Reliable Estimation of Phase Biases of GPS Satellites with a Local Reference Network

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Abstract – Precise Point Positioning with GPS and/or Galileo is becoming increasingly popular as it does not need any measurements from a reference station. However, the resolution of the integer ambiguities of the periodic carrier phases requires precise and accurate estimates of satellite phase biases. This paper describes a new method for the estimation of the receiver and satellites phase biases on all frequencies. It uses a geometry-free approach with a Kalman filter and sequentially fixes the undifferenced integer ambiguities. Several steps are performed to improve the reliability of ambiguity fixing, e.g. the fixing decision over a time-window and the use of both statistical information from the Kalman filter and the actual deviation between the float and nearest integer numbers. The proposed method is applied to dual-frequency L1/L2 GPS measurements from 11 SAPOS stations in Bavaria to analyze the stability of the satellite phase biases. The long time span of 24 hours involves several rises and settings of satellites and, thus, requires parameter mappings and trackings within the estimation. The observed satellite biases vary by only 3 cm over 5 hours, and the receiver phase biases are even more stable.
Keywords – phase biases, Kalman filter, stability

I. Introduction

The positioning of a kinematic receiver in real-time with centimeter-level positioning accuracy can currently only be achieved in differential mode, i.e. a relative positioning of two receivers. Double difference measurements between a pair of satellites and a pair of receivers are performed to eliminate the receiver and satellite biases and, thus, to simplify the resolution of the carrier phase ambiguities.

However, this double differencing requires the exchange of the complete set of measurements, which is a major drawback and a strong motivation for precise point positioning. A prerequisite for resolution of undifferenced integer ambiguities is the knowledge of satellite phase and code biases. Today, the International GNSS Services (IGS) [1] is providing differential P1/C1 code biases, which are computed on the basis of the ionosphere-free linear combination in the course of a global GNSS clock analysis [2] [3].

Ge et al. [4], Gabor and Nerem [5] and Laurichesse and Mercier [6] estimated the L1 and L2 phase biases by combining the fractional bias term of the Melbourne-Wübbena combination [7] and the joint bias/ambiguity term of the geometry-preserving, ionosphere-free phase-only combination. The obtained pseudo-phase biases enable an unbiased estimation of the L1 and L2 integer ambiguities. However, these phase biases also include a weighted combination of code biases on both frequencies. It is shown in [8] that these L1/L2 pseudo-phase biases correspond to a geometry-preserving, ionosphere-free narrowlane combination with a wavelength of only 10.7 cm. There does not exist any geometry-preserving, ionosphere-free combination with the applicability of these biases and a larger wavelength than 10.7 cm.

This paper provides a new method for the estimation of undifferenced and non-combined satellite phase biases with a Kalman filter. Section II includes a general model for undifferenced measurements and a parameter mapping to obtain a full rank equation system. In section III, the estimation of satellite phase biases with a Kalman filter and sequential integer ambiguity resolution is described. The method is applied to real GPS measurements from 11 stations of the German SAPOS network in section IV.

II. Measurement Model and Parameter Mapping

A. Measurement Model

A very general model shall be used for the undifferenced carrier phase and code measurements on frequency m, receiver r, satellite k and time t:

$$\begin{aligned}
\lambda_1\varphi_{1,r}^k(t) &= g_r^k(t) - I_{1,r}^k(t) + \lambda_1 N_{1,r}^k \\
&\quad + \beta_{1,r} + \beta_1^k + \varepsilon_{1,r}^k(t) \\
\lambda_2\varphi_{2,r}^k(t) &= g_r^k(t) - q_{12}^2 I_{1,r}^k(t) + \lambda_2 N_{2,r}^k \\
&\quad + \beta_{2,r} + \beta_2^k + \varepsilon_{2,r}^k(t) \\
\rho_{1,r}^k(t) &= g_r^k(t) + I_{1,r}^k(t) + b_{1,r} + b_1^k + \eta_{1,r}^k(t) \\
\rho_{2,r}^k(t) &= g_r^k(t) + q_{12}^2 I_{1,r}^k(t) + b_{2,r} + b_2^k + \eta_{2,r}^k(t),
\end{aligned} \tag{1}$$

where:

$\lambda_m\varphi_{m,r}^k$:	carrier phase measurement,
$\rho_{m,r}^k$:	code measurement,
g_r^k:	geometry term,
$I_{m,r}^k$:	ionospheric slant delay,
$N_{m,r}^k \in \mathbb{Z}$:	integer ambiguity,
$\beta_{m,r}$:	receiver phase bias,
β_m^k:	satellite phase bias,
$b_{m,r}$:	receiver code bias,
b_m^k:	satellite code bias,
$\varepsilon_{m,r}^k$:	phase noise,
$\eta_{m,r}^k$:	code noise,

and $q_{12} = f_1/f_2$ is the frequency ratio. The multipath errors are included in the phase and code noise.

