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***Abstract*—We focus on the globally optimal linear beamformer design based on *quality-of-service* (QoS) power minimization and balancing in the satellite *downlink* (DL) with perfect and statistical *channel state information* (CSI) users. Contrary to the usual rate requirements for the perfect CSI users, we consider ergodic rate requirements for the statistical CSI users. Assuming that the satellite works at Ka-band, we model the fading channels as zero-mean Gaussian vectors with rank-one covariance matrices. This leads to tractable ergodic mutual information expressions that enable to formulate a simple necessary and sufficient feasibility test for the power minimization under minimum rate constraints. Moreover, we are able to solve the power minimization with a globally optimal branch-and-bound (BB) method via properly reformulating the optimization problem. This BB formulation can also be applied to the rate balancing problem. The results serve as a benchmark for other optimization methods.**

***Index Terms*—QoS power minimization; rate balancing; beamforming; statistical CSI; ergodic rate; branch-and-bound; satellite communications**

I. Introduction

We consider the DL of a satellite communication system as detailed in [1] and [2], where the N-antenna satellite serves a set $\mathbb{K}$ of single-antenna receivers at the earth's surface, with $|\mathbb{K}| = K$. These users are differentiated into the two disjoint groups $\mathbb{P}$ and $\mathbb{S}$: users where we have accurate knowledge of the frequency flat channel states, e.g., static receivers with line of sight to the satellite, and users where we can only estimate the channel statistics, respectively, e.g., moving mobiles in urban environments. Modeling the DL as a vector *broadcast channel* (BC) [3], we focus on the power efficient linear beamformer design based on:

- *(P)* *QoS power minimization*: given QoS requirements, e.g., expressed as minimum rates, shall be fulfilled using minimum total average transmit power;
- *(B)* *Balancing optimization*: the ratios between achievable and target rates shall commonly be maximized under a total average transmit power restriction.

In agreement with recent literature [4], we are interested in a stochastically robust formulation of above QoS power minimization and balancing optimization. In contrast to the usual rate requirements for the users with perfect CSI, *ergodic* rate targets are used as QoS measure for the statistical CSI users. Thereby, we properly exploit the knowledge about the channel statistics to constrain the average rate of these users. Note that the stochastically robust beamformer design based on ergodic rate requirements for the statistical CSI users is commonly known to be less conservative than worst-case robust design strategies [5].

A. Channel Assumptions

Unfortunately, general ergodic rate expressions (e.g., see [6] and [7]) are too complicated for a direct implementation in currently available multi-user beamformer design techniques. Therefore, we exploit the fact that in satellite communications the fading channels can essentially be modeled as zero-mean Gaussian vectors with rank-one covariance matrices when assuming that the mobile users and the satellite work at Ka-band (cf. [1] and [8]). The spatial signatures of the channels remain essentially constant due to the large distance from the satellite to the ground and the slow movement of the mobiles. The signal of far distant large scatterers, e.g., mountains and skyscrapers, can be neglected in most of the considered environments due to the additional path loss and the delay compared to the direct path signal. However, the norm and phase of the statistical CSI users' channels strongly varies with the position of the mobile, the surrounding scatterers, and the shadowing in urban and sub-urban environments.

B. Difficulties

In [1] and [2], we considered the same system setup and proposed efficient but suboptimal approaches for the robust formulations of *(P)* and *(B)* that were based on partial *zero-forcing* (ZF) and bounds on the ergodic rates, respectively. Unlike these attempts, in this paper the focus is on globally optimally solving the ergodic robust problem reformulations of *(P)* and *(B)* under above assumptions. The difficulty is that the problems are non-convex in the BC and, in contrast to the perfect CSI rate constraints, no convex reformulation is known for the ergodic counterparts. Especially, we cannot write the problems with minimum *signal-to-interference-plus-noise-ratio* (SINR) constraints since the ergodic rate expressions cannot be represented in terms of some SINR like term. Thus, employing the convex optimization techniques of [9] and [10] is impossible for the considered system setup.

An additional problem is that *(P)* might not have a solution if an overloaded setup with less transmit antennas than users is considered. In [11], the feasibility region of the perfect CSI vector BC is shown to be a simple polytope when reformulating the minimum rate targets to equivalent upper bounds on the *minimum-mean-square-errors* (MMSEs). This feasibility region was further refined in [12] to singular channels. However, the feasible region of the vector BC with ergodic rate requirements is still unknown. The proof in the perfect CSI case cannot straightforwardly be extended to the considered ergodic robust formulation due to the lack of appropriate lower bounds for the ergodic rates [2].

C. Contributions and Structure

In this work, we overcome both of above problems that are a consequence of the ergodic rate requirements.

- We show that the complete feasibility region of the ergodic robust QoS power minimization problem *(P)* is again a polytope in terms of MMSEs. The proof is based on the results in [11] and a tight upper bound for the ergodic rates that was already introduced in [2]. Moreover, the given proof is constructive in the sense that it shows one way to find strictly feasible beamforming vectors for the robust QoS problem.
- Given feasibility, we globally optimally solve the QoS power minimization via a *branch-and-bound* (BB) method (e.g., see [13]) that is adopted from the framework in [14]. Therein, an optimization formulation is given that takes into account the *partly convex-monotone* structure of vector BC and interference channel problems. The BB formulation in this work is better suited for the considered robust beamformer design problems. That is, also the stochastically robust balancing optimization *(B)* can be incorporated in the given formulation.

We remark, that the BB method is inappropriate for real satellite systems as it is an exhaustive iterative partitioning procedure [13]. The considered formulation has exponential complexity in the number of statistical CSI users. Therefore, this method is merely for benchmarking the suboptimal partial ZF and bounding methods of [1] and [2], respectively, in selected scenarios with relatively small N and $|\mathbb{S}|$.

The remainder of this work is structured as follows. The detailed system model, together with the achievable and ergodic rates of the BC at hand are introduced in Section II. In Section III, *(P)* is recast. The MMSE feasible region of the vector BC with ergodic rates is presented, followed by a detailed proof. As the proof is constructive, a suboptimal but feasible beamformer design method is proposed next. In Section V, the used BB formulation is introduced and problem *(P)* is rewritten to fit into this framework. One way of implementing the BB algorithm is shown in Section V. Problem *(B)* is recast in Section VI and shown to fit to the BB formulation. Finally, numerical results for both problems are presented and used to benchmark the partial ZF scheme of [1] and the bounding methods in [2].

II. System Model and Achievable Rates

In the considered vector BC, independent unit-variance data signals $s_k \sim \mathcal{N}_{\mathbb{C}}(0,1)$ are linearly precoded with the beamformers $\boldsymbol{t}_k \in \mathbb{C}^N$, $k \in \mathbb{K} = \{1,\ldots,K\}$ and then simultaneously transmitted to the K receivers, i.e., the transmit signal is $\boldsymbol{x} = \sum_{k=1}^K \boldsymbol{t}_k s_k \in \mathbb{C}^N$. Assuming zero-mean unit-variance additive Gaussian noise $n_k \sim \mathcal{N}_{\mathbb{C}}(0,1)$ at the receivers, the kth user's received signal reads as

$$y_k = \boldsymbol{h}_k^{\mathrm{H}} \boldsymbol{x} + n_k = \boldsymbol{h}_k^{\mathrm{H}} \boldsymbol{t}_k + \boldsymbol{h}_k^{\mathrm{H}} \sum_{i \neq k}^{K} \boldsymbol{t}_i s_i + n_k,$$

where $\boldsymbol{h}_k^{\mathrm{H}} \in \mathbb{C}^{1\times N}$ denotes the frequency flat fading channel vector. The corresponding mutual information $r_k \triangleq I(\boldsymbol{x}, y_k)$ of user k is given by

$$r_k = \log_2\left(1 + \frac{|\boldsymbol{h}_k^{\mathrm{H}} \boldsymbol{t}_k|^2}{1 + \sum_{i\neq k} |\boldsymbol{h}_k^{\mathrm{H}} \boldsymbol{t}_i|^2}\right). \tag{1}$$

