Extremes of Regularly Varying Lévy Driven Mixed Moving Average Processes

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Abstract

In this paper we study the extremal behavior of stationary mixed moving average processes $Y(t) = \int_{\mathbb{R}_+ \times \mathbb{R}} f(r, t-s)\, d\Lambda(r,s)$ for $t \in \mathbb{R}$, where f is a deterministic function and Λ is an infinitely divisible independently scattered random measure, whose underlying driving Lévy process is regularly varying. We give sufficient conditions for the stationarity of Y and compute the tail behavior of certain functionals of Y. The extremal behavior is modelled by marked point processes at a discrete-time skeleton chosen properly by the jump times of the underlying driving Lévy process and the extremes of the kernel function. The sequences of marked point processes converge weakly to a cluster Poisson random measure and reflect extremes of Y on a high level. We obtain also convergence of partial maxima to the Fréchet distribution. Our models and results cover short and long range dependence regimes.

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1 Introduction

In this paper we investigate the extremal behavior of a stationary continuous-time *mixed moving average (MA) process* of the form

$$Y(t) = \int_{\mathbb{R}_+ \times \mathbb{R}} f(r, t-s)\, d\Lambda(r,s) \quad \text{for } t \in \mathbb{R}, \tag{1.1}$$

where the *kernel function* $f : \mathbb{R}_+ \times \mathbb{R} \to \mathbb{R}$ is measurable and Λ is an *infinitely divisible independently scattered random measure* (i. d. i. s. r. m.). We recall the definition of an i. d. i. s. r. m. on $\mathbb{R}_+ \times \mathbb{R}$: let $\mathcal{A}$ be a δ-ring (i. e. a ring which is closed under countable intersections) of $\mathbb{R}_+ \times \mathbb{R}$ such that there exists an increasing sequence $\{S_n\}_{n\in\mathbb{N}}$ of sets in $\mathcal{A}$ with $\bigcup_{n=1}^{\infty} S_n = \mathbb{R}_+ \times \mathbb{R}$. Moreover, let $\Lambda = \{\Lambda(A) : A \in \mathcal{A}\}$ be a real valued stochastic process defined on some probability space. We call Λ an *independently scattered random measure*, if for every sequence $\{A_n\}_{n\in\mathbb{N}}$ of disjoint sets in $\mathcal{A}$, the random variables (r. v. s) $\Lambda(A_n)$, $n \in \mathbb{N}$, are independent and, if $\bigcup_{n=1}^{\infty} A_n \in \mathcal{A}$, then $\Lambda\left(\bigcup_{n=1}^{\infty} A_n\right) = \sum_{n=1}^{\infty} \Lambda(A_n)$ almost surely (a. s.). We call a random measure *infinitely divisible* (i. d.), if $\Lambda(A)$ is i. d. for every $A \in \mathcal{A}$. The reader is referred to Rajput and Rosinski [29], Urbanik [35] and Kwapień and Woyczyński [22] for more details on i. d. i. s. r. m. and integrals as given in (1.1).

In the following we consider only i. d. i. s. r. m., where the characteristic function of $\Lambda(A)$ has the representation $\mathbb{E}[\exp(iu\Lambda(A))] = \exp(\lambda(A)\psi(u))$ for $u \in \mathbb{R}$, $A \in \mathcal{A}$. Throughout the paper we assume that there exists a probability measure π on $\mathbb{R}_+$ such that $\lambda(d\omega) = \pi(dr) \times dt$ for $\omega = (r,t) \in \mathbb{R}_+ \times \mathbb{R}$. Moreover, ψ is the cumulant generating function of a Lévy process with

$$\psi(u) = ium - \frac{1}{2}u^2\sigma^2 + \int_{\mathbb{R}} \left(\mathrm{e}^{iux} - 1 - iuh(x)\right) \nu(dx) \ \text{ for } u \in \mathbb{R}, \tag{1.2}$$

and $h(x) = \mathbf{1}_{[-1,1]}(x)$. The quantities (m, σ^2, ν, π) are called the *generating quadruple* of the i. d. i. s. r. m. Λ. Here $m \in \mathbb{R}$, $\sigma^2 \geq 0$, and ν is a measure on $\mathbb{R}$, called *Lévy measure*, satisfying $\nu(\{0\}) = 0$ and $\int_{\mathbb{R}} (1 \wedge |x|^2)\, \nu(dx) < \infty$. We denote by $L = \{L(t)\}_{t\in\mathbb{R}}$ the *underlying driving Lévy process* with

$$L(t) = \Lambda(\mathbb{R}_+ \times [0,t]) \quad \text{for } t \in \mathbb{R}, \tag{1.3}$$

whose generating triplet is (m, σ^2, ν).

Typical examples for mixed MA processes are *superpositions of Ornstein-Uhlenbeck* (supOU) processes studied by Barndorff-Nielsen [2] (see Example 3.4), which are used for stochastic volatility modelling. If $f(r,s)$ is independent of r, i. e. $f(r,s) = \widetilde{f}(s)$ for every $r \in \mathbb{R}_+, s \in \mathbb{R}$, and $\widetilde{f} : \mathbb{R} \to \mathbb{R}$ is measurable, then we interpret Y given by (1.1) as the classical *Lévy driven MA process*

$$Y(t)=\int_{\mathbb{R}} f(t-s)\,dL(s) \quad \text{for } t\in\mathbb{R}, \tag{1.4}$$

where we used the same symbol f for the kernel function $\widetilde{f}$. This class includes CARMA, FICARMA processes (cf. Brockwell and Marquardt [9]) and stochastic delay equations (cf. Gushchin and Küchler [17]).

In the present paper we investigate *regularly varying* Lévy driven mixed MA processes with respect to their extremal behavior. We present the precise conditions below. For details on extreme value theory we refer to the monograph of Embrechts et al. [14]. We shall use the following standard notations: $\overline{\mathbb{R}}=\mathbb{R}\cup\{-\infty\}\cup\{\infty\}$, $\mathbb{R}_+=(0,\infty)$, $\stackrel{w}{\Longrightarrow}$ denotes weak convergence and $\stackrel{v}{\Longrightarrow}$ denotes vague convergence. For real functions g and h we abbreviate $g(t)\sim h(t)$ for $t\to\infty$, if $g(t)/h(t)\to 1$ for $t\to\infty$.

Condition (L). *The marginal distribution $L(1)$ of the underlying driving Lévy process L as given in (1.3) is regularly varying of index α for some $\alpha>0$, i. e. there exists a sequence $0<a_n\uparrow\infty$ of constants such that*

$$n\mathbb{P}(a_n^{-1}L(1)\in\cdot)\stackrel{v}{\Longrightarrow}\sigma(\cdot) \quad \text{on } \mathcal{B}(\overline{\mathbb{R}}\backslash\{0\}) \text{ for } n\to\infty, \tag{1.5}$$

where for some $p\in[0,1]$ and $q=1-p$,

$$\sigma(dx)=p\alpha x^{-\alpha-1}\,\mathbf{1}_{(0,\infty)}(x)\,dx+q\alpha(-x)^{-\alpha-1}\,\mathbf{1}_{(-\infty,0)}(x)\,dx. \tag{1.6}$$

Regularly varying distribution functions (d. f. s) include, in particular, stable, Pareto, loggamma and Burr distribution. Notice, that $\mathbb{E}|L(1)|^\delta<\infty$ for $\delta<\alpha$ and $\mathbb{E}|L(1)|^\delta=\infty$ for $\delta>\alpha$.

For studying the extremal behavior of Y we will impose the following condition in Section 4. We define

$$\mathbb{L}^\delta(\pi):=\left\{f:\mathbb{R}_+\times\mathbb{R}\to\mathbb{R} \text{ measurable}, \int_{\mathbb{R}_+}\int_{\mathbb{R}}|f(r,s)|^\delta\,ds\,\pi(dr)<\infty\right\}$$

for $\delta>0$. If $f(r,s)$ is independent of r we write $f\in\mathbb{L}^\delta$ instead of $f\in\mathbb{L}^\delta(\pi)$.

