Reduction and Optimization of Almanac transmission for GNSS Satellites

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Abstract – Almanacs are satellite position and clock data of reduced precision, which are transmitted by navigation satellites to fasten signal acquisition. Currently, each satellite is transmitting the almanacs of all satellites independent of the receiver-satellite geometry. This means that the transmission of the complete almanacs takes 12 minutes for GPS. This paper suggests an optimized almanac transmission scheme, which takes the receiver-satellite geometry into account and thereby reduces the number of almanac transmissions for each satellite. The optimization of the subsets of satellites and of the order of transmissions within each subset reduces the number of almanac transmissions from 27 to 8 for Galileo. Moreover, the optimization of the almanacs also enables an approximately two times faster signal acquisition.
Keywords – satellite navigation, signal acquisition, almanacs

I. Introduction

The almanac is a subset of clock and ephemeris data with reduced precision in the navigation message [1]. The first purpose of the almanac is to initialize signal acquisition when a new satellite rises above the horizont and the user position is approximately known. The second purpose of the almanac is to fasten the signal acquisition of a warm start: The warm start is an acquisition mode which is characterized by a priori information of the last user position, the receiver clock time and the complete almanacs such that the search of the code delay $\Delta\tau$ and the Doppler shift Δf_d is significantly fastened.

On the contrary, a cold start is defined by signal acquisition without any a priori information. The signal acquisition of the first satellite is very time consuming as the two-dimensional search space $(\Delta\tau, \Delta f_d)$ is very large [1]. After acquisition and carrier tracking, the navigation message is demodulated and the almanac is read which helps the signal acquisition of further satellites.

The transmission of full almanacs in GPS has some disadvantages: There exists a considerable redundancy of almanac broadcasts as each satellite transmits the almanacs of *all* satellites including its own almanac. Every satellite uses the same set of almanacs without taking the inter-satellite distances into account. Moreover, the navigation message of each satellite shows the same temporal order of almanacs and no permutation is applied to reduce the time for reading of all almanacs.

In GPS, the almanac data for the i-th satellite are implemented in the 5-th subframe of the i-th page [2]. The transmission of one page takes 30s resulting in 12 minutes of the fundamental GPS constellation with 24 satellites.

II. Galileo system model with reduced almanac

We consider the ideal (27/3/1) Walker constellation proposed for Galileo in [3]. The analysis is restricted to this constellation although our algorithm can be equally applied to any other constellation. The three $56°$ inclined orbits are characterized by a semi-major axis of $r_{\text{S}} = 29600$ km and a Right Ascension of the Ascending Node (RAAN) of

$$\Omega^{(k)} = 120° \cdot \lfloor \frac{k-1}{9} \rfloor \in \{0°, 120°, 240°\}, \tag{1}$$

where $k = \{1, 2, \ldots, 27\}$ denotes the satellite index and $\lfloor z \rfloor$ represents the nearest integer which is equal or smaller than z. The argument of perigee is assumed to be $\omega = 0°$ for all satellites. The true anomaly is given by $\nu^{(k)}(t) = \nu_0^{(k)} + 2\pi \cdot \frac{t}{T_{\text{S}}}$ with the satellite orbit period T_{S} and the initial true anomaly

$$\nu_0^{(k)} = \frac{40°}{3} \cdot \lfloor \frac{k-1}{9} \rfloor + 40° \cdot (k - 1 - 9 \cdot \lfloor \frac{k-1}{9} \rfloor). \tag{2}$$

Equivalently, the satellite index k can be expressed as a function of the initial true anomaly $\nu_0^{(k)}$ and the RAAN $\Omega^{(k)}$:

$$k = 1 + \lfloor \frac{\nu_0^{(k)}}{40°} \rfloor + 9 \cdot \frac{\Omega^{(k)}}{120°}. \tag{3}$$

III. Optimization of the Reduced Almanac

The selection of the almanac sets is a two-step procedure: First, we fix L satellites as previously described and second, we check that any visible constellation can be completely acquired with the reduced almanac data.