To rely on (1), the transmitter needs to know the channel state $\boldsymbol{h}_k$ which is only available for a subset of the users $k \in \mathbb{P}$, i.e., the perfect CSI users. For the other users $k \in \mathbb{S} = \mathbb{K} \setminus \mathbb{P}$, only the statistics of $\boldsymbol{h}_k \sim \mathcal{N}_{\mathbb{C}}(\boldsymbol{0}, \boldsymbol{C}_k)$ are available why we resort to the *ergodic* mutual information $R_k \triangleq \mathrm{E}_{\boldsymbol{h}_k}[r_k]$, i.e., the rate that is achievable on average. Under the assumption that the covariance matrices are essentially rank-one in satellite communications (cf. Subsection I-A), they are characterized by the spatial signatures $\boldsymbol{v}_k \in \mathbb{C}^N$ of the channels to users $k \in \mathbb{S}$. That is, we can write $\boldsymbol{C}_k = \boldsymbol{v}_k \boldsymbol{v}_k^{\mathrm{H}}$. Being aware of $\boldsymbol{v}_k$, the kth user's ergodic rate reads as

$$\begin{aligned} R_k = \frac{1}{\log(2)} \varsigma\left(\frac{1}{\sum_{i=1}^K |\boldsymbol{v}_k^{\mathrm{H}} \boldsymbol{t}_i|^2}\right) \\ - \frac{1}{\log(2)} \varsigma\left(\frac{1}{\sum_{i\neq k} |\boldsymbol{v}_k^{\mathrm{H}} \boldsymbol{t}_i|^2}\right), \end{aligned} \tag{2}$$

where we define $\varsigma(x) \triangleq \mathrm{e}^x \, \mathrm{E}_1(x)$ and $\mathrm{E}_1(x) = \int_x^\infty \frac{\mathrm{e}^{-t}}{t} \, \mathrm{d}t$ denotes the exponential integral function [15].

III. QoS Power Minimization

Given the non-negative QoS requirements ρ_k, expressed as minimum rate targets, $k \in \mathbb{K} = \{1,\ldots,K\}$, the power minimization problem *(P)* can be formulated as

$$\begin{aligned} \min_{\{\boldsymbol{t}_k\}} \quad & \sum_{k=1}^K \|\boldsymbol{t}_k\|_2^2 \\ \text{s.t.:} \quad & \begin{cases} r_k \geq \rho_k & \forall k \in \mathbb{P}, \\ R_k \geq \rho_k & \forall k \in \mathbb{S}. \end{cases} \end{aligned} \tag{3}$$

Here, we write $\{\boldsymbol{t}_k\}$ to explicitly denote that an optimization (or function) depends on <u>all</u> beamformers $\boldsymbol{t}_k$ with $k \in \mathbb{K}$.

As already mentioned in the introduction, (3) might not have a solution in general. A given set of rate targets $\{\rho_k\}_{k=1}^K$ might be infeasible when $K > N$, even though the transmit power is unbounded in this problem formulation (e.g., see the discussion in [16] and [10]). In [11], a simple (non-iterative) feasibility test is given for the purely perfect CSI power minimization problem with *regular* channels.[1] In this case, the QoS feasibility region, formulated as achievable MMSEs, is a polytope with individual box constraints and one half-space constraint for the sum of the MMSEs. For *singular* channels, additional half-space constraints bound the feasible MMSE region [12]. Next, we incorporate the ergodic rate requirements of (3) into this feasibility region framework.

For this purpose, we express the rate targets as maximum MMSE requirements, i.e.,

$$\varepsilon_k = 2^{-\rho_k} \geq \mathrm{MMSE}_k = \begin{cases} 2^{-r_k} & k \in \mathbb{P}, \\ 2^{-R_k} & k \in \mathbb{S}. \end{cases} \tag{4}$$

Moreover, we combine the channel vectors of the perfect CSI receivers and the channel characteristics of the statistical CSI receivers in the set of channel signatures $\mathcal{H} = \{\boldsymbol{v}_k\}_{k\in\mathbb{S}} \cup \{\boldsymbol{h}_k\}_{k\in\mathbb{P}}$. Based on above definition of the MMSEs and regular channel signatures, the following theorem is valid.

Theorem 1. *Let the channel signatures in $\mathcal{H}$ be regular. Then, the maximum MMSE requirements $\{\varepsilon_k\}_{k=1}^K$ are feasible iff they lie inside a polytope with box constraints $0 < \varepsilon_k \leq 1\ \forall k \in \mathbb{K}$ and the single half-space constraint $\sum_{k=1}^K \varepsilon_k > \max(0, K-N)$ (cf.* [11, Theorem 1]*).*

In the remainder of this section, a detailed proof of this statement is given and a suboptimal beamforming scheme is presented that is motivated by the ideas of the proof.

A. Proof of Theorem 1

The proof of Theorem 1 is based on the fact that the ergodic mutual information in (2) is upper bounded by (cf. [2])

$$R_k \leq \overline{R}_k = \log_2\left(1 + \frac{|\boldsymbol{v}_k^{\mathrm{H}}\boldsymbol{t}_k|^2}{1+\sum_{i\neq k}|\boldsymbol{v}_k^{\mathrm{H}}\boldsymbol{t}_i|^2}\right), \tag{5}$$

which has the structure of the usual (non-ergodic) vector BC mutual information expression but with 'channels' $\boldsymbol{v}_k$ instead of $\boldsymbol{h}_k$. The key property of this upper bound is that it is tight for increasing interference $\mathrm{I}_k = \sum_{k\neq i}|\boldsymbol{v}_k^{\mathrm{H}}\boldsymbol{t}_i|^2$ [cf. (19b)], i.e., the following lemma is valid.

Lemma 1. *The ergodic rate R_k is upper bounded by $\overline{R}_k$ and differs from $\overline{R}_k$ by only a small error, i.e., $R_k \geq \overline{R}_k - \delta_k$ with $\delta_k \geq 0$. Here, δ_k is given by*

$$\delta_k = \frac{1}{\log(2)}\varsigma\big(1/\mathrm{I}_k\big) - \log_2\big(1+\mathrm{I}_k\big) + \frac{\gamma}{\log(2)} \leq \frac{\gamma}{\log(2)},$$

where $\gamma \approx 0.5772$ denotes the Euler-Mascheroni constant [15]*, and has the following properties* [2, Proposition 1]*:*

(i) *δ_k is monotonically decreasing with I_k,*
(ii) *$\delta_k = \frac{\gamma}{\log(2)}$ for $I_k = 0$, and $\delta_k = 0$ for $I_k \to \infty$.*

The proof of this lemma can be found in [2, Proof of Proposition 1]. It is based on the fact that the difference $\log(1+x) - \varsigma(1/x) \geq 0$, with $x \geq 0$, is strictly monotonically increasing in x, equals zero for $x = 0$, and becomes γ for $x \to \infty$.