Condition (M). *Let Y as given in (1.1) be a stationary i. d. process and the underlying driving Lévy process L as given in (1.3) satisfy (L). Let the kernel function $f\in\mathbb{L}^\delta(\pi)$ for some $\delta<\alpha$, or let $L(1)$ be α-stable and $f\in\mathbb{L}^\alpha(\pi)$. In both cases assume that $f^+:=\sup_{(r,t)\in\mathbb{R}_+\times\mathbb{R}}f^+(r,t)<\infty$ and $f^-:=\sup_{(r,t)\in\mathbb{R}_+\times\mathbb{R}}f^-(r,t)<\infty$ with $f^+(r,t):=\max\{f(r,t),0\}$, $f^-(r,t):=\max\{-f(r,t),0\}$. Furthermore, let $\int_{\mathbb{R}_+}\int_{\mathbb{R}}p(f^+(r,s))^\alpha+q(f^-(r,s))^\alpha\,ds\,\pi(dr)>0$.*

We shall give sufficient conditions for Y to be a stationary i. d. process and also regularly varying of index α; see Section 3.

Extreme value theory for stable MA processes was derived in Rootzén [31]. We extend Rootzén's results to the much richer class of regularly varying mixed MA processes. Furthermore, we weaken his assumptions on the kernel function. Thus, our model includes also heavy tailed long memory processes.

This paper is organized as follows. We start with preliminaries in Section 2 introducing multivariate regular variation (in Section 2.1) and point processes of multivariate regularly varying sequences (in Section 2.2). An investigation of heavy tailed mixed MA processes follows in Section 3. This includes sufficient conditions for (M), followed by a study of the tail behavior of Y as well as the tail behavior of $M(h) = \sup_{t\in[0,h]} Y(t)$ for $h > 0$. Finally, we introduce supOU processes as examples for heavy tailed mixed MA processes, which can exhibit long range dependence.

The main results of this paper are presented in Section 4. In Section 4.1 our investigation on the extremal behavior of Y is based on marked point processes at a properly chosen discrete-time skeleton, namely by the jump times of the underlying driving Lévy process in combination with extremes of the kernel function. The marked point processes converge to a marked cluster Poisson random measure. In the neighborhood of such an extreme event the behavior of the process is solely determined by the kernel function. Finally, we obtain the limit distribution of running maxima of Y in Section 4.2. The results are applied in particular to supOU processes in Section 4.3. We conclude with the rather technical proofs of Lemma 2.4, Proposition 3.3, Theorem 4.1 and Theorem 4.4 in Section 5.

Throughout the paper we use the following notation. We write $X \stackrel{d}{=} Y$, if the distributions of the r. v. s X and Y coincide. For a vector $\mathbf{x} \in \mathbb{R}^d$ we denote by $\mathbf{x}^t$ the transposed of $\mathbf{x}$ and by $|\mathbf{x}| = \max\{|x_1|, \ldots, |x_d|\}$ the maximum norm. For a matrix $\mathbf{A} \in \mathbb{R}^{d\times r}$ we denote by $\|A\|$ the row-sum-norm. For a measure π we denote by $\operatorname{supp}(\pi)$ the support of π. Further, $\sum_{k=1}^{0} := 0$ and $\bigvee_{k=1}^{0} := 0$. As usual $\mathbb{D}(\mathbb{R})$ is the space of càdlàg functions on $\mathbb{R}$.

2 Preliminaries

2.1 Multivariate regular variation

In Section 4.1, we shall find that the finite dimensional distributions of Y are multivariate regularly varying. We start with the definition.

Definition 2.1 (Multivariate regular variation)
A random vector $\mathbf{X} = (X_1, \ldots, X_d)$ on $\mathbb{R}^d$ is said to be regularly varying with in-

dex α, $\alpha > 0$, if there exists a random vector Θ with values on the unit sphere $\mathbb{S}^{d-1} = \{\mathbf{x} \in \mathbb{R}^d : |\mathbf{x}| = 1\}$ such that for every $x > 0$,

$$\frac{\mathbb{P}(|\mathbf{X}| > ux, \mathbf{X}/|\mathbf{X}| \in \cdot)}{\mathbb{P}(|\mathbf{X}| > u)} \stackrel{w}{\Longrightarrow} x^{-\alpha}\mathbb{P}(\Theta \in \cdot) \quad \text{on } \mathcal{B}(\mathbb{S}^{d-1}) \text{ for } u \to \infty. \tag{2.1}$$

The distribution of Θ is referred to as the *spectral measure* of $\mathbf{X}$. It describes in which direction we are likely to find extreme realizations of $\mathbf{X}$. This definition of regular variation is equivalent to the following:

There exists a Radon measure $\sigma(\cdot)$ on $\overline{\mathbb{R}}^d \backslash \{\mathbf{0}\}$ with $\sigma(\overline{\mathbb{R}}^d \backslash \mathbb{R}^d) = 0$ and $\sigma(E) > 0$ for at least one relatively compact set $E \subseteq \overline{\mathbb{R}}^d \backslash \{\mathbf{0}\}$, where $\mathbf{0} = (0, \ldots, 0) \in \mathbb{R}^d$, and a sequence $0 < a_n \uparrow \infty$ of constants such that

$$n\mathbb{P}(a_n^{-1}\mathbf{X} \in \cdot) \stackrel{v}{\Longrightarrow} \sigma(\cdot) \quad \text{on } \mathcal{B}(\overline{\mathbb{R}}^d \backslash \{\mathbf{0}\}) \text{ for } n \to \infty. \tag{2.2}$$

More about multivariate regular variation can be found e. g. in Basrak et al. [4], Lindskøg [25] and Resnick [30], Chapter 5.

The following Lemma is a multivariate extension of Breiman's [8] classical result on regular variation of products and Proposition A.1 of Basrak et al. [5]. The explicit representation of the spectral measure in Lemma 2.2 follows by straightforward calculations.

Lemma 2.2 *Let $\mathbf{Z} = (Z_1, \ldots, Z_r)$ be a vector of independent r. v. s, which are regularly varying of index α, such that for $j = 1, \ldots, r$ there exists a sequence $0 < a_n \uparrow \infty$ of constants satisfying*

$$n\mathbb{P}(a_n^{-1}Z_j \in \cdot) \stackrel{v}{\Longrightarrow} \sigma_j(\cdot) \quad \text{on } \mathcal{B}(\overline{\mathbb{R}} \backslash \{0\}) \text{ for } n \to \infty,$$

where $\sigma_j(dx) = p_j \alpha x^{-\alpha-1}\, \mathbf{1}_{(0,\infty)}(x)\, dx + q_j \alpha(-x)^{-\alpha-1}\, \mathbf{1}_{(-\infty,0)}(x)\, dx$ with $p_j, q_j \geq 0$, $p_j + q_j > 0$. Furthermore, let $\mathbf{A} = (\mathbf{a}_1, \ldots, \mathbf{a}_r)$ be a random $d \times r$-matrix, independent of $\mathbf{Z}$. If $0 < \mathbb{E}\|\mathbf{A}\|^\gamma < \infty$ for some $\gamma > \alpha$, then $\mathbf{Y} = \mathbf{AZ}$ is regularly varying of index α and has spectral measure,

$$\mathbb{P}(\Theta \in \cdot) = \frac{\sum_{j=1}^r \left[p_j \mathbb{E}(|\mathbf{a}_j|^\alpha\, \mathbf{1}_{\{\mathbf{a}_j/|\mathbf{a}_j| \in \cdot\}}) + q_j \mathbb{E}(|\mathbf{a}_j|^\alpha\, \mathbf{1}_{\{-\mathbf{a}_j/|\mathbf{a}_j| \in \cdot\}})\right]}{\sum_{j=1}^r (p_j + q_j)\mathbb{E}|\mathbf{a}_j|^\alpha}. \tag{2.3}$$