A. Maximum Likelihood Approach based on Inter-Satellite Distances

Let us consider all possible user positions $\boldsymbol{x}_{\text{u}}$ from which a fixed satellite k at position $\boldsymbol{x}^{(k)}(t)$ can be observed at time t. We search for the satellite l at position $\boldsymbol{x}^{(l)}(t)$ which can be seen from as many user positions as possible, i.e.

$$\max_{l} \left| \left\{ \boldsymbol{x}_{\text{u}} | \gamma(\boldsymbol{x}_{\text{u}}, \boldsymbol{x}^{(l)}(t)) > \alpha \wedge \gamma(\boldsymbol{x}_{\text{u}}, \boldsymbol{x}^{(k)}(t)) > \alpha \right\} \right|, \tag{4}$$

where $\gamma(\boldsymbol{x}_{\text{u}}, \boldsymbol{x}^{(k)}(t))$ denotes the elevation angle of satellite k from $\boldsymbol{x}_{\text{u}}$ and α the elevation mask.

Equation (4) maximizes the intersection area of the spherical calottes which represent the visibility regions of the two satellites (Fig. 1). This is equivalent to the selection of the satellite with minimum distance to the fixed satellite.

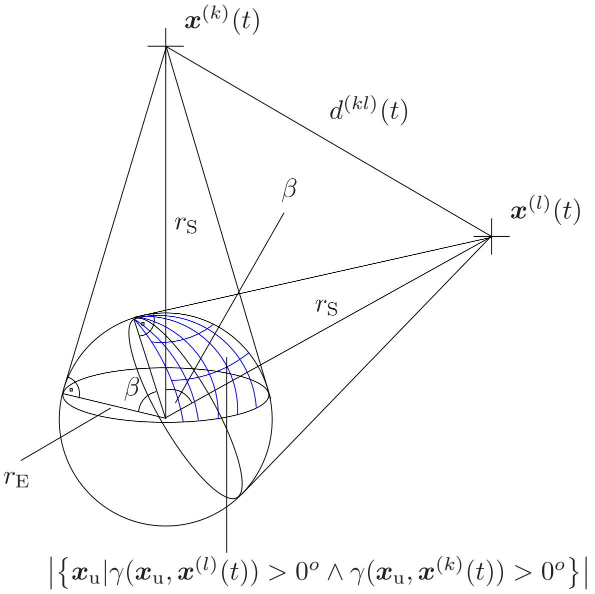

Fig. 1. Geometric properties of neigboured satellites: Visibility region, inter-satellite distance and grid angle between two users at the edge of the visibility regions

The generalization to L almanac data per satellite is also based on inter-satellite distances, i.e. each satellite should contain at time t the almanacs of the L nearest satellites.

Inter-satellite distances are independent of the earth rotation so that we define the position of satellite k in an earth centered but not earth fixed coordinate system as

$$\boldsymbol{x}^{(k)}(t) = \boldsymbol{R}_3(-\Omega^{(k)})\boldsymbol{R}_1(-i)\begin{bmatrix} r_S\cos(\nu^{(k)}(t)) \\ r_S\sin(\nu^{(k)}(t)) \\ 0 \end{bmatrix}, \quad (5)$$

where $i = 56°$ denotes the inclination angle of all satellites. The rotation matrices are defined as in [1], i.e.

$$\boldsymbol{R}_1(\theta) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos(\theta) & \sin(\theta) \\ 0 & -\sin(\theta) & \cos(\theta) \end{bmatrix}$$

and

$$\boldsymbol{R}_3(\theta) = \begin{bmatrix} \cos(\theta) & \sin(\theta) & 0 \\ -\sin(\theta) & \cos(\theta) & 0 \\ 0 & 0 & 1 \end{bmatrix},$$

with the rotation angle θ. The inter-satellite distance between satellites k and l is obtained from equation (5) as

$$d^{(kl)}(t) = \|\boldsymbol{x}^{(k)}(t) - \boldsymbol{x}^{(l)}(t)\|. \quad (6)$$

