the BBC with common messages.

Furthermore, it shows that the normalized capacity achieving beamforming vector for the BBC without common messages can be expressed as a linear combination of the two channel directions with a fixed phase difference between the coefficients. This transfers to the BBC with common messages in a similar fashion. Interestingly, it shows that correlation between the channels is advantageous.

Moreover, this allows to characterize the transmit strategy that realizes equal sum rates. Again, the previous section determines when the corresponding transmit strategy for the BBC without common messages transfers to one with common messages. In particular, this is an interesting transmit strategy since it characterizes the rate region that is achievable using suboptimal network coding strategies such as the XOR coding approach [2, 52] as depicted in Figure 3(b).

For the MIMO scenario, the situation is much more complicated since in general there exist different equivalent transmit strategies with different ranks. But for the special case where the rank of the channels is equal to the number of antennas at the relay node and a full-rank transmission is optimal, the optimal transmit covariance matrix for the BBC without common messages can be obtained in closed-form from [51]. Accordingly, once we obtained the optimal covariance matrix, it immediately transfers to the BBC with common messages under certain weight conditions. The same is true for the case of parallel channels. In particular this is a relevant scenario since it immediately provides also solutions for the power allocation problem of a single-antenna OFDM system.

VI. Conclusion

In this work we studied the MIMO Gaussian bidirectional broadcast channel where the relay added an own common message to the communication. We derived the capacity region and characterized the optimal transmit strategies in detail. Interestingly, we showed that there are strong connections between the MIMO Gaussian BBC with and without common messages. Therefore, an additional multicast communication can easily be enabled. This is based on the fact that existing optimization policies or algorithms must only be slightly extended by including an additional discussion for the weight of the common message. Consequently, this underlines the ability of the decode-and-forward bidirectional relaying concept to be a promising candidate for the efficient integration of different services on the same resources.

The strong connection begins with the optimal coding strategy for the BBC with common messages, since basically, it reduces to coding without common messages. The common message is treated as a part of both individual messages and as a result, the coding idea of the case without common messages is applicable. All messages are combined into a single data stream which allows the receiving nodes to decode the intended individual and common message using their own message as side information.

In retrospect it is not surprising that the connection carries over to the transmit covariance optimization problem. In contrast to suboptimal strategies such as superposition coding approaches [53], where the messages are associated with several transmit covariance matrices, in the optimal strategy there is only one transmit covariance matrix that has to be optimized. This is similar to the BBC without common messages, and as expected, the optimal transmit strategies transfer as well.

Appendix

A. *Proof of Lemma 1*

Here, we present the analysis for receiving node 1. Then, the other case follows accordingly using exactly the same arguments. From the received sequence $\mathbf{Y}_1^n$ and its own

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message M_1 node 1 estimates the indices M_0 and M_2 from the sent codeword $\mathbf{X}^n(\mathrm{M}_0, \mathrm{M}_1, \mathrm{M}_2)$. We define the event of an error at node 1 as

$$\mathrm{E}_1 := \begin{cases} 1, & \text{if } g_1(\mathbf{Y}_1^n, \mathrm{M}_1) \neq (\mathrm{M}_0, \mathrm{M}_2) \\ 0, & \text{if } g_1(\mathbf{Y}_1^n, \mathrm{M}_1) = (\mathrm{M}_0, \mathrm{M}_2) \end{cases}$$

so that we have for the average probability of error $\mu_1^{(n)} = \mathbb{P}\{\mathrm{E}_1 = 1\}$. From the chain rule for entropies [54, Lemma 8.3.2] we have

$$\begin{aligned} &H(\mathrm{E}_1, \mathrm{M}_0, \mathrm{M}_2|\mathbf{Y}_1^n, \mathrm{M}_1) \\ &\quad = H(\mathrm{M}_0, \mathrm{M}_2|\mathbf{Y}_1^n, \mathrm{M}_1) + H(\mathrm{E}_1|\mathbf{Y}_1^n, \mathrm{M}_0, \mathrm{M}_1, \mathrm{M}_2) \\ &\quad = H(\mathrm{E}_1|\mathbf{Y}_1^n, \mathrm{M}_1) + H(\mathrm{M}_0, \mathrm{M}_2|\mathbf{Y}_1^n, \mathrm{M}_1, \mathrm{E}_1). \end{aligned}$$

Since E_1 is a function of M_0, M_1, M_2, and $\mathbf{Y}_1^n$, we have $H(\mathrm{E}_1|\mathbf{Y}_1^n, \mathrm{M}_0, \mathrm{M}_1, \mathrm{M}_2) = 0$. Further, since E_1 is a binary-valued random variable, we get $H(\mathrm{E}_1|\mathbf{Y}_1^n, \mathrm{M}_1) \leq H(\mathrm{E}_1) \leq 1$. So that finally with the next inequality

$$\begin{aligned} &H(\mathrm{M}_0, \mathrm{M}_2|\mathbf{Y}_1^n, \mathrm{M}_1, \mathrm{E}_1) \\ &\quad = \mathbb{P}\{\mathrm{E}_1 = 0\} H(\mathrm{M}_0, \mathrm{M}_2|\mathbf{Y}_1^n, \mathrm{M}_1, \mathrm{E}_1 = 0) + \\ &\qquad\qquad \mathbb{P}\{\mathrm{E}_1 = 1\} H(\mathrm{M}_0, \mathrm{M}_2|\mathbf{Y}_1^n, \mathrm{M}_1, \mathrm{E}_1 = 1) \\ &\quad \leq (1 - \mu_1^{(n)})0 + \mu_1^{(n)} \log((M_0^{(n)} - 1)(M_2^{(n)} - 1)) \\ &\quad \leq \mu_1^{(n)} \log(M_0^{(n)} M_2^{(n)}) \end{aligned}$$

we get the desired version of Fano's inequality for the MIMO Gaussian BBC with common messages. ■

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