For $x > 0$ we have $\lim_{n\to\infty} n\mathbb{P}(|\mathbf{Y}| > a_n x) = x^{-\alpha} \sum_{j=1}^r (p_j + q_j)\mathbb{E}|\mathbf{a}_j|^\alpha$, giving for $d = 1$,

$$\lim_{n\to\infty} n\mathbb{P}(Y > a_n x) = x^{-\alpha} \sum_{j=1}^r [p_j \mathbb{E}a_j^{+\,\alpha} + q_j \mathbb{E}a_j^{-\,\alpha}].$$

Remark 2.3 Let $\mathbf{A}$ be a deterministic matrix and $\rho := \sum_{j=1}^r (p_j + q_j)\mathbb{E}|\mathbf{a}_j|^\alpha$. An interpretation of (2.3) is that the spectral measure Θ reaches the values $\mathbf{a}_j/|\mathbf{a}_j|$ with probability $p_j|\mathbf{a}_j|^\alpha/\rho$ and $-\mathbf{a}_j/|\mathbf{a}_j|$ with probability $q_j|\mathbf{a}_j|^\alpha/\rho$. Thus, only in the directions $\mathbf{a}_j/|\mathbf{a}_j|$ and $-\mathbf{a}_j/|\mathbf{a}_j|$, $j = 1, \ldots, r$, extremes are likely to occur. □

2.2 Point process convergence

We follow Resnick [30] and introduce point processes to describe the extremal behavior of Y. In order to achieve distributional stability of a sequence of point processes, it is necessary to allow for a build up of infinite mass at $[s,t)\times\{\mathbf{0}\}$. For our problem this is achieved by defining the state space $S=[0,\infty)\times\overline{\mathbb{R}}^d\backslash\{\mathbf{0}\}$, $d\in\mathbb{N}$. Then S can be metricized as a locally compact, complete and separable Hausdorff space. Compact sets in S are closed sets, which are bounded away from $\mathbf{0}$ and $\pm\infty$. Furthermore, $\mathcal{B}(S)$ denotes the Borel σ-field on S and $M_P(S)$ the class of point measures on S, where $M_P(S)$ is equipped with the metric ρ that generates the topology of vague convergence. The space $(M_P(S),\rho)$ is a complete and separable metric space with Borel σ-field $\mathcal{M}_P(S)$. The zero measure is denoted by $\mathbf{0}$. A *point process* in S is a random element in $(M_P(S),\mathcal{M}_P(S))$, i. e. a measurable map from a probability space $(\Omega,\mathcal{F},\mathbb{P})$ into $(M_P(S),\mathcal{M}_P(S))$. A typical example for a point process in extreme value theory is a Poisson random measure, i. e. given a Radon measure ϑ on $\mathcal{B}(S)$, a point process κ is called *Poisson random measure* with intensity measure ϑ, denoted by PRM(ϑ), if

(a) $\kappa(A)$ is Poisson distributed with intensity $\vartheta(A)$ for every $A\in\mathcal{B}(S)$,

(b) for mutually disjoint sets $A_1,\ldots,A_n\in\mathcal{B}(S)$, $n\in\mathbb{N}$, the r. v. s $\kappa(A_1),\ldots,\kappa(A_n)$ are independent.

More about point process theory can be found in Daley and Vere-Jones [10] and Kallenberg [21]. Furthermore, results of Davis and Hsing [11] about the point process behavior of a stationary sequence of regularly varying r. v. s under weak dependence are of vital importance for our studies. Their results were generalized by Davis and Mikosch [12] to multidimensional regularly varying stationary processes, which are used in Section 4.1.

The following Lemma shows that adding a sequence of small random vectors to a sequence of multivariate regularly varying random vectors has no influence on the point process behavior. The meaning of "small random vector" is that the tail of the norm value of the random vector decreases faster than the tail of the norm value of the multivariate regularly varying random vectors. Let N be a point process with jump times $\{\Gamma_k\}_{k\in\mathbb{N}}$ labelled such that $0<\Gamma_1<\Gamma_2<\ldots<\infty$. If the inter-arrival times $\{\Gamma_{k+1}-\Gamma_k\}_{k\in\mathbb{N}}$ are i. i. d. the counting process N is said to be a *renewal process* with intensity $\mu:=\mathbb{E}(\Gamma_2-\Gamma_1)$.

Lemma 2.4 *Let $\mathbf{Z}=\{\mathbf{Z}_k\}_{k\in\mathbb{N}}$ and $\mathbf{\Psi}=\{\mathbf{\Psi}_k\}_{k\in\mathbb{N}}$ be sequences of random vectors in $\mathbb{R}^d$. Furthermore, let $\{\Gamma_k\}_{k\in\mathbb{N}}$ be the jump times of a renewal process N with intensity $\mu>0$, $h\in\mathbb{R}$ be arbitrary and $s_k\in[\Gamma_{k-1}+h,\Gamma_{k+1}+h)$ for $k\in\mathbb{N}$, setting*

$\Gamma_0 := 0$. *Denote by* $0 < a_n \uparrow \infty$ *a sequence of constants and define the point processes*

$$\widetilde{\kappa}_n = \sum_{k=1}^{\infty} \varepsilon_{(k/n, \mathbf{Z}_k/a_n)},\ n \in \mathbb{N},\ \text{ and } \kappa_T = \sum_{k=1}^{\infty} \varepsilon_{\left(s_k\mu/T, (\mathbf{Z}_k+\mathbf{\Psi}_k)/a_{\lfloor T\rfloor}\right)},\ T > 0,\ \text{ in } M_P(S).$$

Suppose there exists a point process κ *in* $M_P(S)$ *with* $\kappa([s,t) \times \{\mathbf{x}\}) = 0$ *a.s. for* $\mathbf{x} \in \overline{\mathbb{R}}^d \backslash \{\mathbf{0}\}$, $t > s \geq 0$, *such that* $\widetilde{\kappa}_n \stackrel{w}{\Longrightarrow} \kappa$ *for* $n \to \infty$. *Furthermore, assume that for every* $\epsilon, t > 0$,

$$\sum_{k=1}^{\lfloor nt\rfloor} \mathbb{P}(|\mathbf{\Psi}_k| > a_n\epsilon) \stackrel{n\to\infty}{\longrightarrow} 0. \tag{2.4}$$

Moreover, we suppose that there exists a r. v. W *such that*

$$\mathbb{P}(|\mathbf{Z}_k + \mathbf{\Psi}_k| > x) \leq \mathbb{P}(W > x) \text{ for } x > 0 \text{ and } \mathbb{P}(W > a_n x) = O(1/n) \text{ for } n \to \infty.$$

Let $I = [s,t) \times \prod_{i=1}^{d} (c_i, d_i] \subseteq S$ *be bounded away from* $\mathbf{0}$ *and* $\boldsymbol{\infty}$. *Then*

$$\lim_{T\to\infty} \mathbb{P}(\kappa_T(I) \neq \widetilde{\kappa}_{\lfloor T\rfloor}(I)) = 0$$

and $\kappa_T \stackrel{w}{\Longrightarrow} \kappa$ *for* $T \to \infty$.

Note, that $\{\mathbf{Z}_k\}_{k\in\mathbb{N}}$ and $\{\mathbf{Z}_k + \mathbf{\Psi}_k\}_{k\in\mathbb{N}}$ need not to be stationary sequences. To provide some intuition for random vectors $\{\mathbf{\Psi}_k\}_{k\in\mathbb{N}}$ to satisfy (2.4), we give some examples.

Example 2.5 *(a)* Assume there exists a r. v. ψ such that for some $x_0 \geq 0$ and any $\epsilon > 0$, $k \in \mathbb{N}$,

$$\mathbb{P}(|\mathbf{\Psi}_k| > x) \leq \mathbb{P}(\psi > x) \quad \text{for } x \geq x_0 \quad \text{and} \quad \mathbb{P}(\psi > a_n\epsilon) = o(1/n) \quad \text{for } n \to \infty,$$

then (2.4) is satisfied.

