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Computational Complexity Analysis of Advanced Physical Layers based on Multicarrier Modulation

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Abstract: In this contribution we thoroughly analyze the computational effort necessary to implement a multicarrier based physical layer for wireless communications. We consider two variations of this modulation concept, namely traditional cyclic pre- x based orthogonal frequency division multiplexing (CP-OFDM) and lter bank based multicarrier (FBMC) systems. First, we give the analytical formulas for the computational complexity of both systems by taking into account the turbo channel decoding and the multicarrier modulation, demodulation and equalization. Based on the speci cations of WiMAX, we present a numerical comparison between both systems, where they occupy the same bandwidth and possess the same throughput. From the examples, we can conclude that, for a certain bit error ratio, the two alternatives present a similar computational complexity, but CP-OFDM still has to deploy 1.7 dB more energy per information bit than FBMC in order to achieve the same performance.

Keywords: Multicarrier modulation, computational complexity, OFDM, Filter banks

1. Introduction

Multicarrier modulation is the choice for the physical layer of many current high data rate wireless communications systems. The widely spread scheme employed is the so called cyclic prefix orthogonal frequency division multiplexing (CP-OFDM). There are many advantages of using CP-OFDM in wireless systems, amongst others its simple implementation with reduced computational complexity and simple equalization. The main drawbacks are the loss in spectral efficiency incurred by the insertion of the cyclic prefix (CP), the high out-of-band radiation and the high peak-to-average power ratio (PAPR).

Filter bank based multicarrier systems (FBMC) represents a generalization of the concept upon which OFDM is based. Specifically, they do not require the use of any prefix if certain requirements are fulfilled, what maximizes the spectral efficiency, they contain a linear filtering in addition to the fast Fourier transform (FFT) and a multi-tap equalizer per subcarrier has to be usually employed, resulting in an increased computational complexity of the multicarrier processing. An FBMC based advanced physical layer for cognitive radios and dynamical spectrum access was thoroughly explored in the context of the FP7 ICT project PHYDYAS [1].

In order to perform a fair comparison of the computational effort necessary in CP-OFDM and in FBMC we consider in this work not only the signal processing blocks but also the channel decoder, which considerably contributes to the overall complexity compared to the multicarrier modulation and demodulation.

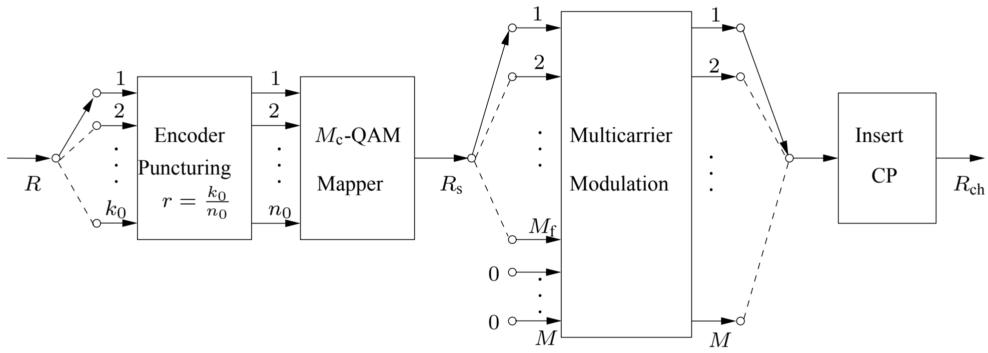

Figure 1: Transmitter model

The evaluation of the complexity is made in terms of the number of real multiplications and additions per transmitted information bit. We assume that the multicarrier modulation is the only signal processing at the transmit side. That is to say there is no channel matched transmit processing.

In the CP-OFDM case we make the assumption that no extra filtering in the time domain is necessary neither at the transmitter nor at the receiver. However, in dynamic spectrum access scenarios, additional processing at both transmitter and receiver would be necessary to reduce the out-of-band radiation and to suppress possible strong narrow-band interference within the signal band. Due to high spectral containment of subcarrier processing, FBMC systems do not require such additional filtering, which would contribute significantly to the baseband signal processing complexity in CP-OFDM based systems.