Let us determine the period of $d^{(kl)}$: From Newton's law of universal gravitation [1], the satellite orbit period can be derived as $T_S = \sqrt{4\pi^2 \frac{r_S^3}{G \cdot m_E}}$, where G and m_E denote the gravitational constant and the mass of the earth. All satellites are in the opposite position of their orbit after a half cycle. This results in a repetition of the inter-satellite distances after

$$T_d = 1/2 \cdot T_S \approx 7.04\,\text{h}. \quad (7)$$

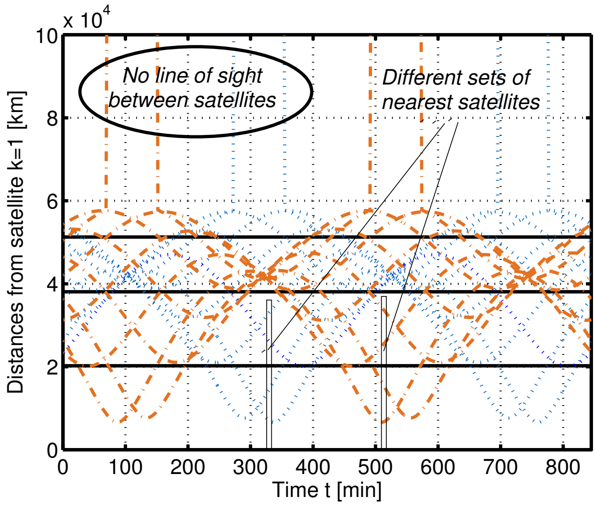

Fig. 2. Inter-satellite distances $d^{(kl)}(t)$ with $k = 1$ and $l = 2, \ldots, 27$

Fig. 2 shows the time dependency of the inter-satellite distances between the satellite $k = 1$ and the other satellites of the ideal Walker constellation. Obviously, the distance of a satellite pair of the same orbit is independent of time. There exist two satellites that are never visible and two further satellites that are seen only occasionally with one interruption in T_d.

Fig. 2 also visualizes that the set of L nearest satellites of the $k = 1$ satellite changes frequently over time, e.g. selecting the $L = 8$ nearest satellites of the first satellite means 50 changes of the almanac per day. This makes the standardization of the maximum likelihood approach more difficult.

B. Almanac selection based on Time-averaged Inter-Satellite Distances

We investigate a suboptimal approach by considering time-averaged inter-satellite distances to overcome the problem of frequently changing almanacs. The almanac of satellite k consists of a set of L distinct satellites which is obtained from

$$\min_{\substack{\boldsymbol{s} \\ \dim(\boldsymbol{s})=L}} \sum_{\substack{l\in \boldsymbol{s} \\ l\neq k}} \overline{d}^{(kl)} = \min_{\substack{\boldsymbol{s} \\ \dim(\boldsymbol{s})=L}} \sum_{\substack{l\in \boldsymbol{s} \\ l\neq k}} \frac{1}{T_d}\int_0^{T_d} d^{(kl)}(t)dt. \quad (8)$$

The almanac information of satellite k is restricted to permanently visible satellites, i.e. the inter-satellite distance is upper bounded by

$$d^{(kl)}(t) \overset{!}{<} 2\sqrt{r_S^2 - r_E^2} \approx 57815\,\text{km}. \quad (9)$$

Note that all satellites travel on the ground tracks from west to east so that some satellite pairs have always considerably larger distances than other ones.

The result of the optimization (8) is depicted in Tab. I which shows the neighbours of each satellite sorted according to the mean inter-satellite distances in an increasing order. Once we have determined all almanac sets, we verify that any visible constellation can be completely acquired for any initially acquired satellite. This validation requires the introduction of a spatio-temporal grid to define all possible sets of simultaneously visible satellites for any position $\boldsymbol{x}_{\mathrm{u}}$ at any time t.

C. Almanac selection based on permutation of the "Satellite neighbour matrix"

Our third approach is based on the result of the previous optimization, i.e. the sorting of satellites in matrix $\boldsymbol{S}$ according to their mean inter-satellite distances (Tab. I). In contrast to the previous approach, we do not select the L nearest satellites but search for a permutation $\boldsymbol{p}$ of L columns of $\boldsymbol{S}$ such that any visible satellite is always announced by the almanac of at least one other visible satellite. We add a constraint to the set selection to prevent an empty intersection between a visible satellite k and the set spanned by the almanacs of all other visible satellites, i.e.