(b) Let $\{\widetilde{Z}_k\}_{k\in\mathbb{N}}$ be a sequence of identically distributed r. v. s, which are regularly varying of index α in the sense of (2.2) and with the same a_n as given in Lemma 2.4. Suppose $\{\widetilde{Z}_k\}_{k\in\mathbb{N}}$ is independent of the sequence of random vectors $\{\widetilde{\mathbf{\Psi}}_k\}_{k\in\mathbb{N}}$ in $\mathbb{R}^d$, which have support on $[-f^+, f^+]^d$. Define $\mathbf{\Psi}_k := \widetilde{\mathbf{\Psi}}_k \widetilde{Z}_k$ and assume that there exists a $0 < \delta < \alpha$, such that

$$\sum_{k=-\infty}^{\infty} \mathbb{E}|\widetilde{\mathbf{\Psi}}_k|^\delta < \infty. \tag{2.5}$$

Denote by F_k the d. f. of $\widetilde{\mathbf{\Psi}}_k$. By Potter's Theorem (Bingham et al. [6], Theorem 1.5.6) there exists an $n_0 \in \mathbb{N}$ and $K > 1$, such that for $k \in \mathbb{N}, n \geq n_0$,

$$\begin{aligned} \mathbb{P}(|\widetilde{\mathbf{\Psi}}_k \widetilde{Z}_k| > a_n\epsilon) &= \int_{\mathbb{R}^d\setminus\{\mathbf{0}\}} \mathbb{P}(f^+|\widetilde{Z}_k| > a_n\epsilon f^+/|\mathbf{t}|)\, F_k(d\mathbf{t}) \\ &\leq K\mathbb{P}(f^+|\widetilde{Z}_1| > a_n\epsilon)\mathbb{E}|\widetilde{\mathbf{\Psi}}_k|^\delta. \end{aligned} \tag{2.6}$$

Regarding (2.6) and $a_n \to \infty$ as $n \to \infty$, we obtain

$$\lim_{n\to\infty} \sum_{k=1}^{\lfloor nt \rfloor} \mathbb{P}(|\mathbf{\Psi}_k| > a_n\epsilon) \leq K \lim_{n\to\infty} \mathbb{P}(f^+|\widetilde{Z}_1| > a_n\epsilon) \sum_{k=1}^{\infty} \mathbb{E}|\widetilde{\mathbf{\Psi}}_k|^\delta = 0. \tag{2.7}$$

Thus, $\{\mathbf{\Psi}_k\}_{k\in\mathbb{N}}$ satisfies (2.4). □

3 Stationarity and tail behavior of Y

This paper is concerned with extremes of *regularly varying* mixed MA processes Y as given in (1.1), which means that the underlying driving Lévy process satisfies (L). Under certain conditions Y is well-defined as a limit in probability of integrals of step functions approximating f. This has been shown by Rajput and Rosinski [29], Theorem 2.7 (see also Kwapień and Woyczyzński [22]). They give necessary and sufficient conditions, which are formulated in terms of the kernel function f and the generating quadruple (m, σ^2, ν, π) of the i. d. i. s. r. m. Λ. Under these assumptions Y is i. d., and by the structure of a mixed MA process Y is stationary. The following Proposition gives sufficient conditions to ensure that these assumptions are satisfied. For details of the proof, we refer to Fasen [15], Proposition 2.2.3.

Proposition 3.1 (Existence) *Let Λ be an i. d. i. s. r. m. with generating quadruple (m, σ^2, ν, π), where the underlying driving Lévy process L as defined in (1.3) satisfies (L), and let f be bounded. Then Y given by (1.1) is well-defined, i. d. and stationary, if one of the following conditions is satisfied:*

(a) $L(1)$ *is* α*-stable,* $\alpha \in (0,1) \cup (1,2)$*, and* $f \in \mathbb{L}^\alpha(\pi)$.

(b) $f \in \mathbb{L}^\delta(\pi)$ *for some* $\delta < \alpha$, $\delta \leq 1$.

(c) $\mathbb{E}L(1) = 0$, $\alpha > 1$, *and* $f \in \mathbb{L}^\delta(\pi)$ *for some* $\delta < \alpha$, $\delta \leq 2$.

Remark 3.2 *(i)* For a Lévy driven MA process as given in (1.4), Proposition 3.1 provides sufficient conditions for Y to be stationary and the marginal distribution to be i. d. Then $\mathbb{L}^\delta(\pi)$ can be replaced by $\mathbb{L}^\delta$. Typical examples for functions in $\mathbb{L}^\delta$ are bounded functions f with $f(t) \sim K_1 t^{-\delta+\epsilon}$, $f(-t) \sim K_2 t^{-\delta+\epsilon}$ for $t \to \infty$ and for some $\epsilon \in (0,\delta)$, K_1, $K_2 \in \mathbb{R}$. OU-processes, CARMA processes and stochastic delay equations, which have exponentially decreasing kernel functions, as well as FICARMA processes satisfy this condition.

(ii) Let Y be a stationary mixed MA process given by (1.1) with kernel function f and generating quadruple (m,σ^2,ν,π) of Λ. Then $f \in \mathbb{L}^{\alpha+\epsilon}(\pi)$ for some $\epsilon > 0$. □

Proposition 3.3 (Tail behavior) *Let Y be a mixed MA process given by (1.1) satisfying (M), and let $x > 0$. Then for $t \in \mathbb{R}$,*

$$\lim_{n\to\infty} n\mathbb{P}(Y(t) > a_n x) = x^{-\alpha} \int_{\mathbb{R}_+} \int_{\mathbb{R}} p(f^+(r,s))^\alpha + q(f^-(r,s))^\alpha \, ds\, \pi(dr). \tag{3.1}$$

Moreover, for $t_i \in \mathbb{R}$, $i = 1,\ldots,k$, $k \in \mathbb{N}$,

$$\lim_{n\to\infty} n\mathbb{P}\left(\max_{i=1,\ldots,k} |Y(t_i)| > a_n x\right) = x^{-\alpha} \int_{\mathbb{R}_+} \int_{\mathbb{R}} \max_{i=1,\ldots,k} |f(r,t_i - s)|^\alpha \, ds\, \pi(dr). \tag{3.2}$$

Furthermore, let Y has a. s. sample paths in $\mathbb{D}(\mathbb{R})$, $f_h(r,s) := \sup_{t\in[0,h]} |f(r,t+s)| \in \mathbb{L}^{\alpha-\epsilon}(\pi)$ for some $0 < \epsilon < \alpha$, and define $M(h) = \sup_{t\in[0,h]} Y(t)$. Then

$$\begin{aligned} &\lim_{n\to\infty} n\mathbb{P}(M(h) > a_n x) \\ &\quad = x^{-\alpha} \int_{\mathbb{R}_+} \int_{\mathbb{R}} p \sup_{t\in[0,h]} (f^+(r,t+s))^\alpha + q \sup_{t\in[0,h]} (f^-(r,t+s))^\alpha \, ds\, \pi(dr). \end{aligned} \tag{3.3}$$

From (3.1) we see that $Y(t)$ is again regularly varying in the sense of (2.2).

Example 3.4 (supOU process) We consider the mixed MA process as given in (1.1), where the kernel function is $f(r,s) = \mathbf{1}_{[0,\infty)}(s)\mathrm{e}^{-rs}$ for $r \in \mathbb{R}_+$, $s \in \mathbb{R}$. Then

$$Y(t) = \int_{\mathbb{R}_+\times\mathbb{R}} \mathbf{1}_{[0,\infty)}(t-s)\mathrm{e}^{-r(t-s)} \, d\Lambda(r,s) \quad \text{for } t \in \mathbb{R} \tag{3.4}$$

is called supOU (*superposition of Ornstein-Uhlenbeck*) process. An important special case of (3.4) is the OU (Ornstein-Uhlenbeck) process, for which π has only support at some $\lambda > 0$, i. e. $\pi(\{\lambda\}) = 1$.