Being aware of the ergodic rate upper bound in (5) and Lemma 1, we are in the position to prove Theorem 1 in two main steps. First, we show that the MMSE conditions in Theorem 1 are necessary, i.e., we cannot achieve MMSE values outside of the polytope with box constraints $1 \geq \mathrm{MMSE}_k > 0$ and the half-space constraint $\sum_{k=1}^K \mathrm{MMSE}_k > K - N$. Then, we demonstrate that the MMSE conditions in Theorem 1 are also sufficient, i.e., all MMSE-tuples inside the polytope can be reached with finite transmit power.

The necessity of the individual MMSE box constraints $1 \geq \mathrm{MMSE}_k > 0$, $k \in \mathbb{K}$ directly follows from the boundedness of the rate expressions, i.e., $0 \leq r_k, R_k < \infty$, and the MMSE definition in (4). Here, zero for MMSE_k is only achieved for $R_k, r_k \to \infty$ which requires infinite transmit power. To show necessity of the half-space constraint, we state the following series of inequalities:

$$\begin{aligned}\sum_{k=1}^K \mathrm{MMSE}_k &= \sum_{k\in\mathbb{P}} 2^{-r_k} + \sum_{k\in\mathbb{S}} 2^{-R_k} \\ &\geq \sum_{k\in\mathbb{P}} 2^{-r_k} + \sum_{k\in\mathbb{S}} 2^{-\overline{R}_k} \\ &> K - N,\end{aligned} \tag{6}$$

where the first inequality immediately follows from (5). For the second inequality, we apply the common uplink-downlink duality-principle, using $\overline{R}_k$ instead of R_k (cf. [2]). Based on an MMSE reformulation in the dual uplink, we can exploit the MMSE-feasibility results of [11, Theorem 1].

In the dual vector MAC, the data signals $d_k \sim \mathcal{N}_{\mathbb{C}}(0,1)$ are transmitted with power p_k over the vector channels $\boldsymbol{b}_k = \boldsymbol{h}_k$ for $k \in \mathbb{P}$ and $\boldsymbol{b}_k = \boldsymbol{v}_k$ for $k \in \mathbb{S}$. The MAC receiver passes the observed superposition of the channel outputs and the additive noise $\boldsymbol{\eta} \sim \mathcal{N}_{\mathbb{C}}(\boldsymbol{0}, \mathbf{I}_N)$ through the equalizer $\boldsymbol{f}_k^{\mathrm{H}} \in \mathbb{C}^{1\times N}$ to get the estimate $\hat{d}_k \in \mathbb{C}$ for d_k. Defining the MAC MMSE of user k as $\mathrm{MMSE}_k^{\mathrm{MAC}} = \min_{\boldsymbol{f}_k^{\mathrm{H}}} \mathrm{E}\left[|\hat{d}_k - d_k|^2\right]$, the optimum MMSE equalizer reads as (e.g., [17])

$$\boldsymbol{f}_{\mathrm{MMSE},k}^{\mathrm{H}} = \sqrt{p_k}\boldsymbol{b}_k^{\mathrm{H}}\left(\mathbf{I}_N + \sum_{i=1}^K \boldsymbol{b}_i\boldsymbol{b}_i^{\mathrm{H}}p_i\right)^{-1}$$

and results in the MAC MMSE expression

$$\mathrm{MMSE}_k^{\mathrm{MAC}} = 1 - p_k\boldsymbol{b}_k^{\mathrm{H}}\left(\mathbf{I}_N + \sum_{i=1}^K \boldsymbol{b}_i\boldsymbol{b}_i^{\mathrm{H}}p_i\right)^{-1}\boldsymbol{b}_k. \tag{7}$$

Adding up these terms for all $k \in \mathbb{K}$, the sum MMSE in the considered vector MAC can be formulated as

$$\begin{aligned}\sum_{k=1}^K \mathrm{MMSE}_k^{\mathrm{MAC}} &= K - \mathrm{tr}\left(\boldsymbol{P}\boldsymbol{B}^{\mathrm{H}}(\mathbf{I}_N + \boldsymbol{B}\boldsymbol{P}\boldsymbol{B}^{\mathrm{H}})^{-1}\boldsymbol{B}\right) \\ &= K - N + \mathrm{tr}\left((\mathbf{I}_N + \boldsymbol{B}\boldsymbol{P}\boldsymbol{B}^{\mathrm{H}})^{-1}\right),\end{aligned} \tag{8}$$

where $\boldsymbol{B} = [\boldsymbol{b}_1, \ldots, \boldsymbol{b}_K]$ and $\boldsymbol{P} = \mathrm{diag}(p_1, \ldots, p_K)$. Since the inverse in (8) is positive semidefinite, its trace is lower

[1] A set of vectors $\mathcal{A} = \{\boldsymbol{a}_k\}_{k=1}^K$ with $\boldsymbol{a}_k \in \mathbb{C}^N$ is called *regular* iff the vectors of any subset $\mathcal{B} \subseteq \mathcal{A}$ with $|\mathcal{B}| \leq N$ are linearly independent. Alternatively, we can write that the matrix $\boldsymbol{A}_{\mathcal{B}}$ that combines the vectors in $\mathcal{B}$ satisfies $\mathrm{rank}(\boldsymbol{A}_{\mathcal{B}}) = \min(N, |\mathcal{B}|)$. If $\mathrm{rank}(\boldsymbol{A}_{\mathcal{B}}) < \min(N, |\mathcal{B}|)$ for at least one $\mathcal{B} \subseteq \mathcal{A}$, the vectors in $\mathcal{A}$ are *singular*.

bounded by $N - \mathrm{rank}(\boldsymbol{B})$. Therefore, any MAC power allocation with finite sum transmit power $\mathrm{tr}(\boldsymbol{P}) < \infty$ satisfies

$$\sum_{k=1}^{K} \mathrm{MMSE}_k^{\mathrm{MAC}} > K - \mathrm{rank}(\boldsymbol{B}) \geq K - N. \tag{9}$$

We remark that the latter of the two inequalities is strict for $K < N$ as the sum MMSE is clearly bounded below by zero, whereas the inequality becomes an equality for $K \geq N$ when assuming regular channels, i.e., when $\mathrm{rank}(\boldsymbol{B}) = \min(N, K)$.

Note that the bounded vector BC (with $\overline{R}_k$ for $k \in \mathbb{S}$) and the dual MAC share the same feasibility region because of the duality principle. Precisely, the same MMSEs are achievable in the MAC and the bounded BC, i.e., $\mathrm{MMSE}_k^{\mathrm{MAC}} = 2^{-r_k}$ for $k \in \mathbb{P}$ and $\mathrm{MMSE}_k^{\mathrm{MAC}} = 2^{-\overline{R}_k}$ for $k \in \mathbb{S}$, using the same total transmit power $\mathrm{tr}(\boldsymbol{P}) = \sum_{k=1}^{K} \|\boldsymbol{t}_k\|_2^2$ (e.g., see [18]). Therefore, we can conclude that whenever an MMSE-tuple is achievable in the vector MAC with finite transmit power, this also holds for the bounded BC. Thus, the second inequality in (6) is valid.