$$\min_{\boldsymbol{p}} \quad L \;\; \text{s.t.} \bigcup_{\substack{i\in \boldsymbol{c} \\ i\neq k}} \bigcup_{\substack{j\in \boldsymbol{p} \\ |\boldsymbol{p}|=L}} \bigcap (\boldsymbol{S}[i,j], k) \neq \emptyset \quad \forall \quad k, \boldsymbol{c}, \alpha, \tag{10}$$

where $\emptyset$ denotes the empty set and $\boldsymbol{c}$ represents a set of visible satellites. The optimization (10) can be rewritten as

$$\min_{\boldsymbol{p}} \quad L \;\; \text{s.t.} \bigcap_{\alpha} \bigcap_{\boldsymbol{c}} \bigcap_{k \in \boldsymbol{c}} \bigcup_{\substack{i\in \boldsymbol{c} \\ i\neq k}} \bigcup_{\substack{j\in \boldsymbol{p} \\ |\boldsymbol{p}|=L}} \bigcap (\boldsymbol{S}[i,j], k) \neq \emptyset. \tag{11}$$

Each Galileo satellite has a permanent line of sight to 22 other Galileo satellites resulting in $\binom{22}{L}$ permutations of $\boldsymbol{p}$. We obtained $L = 8$ as minimum almanac length, i.e. we tested 319770 different almanac constellations and considered the following elevation masks $\alpha = \{0^o, 5^o, \ldots, 30^o\}$. The optimum permutation is given by $\boldsymbol{p} = [1, 2, 3, 4, 5, 6, 9, 10]$, i.e. the 6 nearest and the 9,10-th nearest satellites are suggested for the almanac of each satellite. Thus, the length of the almanac is reduced to $L = 8$ compared to 27 of actual signal specification.

IV. Acquisition Time of Cold start with Reduced Almanac

We consider the cold start where no ephemeris nor almanac data are available for signal acquisition.

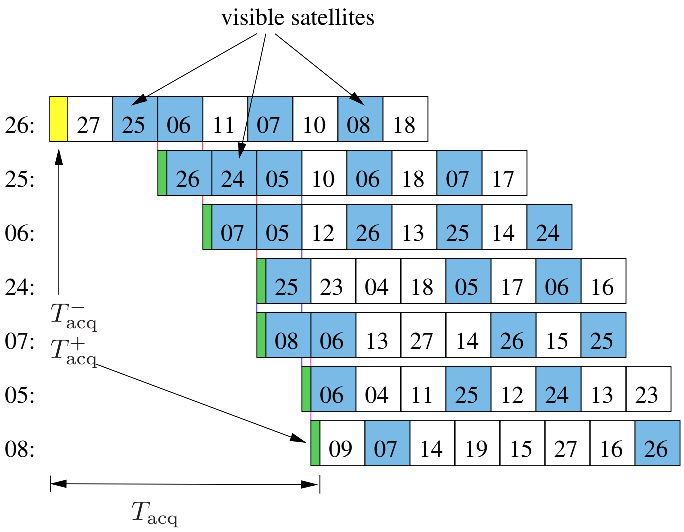

Fig. 3. Acquisition scheme for a visible constellation with elevation mask $\alpha = 10^{\circ}$ and a reduced almanac of length $L = 8$

The signal acquisition is split into three parts: First, an initial satellite k_{i} is acquired without any almanac data in T_{acq}^{-}. Afterwards, the set of almanacs of this satellite is read which lasts T_{A} per almanac. As soon as one almanac of a visible satellite is received completely, signal acquisition of this satellite is started and takes T_{acq}^{+} with $T_{\mathrm{acq}}^{+} \ll T_{\mathrm{acq}}^{-}$ due to additional almanac information. The number K of serially