The geometry term $g_r^k(t)$ is composed of all non-dispersive terms including the range $||\boldsymbol{x}_r - \boldsymbol{x}^k||$, the receiver and satellite clock offsets $c\delta\tau_r$, $c\delta\tau^k$, and tropospheric delays T_r^k, i.e.

$$\begin{aligned} g_r^k(t) &= ||\boldsymbol{x}_r - \boldsymbol{x}^k(t-\Delta\tau_r^k(t))|| + T_r^k(t) \\ &\quad + c\left(\delta\tau_r(t) - \delta\tau^k(t-\Delta\tau_r^k(t))\right), \end{aligned} \tag{2}$$

where $\Delta\tau_r^k(t)$ is the signal travel time from satellite k to receiver r. The treatment of $g_r^k(t)$ as a single term makes the phase bias estimation robust against orbital errors and tropospheric modeling errors.

B. Parameter Mapping

The system of equations of (1) is rank-deficient, i.e. it is not possible to directly estimate all g_r^k, $I_{1,r}^k$, $\beta_{m,r}$, β_m^k and $N_{m,r}^k$. Therefore, a set of mappings is applied to remove rank deficiency of the system as described by Henkel et al. in [8].

1. Mapping of code biases

First, the code biases are combined with the geometry and ionospheric terms, i.e.

$$\tilde{g}_r^k(t) = g_r^k(t) + b_{g_r} + b_{g^k}, \quad \tilde{I}_{1,r}^k(t) = I_{1,r}^k(t) + b_{I_r} + b_{I^k}, \tag{3}$$

with $b_{g_r} = -\frac{b_{2,r} - q_{12}^2 b_{1,r}}{q_{12}^2 - 1}$ and $b_{I_r} = \frac{b_{1,r} - b_{2,r}}{q_{12}^2 - 1}$. The satellite dependant biases b_{g^k} and b_{I^k} are obtained by replacing the lower index r in b_{g_r} and b_{I_r} by an upper index k. The phase biases shall be changed accordingly to compensate for the terms mapped to the geometry and ionospheric parameters, i.e.

$$\begin{aligned} \tilde{\beta}_{1,r} &= \beta_{1,r} - b_{g_r} + q_{11}^2 b_{I_r}, \quad \tilde{\beta}_1^k = \beta_1^k - b_{g^k} + q_{11}^2 b_{I^k} \\ \tilde{\beta}_{2,r} &= \beta_{2,r} - b_{g_r} + q_{12}^2 b_{I_r}, \quad \tilde{\beta}_2^k = \beta_2^k - b_{g^k} + q_{12}^2 b_{I^k}. \end{aligned} \tag{4}$$

2. Mapping of one satellite phase bias

Secondly, one of the satellite phase biases on each frequency is mapped to the receiver phase biases, i.e.

$$\begin{aligned} \tilde{\tilde{\beta}}_{1,r} &= \tilde{\beta}_{1,r} + \tilde{\beta}_1^1, \quad \tilde{\tilde{\beta}}_1^k = \tilde{\beta}_1^k - \tilde{\beta}_1^1 \\ \tilde{\tilde{\beta}}_{2,r} &= \tilde{\beta}_{2,r} + \tilde{\beta}_2^1, \quad \tilde{\tilde{\beta}}_2^k = \tilde{\beta}_2^k - \tilde{\beta}_2^1, \end{aligned} \tag{5}$$

where the first satellite has been chosen as reference satellite. There exists some degrees of freedom for the choice of this best mapping.