So far, we have only shown that no MMSE-tuples are achievable that lie outside the polytope. Next, we prove that the given constraints are also sufficient for feasibility, i.e., we show that all tuples $\{\mathrm{MMSE}_k\}_{k=1}^{K}$ in the interior of the polytope are achievable. For this purpose, we consider an arbitrary tuple of MMSE targets $\{\varepsilon_k\}_{k=1}^{K}$ that satisfy $1 \geq \varepsilon_k > 0 \ \forall k \in \mathbb{K}$ and $\sum_{k=1}^{K} \varepsilon_k > K - N$. As these target MMSEs reside in the interior of the polytope, we know from the discussion above and the results in [11] that, if we restrict to regular channels $\{\boldsymbol{b}_k\}_{k=1}^{K}$, *there exists* a tuple of beamformers $\{\hat{\boldsymbol{t}}_k\}_{k=1}^{K}$ having finite norm that simultaneously satisfy the next set of inequalities:

$$\begin{aligned} 2^{-r_k(\{\hat{\boldsymbol{t}}_k\})} &\leq \varepsilon_k \quad \forall k \in \mathbb{P} \\ 2^{-\overline{R}_k(\{\hat{\boldsymbol{t}}_k\})} &\leq \varepsilon_k \quad \forall k \in \mathbb{S}, \end{aligned}$$

where $r_k(\{\boldsymbol{t}_k\})$ and $\overline{R}_k(\{\boldsymbol{t}_k\})$ define the mapping of the beamformers $\{\boldsymbol{t}_k\}_{k=1}^{K}$ to the corresponding (bounded) rate values [see (1) and (5)]. Note that such a set of beamformers (that additionally minimizes the total transmit power) may be found either via a fixed-point based power minimization in above dual MAC and a transformation of the results to the bounded BC [2], or directly via convex optimization methods in the bounded BC [10], for example.

Now, in order to show that the same MMSE targets are achievable in the vector BC with ergodic rates, but with different transmit power, we have to find beamformers $\{\boldsymbol{t}'_k\}_{k=1}^{K}$ with finite norm such that

$$r_k(\{\boldsymbol{t}'_k\}) \geq r_k(\{\hat{\boldsymbol{t}}_k\}) \quad \forall k \in \mathbb{P} \tag{10a}$$

$$R_k(\{\boldsymbol{t}'_k\}) \geq \overline{R}_k(\{\hat{\boldsymbol{t}}_k\}) \quad \forall k \in \mathbb{S}. \tag{10b}$$

These beamformers may be constructed via a proper scaling of the vectors $\{\hat{\boldsymbol{t}}_k\}_{k=1}^{K}$, for example. That is, we choose

$$\boldsymbol{t}'_k = \sqrt{\alpha}\hat{\boldsymbol{t}}_k \qquad \forall k \in \mathbb{K}, \tag{11}$$

with $\alpha \geq 1$. Since $r_k(\{\boldsymbol{t}'_k\}) = r_k(\{\sqrt{\alpha}\hat{\boldsymbol{t}}_k\})$ is strictly monotonically increasing with α [see (1)], above choice $\alpha \geq 0$ always satisfies (10a). For (10b) to hold, we require a large enough α that additionally satisfies

$$\begin{aligned} R_k\big(\{\sqrt{\alpha}\hat{\boldsymbol{t}}_k\}\big) &\geq \overline{R}_k\big(\{\sqrt{\alpha}\hat{\boldsymbol{t}}_k\}\big) - \delta_k\big(\alpha\hat{I}_k\big) \\ &\geq \overline{R}_k\big(\{\hat{\boldsymbol{t}}_k\}\big) \qquad \forall k \in \mathbb{S}, \end{aligned} \tag{12}$$

with the interference term $\hat{I}_k = \sum_{i \neq k} |\boldsymbol{v}_k^{\mathrm{H}} \hat{\boldsymbol{t}}_i|^2$. The first inequality in this series stems from Lemma 1. For the second inequality in (12), we differentiate two cases with respect to the interference, namely $\hat{I}_k > 0$ and $\hat{I}_k = 0$.

If the interference term is strictly positive, i.e, $\hat{I}_k > 0$, then

$$\overline{R}_k(\{\sqrt{\alpha}\hat{\boldsymbol{t}}_k\}) = \log_2\left(1 + \frac{\alpha|\boldsymbol{v}_k^{\mathrm{H}}\hat{\boldsymbol{t}}_k|^2}{1 + \alpha\hat{I}_k}\right)$$

is monotonically increasing with α but bounded, i.e., we have

$$\lim_{\alpha\to\infty} \overline{R}_k(\{\sqrt{\alpha}\hat{\boldsymbol{t}}_k\}) = \log_2\left(1 + |\boldsymbol{v}_k^{\mathrm{H}}\hat{\boldsymbol{t}}_k|^2/\hat{I}_k\right).$$

Moreover, the resulting maximal error term is

$$\delta_k(\alpha\hat{I}_k) = \frac{1}{\log(2)}\varsigma\left(\frac{1}{\alpha\hat{I}_k}\right) - \log_2\left(1 + \alpha\hat{I}_k\right) + \frac{\gamma}{\log(2)}.$$

Especially, $\delta_k(\alpha\hat{I}_k)$ is monotonically decreasing with α and $\lim_{\alpha\to\infty}\delta_k(\alpha\hat{I}_k) = 0$ according to Lemma 1. As $\overline{R}_k(\{\sqrt{\alpha}\hat{\boldsymbol{t}}_k\}) > \overline{R}_k(\{\hat{\boldsymbol{t}}_k\})$ for $\alpha > 1$ and as $\delta_k(\alpha\hat{\mathrm{I}}_k)$ converges to zero for large α, it is always possible to find an $\alpha^\star$ such that $\overline{R}_k(\{\sqrt{\alpha^\star}\hat{\boldsymbol{t}}_k\}) - \overline{R}_k(\{\hat{\boldsymbol{t}}_k\}) \geq \delta_k(\alpha^\star\hat{\mathrm{I}}_k)$.

Otherwise, if $\hat{I}_k = 0$ exactly, then

$$\overline{R}_k(\{\sqrt{\alpha}\hat{\boldsymbol{t}}_k\}) = \log_2\left(1 + \alpha|\boldsymbol{v}_k^{\mathrm{H}}\hat{\boldsymbol{t}}_k|^2\right)$$

unboundedly increases with α, while the maximal error term is bounded from above, i.e., $\delta_k(0) = \frac{\gamma}{\log(2)}$. For these reasons, it is always possible to find an $\alpha^\star$ such that $\overline{R}_k(\{\sqrt{\alpha^\star}\hat{\boldsymbol{t}}_k\}) - \overline{R}_k(\{\hat{\boldsymbol{t}}_k\}) \geq \frac{\gamma}{\log(2)}$ in this case.

Hence, a suitable $\alpha^\star > 1$ can be found such that the second inequality in (12) is valid simultaneously for all $k \in \mathbb{S}$. Therewith, we have proven that we can construct a set of beamformers $\{\boldsymbol{t}'_k\}_{k=1}^{K}$ according to (11) that satisfies (4) simultaneously for all $k \in \mathbb{K}$, which completes the proof for the converse of Theorem 1.

B. Suboptimal Beamforming

According to above proof, the feasible MMSE region remains the same if we use the ergodic rate upper bound in (5) instead of the actual ergodic rate in (2). Moreover, we remark that the converse part of above proof is constructive. Feasible beamforming vectors $\{\boldsymbol{t}_k\}_{k=1}^{K}$ for given rate requirements $\{\rho_k\}_{k=1}^{K}$—reformulated as MMSE targets that satisfy Theorem 1—are found in two main steps.

1) First, a set of feasible beamforming vectors $\{\hat{\boldsymbol{t}}_k\}_{k=1}^{K}$ has to be determined for the case when we replace the ergodic rate constraints with requirements on the upper bounds, i.e., we require $r_k \geq \rho_k \ \forall k \in \mathbb{P}$ and $\overline{R}_k \geq \rho_k \ \forall k \in \mathbb{S}$.
2) Based on $\{\hat{\boldsymbol{t}}_k\}_{k=1}^{K}$, strictly feasible beamforming vectors $\{\boldsymbol{t}_k\}_{k=1}^{K}$ for the constraints in (4) may be found via $\boldsymbol{t}_k = \sqrt{\alpha_k}\hat{\boldsymbol{t}}_k$ and proper choices of $\alpha_k \geq 1$, $k \in \mathbb{K}$.