2. System model

In Fig. 1 a simplified physical layer model of a multicarrier transmitter is depicted. The information bits arrive at the input at a rate of R bits/s. The complete channel encoder, eventually also including puncturing, is represented by the first block. We consider a parallel concatenated convolutional turbo encoder with two constituent codes, each with code memory ν and it is reasonable to assume that its complexity is negligible compared to the other blocks. The information bits are encoded in groups of k_0 bits that generate n_0 encoded bits, thus giving a code rate of $r = k_0/n_0$. In the second block, the n_0 bits are collected in groups of $\log_2 M_c$ bits and mapped onto symbols from an M_c-QAM constellation and forwarded to the next block at a symbol rate of $R_s = \frac{n_0 R}{k_0 \log_2 M_c}$ symbols/second. In the following step the multicarrier modulation is performed. From the M subcarriers, M_f are data filled with the QAM symbols and the remaining have zero input. After the multicarrier modulation M complex samples are generated. Those samples are serialized and the cyclic prefix (CP) is inserted. After the insertion of the CP the complex samples are forwarded to the DA conversion at a rate of $R_{ch} = R_s \frac{M}{M_f}(1 + L_{CP})$, where L_{CP} is the relative length of the cyclic prefix (a fraction of the number of subcarriers M). For a combination of code rate r and modulation alphabet M_c-QAM the number of information bits per complex symbol is $b_i = r \log_2 M_c$. For example, for QPSK, $r = 3/4$ there are $b_i = 1.5$ information bits/complex symbol and for 16-QAM, $r = 1/2$ there are $b_i = 2$ information bits/complex symbol. Due to the addition of the CP and the need of frequency guards, the normalized number of information bits per multicarrier symbol[1] is $N_i = \frac{M_f b_i}{(1+L_{CP})}$. WiMAX uses the encoder

[1]Normalized to the relative length (i.e. $(1 + L_{CP})$ for OFDM and 1 for FBMC) of the multicarrier symbol.

from section 8.4.9.2.3.1 of the IEEE Standard 802.16 [2], where $k_0 = 2$ and $\nu = 3$.

The main differences between CP-OFDM and FBMC are in the blocks called multicarrier modulation and insert CP. In the first system the symbols are pulse shaped by a trivial filter, a rectangular window, and then exponentially modulated. The efficient implementation of this windowing and modulation is achieved by only performing an M point FFT. In the latter system the filter is not trivial and the so called prototype filter, that is longer than the number of subcarriers M has to be designed and implemented accordingly[3]. After linear filtering the symbol streams have also to be exponentially modulated. The efficient implementation of this structure is obtained by deploying an Offset-QAM staggering, an FFT and a polyphase filtering [3]. We assume that the prototype has length $KM + 1$, where K is called the time overlapping factor. It is worth noting that FBMC needs no prefix or equivalently $L_{\text{CP}} = 0$.

At the receive side the inverse operations are executed in reverse order. First, the CP is removed in the case of CP-OFDM and then the samples are transformed into the subcarrier symbols. In both systems an equalizer has to be deployed in order to compensate for the frequency selectivity of the channel. For CP-OFDM this equalizer has length 1 and for FBMC it has usually length $L_{\text{eq}} \geq 1$. After the demodulation process is finished, the complex symbols are detected, soft bits are generated and forwarded to the channel decoder. We ignore here the computations involved in the process of generating soft bits (or log-likelihood ratios) from the noise contaminated received symbols.

3. Computational complexity

In this section we will give the formulas for the computational complexity of each part of our simplified model of a wireless communication system. In this model we divide the whole system into three blocks: the channel encoder, the signal processing that includes the whole transmit and receive processing, and the channel decoder. As already mentioned, the complexity of the channel encoder is considered negligible.

3.1 Signal Processing

The number of real multiplications and additions for an M-point DFT/IDFT using an FFT Split-Radix algorithm is given by

$$\mathcal{C}_{\text{FFT}} = M(\log_2(M) - 3) + 4 \tag{1}$$

$$\mathcal{A}_{\text{FFT}} = 3M(\log_2(M) - 1) + 4 \tag{2}$$

respectively.