3. Mapping of a subset of ambiguities

In a last step, a subset of ambiguities is absorbed by phase biases and ambiguities, so that the equation system of (1) is transformed to a full rank system, i.e.

$$\begin{aligned} &\tilde{\tilde{\tilde{\beta}}}_r = \tilde{\tilde{\beta}}_r + \sum_{N_i \in N_{\mathrm{sub}}} c_{i,r} N_i, \quad \tilde{N}_r^k = N_r^k + \sum_{N_i \in N_{\mathrm{sub}}} c_{i,r}^k N_i, \\ &\tilde{\tilde{\tilde{\beta}}}^k = \tilde{\tilde{\beta}}^k + \sum_{N_i \in N_{\mathrm{sub}}} c_i^k N_i, \end{aligned} \tag{6}$$

where the subset is denoted by N_{sub} and $c_{i,r}$, c_i^k and $c_{i,r}^k$ denote the coefficients generated by Gaussian elimination as described by Wen in [9].

III. Satellite Phase Bias Estimation with a Kalman filter

A Kalman filter is optimal for processes with a linear behavior over time. Since the geometry in equation (2) contains an elliptical orbit for GPS satellites, a rough estimate of the range is computed from the known position of the reference station and the broadcast satellite ephemerids, and then subtracted from the measurements to remove the nonlinearity to a substantial amount (except for the clock offsets):

$$\begin{aligned} \Delta\rho_{m,r}^k(t) &= \rho_{m,r}^k(t) - \|\hat{\boldsymbol{x}}_r - \hat{\boldsymbol{x}}^k(t - \Delta\hat{\tau}_r^k(t))\| \\ \lambda_m \Delta\varphi_{m,r}^k(t) &= \lambda_m \varphi_{m,r}^k(t) - \|\hat{\boldsymbol{x}}_r - \hat{\boldsymbol{x}}^k(t - \Delta\hat{\tau}_r^k(t))\|. \end{aligned} \tag{7}$$

The bias estimation is then based on these "differenced" measurements, which are sampled at time $t = nT$, i.e. the continuous time t shall be replaced from now on by a discrete index n. The "differential" geometry term in (7) can be well modeled by a random walk process for the differential range acceleration, i.e. the following state space model is proposed:

$$\Delta\tilde{g}_{r,n}^k = \Delta\tilde{g}_{r,n-1}^k + \Delta t \Delta\dot{\tilde{g}}_{r,n-1}^k + \frac{1}{2}\Delta t^2 \Delta\ddot{\tilde{g}}_{r,n-1}^k + w_{g_{r,n}^k}, \tag{8}$$

where Δt denotes the time interval between consecutive states, and $w_{g_r^k}(t) \sim \mathcal{N}(0, \sigma^2_{w_{g_r^k}})$ denotes the process noise to model accelerations.

The slant ionospheric delays, receiver and satellite phase biases shall also be modeled by a random walk process, while the ambiguities are assumed to be constant over time.

The system of equations (1) can be solved with a Kalman filter after the parameter mapping. The phase and code measurements are combined in vector notation as

$$\boldsymbol{z}_n = \left[\lambda_1 \Delta\boldsymbol{\varphi}_{1,n}^{\mathrm{T}}, \lambda_2 \Delta\boldsymbol{\varphi}_{2,n}^{\mathrm{T}}, \Delta\boldsymbol{\rho}_{1,n}^{\mathrm{T}}, \Delta\boldsymbol{\rho}_{2,n}^{\mathrm{T}}\right]^{\mathrm{T}} = \boldsymbol{H}_n \boldsymbol{x}_n + \boldsymbol{v}_n, \tag{9}$$

where the $\boldsymbol{H}_n$ matrix can be obtained from Equations (1), (3), (5) and (6) and the state vector $\boldsymbol{x}_n$ includes the geometry term, the first and second order derivatives of geometry terms, the ionospheric slant delays, the receiver and satellite phase biases and the integer ambiguities, i.e.

$$\boldsymbol{x}_n = \left[\Delta\tilde{\boldsymbol{g}}_n^{\mathrm{T}}, \Delta\dot{\tilde{\boldsymbol{g}}}_n^{\mathrm{T}}, \Delta\ddot{\tilde{\boldsymbol{g}}}_n^{\mathrm{T}}, \tilde{\boldsymbol{I}}_n^{\mathrm{T}}, \tilde{\tilde{\tilde{\boldsymbol{\beta}}}}_{\mathrm{rec}}^{\mathrm{T}}, \tilde{\tilde{\tilde{\boldsymbol{\beta}}}}_{\mathrm{sat}}^{\mathrm{T}}, \tilde{\boldsymbol{N}}^{\mathrm{T}}\right]^{\mathrm{T}}, \tag{10}$$