This motivates suboptimal two step approaches to obtain reasonable beamformers for problem (3), as detailed in [2] for example. Therein, we find the beamformers $\{\hat{\boldsymbol{t}}_k\}_{k=1}^K$ via solving an approximation of (3), where we replace the ergodic constraints with the constraints in *1)*, i.e.,

$$\begin{aligned} \min_{\{\hat{\boldsymbol{t}}_k\}} \quad & \sum_{k=1}^K \|\hat{\boldsymbol{t}}_k\|_2^2 \\ \text{s. t.:} \quad & \begin{cases} r_k(\{\hat{\boldsymbol{t}}_k\}) \geq \rho_k & \forall k \in \mathbb{P}, \\ \overline{R}_k(\{\hat{\boldsymbol{t}}_k\}) \geq \rho_k & \forall k \in \mathbb{S}. \end{cases} \end{aligned} \tag{13}$$

This approximate problem can equivalently be formulated as a power minimization with minimum SINR requirements, that is commonly known to be a convex optimization problem [10]. That is, (13) is efficiently solvable via SINR uplink-downlink duality and the standard interference function framework [19].

Then, instead of applying equal scaling for all $\hat{\boldsymbol{t}}_k$, $k \in \mathbb{K}$ as in (11), another optimization is performed in [2] for finding those scalars $\{\alpha_k\}_{k=1}^K$ that satisfy $r_k = \rho_k \; \forall k \in \mathbb{P}$ and $R_k = \rho_k \; \forall k \in \mathbb{S}$ and additionally minimize the transmit power, i.e.,

$$\begin{aligned} \min_{\{\alpha_k\}} \quad & \sum_{k=1}^K \alpha_k \|\hat{\boldsymbol{t}}_k\|_2^2 \\ \text{s. t.:} \quad & \begin{cases} r_k(\{\alpha_k \hat{\boldsymbol{t}}_k\}) \geq \rho_k & \forall k \in \mathbb{P}, \\ R_k(\{\alpha_k \hat{\boldsymbol{t}}_k\}) \geq \rho_k & \forall k \in \mathbb{S}. \end{cases} \end{aligned} \tag{14}$$

Solving (14), efficient beamformers $\boldsymbol{t}'_k = \alpha_k \hat{\boldsymbol{t}}_k$, $k \in \mathbb{K}$ may be obtained that are feasible for (3) but suboptimal in general. Note that the optimum of (14) is in general a upper bound for the optimum of (3).

IV. Problem Reformulation

In what follows, we consider $\{\rho_k\}_{k=1}^K$ that are feasible. Given feasibility, (3) is still a non-convex problem. The rates are neither convex nor concave functions of $\{\boldsymbol{t}_k\}_{k=1}^K$ and, unlike the perfect CSI rate constraints, no convex reformulation is known for the ergodic counterparts. This observation motivated us to apply a globally optimal BB approach (e.g., see [13]). To this end, we use a BB formulation that is similar to that in [14] and exploits the *partly convex monotone structure* of the problem. The objective and the perfect CSI rate requirements can equivalently be represented as convex *second-order-cones* (SOCs), while the statistical CSI rate requirements may be represented as a *difference of monotonic* functions in the experienced useful signal and interference. In the considered BB formulation, branching and bounding will be performed on the reduced space of experienced interference levels at the statistical CSI users.

For this purpose, we rewrite the power minimization problem into the following form:

$$\begin{aligned} \min_{(b,\{\boldsymbol{t}_k\})\in\mathcal{Q},\boldsymbol{i}} \quad & b \\ \text{s. t.:} \quad & \boldsymbol{f}(\{\boldsymbol{t}_k\},\boldsymbol{i}) \leq \boldsymbol{0}, \\ & \boldsymbol{f}_{\boldsymbol{i}}(\{\boldsymbol{t}_k\}) \leq \boldsymbol{i}. \end{aligned} \tag{15}$$

Here, $\mathcal{Q}$ defines a tractable (convex) constraint set that will be defined by a transmit power constraint and the convex reformulations of the perfect CSI rate requirements. The function $\boldsymbol{f}(\{\boldsymbol{t}_k\},\boldsymbol{i})$ is entry-wise convex in the beamformers $\{\boldsymbol{t}_k\}_{k=1}^K$ for fixed $\boldsymbol{i}$ and monotonically increasing in $\boldsymbol{i}$ for fixed $\{\boldsymbol{t}_k\}_{k=1}^K$. Reformulations of the statistical CSI rate requirements will be the basis of this function. Finally, $\boldsymbol{f}_{\boldsymbol{i}}(\{\boldsymbol{t}_k\}) \leq \boldsymbol{i}$ will combine the definitions of the experienced interference from statistical CSI users, where equality holds in the optimum.

To arrive at (15), we first introduce the power constraint

$$\sum_{k=1}^K \|\boldsymbol{t}_k\|_2^2 \leq b^2 \tag{16}$$

where the new slack variable $b \in \mathbb{R}_+$ is additionally the objective of the QoS power minimization problem. Note that equality holds in the optimum for this constraint. A *standard convex formulation* of (16) is the SOC representation (cf. [10])

$$\|\mathrm{vec}(\boldsymbol{T})\|_2 \leq b, \tag{17}$$

with $\mathrm{vec}(\cdot)$ denoting the column stacking operator and the matrix $\boldsymbol{T} = [\boldsymbol{t}_1, \ldots, \boldsymbol{t}_K]$ combines all beamforming vectors.

Next, we (convex) reformulate the rate requirements. To this end, we express the required useful signal power at user $k \in \mathbb{P}$ in terms of the experienced interference, i.e.,

$$|\boldsymbol{h}_k^{\mathrm{H}} \boldsymbol{t}_k|^2 \geq g_k(\mathrm{I}_k) = (2^{\rho_k} - 1)(1 + \mathrm{I}_k), \tag{18a}$$

where we defined the interference to be

$$\mathrm{I}_k = \sum\nolimits_{i \neq k} |\boldsymbol{h}_k^{\mathrm{H}} \boldsymbol{t}_i|^2 \qquad k \in \mathbb{P}. \tag{18b}$$

A similar reformulation for the ergodic rate requirements of the statistical CSI users $k \in \mathbb{S}$ results in

$$|\boldsymbol{v}_k^{\mathrm{H}} \boldsymbol{t}_k|^2 \geq g_k(\mathrm{I}_k) = \frac{1}{\varsigma^{-1}\big(\rho_k \log(2) + \varsigma(1/\mathrm{I}_k)\big)} - \mathrm{I}_k, \tag{19a}$$

where the experienced interference is denoted as

$$\mathrm{I}_k = \sum\nolimits_{i \neq k} |\boldsymbol{v}_k^{\mathrm{H}} \boldsymbol{t}_i|^2 \qquad k \in \mathbb{S}. \tag{19b}$$

We remark that $g_k(\mathrm{I}_k)$ is positive and linear (and increasing) in I_k for $k \in \mathbb{P}$. Moreover, as (18a), (19a), and (17) are independent with respect to scalar multiplications $\{\mathrm{e}^{\mathrm{j}\phi_k} \boldsymbol{t}_k\}_{k=1}^K$ of the beamformers, we can require the useful signals to be real valued, i.e., $\mathrm{Im}\{\boldsymbol{h}_k^{\mathrm{H}} \boldsymbol{t}_k\} = 0$ for $k \in \mathbb{P}$ and $\mathrm{Im}\{\boldsymbol{v}_k^{\mathrm{H}} \boldsymbol{t}_k\} = 0$ for $k \in \mathbb{S}$. Exploiting these properties, we can equivalently formulate (18a) as SOCs [10], i.e,

$$\boldsymbol{h}_k^{\mathrm{H}} \boldsymbol{t}_k \geq \sqrt{2^{\rho_k} - 1} \big\| \big[\boldsymbol{h}_k^{\mathrm{H}} \boldsymbol{T}_{\bar{k}}, 1\big] \big\|_2 \qquad k \in \mathbb{P}, \tag{20}$$

where $\boldsymbol{T}_{\bar{k}} = [\boldsymbol{t}_1, \ldots, \boldsymbol{t}_{k-1}, \boldsymbol{t}_{k+1}, \ldots, \boldsymbol{t}_K]$ combines all beamformers except for the kth one.