At the transmitter of a CP-OFDM system the total number of real multiplications and additions is only $\mathcal{C}_{\text{FFT}}$ and $\mathcal{A}_{\text{FFT}}$. If we consider M_{f} occupied subcarriers and the one-tap per-subcarrier channel equalizer, the complexity at the receiver is $\mathcal{C}_{\text{FFT}}(M) + 4M_{\text{f}}$ multiplications and $\mathcal{A}_{\text{FFT}}(M) + 2M_{\text{f}}$ additions. Consequently, the total number of real operations per information bit is given by

$$\mathcal{C}_{\text{OFDM}} = \frac{2\mathcal{C}_{\text{FFT}} + 4M_{\text{f}}}{N_{\text{i}}} = \frac{2M(\log_2(M) - 3) + 8 + 4M_{\text{f}}}{N_{\text{i}}} \tag{3}$$

$$\mathcal{A}_{\text{OFDM}} = \frac{2\mathcal{A}_{\text{FFT}} + 2M_{\text{f}}}{N_{\text{i}}} = \frac{6M(\log_2(M) - 1) + 4 + 2M_{\text{f}}}{N_{\text{i}}}. \tag{4}$$

Both efficient transmitter and receiver structures of an FBMC system are composed of the filtering through M parallel polyphase components working at twice the symbol rate, by an FFT/IFFT also working at twice the symbol rate and, at the receiver, by the linear equalizers also working in $T/2$, where $1/T$ is the QAM symbol rate. At the transmitter the FFT has in its input pure real or pure imaginary symbols due to the OQAM modulation and for that reason the transform has a reduced complexity when compared to the case where all inputs are complex numbers. As a result the number of real multiplications needed to generate one complex output sample at the transmitter is given by $\mathcal{C}_{\mathrm{SFB}} = M(\log_2(M/2) - 3) + 8 + 4(MK + 1)$. The number of real additions is $\mathcal{A}_{\mathrm{SFB}} = 3M(\log_2(M/2) - 1) + 8 + 4(MK - M + 1)$ correspondingly.

The receiver uses an FFT with complex inputs and outputs, resulting in a multiplication complexity equal to $\mathcal{C}_{\mathrm{AFB}} = 2M(\log_2(M) - 3) + 8 + 4(MK + 1) + 4L_{\mathrm{eq}}$, where L_{eq} is the length of the equalizer. The number of real additions is in this case $\mathcal{A}_{\mathrm{AFB}} = 6M(\log_2(M) - 1) + 8 + 4(MK - M + 1)$. That said the total number of real multiplications for each information bit is

$$\mathcal{C}_{\mathrm{FBMC}} = \frac{\mathcal{C}_{\mathrm{SFB}} + \mathcal{C}_{\mathrm{AFB}} + 4L_{\mathrm{eq}}M_{\mathrm{f}}}{N_{\mathrm{i}}} \tag{5}$$

$$= \frac{M(3\log_2(M) - 10) + 16 + 8(MK + 1)4L_{\mathrm{eq}}M_{\mathrm{f}}}{N_{\mathrm{i}}}. \tag{6}$$

Similarly, the number of real additions is

$$\mathcal{A}_{\mathrm{FBMC}} = \frac{\mathcal{A}_{\mathrm{SFB}} + \mathcal{A}_{\mathrm{AFB}} + (4L_{\mathrm{eq}} - 2)M_{\mathrm{f}}}{N_{\mathrm{i}}} \tag{7}$$

$$= \frac{M(9\log_2(M) - 12) + 16 + 8(MK - M + 1) + (4L_{\mathrm{eq}} - 2)M_{\mathrm{f}}}{N_{\mathrm{i}}}. \tag{8}$$

3.2 Channel decoder

The optimal convolutional turbo decoder is based on the BCJR [4] algorithm, because it delivers the minimal symbol error probability or, more generally, it is the maximum a posteriori probability (MAP) estimator of the encoded bits. If the original BCJR algorithm were to be implemented, the computational complexity of the decoder would become impractical because of the high number of additions and multiplications. For that reason, in most of the implementations of the decoder an approximation of the BCJR algorithm is employed. We assume here that the turbo decoder is based on the log-MAP algorithm [5], because of its relevance.