and the measurement noise $\boldsymbol{v}_n$ follows a Gaussian distribution $\mathcal{N}(0, \boldsymbol{\Sigma}_{\mathrm{R}})$. The state transition model for $\boldsymbol{x}_n$ is given by:

$$\boldsymbol{x}_n = \boldsymbol{\Phi}_{n-1}\boldsymbol{x}_{n-1} + \boldsymbol{w}_{n-1}, \tag{11}$$

with the process noise $\boldsymbol{w}_n \sim \mathcal{N}(0, \Sigma_{\mathrm{Q}})$. The state estimates and their covariance matrix are predicted as

$$\begin{aligned} \hat{\boldsymbol{x}}_n^- &= \boldsymbol{\Phi}_{n-1}\hat{\boldsymbol{x}}_{n-1}^+ \\ \boldsymbol{P}_n^- &= \boldsymbol{\Phi}_{n-1}\boldsymbol{P}_{n-1}^+\boldsymbol{\Phi}_{n-1}^{\mathrm{T}} + \boldsymbol{\Sigma}_{\mathrm{Q},n-1}, \end{aligned} \tag{12}$$

and updated by the new measurements at epoch n [10], i.e.

$$\begin{aligned} \hat{\boldsymbol{x}}_n^+ &= \hat{\boldsymbol{x}}_n^- + \boldsymbol{K}_n(\boldsymbol{z}_n - \boldsymbol{H}_n\hat{\boldsymbol{x}}_n^-) \\ \boldsymbol{K}_n &= \boldsymbol{P}_n^-\boldsymbol{H}_n^{\mathrm{T}}(\boldsymbol{H}_n\boldsymbol{P}_n^-\boldsymbol{H}_n^{\mathrm{T}} + \boldsymbol{\Sigma}_{\mathrm{R}})^{-1} \\ \boldsymbol{P}_n^+ &= (\boldsymbol{I} - \boldsymbol{K}_n\boldsymbol{H}_n)\boldsymbol{P}_n^-, \end{aligned} \tag{13}$$

with $\boldsymbol{\Phi}_n$ being the state transition matrix.

A. Sequential Ambiguity Fixing

In this work, a new fixing decision criterion is introduced: The float ambiguities are fixed to integer numbers only if the offsets between float and integer numbers are below a certain threshold during a time window, i.e.

$$\sum_{i=1}^{T_\mathrm{p}} f(\hat{N}^{+,k}_{m,r,n-i}) \leq T_\mathrm{p} \cdot w, \tag{14}$$

where

$$f(\hat{N}^{+,k}_{m,r,n-i}) = \begin{cases} 1 & \text{if } \left|\hat{N}^{+,k}_{m,r,n-i} - [\hat{N}^{+,k}_{m,r,n-i}]\right| > e_\mathrm{th} \\ 0 & \text{else.} \end{cases}, \tag{15}$$

with T_p denoting the length of the time window in which the convergence of the ambiguities is observed, w being the probability (allowing some outliers), and e_th being the error threshold. In the implementation, the parameters T_p, w and e_th are chosen to be respectively 600 epochs, 0.95, and 0.08 cycles. A motivation for this fixing criterion is given by Fig. 1: The float ambiguity estimate first varies around -20 (due to multipath) before converging to the true integer number -22.

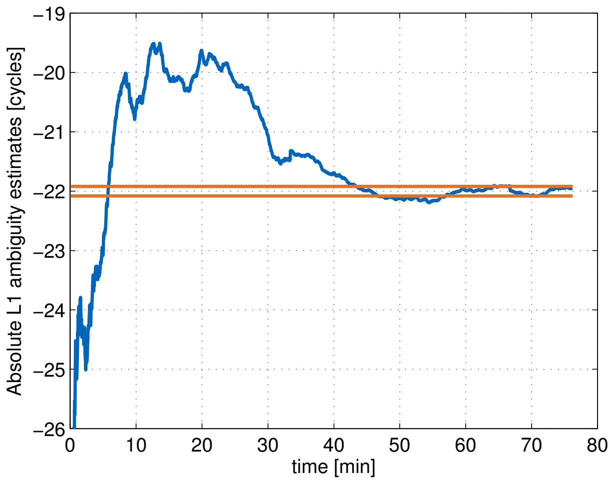

Fig. 1. The convergence behavior of one ambiguity estimate on L1. The ambiguity estimate might be fixed to the integer -20 after 20 minutes. However, the ambiguity converges to -22 within the next 30 minutes, and then remains very stable inside the threshold of 0.08 cycles (orange horizontal lines) for 10 minutes.