Unfortunately, a similar reformulation is impossible for (19a) as $g_k(\mathrm{I}_k)$ is non-linear in I_k, $k \in \mathbb{S}$ and it is non-convex. However, noting that $g_k(\mathrm{I}_k)$ is monotonically increasing in I_k, $k \in \mathbb{S}$ and introducing $\{\mathrm{I}_k\}_{k \in \mathbb{S}}$ as additional slack variables, we recast (19a) with (19b) as

$$\boldsymbol{v}_k^{\mathrm{H}} \boldsymbol{t}_k \geq \sqrt{g_k(\mathrm{I}_k)}, \quad \|\boldsymbol{v}_k^{\mathrm{H}} \boldsymbol{T}_{\bar{k}}\|_2 = \sqrt{\mathrm{I}_k} \quad k \in \mathbb{S}. \tag{21}$$

Then, we relax the equality constraint in (21) to inequality which is valid as $\|\boldsymbol{v}_k^{\mathrm{H}}\boldsymbol{T}_{\bar{k}}\|_2 \leq \sqrt{\mathrm{I}_k}$ will be satisfied with equality in the optimum. Note, that the search space of the optimization under consideration is increased by this notation. However, assuming a tuple of interference values $\{\mathrm{I}_k\}_{k\in\mathbb{S}}$ given, the resulting optimization is a standard *second-order-cone-programming* (SOCP) problem and efficiently solvable via standard *interior-point methods* [20]. Thus, it remains to search over the space spanned by $\{\mathrm{I}_k\}_{k\in\mathbb{S}}$, which will be done with the BB algorithm (cf. Section V).

Now, to arrive at the general formulation in (15) for the BB method, we define the ℓth element of $\boldsymbol{i}$ as $i_{\ell(k)} = \sqrt{\mathrm{I}_k}$ and $k \in \mathbb{S}$. Here, $\ell(\cdot)$ maps the user index $k \in \mathbb{S} \subseteq \mathbb{K}$ to the entry index of the vector $\boldsymbol{i}$, i.e., $\ell : \mathbb{S} \to \{1, \ldots, |\mathbb{S}|\}$. With this generic interference vector, the ℓth entries of the vector functions $\boldsymbol{f}(\{\boldsymbol{t}_k\}, \boldsymbol{i})$ and $\boldsymbol{f}_{\boldsymbol{i}}(\{\boldsymbol{t}_k\})$, from the explicitly written constraints in (15), are $f_{\ell(k)}(\{\boldsymbol{t}_k\}, i_{\ell(k)}) = \sqrt{g_k(i^2_{\ell(k)})} - \boldsymbol{v}_k^{\mathrm{H}}\boldsymbol{t}_k$ and $f_{\boldsymbol{i},\ell(k)} = \|\boldsymbol{v}_k^{\mathrm{H}}\boldsymbol{T}_{\bar{k}}\|_2$, respectively. The tractable SOCs for the transmit power (17) and the useful signal of the perfect CSI users (20) explicitly represent the convex set $\mathcal{Q}$ in (15).

V. Branch and Bound Method

Based on the formulation in (15), the typical BB optimization procedure can be employed (e.g., [13]), where branching and bounding is performed on the non-convex variables in $\boldsymbol{i}$. To this end, we require an initial index set $\mathbb{A} = \{1\}$ and the interference box $\mathcal{B}_1 = [\boldsymbol{i}^{\min}, \boldsymbol{i}^{\max}]$ that contains the optimal interference vector $\boldsymbol{i}^{\mathrm{opt}}$ and is labeled with a lower bound $\mathcal{L}(\mathcal{B}_1)$ and an upper bound $\mathcal{U}(\mathcal{B}_1)$ on the achievable objective. Given this initialization, the following branching and bounding steps are repeated until some ϵ-accuracy is met (cf. [14]).

1) *Branching:* In each iteration, the box $\mathcal{B}_l$ with minimum lower bound (15), i.e.,
$$\mathcal{B}_l = \underset{\{\mathcal{B}_m\}, m\in\mathbb{A}}{\operatorname{argmin}}\ \mathcal{L}(\mathcal{B}_m),$$
is devided into disjoint sub-boxes $\{\mathcal{B}_j\}_{j\notin\mathbb{A}}$. A standard bisection along the longest edge of $\mathcal{B}_l$ may be applied for example, such that the two sub-boxes are of equal size.

2) *Bounding:* For each new sub-box $\mathcal{B}_j = [\boldsymbol{i}^{\min}, \boldsymbol{i}^{\max}]$, we calculate the lower bound $\mathcal{L}(\mathcal{B}_j)$ of the objective as
$$\begin{aligned} \mathcal{L}(\mathcal{B}_j) = \min_{(b,\{\boldsymbol{t}_k\})\in\mathcal{Q}}\ & b \\ \text{s.t.:}\ & \boldsymbol{f}(\{\boldsymbol{t}_k\}, \boldsymbol{i}^{\min}) \leq \boldsymbol{0}, \\ & \boldsymbol{f}_{\boldsymbol{i}}(\{\boldsymbol{t}_k\}) \leq \boldsymbol{i}^{\max} \end{aligned} \tag{22a}$$
and the upper bound $\mathcal{U}(\mathcal{B}_j)$ as
$$\begin{aligned} \mathcal{U}(\mathcal{B}_j) = \min_{(b,\{\boldsymbol{t}_k\})\in\mathcal{Q}}\ & b \\ \text{s.t.:}\ & \boldsymbol{f}(\{\boldsymbol{t}_k\}, \boldsymbol{i}^{\max}) \leq \boldsymbol{0}, \\ & \boldsymbol{f}_{\boldsymbol{i}}(\{\boldsymbol{t}_k\}) \leq \boldsymbol{i}^{\max}. \end{aligned} \tag{22b}$$
These bounds guarantee that $\mathcal{L}(\mathcal{B}_j) \to \mathcal{U}(\mathcal{B}_j)$ for $\boldsymbol{i}^{\min} \to \boldsymbol{i}^{\max}$. Finally, we update the index set $\mathbb{A}$ of the active boxes according to the following rule:
$$\mathbb{A} \leftarrow \big\{m \in (\mathbb{A}\setminus\{l\}) \cup \{j\} \big|\ \mathcal{L}(\mathcal{B}_m) + \epsilon < \min_n \mathcal{U}(\mathcal{B}_n)\big\}.$$

Note that we have $\min_{l\in\mathbb{A}} \mathcal{L}(\mathcal{B}_l) + \epsilon \geq \min_{l\in\mathbb{A}} \mathcal{U}(\mathcal{B}_l)$ at the convergence point of this BB procedure. If the chosen tolerance is $\epsilon > 0$, the algorithm converges in finitely many iterations. However, being essentially an exhaustive search strategy, the complexity of the BB method is exponential in the dimension of $\boldsymbol{i}$ and the basis increases unboundedly with decreasing ϵ [21, Theorem 4]. Therefore, this technique is only tractable for small dimensions and may serve as a benchmark.