The log-MAP decoder uses only additions, maximum operations for two variables, and look-ups from a small table (e. g. , 8 elements). Based on [5] and by assuming that max-operations and the look-up operations have the same complexity cost as additions, the number of additions for each constituent code per time step for the log-MAP decoder is given by $\mathcal{A}_{\mathrm{stage}} = 25 \times 2^{\nu} + 13$. Consequently, the total number of additions of the iterative turbo decoder with two constituent codes is given by

$$\mathcal{A}_{\mathrm{logMAP}} = \frac{2I\mathcal{A}_{\mathrm{stage}}}{k_0} \quad \text{additions/information bit.} \tag{9}$$

Let us assume that the encoder of WiMAX is employed. Table 1 shows the number of real additions per information bit of the log-MAP turbo decoder for different number of iterations.

Iterations	1	2	3	4	5
$\mathcal{A}_{\text{logMAP}}$	213	426	639	852	1065

Table 1: Number of real additions per information bit for the log-MAP turbo decoder for $k_0 = 2$ and $\nu = 3$

4. Numerical Examples

In this section we will give first some examples of the computational complexity for the signal processing, i.e. for the multicarrier modulation, demodulation and channel equalization for 3 compatible configurations of CP-OFDM and FBMC.

4.1 Signal processing

In the numerical examples we consider two different propagation channels. ITU-R Vehicular A model has typical characteristics, which are assumed in WiMAX and 3GPP-LTE system development. The ITU-R Vehicular B model has longer delay spread and is therefore much more frequency selective. We also assume in the examples that the prototype has an overlapping factor of $K = 4$. 10 MHz transmission bandwidth is assumed and we focus on the down-link, in which case there is no overhead due to guard bands between different users. We include here only the computations due to processing the data symbols; the synchronization and channel estimation/equalizer adaptation parts are not included, neither the pilot symbol generation in FBMC.

In a first case we consider that the same total number of subcarriers $M = 1024$ is used for both CP-OFDM and FBMC, but the FBMC system has a higher number of data filled subcarriers $M_{\text{f}} = 768$ compared to CP-OFDM $M_{\text{f}} = 720$, because of the reduced guard bands between transmission channels, which is possible due to the better filter bank selectivity. In OFDM, the CP length is assumed to be $L_{\text{CP}} = 1/8$ of the useful symbol duration. With Vehicular A-model, it is sufficient to use 1-tap equalizers in FBMC, i.e. $L_{\text{eq}} = 1$, assuming that the receiver is well synchronized both in time and frequency [6]. Longer equalizers would be able to compensate synchronization imperfections and might be preferred in practice, but this choice gives a lower bound for the FBMC complexity.

In the second case we assume the more frequency-selective Vehicular B channel and $M = 1024$ subcarriers in both OFDM and FBMC systems. Now we have to use longer subchannel equalizer in FBMC with $L_{\text{eq}} = 5$ and the guard interval in OFDM has to be increased to $L_{\text{CP}} = 1/4$ of the useful symbol.

In the third case we consider a FBMC system with wider subcarriers and $M = 256$, and a CP-OFDM system with $M = 1024$. In this case $M_{\text{fill}} = 189$ filled subcarriers and $L_{\text{eq}} = 21$ in FBMC. The computational complexity per information bit for different system setups are shown in Table 2.

Case	$\mathcal{C}_{\text{OFDM}}$	$\mathcal{C}_{\text{FBMC}}$	$\mathcal{A}_{\text{OFDM}}$	$\mathcal{A}_{\text{FBMC}}$
1	14	49	45	95
2	15	60	50	105
3	15	98	50	133

Table 2: Number of multiplications and additions of the multicarrier signal processing for 3 different con gu-rations of FBMC and 2 con gurations of OFDM

It is clear that FBMC processing has considerably higher complexity compared to OFDM. However, when we consider the turbo channel decoding, even though the