TABLE I
Sorting of satellite indices according to mean inter-satellite distances

Satellite k	The neighbour matrix $\boldsymbol{S}$: Sorting of satellite indices w.r.t. $\overline{d}^{(kl)}$																				
1	2	9	16	21	17	20	15	22	18	19	3	8	23	14	27	10	24	13	26	11	4	7
2	3	1	17	22	18	21	16	23	10	20	4	9	24	15	19	11	25	14	27	12	5	8
3	4	2	18	23	10	22	17	24	11	21	5	1	25	16	20	12	26	15	19	13	6	9
4	5	3	10	24	11	23	18	25	12	22	6	2	26	17	21	13	27	16	20	14	7	1
5	6	4	11	25	12	24	10	26	13	23	7	3	27	18	22	14	19	17	21	15	8	2
6	7	5	12	26	13	25	11	27	14	24	8	4	19	10	23	15	20	18	22	16	9	3
7	8	6	13	27	14	26	12	19	15	25	9	5	20	11	24	16	21	10	23	17	1	4
8	9	7	14	19	15	27	13	20	16	26	1	6	21	12	25	17	22	11	24	18	2	5
9	1	8	15	20	16	19	14	21	17	27	2	7	22	13	26	18	23	12	25	10	3	6
10	11	18	25	4	26	3	24	5	27	2	12	17	6	23	1	19	7	22	9	20	13	16
11	12	10	26	5	27	4	25	6	19	3	13	18	7	24	2	20	8	23	1	21	14	17
12	13	11	27	6	19	5	26	7	20	4	14	10	8	25	3	21	9	24	2	22	15	18
13	14	12	19	7	20	6	27	8	21	5	15	11	9	26	4	22	1	25	3	23	16	10
14	15	13	20	8	21	7	19	9	22	6	16	12	1	27	5	23	2	26	4	24	17	11
15	16	14	21	9	22	8	20	1	23	7	17	13	2	19	6	24	3	27	5	25	18	12
16	17	15	22	1	23	9	21	2	24	8	18	14	3	20	7	25	4	19	6	26	10	13
17	18	16	23	2	24	1	22	3	25	9	10	15	4	21	8	26	5	20	7	27	11	14
18	10	17	24	3	25	2	23	4	26	1	11	16	5	22	9	27	6	21	8	19	12	15
⋮										⋮												⋮
27	19	26	7	12	8	11	6	13	9	10	20	25	14	5	18	1	15	4	17	2	21	24

read almanacs for acquisition of the whole visible constellation depends on the set of visible satellites $\boldsymbol{c}$, the initial satellite k_{i}, the elevation mask α and the set length L. The number M of serial signal acquisitions with almanac information also depends on $\boldsymbol{c}, k_{\mathrm{i}}, \alpha$ and L.

Fig. 3 shows the acquisition scheme for a visible constellation with elevation mask $\alpha = 10°$. All visible satellites are marked coloured in the almanac sets of length $L = 8$. The acquisition of all visible satellites requires the consecutive reading of $K = 5$ almanacs and the consecutive acquisition of $M = 3$ satellites. The total acquisition time yields

$$T_{\mathrm{acq}} = T_{\mathrm{acq}}^{-} + K(\boldsymbol{c}, k_{\mathrm{i}}, \alpha, L) \cdot T_{\mathrm{A}} + M(\boldsymbol{c}, k_{\mathrm{i}}, \alpha, L) \cdot T_{\mathrm{acq}}^{+}, \quad (12)$$

where the set $\boldsymbol{c}$ of visible satellites is a function of the user location $\boldsymbol{x}_{\mathrm{u}}$ and the time t. Note that not every signal acquisition plays a role in T_{acq}.

We compare the average number of serially read almanacs $\mathrm{E}_{\boldsymbol{c}}\{\mathrm{E}_{k_{\mathrm{i}}}\{K\}\}$ and acquisition processes $\mathrm{E}_{\boldsymbol{c}}\{\mathrm{E}_{i}\{M\}\}$ for the reduced and full almanac in Tab. II. The input parameters for the computation of these almanac acquisition statistics are all possible sets of visible satellites (which have been derived in the previous section) and the reduced almanacs $(\boldsymbol{S}, \boldsymbol{p})$.

TABLE II

COMPARISON OF ACQUISITION TIME PARAMETERS FOR $\alpha = 0°$ AND SET LENGTH $L = \{8, 27\}$

L	8	27
$\mathrm{E}_{\boldsymbol{c}}\{\mathrm{E}_{k_{\mathrm{i}}}\{K\}\}$	9.46	24.56
$\mathrm{E}_{\boldsymbol{c}}\{\mathrm{E}_{k_{\mathrm{i}}}\{M\}\}$	3.47	1.00

The optimized almanac sets reduce the required number of serially read almanacs by a factor 2.6. The number of acquisition processes is increased but its duration is much smaller than the reading of one almanac in T_{A}.