Furthermore, we remark that finding an initial box $\mathcal{B}_1$ that contains $\boldsymbol{i}^{\mathrm{opt}}$ is not a straightforward task. While we can set $\boldsymbol{i}^{\min} = \boldsymbol{0}$, $\boldsymbol{i}^{\max}$ is unbounded in general since the transmit power b^2 is unbounded for the power minimization *(P)*. Here, we can exploit the method in Subsection III-B for finding suboptimal but initial feasible beamforming vectors and, therewith, an upper bound for b^{opt}. Being aware of $b^{\mathrm{upper}} > b^{\mathrm{opt}}$, the entries in $\boldsymbol{i}^{\max}$ of $\mathcal{B}_1$ may be obtained via $(i^{\max}_{\ell(k)})^2 = (K-1)(b^{\mathrm{upper}})^2\|\boldsymbol{v}_k\|_2^2 > \mathrm{I}_k^{\mathrm{opt}}$, assuming that all $\boldsymbol{t}_i$, $i \neq k$ are colinear with $\boldsymbol{v}_k$ and $\|\boldsymbol{t}_i\|_2^2 = b^{\mathrm{upper}}$.

VI. Rate Balancing

A problem that is closely related to *(P)* is the rate balancing optimization *(B)* that reads as
$$\begin{aligned} \max_{\beta,\{\boldsymbol{t}_k\}}\ & \beta \\ \text{s.t.:}\ & \sum_{k=1}^{K} \|\boldsymbol{t}_k\|_2^2 \leq P_{\mathrm{tx}}, \\ & \begin{cases} r_k \geq \beta\rho_k & \forall k \in \mathbb{P}, \\ R_k \geq \beta\rho_k & \forall k \in \mathbb{S}, \end{cases} \end{aligned} \tag{23}$$
where the transmit power is P_{tx} in the optimum and the resulting rates are balanced, i.e., the rate of user k is $\beta_{\mathrm{opt}}\rho_k$. Actually, (3) and (23) are inverse problems (cf. [16] and [10]). As a consequence, (23) may be solved via a series of QoS problems (3) until the optimum equals P_{tx}. Alternatively, the balancing problem can also be solved via above BB algorithm when reformulating (23) to (15) as detailed next.

In order to arrive at (15), we first remark that β is maximized in (23). Therefore, we set $b = -\beta$ in this case, while the constraints are formulated equal to Section IV except for replacing ρ_k with $-b\rho_k$. That is, the transmit power restriction is recast as [cf. (17)]
$$\|\mathrm{vec}(\boldsymbol{T})\|_2 \leq \sqrt{P_{\mathrm{tx}}} \tag{24}$$
and the perfect CSI rate requirements are reformulated into [cf. (20)]
$$\boldsymbol{h}_k^{\mathrm{H}}\boldsymbol{t}_k \geq \sqrt{2^{-b\rho_k} - 1}\,\big\|\big[\boldsymbol{h}_k^{\mathrm{H}}\boldsymbol{T}_{\bar{k}}, 1\big]\big\|_2 \qquad k \in \mathbb{P}, \tag{25}$$
where in addition $\mathrm{Im}\{\boldsymbol{h}_k^{\mathrm{H}}\boldsymbol{t}_k\} = 0$. Again, (24) and (25) are the explicit contraints that shape $\mathcal{Q}$ in (15). For the ℓth element in $\boldsymbol{f}(\{\boldsymbol{t}_k\}, \boldsymbol{i})$ and $\boldsymbol{f}_{\boldsymbol{i}}(\{\boldsymbol{t}_k\})$ of (15), we write
$$f_{\ell(k)}(\{\boldsymbol{t}_k\}, i_{\ell(k)}) = \sqrt{g_k(-b, i^2_{\ell(k)})} - \boldsymbol{v}_k^{\mathrm{H}}\boldsymbol{t}_k \quad k \in \mathbb{S}, \tag{26}$$
with $\mathrm{Im}\{\boldsymbol{v}_k^{\mathrm{H}}\boldsymbol{t}_k\} = 0$, and $f_{\boldsymbol{i},\ell(k)} = \|\boldsymbol{v}_k^{\mathrm{H}}\boldsymbol{T}_{\bar{k}}\|_2$, respectively, where we defined [cf. (19a)]
$$g_k(x, y) = \frac{1}{\varsigma^{-1}\big(x\rho_k\log(2) + \varsigma(1/y)\big)} - y.$$

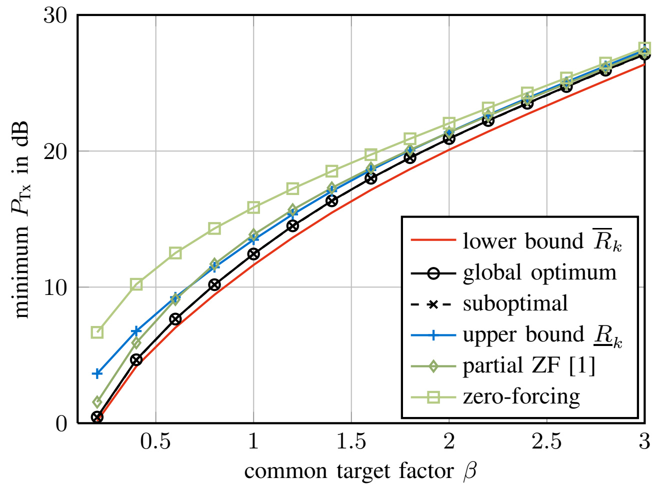

Figure 1. Minimum transmit power over β for an exemplary channel realization in the fully loaded system with $K = N = 4$.

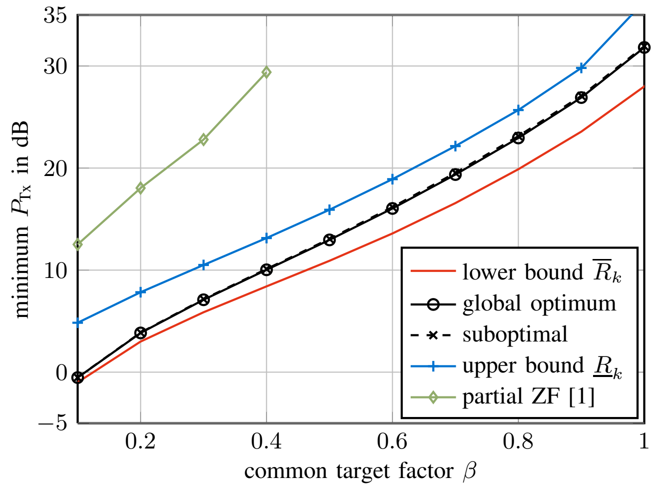

Figure 2. Minimum transmit power over β for an exemplary channel realization in an overloaded system with $K = 6$ and $N = 4$.