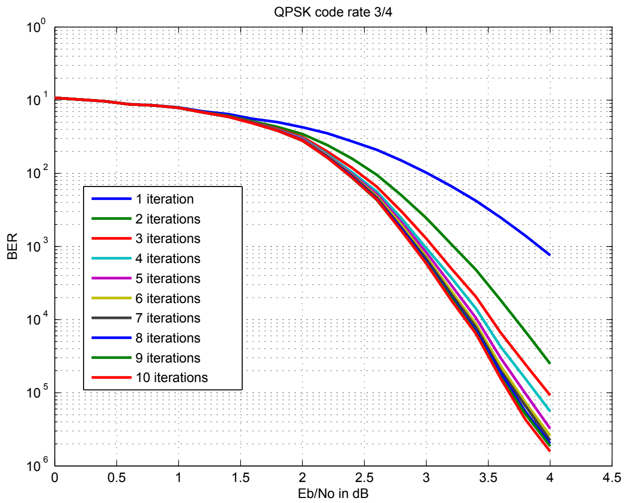

Figure 2: BER for QPSK, $r = 3/4$ for a log-MAP based turbo decoder.

log-MAP algorithm does not require any multiplications, the number of additions/max-operations is so tremendous (recall Table 1), that the turbo decoder implementation dominates the overall system complexity. We will see this effect in the next example.

4.2 Signal processing and turbo decoding

In the next comparison we consider that both systems occupy the same bandwidth and have the same throughput or same net bit rate and consequently the same spectral efficiency. To reach this we assume the same parameters of the case 2 in the former section, i.e. the same number of data-filled subcarriers M_f and the same length of the CP $L_\text{CP} = 1/4$ and equalizer $L_\text{eq} = 5$. Moreover, to reach the same information throughput we consider that the systems use different modulation alphabet and different code rates. For FBMC we consider a QPSK modulation and $r = 3/4$ and because OFDM has less data-filled subcarrriers and the overhead of the CP, we consider in the latter a 16-QAM modulation and $r = 1/2$.

Since we evaluate the systems with different combinations of modulation alphabet and code rate, we first need to take a look at the performance of the turbo decoder for the case studied. In Figs. 2 and 3 the Bit Error Ratio (BER) curves as a function of the E_b/N_0 (energy per bit to noise power spectral density ratio) for an AWGN channel are depicted for different number of iterations of the receiver. Those curves were obtained from the open source toolbox Coded Modulation Library (CML) from Iterative Solutions [7], where some convolutional turbo decoding results for the encoder used in WiMAX are made available.

If we take, for example, a BER=10^{-3} it is possible to see that independent of the number of iterations, the combination 16-QAM, $r = 1/2$ will never reach the same performance as the combination QPSK, $r = 3/4$. In other words, to make the complexity comparisons, also some loss in E_b/N_0 will have to be accepted for CP-OFDM.

In Table 3 we have listed the E_b/N_0 loss for different number of iterations for the 16-QAM, $r = 1/2$ in comparison to different number of iterations of the QPSK, $r = 3/4$. Because part of the energy per symbol for CP-OFDM has to be consumed by the CP, the total E_b/N_0 loss for CP-OFDM is

$$\Delta E_\text{b}/N_0 = (E_\text{b}/N_0)_\text{QPSK-3/4} - (E_\text{b}/N_0)_\text{16-QAM-1/2} + 10\log_{10}(1 + L_\text{CP}) \quad (10)$$

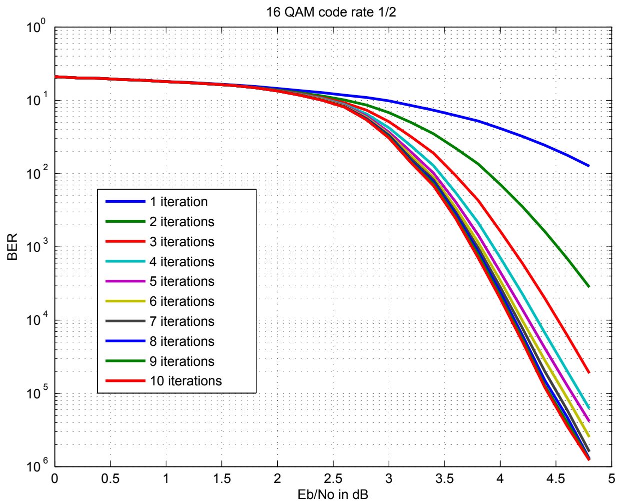

Figure 3: BER for 16-QAM, $r = 1/2$ for a log-MAP based turbo decoder.