B. Rising and Setting Satellite

Since the bias estimation needs a network of receivers over a period of several hours, rising and setting satellites have to be considered during the Kalman filtering, so that the bias estimates can be observed for an arbitrary long time period. For a setting satellite at one receiver, the visibility of this satellite at other receivers has an impact on the parameter mapping. If a setting satellite is invisible at only some of the receivers, the measurements on the vanishing links do not exist any more, which means the corresponding rows of the $\boldsymbol{H}$ matrix should be deleted, so do the relevant states $\Delta\tilde{g}$, $\Delta\dot{\tilde{g}}$, $\Delta\ddot{\tilde{g}}$, and $\tilde{\boldsymbol{I}}$. However, the corresponding satellite bias shall be still kept to be estimated because the other receivers could still "see" that satellite. In this case, the estimation of that satellite bias could become worse because of a decreasing number of measurements from that satellite.

If a setting satellite moves out of the sight of all receivers, not only the measurements from that satellite but also the satellite bias along with the relevant states $\Delta\tilde{g}$, $\Delta\dot{\tilde{g}}$, $\Delta\ddot{\tilde{g}}$, and $\tilde{\boldsymbol{I}}$ should be eliminated. Moreover, if the setting satellite was *the* mapped reference satellite in Equation (5), the bias of another visible satellite has to be mapped into the other biases to keep the full rank of $\boldsymbol{H}$. If the ambiguities of a setting satellite were already mapped to other ambiguities in section II, an additional parameter mapping is required at the setting epoch. These mappings are continuously tracked to observe the stability over time.

For the rising case, new links between receivers and the rising satellites come up. Therefore, the state vector is extended by the new states of $\Delta\tilde{g}$, $\Delta\dot{\tilde{g}}$, $\Delta\ddot{\tilde{g}}$, $\tilde{\boldsymbol{I}}$, $\tilde{\tilde{\boldsymbol{\beta}}}$ and $\tilde{\boldsymbol{N}}$. If the rising satellites are only rising at some of the receivers, which means the measurements from that satellite increase, the satellite biases do not have to be initialized any more. In the other case, the new satellite phase biases are initialized, i.e. a least-squares estimation of the ranges, slant ionospheric delays, and new combined ambiguity/ bias terms is performed with measurements of at least three epochs.

TABLE I
REFERENCE STATIONS OF SAPOS NETWORK IN BAVARIA, GERMANY.

ID	0261	0265	0268	0269	0272	0273
ϕ [°]	48.568	48.429	49.737	47.602	47.868	48.042
λ [°]	13.443	12.933	10.162	10.416	12.107	10.494
ID	0274	0281	0285	0286	0292	
ϕ [°]	48.453	49.512	47.509	48.936	47.559	
λ [°]	10.279	12.546	11.143	13.564	9.708	

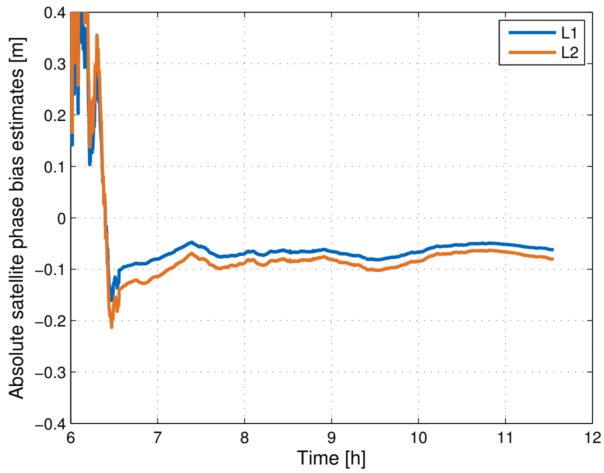

Fig. 2. Absolute satellite phase bias estimates of PRN 24.