For a general probability measure π and for some $\delta > 0$ we have $f \in \mathbb{L}^\delta(\pi)$ if and only if

$$\int_{\mathbb{R}_+} \int_{\mathbb{R}_+} \mathrm{e}^{-rs\delta} \, ds \pi(dr) = \delta^{-1} \int_{\mathbb{R}_+} r^{-1}\, \pi(dr) < \infty.$$

We assume in the following that $\lambda^{-1} := \int_{\mathbb{R}_+} r^{-1}\, \pi(dr) < \infty$. Hence, $f \in \mathbb{L}^\delta(\pi)$ for every $\delta > 0$. The necessary and sufficient conditions for supOU processes to exist and be i. d. (see Rajput and Rosinski [29], Theorem 2.7) reduce to the necessary and sufficient conditions of a simple OU process with parameter λ to exist. Then by Sato [34], Theorem 17.5, the supOU process exists and is i. d. if and only if $\int_{|x|>2} \log|x|\,\nu(dx) < \infty$. We obtain the generating triplet

$$\begin{aligned} m_Y &= \frac{1}{\lambda}\left[m + \int_{|y|>1} \frac{y}{|y|}\,\nu(dy)\right], \qquad \sigma_Y^2 = \frac{\sigma^2}{2\lambda}, \\ \nu_Y\,[x,\infty) &= \frac{1}{\lambda}\int_x^\infty \frac{\nu\,[y,\infty)}{y}\,dy\,, \quad x>0. \end{aligned} \tag{3.5}$$

Note, that the finite dimensional distributions of Y are that of an OU process with parameter λ, whose driving Lévy process has characteristic triplet (m,σ^2,ν), i. e. the marginal distribution of Y is selfdecomposable. Furthermore, for any regularly varying Lévy process satisfying (L), Y is a stationary i. d. process and

$$\lim_{n\to\infty} n\mathbb{P}(Y(t) > a_n x) = \frac{1}{\lambda\alpha}x^{-\alpha} \quad \text{ for } t\in\mathbb{R}.$$

Define the probability measure $\overline{\pi}(dr) := \lambda r^{-1}\pi(dr)$ and the i. d. i. s. r. m. $\overline{\Lambda}$ with generating quadruple $(m/\lambda, \sigma^2/\lambda, \nu/\lambda, \overline{\pi})$. Then the finite dimensional distributions of the stochastic process

$$X(t) = \int_{-\infty}^{\infty} \mathrm{e}^{-rt} \int_{-\infty}^{rt} \mathrm{e}^{s}\,d\overline{\Lambda}(r,s) \quad \text{ for } t\in\mathbb{R} \tag{3.6}$$

coincide with those of Y, i. e. $X \stackrel{d}{=} Y$ (Barndorff-Nielsen [2], Theorem 3.1). Since

$$dX(t) = \int_{\mathbb{R}_+} \{-rX(t,dr)\,dt + d\overline{\Lambda}(t,r)\} \quad \text{ for } t\in\mathbb{R}$$

with $X(t,B) = \int_B \mathrm{e}^{rt}\int_{-\infty}^{rt} \mathrm{e}^{s}\,d\overline{\Lambda}(r,s)$ for $t\in\mathbb{R}$, $B\in\mathcal{B}(\mathbb{R})$, the process X, respectively Y, is called supOU process.

By a proper choice of π the correlation function $\rho(h) = \lambda\int_0^\infty r^{-1}\mathrm{e}^{-hr}\,\pi(dr)$ for $h\in\mathbb{R}$, can model long memory processes. For example, if π is gamma distributed with density $\pi(dr) = \Gamma(2H+1)^{-1}r^{2H}\mathrm{e}^{-r}\,dr$ for $r>0$, $H>0$, then $\rho(h) = \Gamma(2H)^{-1}\int_0^\infty r^{2H-1}\mathrm{e}^{-r(h+1)}\,dr = (h+1)^{-2H}$ for $h\in\mathbb{R}$. More about supOU models and their relevance for applications to financial data can be found in Barndorff-Nielsen and Shephard [3]. □

4 Extremal behavior

4.1 The point process of a discrete-time skeleton

In this section we study the extremal behavior of a regularly varying mixed MA process. To this end we use a discrete-time skeleton for Y as given in (1.1) satisfying

(M). This means we investigate the extremal behavior of a discrete-time sequence $\{Y(t_n)\}_{n\in\mathbb{N}}$, where the discrete-time random sequence $\{t_n\}_{n\in\mathbb{N}}$ is chosen properly by the jump times of the underlying driving Lévy process L as given in (1.3) and the extremes of the kernel function. We shall show that the extremes of $\{Y(t_n)\}_{n\in\mathbb{N}}$ coincide with extremes of Y on high levels.

Therfore, we decompose Λ into two independent i. d. i. s. r. m. according to the jump sizes of the underlying driving Lévy process L, which are represented by ν. We define

$$\Lambda = \Lambda_1 + \Lambda_2 \quad \text{and} \quad \Lambda_1(A) = \int_{\mathbb{R}} x\, d\widetilde{N}_1(A,x) \quad \text{for } A \in \mathcal{A}, \tag{4.1}$$

where $\widetilde{N}_1$ is a *Poisson random measure* with intensity

$$\vartheta(dr \times dt \times dx) = \pi(dr) \times dt \times \nu_1(dx),$$

(denoted by PRM(ϑ)), and ν_1 is the Lévy measure

$$\nu_1(A) = \nu(A \cap (1,\infty)) + \nu(A \cap (-\infty,-1)) \quad \text{for } A \in \mathcal{B}(\mathbb{R}).$$

The generating quadruple of Λ_1 is $(0,0,\nu_1,\pi)$. Furthermore, Λ_1 is called *compound Poisson random measure*. The i. d. i. s. r. m. Λ_2 has the generating quadruple (m,σ^2,ν_2,π) with Lévy measure $\nu_2 = \nu - \nu_1$, i. e. it has finite support. We refer to Pedersen [28] for the Lévy-Ito decomposition of i. d. i. s. r. m. s. The underlying driving Lévy process of Λ_1 with generating triplet $(0,0,\nu_1)$ has jumps with modulus larger than one, and the underlying driving Lévy process of Λ_2 with generating triplet (m,σ^2,ν_2) has jumps with modulus smaller than one. Furthermore, $\widetilde{N}_1$ has the representation

$$\widetilde{N}_1 = \sum_{k=-\infty}^{\infty} \varepsilon_{(R_k,\Gamma_k,Z_k)}, \tag{4.2}$$

where $-\infty < \ldots < \Gamma_{-1} < \Gamma_0 \le 0 < \Gamma_1 < \ldots < \infty$ are the jump times of a Poisson process $N = \{N(t)\}_{t\in\mathbb{R}}$ with intensity $\mu = \nu_1(\mathbb{R}) > 0$, $Z = \{Z_k\}_{k\in\mathbb{Z}}$ is an i. i. d. sequence with d. f. $\mathbb{P}(Z_1 \le x) = \nu_1(-\infty,x]/\mu$ for $x \in \mathbb{R}$ and $R = \{R_k\}_{k\in\mathbb{Z}}$ is an i. i. d. sequence with d. f. π. The processes N, Z and R are independent. It is also possible to choose a different decomposition in (4.1) by a Poisson random measure and an i. d. i. s. r. m., whose underlying driving Lévy process has bounded support in a neighborhood of the origin.