For $L_{\text{CP}} = 1/4$, we have that $10\log_{10}(1 + L_{\text{CP}}) = 0.97$ dB.

Modulation r	I (E_b/N_0)	QPSK 3/4 2(3.22)	3 (3.06)	4 (2.99)	5 (2.96)	6 (2.93)
16-QAM 1/2	2 (4.52)	2.27	2.43	2.50	2.53	2.56
	3 (4.1)	1.85	2.01	2.08	2.11	2.14
	4 (3.94)	1.69	1.85	1.92	1.95	1.98
	5 (3.86)	1.61	**1.77**	1.84	1.87	1.90
	6 (3.82)	1.57	1.73	1.80	1.83	1.86

Table 3: E_b/N_0 loss ($\Delta E_b/N_0$) in dB for the different number of iterations for BER= 10^{-3} and including the energy per bit lost in the CP

As mentioned before the log-MAP decoder has only max operations and additions. For this reason we have to convert the number of multiplications for the signal processing into an equivalent number of additions. We assume that all multiplications occur with the same word length and consider 12 bits as a realistic value. In this case each multiplication can be approximated by 10 additions.

The complexity can now be calculated using (3), (5), (7) and (9). Table 4 exhibits the complexity in additions for both systems and for different number of iterations.

Iterations	OFDM (16-QAM 1/2) $\mathcal{A}_{\text{OFDM}} + 10\mathcal{C}_{\text{OFDM}} + \mathcal{A}_{\text{logMAP}}$	FBMC (QPSK 3/4) $\mathcal{A}_{\text{FBMC}} + 10\mathcal{C}_{\text{FBMC}} + \mathcal{A}_{\text{logMAP}}$
1	412	914
2	625	1127
3	838	**1340**
4	1051	1553
5	**1264**	1766

Table 4: Comparison of the total number of additions for case 2 and log-MAP decoder, where the OFDM system has an E_b/N_0 loss depending on the number of iterations

By analyzing the numbers in Table 4 one can see that as the number of iterations increases the relative difference between the complexity of the systems decreases. That is, the turbo decoder becomes dominant over the signal processing. At a first glance, it appears that FBMC has a higher complexity than CP-OFDM. But we have to observe that for each row of the table the performance of the systems are not the same. As we saw before, the combination of modulation and code assumed for CP-OFDM will never reach the same performance as the one for FBMC. In other words, in CP-OFDM some loss in performance has to be accepted or an increased energy per bit to achieve the same performance is necessary. A realistic approach would be to increase the number of iterations in order to reduce the E_b/N_0 loss. For this, we could assume the lowest number of iterations for FBMC and the highest number of iterations for CP-OFDM, i.e. we take the two boldfaced values in Table 4. In this case we can see that both systems will have a similar computational complexity and nevertheless, by taking the boldfaced value in Table 3, CP-OFDM will have a E_b/N_0 loss of 1.77 dB.

5. Conclusions

The computational complexity of FBMC systems seems much larger (3 to 7 times) than that of CP-OFDM at a first glance, i.e. when only looking at the multicarrier modulation and equalization effort. But by assuming that the systems have to use different modulation formats and code rates to achieve the same throughput in the same bandwidth, it turns out that the iterative turbo decoding changes the picture significantly. Even if we are willing to utilize more iterations in the CP-OFDM system than in the FBMC system, we still have to consider an E_b/N_0 penalty, which has to be offset with a higher transmit power. For the typical range of iterations used in turbo decoders and similar channel conditions for both systems, and if we assume that in CP-OFDM 5 iterations should be used and FBMC 3 iterations, it can be realized that the systems have a similar complexity, but CP-OFDM would require an increase of around 1.7 dB in the energy per information bit at the transmit side. The conclusion is that FBMC can achieve the same performance as CP-OFDM with almost the same complexity but with a lower transmit power.

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