IV. Results

In this section, dual frequency GPS measurements from 11 SAPOS (Satellitenpositionierungsdienst der deutschen Landesvermessung) stations in Bavaria, Germany, are processed from 24 hours of March 14, 2011. All stations are using the same type of Trimble receivers. Their coordinates are listed in Tab. I. The process noise standard deviations were set to $\sigma_{w_{gr}} = 1$ m for the range and to $\sigma_{w_I} = 1$ cm for the slant ionospheric delay. Fig. 2 - 5 show the stability of the obtained satellite and receiver phase bias estimates over several hours. The temporal variations of the converged bias estimates are around 3 cm, and show a high correlation between both frequencies.

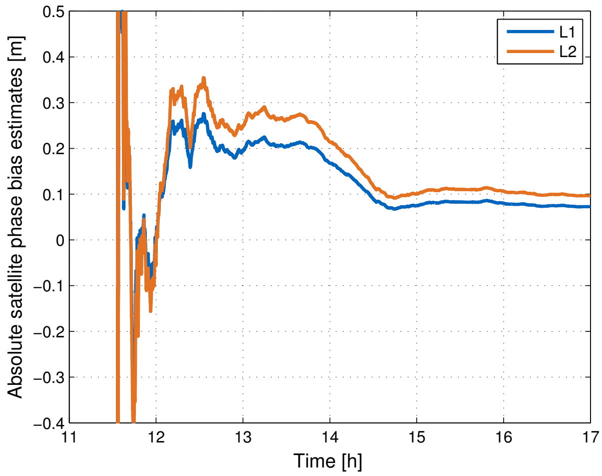

Fig. 3. Absolute satellite phase bias estimates of PRN 10.

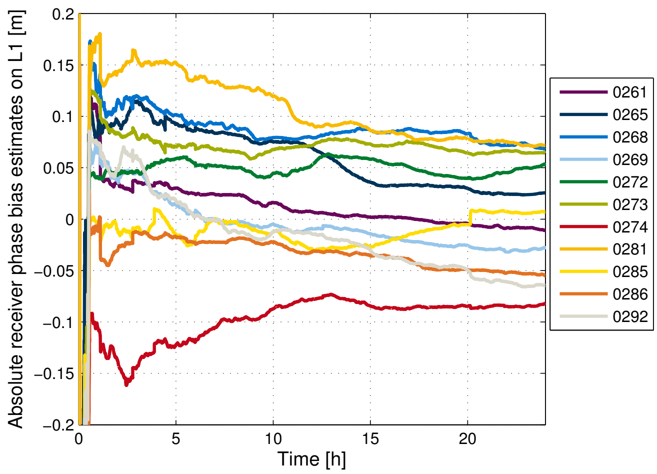

Fig. 4. Absolute receiver phase bias estimates on L1.

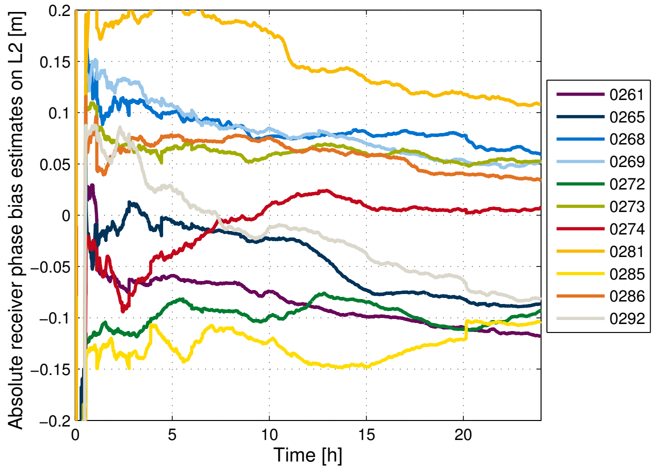

Fig. 5. Absolute receiver phase bias estimates on L2.

V. Conclusion

A new method for the reliable estimation of receiver and satellite phase biases was presented in this paper. A Kalman filter has been used in the approach to estimate the phase biases, while the integer ambiguities were fixed sequentially and reliably due to a new fixing decision criterion over a time-window. The method has been applied to real GPS measurements taken from the SAPOS network in Bavaria to analyze the stability of the phase biases. The results have shown 3 cm variation over 5 hours of the observed satellite phase biases and even less variation for the receiver phase biases. Future work will focus on the stability and repeatability of the satellite phase biases for days using a global network.

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