This decomposition of Λ induces a decomposition of $Y = Y_1 + Y_2$, where for $i = 1, 2$,

$$Y_i(t) = \int_{\mathbb{R}_+\times\mathbb{R}} f(r, t-s)\, d\Lambda_i(r,s) \quad \text{for } t \in \mathbb{R}, \tag{4.3}$$

are independent mixed MA processes. W. l. o. g. we assume Y_1 and Y_2 are stationary i. d. processes, else we choose Λ_i, $i = 1, 2$, properly. We shall see that the extremal behavior of a mixed MA process Y satisfying (M) is completely determined by the extremes of the *mixed Poisson shot noise process* Y_1 with representation

$$Y_1(t) = \sum_{k=-\infty}^{\infty} f(R_k, t - \Gamma_k) Z_k \quad \text{for } t \in \mathbb{R}. \tag{4.4}$$

First we give a short motivation for the choice of the discrete-time random sequence $\{t_n\}_{n \in \mathbb{N}}$. Suppose there exists an $\eta^{(1)} \in \mathbb{R}$ with $f(r, \eta^{(1)}) = f^+$ for every $r \in \operatorname{supp}(\pi)$. Consider the mixed Poisson shot noise process Y_1, then

$$Y_1(\Gamma_k + t) = f(R_k, t) Z_k + \sum_{\substack{j=-\infty \\ j \neq k}}^{\infty} f(R_j, t + \Gamma_k - \Gamma_j) Z_j \quad \text{for } k \in \mathbb{N},\, t \in \mathbb{R}.$$

In the case that the $\{Z_k\}_{k \in \mathbb{Z}}$ are regularly varying one of the Z_k is likely to be large in comparison to $\{Z_j\}_{j \in \mathbb{Z} \setminus \{k\}}$. Then $Y_1(\Gamma_k + t)$ behaves roughly like $f(R_k, t) Z_k$. The process $\{f(R_k, t) Z_k\}_{t \geq 0}$ achieves a maximum in $\eta^{(1)}$. Similar results hold for large negative jumps and a minimum of the kernel function $\eta^{(2)}$ with $f(r, \eta^{(2)}) = -f^-$ for every $r \in \operatorname{supp}(\pi)$. This suggests that $Y_1(t_n)$ with

$$t_n \in \{\Gamma_k + \eta^{(1)} : k \in \mathbb{N}\} \cup \{\Gamma_k + \eta^{(2)} : k \in \mathbb{N}\}$$

is a local extreme value of Y_1, if the absolute value of the jump of the underlying driving Lévy process is large.

For the rest of the paper we use the following assumptions and notations: let $t_1, \ldots, t_{d-1} \in \mathbb{R}$ for $d \in \mathbb{N}$ be fixed and $\eta^{(1)}$ as above, then we define for $t \in \mathbb{R}$,

$$\mathbf{f}(r, t) := (f(r, t + t_1), \ldots, f(r, t + t_{d-1}), f(r, t + \eta^{(1)})), \tag{4.5}$$

$$\mathbf{Y}(t) := (Y(t + t_1), \ldots, Y(t + t_{d-1}), Y(t + \eta^{(1)})). \tag{4.6}$$

The extremal behavior of Y is described by the multivariate point processes

$$\kappa_n = \sum_{k=1}^{\infty} \varepsilon_{(\Gamma_k/n, \mathbf{Y}(\Gamma_k)/a_n)} \quad \text{in } M_P(S) \text{ for } n \in \mathbb{N}. \tag{4.7}$$

Such point processes can be interpreted as *marked point processes* (Daley and Vere-Jones [10], Section 6.4). Let $Y_{k,i} = Y(\Gamma_k + t_i)$ for $i \in \{1, \ldots, d\}$ be the i^{th} coordinate of $\mathbf{Y}(\Gamma_k)$, where $t_d := \eta_1$. Marked point process means that we consider the point process behavior of $\sum_{k=1}^{\infty} \varepsilon_{\left(\Gamma_k/n, Y_{k,i}/a_n\right)}$ for some fixed $i \in \{1, \ldots, d\}$, and the remaining coordinates of $\mathbf{Y}(\Gamma_k)$ describe the behavior of the process, when an excess of $Y_{k,i}$

over a high threshold occurs. In our setting $(Y(\Gamma_k+t_1),\ldots,Y(\Gamma_k+t_{d-1}))/a_n$ are the marks, which describe the sample path behavior of the continuous-time process Y, if $Y(\Gamma_k+\eta^{(1)})$ exceeds a high level. They characterize clearly the location of extremes on high levels.

Throughout the rest of this paper the support of f in its second coordinate is $[a,b]$, $(-\infty,a]$, $[a,\infty)$, respectively for $a<b$. Furthermore, f is in the second coordinate continuous on its support and has one-sided limits at the boundaries. We work with the sequence $T=\{T_k\}_{k\in\mathbb{Z}}$, where

$$T_k := \Gamma_{k+1}-\Gamma_1 \quad \text{and} \quad T_{-k} := \Gamma_{-k}-\Gamma_0 \quad \text{for } k\in\mathbb{N}_0. \tag{4.8}$$

Hence $\{T_k-T_{k-1}\}_{k\in\mathbb{Z}\backslash\{0\}}$ is an i. i. d. sequence with $T_k-T_{k-1}\stackrel{d}{=}\Gamma_1$ and $T_0=0$.

Theorem 4.1 *Let Y be a mixed MA process as given in (1.1) satisfying (M) and the kernel function f satisfies $f(r,\eta^{(1)})=f^+\geq f^-$ for every $r\in\operatorname{supp}(\pi)$. Let $\sum_{k=1}^\infty \varepsilon_{(s_k,P_k)}$ be a $\mathrm{PRM}(\vartheta)$ with $\vartheta(dt\times dx)=dt\times\alpha x^{-\alpha-1}\mathbf{1}_{(0,\infty)}(x)\,dx$. Suppose $\{T^{(k)}\}_{k\in\mathbb{N}}$ are i. i. d. with $T^{(k)}=\{T_{k,j}\}_{j\in\mathbb{Z}}\stackrel{d}{=}T$, where T is given by (4.8), and $R=\{R_k\}_{k\in\mathbb{N}}$ is an i. i. d. sequence with d. f. π. Let $\chi=\{\chi_k\}_{k\in\mathbb{N}}$ be an i. i. d. sequence with $\mathbb{P}(\chi_k=1)=p$ and $\mathbb{P}(\chi_k=-1)=q$. Furthermore, suppose the random elements $\sum_{k=1}^\infty\varepsilon_{(s_k,P_k)}, \{T^{(k)}\}_{k\in\mathbb{N}}, R$ and χ are independent. Then for $T\to\infty$,*

$$\sum_{k=1}^\infty \varepsilon_{(\Gamma_k/T,\mathbf{Y}(\Gamma_k)/a_{\lfloor T\rfloor})} \stackrel{w}{\Longrightarrow} \sum_{k=1}^\infty\sum_{j=-\infty}^\infty \varepsilon_{(s_k,\mathbf{f}(R_k,T_{k,j})\chi_k P_k)} =:\kappa \quad \text{in } M_P(S).$$

In particular, for t with $f(r,t)\neq 0$ for $r\in\operatorname{supp}(\pi)$ and $T\to\infty$,

$$\sum_{k=1}^\infty \varepsilon_{(\Gamma_k/T,Y(\Gamma_k+t)/a_{\lfloor T\rfloor})} \stackrel{w}{\Longrightarrow} \sum_{k=1}^\infty\sum_{j=-\infty}^\infty \varepsilon_{(s_k,f(R_k,T_{k,j}+t)\chi_k P_k)} \quad \text{in } M_P([0,\infty)\times\overline{\mathbb{R}}\backslash\{0\}).$$

The assumption $f(r,\eta^{(1)})=f^+\geq f^-$ for every $r\in\operatorname{supp}(\pi)$ can be replaced by $f(r,\eta^{(2)})=-f^-\leq -f^+$ for every $r\in\operatorname{supp}(\pi)$.

Remark 4.2 *(a)* The properly chosen discrete-time points, where exceedances of the underlying driving Lévy process occur in combination with extremes of the kernel function, result in exceedances of the mixed MA process. These exceedances are carried on in time by the kernel function and result in the limiting process in clusters of exceedances. Furthermore, they reflect also local extremes of the process on high levels.

(b) Regularly varying d. f. s are a subclass of subexponential d. f. s. Subexponential models are typical models for situations, where extremely large values are likely to occur in comparison to the mean size of the data. Regularly varying d. f. s agree

with subexponential d. f. s in the maximum domain of attraction of the *Fréchet* distribution. Extremes of subexponential Lévy driven MA processes, which are in the maximum domain of attraction of the *Gumbel* distribution have been studied in Fasen [16]. In both classes of subexponential distributions the large jumps of the Lévy process affect the extremal behavior, which can be modelled by a properly chosen discrete-time skeleton. But in contrast to the Fréchet case as described in *(a)*, in the Gumbel case exceedances over high thresholds collapse into single points, which are described by the extremes of the kernel function, so that the marked point processes converge to a cluster Poisson random measure with constant cluster sizes.