We remark that, in contrast to the power minimization formulation in Section IV, problem (15) is not a SOCP problem for fixed interference $\boldsymbol{i}$. Neither $\mathcal{Q}$ nor $\boldsymbol{f}(\{\boldsymbol{t}_k\}, \boldsymbol{i})$ are convex in $(b, \{\boldsymbol{t}_k\})$ due to their dependence on b for the balancing optimization [cf. (25) and (26)]. However, additionally fixing b, the resulting constraints are the same SOCs as for the power minimization. Moreover, noting that b is monotonically decreasing in P_{tx}, the lower and upper bound calculation in step **2)** *Bounding* of the BB algorithm, i.e., (22a) and (22b), may be done via a bisection with respect to b. In each step of the bisection, a feasibility test has to be performed to check wether b is achievable with transmit power less than or equal to P_{tx}. This test may be realized with the corresponding SOCP power minimization problems in **2)** *Bounding* of the BB algorithm in Section V that stem from the reformulation in Section IV. Denoting the optimal transmit power as $P(-b\rho_1, \ldots, -b\rho_K)$, the bisection is performed until $P(-b\rho_1, \ldots, -b\rho_K) = P_{\text{tx}}$.

VII. Numerical Results

For numerical evaluations, we consider a GEO-stationary satellite that is directed to Munich (11°east and 48°north). The satellite is equipped with a rectangular antenna array of N elements. The K users, $|\mathbb{P}| = |\mathbb{S}| = K/2$, are randomly placed within 1° to 21°east and 40° to 56°north. Within this geometric model, we used the free space path loss model for determining properly normalized values of $\{\boldsymbol{h}_k\}_{k\in\mathbb{P}}$ and $\{\boldsymbol{v}_k\}_{k\in\mathbb{S}}$.

Since the BB algorithm in Section V is exponential in $|\mathbb{S}|$, we have performed simulations in relatively small systems to be able to compute the optimal solutions. For this work, we considered two systems in more detail: a *fully loaded system* where $K = N = 4$ and an *overloaded system* with $K = 6$ users but only $N = 4$ transmit antennas. In both scenarios, the basic targets of the users are chosen to be $\rho'_{2i-1} = 1$ and $\rho'_{2i} = 2$, $i \in \{1, \ldots, K/2\}$, which we scaled with some common factor β for the plots to the power minimization and the rate balancing optimization results, i.e., $\rho_k(\beta) = \beta\rho'_k$.

A. Power Minimization Results

In Fig. 1 and Fig. 2, simulation results for the power minimization are depicted for the *fully loaded system* and the *overloaded system*, respectively. We remark that all finite rate requirements are feasible in the fully loaded system. In contrast, the achievable rates in the overloaded system are limited, why we show only a small range for β in Fig. 1.

In both figures, we show an *upper bound* and a *lower bound* for the achievable minimum transmit power. For the lower bound, we solved (13), where the ergodic rates R_k are replaced with their upper bounds $\overline{R}_k$ in (5). The upper bound is the predicted minimum transmit power when replacing the ergodic rates R_k in (3) with the following lower bound in [2]:

$$R_k \geq \underline{R}_k = \log_2\left(1 + \frac{|\boldsymbol{v}_k^{\mathrm{H}}\boldsymbol{t}_k|^2}{\mathrm{e}^{\gamma} + \sum_{i\neq k}|\boldsymbol{v}_k^{\mathrm{H}}\boldsymbol{t}_i|^2}\right) - \frac{d}{\log(2)}, \tag{27}$$

where $d \approx 0.1709$ is the maximal distance between $\varsigma(1/x)$ and $\log(1 + \mathrm{e}^{-\gamma}x)$ for $x \geq 0$ (cf. [2]).

Between the two bounds, we can find the *global optimum* that corresponds to the power minimization results for performing the BB algorithm and the curve for the proposed *suboptimal* strategy in Subsection III-B. Astonishingly, only a small difference is visible between these results for the considered channel realizations. That means, the suboptimal method has the potential to achieve a close to optimal performance with comparingly small complexity. Only two fixed point algorithms have to be employed for solving (13) and (14).

In addition, the power minimization results for the *partial ZF* strategy of [1] (and ZF over all users) are shown here. However, note that the ZF methods strongly restrict the beamformer directions. In the overloaded system (see Fig. 2), the partial ZF strategy considerably reduces the available degrees of freedom for the beamformers to the perfect CSI users, i.e., the beamformers are constrained to cause no interference to the statistical CSI users. Considering a system with $|\mathbb{S}| = |\mathbb{P}| = 3$

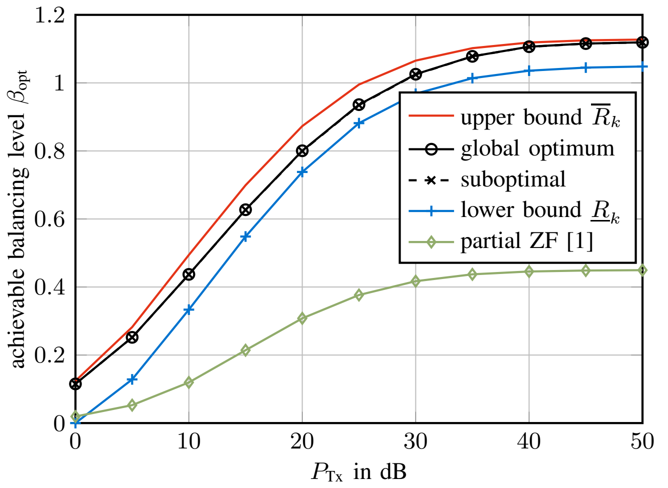

Figure 3. Average achievable balancing level $\beta_{\rm opt}$ in an overloaded system with $K = 6$ users and $N = 4$ transmit antennas for 10 channel realizations.

and $N = 4$, there is only one remaining degree of freedom available for three perfect CSI users. Thus, the achievable rate region (and MMSE region) is considerably reduced. Therefore, these ZF methods clearly lack in performance and we were not able to achieve a higher factor than $\beta = 0.4$ in the power minimization. This effect is also visible for the balancing optimization results detailed next.

B. Rate Balancing Results

In Fig. 3, we plotted the average (optimal) balancing level $\beta_{\rm opt}$ versus $P_{\rm tx}$ in dB for the balancing optimization *(B)*. For the figure, we generated 10 channel realizations $\boldsymbol{h}_k$, $k \in \mathbb{P}$, and $\boldsymbol{v}_k$, $k \in \mathbb{S}$ for the *overloaded* system and calculated the mean of the outcomes $\beta_{\rm opt}$. We clearly see that $\beta_{\rm opt}$ saturates in the high $P_{\rm tx}$ regime. Note that, using $\overline{R}_k$ instead of R_k, we obtain an upper bound for $\beta_{\rm opt}$, whereas a lower bound is obtained for performing the optimization with $\underline{R}_k$ instead of R_k.

Also here, the global optimal curve based on the BB results *and* the curve based on the suboptimal strategy lie between the bounds as expected. Surprisingly, there is no difference visible between the suboptimal method (see Subsection III-B) and the globally optimal BB method in Fig. 3. In fact, the performance degradation of the suboptimal method is negligible for the considered scenarios. Moreover, we see that the partial ZF curve saturates to some value smaller than 0.5 in Fig. 3 as expected from the power minimization results. This means that the achievable (ergodic) rate region is considerably reduced by imposing ZF with respect to the statistical CSI users' channels.

VIII. Conclusions

In this work, the complete feasibility region of the QoS optimization in the vector BC with perfect and statistical CSI users was presented for the assumption of rank-one channel covariance matrices. The corresponding proof motivated a sub-optimal two step beamforming and power allocation scheme which has shown to achieve close to optimal performance in satellite communications. For benchmarking issues, the QoS power minimization with ergodic rate constraints and the ergodic robust rate balancing optimization were reformulated to fit into a known BB global optimization framework.

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