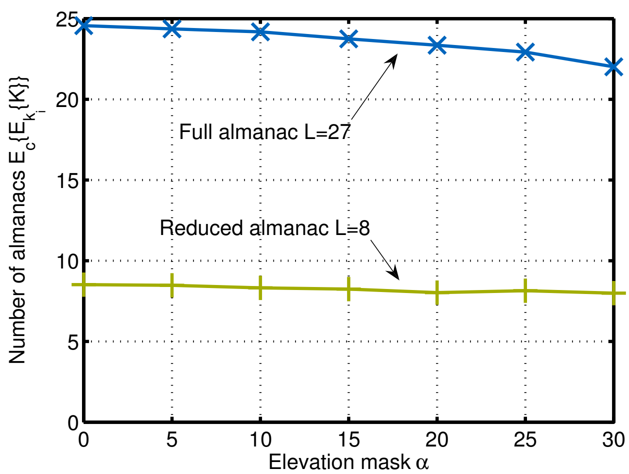

Fig. 4. Average number of almanacs $\mathrm{E}_{\boldsymbol{c}}\{\mathrm{E}_{k_{\mathrm{i}}}\{K\}\}$ for complete acquisition with $L = 8$ and $P_{\mathrm{n}} = 0$

Fig. 4 shows the average number of read almanacs $\mathrm{E}_{\boldsymbol{c}}\{\mathrm{E}_{k_{\mathrm{i}}}\{K(\boldsymbol{c}, k_{\mathrm{i}}, \alpha, L)\}\}$ as a function of the elevation mask. Increasing α reduces the number of visible satellites and, thus, shortens the acquisition process.

The histogram of the number of serially read almanacs for complete acquisition (Fig. 5) visualizes the dependency of the acquisition time T_{acq} on the current set of visible satellites $\boldsymbol{c}$. We assume the worst-case initial satellite k_{i} of each set, i.e. $\max_{k_{\mathrm{i}}}(K(\boldsymbol{c}, k_{\mathrm{i}}, \alpha, L))$. In this case, the initial satellite usually moves slightly above the horizont. We observe a stronger impact of k_{i} on T_{acq} for the reduced almanac compared to the full almanac.

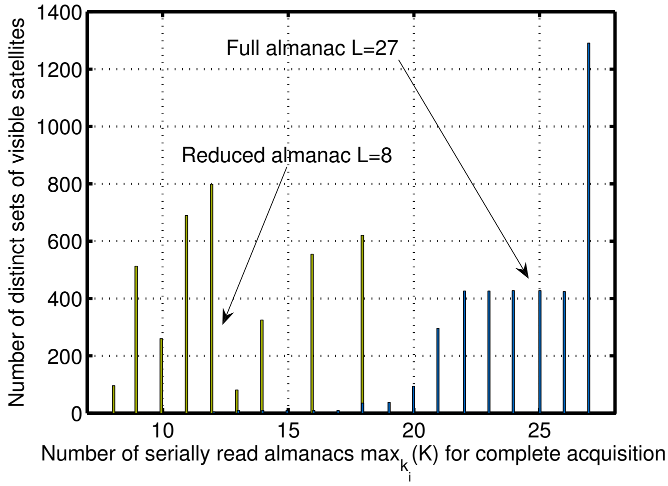

Fig. 5. Comparison of number of read almanacs $\max_{k_{\mathrm{i}}}(K)$ for complete acquisition, elevation mask $\alpha = 10°$ and $L = \{8, 27\}$

V. CONCLUSION

In this paper, we have suggested a reduced almanac transmission scheme to fasten signal acquisition of GNSS satellites. The receiver-satellite geometry was taken into account for the choice of an individual subset of satellites for each satellite almanac as well as for the optimization of the order of satellites within each subset. We have shown that the optimized almanac information shortens the acquisition time of a cold start by a factor of 2.6. The achieved reduction of the navigation message might also be used for additional services, e.g. the transmission of satellite phase and code biases.

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