(c) We obtain in Theorem 4.1 also information about local minima of Y, since the point process convergence is in $M_P([0,\infty)\times\overline{\mathbb{R}}^d\backslash\{\mathbf{0}\})$. The interpretation of small minima is analog to large maxima. They occur in clusters and are caused by large jumps of the underlying driving Lévy process.

(d) It should be possible to extend Theorem 4.1 to an infinite-dimensional setting, where we use as marks the stochastic processes $\{Y(\Gamma_k+t)\}_{t\in[0,m]}$ in $\mathbb{D}[0,m]$, $m>0$, instead of multi-dimensional random vectors $\mathbf{Y}(\Gamma_k)\in\mathbb{R}^d$ for $k\in\mathbb{N}$. The formulation of such results requires the definition of regular variation of stochastic processes with a. s. càdlàg sample paths as given in Hult and Lindskøg [20]. Moreover, since $\mathbb{D}$ is not locally compact, a special definition of convergence (convergence on bounded Borel sets) as given in Daley and Vere-Jones [10], Section A2.6 is needed.

Corollary 4.3 (Point process of exceedances) *Let Y be given as in Theorem 4.1 with $f^+\leq 1$. Suppose $\{\widetilde{s}_k\}_{k\in\mathbb{N}}$ are the jump times of a Poisson process with intensity $x^{-\alpha}$, $x>0$, independent of the i. i. d. sequence $\{\zeta_k\}_{k\in\mathbb{Z}}$ with d. f.*

$$\pi_k=\mathbb{P}(\zeta_1=k)=p\left[\mathbb{E}f_k^{(1)\,\alpha}-\mathbb{E}f_{k+1}^{(1)\,\alpha}\right]+(1-p)\left[\mathbb{E}f_k^{(2)\,\alpha}-\mathbb{E}f_{k+1}^{(2)\,\alpha}\right]\qquad\text{for } k\in\mathbb{N},$$

where $f_1^{(1)}>f_2^{(1)}>\ldots$ are the order statistics of $\{f^+(R_1,T_j+t)\}_{j\in\mathbb{Z}}$ and $f_1^{(2)}>f_2^{(2)}>\ldots$ are the order statistics of $\{f^-(R_1,T_j+t)\}_{j\in\mathbb{Z}}$. Then for $T\to\infty$,

$$\sum_{k=1}^{\infty}\varepsilon_{(\Gamma_k/T,Y(\Gamma_k+t)/a_{\lfloor T\rfloor})}(\cdot\times(x,\infty))\overset{w}{\Longrightarrow}\sum_{k=1}^{\infty}\zeta_k\varepsilon_{\widetilde{s}_k}.$$

In the case of a positive shot noise process with non-increasing kernel function and $t=\eta^{(1)}$, the last result represents the cluster intensities among local extremes of the process.

4.2 Normalizing constants of running maxima

With the results of the previous section we calculate the normalizing constants of running maxima. Already Lebedev [24] calculated the limit distribution of running maxima of subexponential positive shot noise processes restricting his attention to non-decreasing kernel functions with unbounded support. In our result the assumption of a positive process with non-increasing kernel function is not necessary.

Theorem 4.4 *Let Y be a mixed MA process as given in (1.1) with a. s. càdlàg sample paths, satisfying (M) and one of the following conditions:*

(a) Let f be a positive kernel function. Assume there exists a measurable function $\widetilde{f}: \mathbb{R}_+ \times \mathbb{R} \to \mathbb{R}_+$ with the following properties: $f(r,s) \le \widetilde{f}(r,s)$ for $(r,s) \in \mathbb{R}_+ \times \mathbb{R}$, where $\widetilde{f}(r,\cdot)$ is non-increasing on $\left[\eta^{(1)}, \infty\right)$ and $\widetilde{f}(r,\eta^{(1)}) = f^+$ for every $r \in \operatorname{supp}(\pi)$. Furthermore, let the support of $\widetilde{f}$ be contained in $\mathbb{R}_+ \times \left[\eta^{(1)}, \infty\right)$ and $\widetilde{f} \in \mathbb{L}^\delta(\pi)$ for some $\delta < \min\{1, \alpha\}$.

(b) Let Y be a Lévy driven MA process with $\int_{-\infty}^{\infty} \sup_{0 \le s \le 1} |f(s+t)|^\delta \, dt < \infty$ for some $\delta < \min\{1,\alpha\}$.

Define $M(T) := \sup_{t\in[0,T]} Y(t)$ for $T > 0$. Then

$$\lim_{T\to\infty} \mathbb{P}\left(a_{\lfloor T\rfloor}^{-1} M(T) \le x\right) = \exp\left(-x^{-\alpha}\left[p{f^+}^{\alpha} + q{f^-}^{\alpha}\right]\right) \quad \text{for } x > 0. \tag{4.9}$$

Notice, that by the integrability assumption on f this result rules out MA processes, which exhibits long range dependence, only mixed MA processes with long range dependence are included. In this case the long range dependence is caused by the distribution of π and not by the asymptotically behavior of the kernel function for fixed r.

Theorem 4.4 requires $pf^+ + qf^- > 0$ by condition (M) (otherwise the limit in (4.9) is 1). More about the extremal behavior of totally skewed α-stable MA processes, which satisfy $pf^+ + qf^- = 0$ (hence the right tail is not regularly varying) can be found in Albin [1].

Remark 4.5 *(a)* The results of this paper can be extended to mixed MA processes driven by a multivariate i. d. i. s. r. m. Λ in $\mathbb{R}_+^m \times \mathbb{R}$, whose stationary distribution has the cumulant generating function $\psi_A(u) = \lambda(A)\psi(u)$, where ψ is the cumulant generating function of a Lévy process and $\lambda(d\omega) = \pi_1(dr_1) \times \cdots \times \pi_m(dr_m) \times dt$ for $\omega = (r_1, \ldots, r_m, t) \in \mathbb{R}_+^m \times \mathbb{R}$ and π_i, $i = 1, \ldots, m$, are probability measures on $\mathbb{R}_+$.

(b) In particular the results hold for stationary renewal shot noise processes. This processes have the structure of a Poisson shot noise process, but the jump times of the Poisson process are replaced by the jump times of a stationary renewal shot noise process; for more details we refer to Fasen [15]. The extremal behavior of heavy tailed renewal shot noise processes, where $f : [0,1] \to [0,1]$ is strictly decreasing and the jump sizes are positive, have been thoroughly investigated by McCormick [26]. □

4.3 Examples

Example 4.6 (Discrete-time MA process) Let $\xi = \{\xi_k\}_{k\in\mathbb{Z}}$ be an i. i. d. sequence of r. v. s, which are regularly varying in the sense of (2.2) with measure σ given by (1.6), and let $\{c_k\}_{k\in\mathbb{Z}}$ be a sequence in $\mathbb{R}$. Define the discrete-time MA process $Y_n = \sum_{k=-\infty}^{\infty} c_{n-k}\xi_k$ for $n \in \mathbb{Z}$. Suppose $\sum_{k=-\infty}^{\infty} |c_k|^\delta < \infty$ for $\delta < \alpha$, $\delta \leq 2$, with either $\delta < 1$, or $\alpha > 1$ and $\mathbb{E}\xi_k = 0$. This class includes MA processes with the long memory property. By Mikosch and Samorodnitsky [27], Lemma A.3, Y is a stationary process, whose one-dimensional marginal distribution is regularly varying with

$$\lim_{n\to\infty} n\mathbb{P}(Y_k > a_n x) = x^{-\alpha}\left[p\sum_{k=-\infty}^{\infty} c_k^{+\,\alpha} + q\sum_{k=-\infty}^{\infty} c_k^{-\,\alpha}\right].$$

As in the proof of Theorem 4.1 we have for $i_1, \ldots, i_d \in \mathbb{N}$ and $n \to \infty$,

$$\sum_{k=1}^{\infty} \varepsilon_{(k/n, a_n^{-1}(Y_k, Y_{k+i_1}, \ldots, Y_{k+i_d}))} \stackrel{w}{\Longrightarrow} \sum_{k=1}^{\infty}\sum_{j=-\infty}^{\infty} \varepsilon_{(s_k, (c_j, c_{j-i_1}, \ldots, c_{j-i_d})P_k)}. \tag{4.10}$$

This is a supplement of the well known result of Davis and Resnick [13], Theorem 2.4, (Rootzén [31] for stable MA processes) in the case of long memory processes. □

Example 4.7 (Continuation of Example 3.4) We investigate the extremal behavior of the supOU process Y given by (3.4) driven by an i. d. i. s. r. m. Λ with generating quadruple (m, σ^2, ν, π), where $\int_{\mathbb{R}_+} r^{-1}\,\pi(dr) < \infty$ and (m, σ^2, ν) is the generating triplet of the underlying driving Lévy process L as given in (1.3), which satisfies (L) with $p > 0$. Let $0 = t_1 < \cdots < t_d$. Then by Theorem 4.1,

$$\sum_{k=1}^{\infty} \varepsilon_{(\Gamma_k/T, \{Y(\Gamma_k + t_i)/a_{\lfloor T\rfloor}\}_{i=1,\ldots,d})} \stackrel{w}{\Longrightarrow} \sum_{k=1}^{\infty}\sum_{j=0}^{\infty} \varepsilon_{(s_k, \{\exp(-R_k(T_{k,j}+t_i))\chi_k P_k\}_{i=1,\ldots,d})}$$

holds for $T \to \infty$. If Y has an exceedance over a high level at the discrete-time skeleton $\{\Gamma_k + t_i : k \in \mathbb{N}, i = 1, \ldots, d\}$, then we have an extreme at $Y(\Gamma_k)$ for some $k \in \mathbb{N}$. Furthermore, if Y has a. s. sample paths in $\mathbb{D}(\mathbb{R})$, e. g. if $\int_{-\infty}^{\infty} 1 \wedge |x|\,\nu(dx) < \infty$, then by Theorem 4.4 the running maxima are in the maximum domain of attraction of the Fréchet distribution with

$$\lim_{T\to\infty} \mathbb{P}(a_{\lfloor T\rfloor}^{-1} M(T) \leq x) = \exp(-px^{-\alpha}) \quad \text{for } x > 0.$$ □

5 Proofs

Proof of Lemma 2.4. Let $\epsilon > 0$. Denote by $\zeta_n := \sum_{k=1}^{\infty} \varepsilon_{\left(k/n, a_n^{-1}(\mathbf{Z}_k + \mathbf{\Psi}_k)\right)}$ a point process in $M_P(S)$ for $n \in \mathbb{N}$. Define the sets

$$I_\epsilon^{(1)} = \prod_{i=1}^{d} (c_i - \epsilon, d_i + \epsilon], \quad I_\epsilon^{(2)} = \prod_{i=1}^{d} (c_i + \epsilon, d_i - \epsilon] \quad \text{and} \quad I_\epsilon = I_\epsilon^{(1)} \backslash I_\epsilon^{(2)}.$$

We obtain

$$\begin{aligned}\{\widetilde{\kappa}_n(I) \neq \zeta_n(I)\} \quad \subseteq \quad & \{\widetilde{\kappa}_n(I_\epsilon) > 0\} \cup \bigcup_{k \in (ns, nt]} \left\{a_n^{-1}(\mathbf{Z}_k + \mathbf{\Psi}_k) \in I, a_n^{-1}\mathbf{Z}_k \in I_\epsilon^{(1)\,c}\right\} \cup \\ & \bigcup_{k \in (ns, nt]} \left\{a_n^{-1}(\mathbf{Z}_k + \mathbf{\Psi}_k) \in I^c, a_n^{-1}\mathbf{Z}_k \in I_\epsilon^{(2)}\right\}. \end{aligned} \tag{5.1}$$

On the one hand,

$$\sum_{k \in (ns, nt]} \mathbb{P}\left(a_n^{-1}(\mathbf{Z}_k + \mathbf{\Psi}_k) \in I, a_n^{-1}\mathbf{Z}_k \in I_\epsilon^{(1)\,c}\right) \leq \sum_{k \in (ns, nt]} \mathbb{P}(|\mathbf{\Psi}_k| > a_n \epsilon) \stackrel{n \to \infty}{\longrightarrow} 0, \tag{5.2}$$

$$\sum_{k \in (ns, nt]} \mathbb{P}\left(a_n^{-1}(\mathbf{Z}_k + \mathbf{\Psi}_k) \in I^c, a_n^{-1}\mathbf{Z}_k \in I_\epsilon^{(2)}\right) \leq \sum_{k \in (ns, nt]} \mathbb{P}(|\mathbf{\Psi}_k| > a_n \epsilon) \stackrel{n \to \infty}{\longrightarrow} 0, \tag{5.3}$$

and on the other hand,

$$\lim_{\epsilon \downarrow 0} \lim_{n \to \infty} \mathbb{P}(\widetilde{\kappa}_n(I_\epsilon) > 0) = \lim_{\epsilon \downarrow 0} \mathbb{P}(\kappa(I_\epsilon) > 0) = 0. \tag{5.4}$$

Thus, by (5.1)-(5.4) we get $\lim_{n \to \infty} \mathbb{P}(\widetilde{\kappa}_n(I) \neq \zeta_n(I)) = 0$. Applying Rootzén [32], Lemma 3.3, we conclude $\zeta_n \stackrel{w}{\Longrightarrow} \kappa$ as $n \to \infty$. A modification of an argument of Hsing and Teugels [19] (the proofs of their Theorem 4.2 and Lemma 2.1; see also Fasen [15], Lemma 1.2.4) yields $\lim_{T \to \infty} \mathbb{P}(\zeta_{\lfloor T \rfloor}(I) \neq \kappa_T(I)) = 0$. Then

$$\lim_{T \to \infty} \mathbb{P}(\kappa_T(I) \neq \widetilde{\kappa}_{\lfloor T \rfloor}(I)) \leq \lim_{T \to \infty} \mathbb{P}(\kappa_T(I) \neq \zeta_{\lfloor T \rfloor}(I)) + \lim_{T \to \infty} \mathbb{P}(\zeta_{\lfloor T \rfloor}(I) \neq \widetilde{\kappa}_{\lfloor T \rfloor}(I)) = 0.$$

□

Proof of Proposition 3.3. By Rajput and Rosinski [29], Theorem 2.7, the Lévy measure of Y is

$$\nu_Y(x, \infty) = \int_{f(r,s) > 0} \nu\left(\frac{x}{f(r,s)}, \infty\right) ds\, \pi(dr) + \int_{f(r,s) < 0} \nu\left(-\infty, \frac{x}{f(r,s)}\right) ds\, \pi(dr) \tag{5.5}$$

for $x > 0$. By Potter's Theorem (Bingham et al. [6], Theorem 1.5.6) there exists for every $x > 0$, $K > 1$ an $n_0(x) \in \mathbb{N}$ such that $\nu(a_n xy, \infty)/\nu(a_n x, \infty) \leq K y^{-\delta}$

for $y \geq 1, n \geq n_0$. Taking $f \in \mathbb{L}^\delta(\pi)$ into account, dominated convergence and the boundedness of f yields for $n \to \infty$,

$$\lim_{x\to\infty} \frac{\nu_Y(x,\infty)}{\nu(x,\infty)} = \left[\int_{\mathbb{R}_+}\int_{\mathbb{R}} p(f^+(r,s))^\alpha + q(f^-(r,s))^\alpha\, ds\, \pi(dr)\right].$$

The result (3.1) follows then by the tail-equivalence of Lévy measure and probability measure for regularly varying d. f. s. An application of Rosinski and Samorodnitsky [33], Theorem 3.1 (cf. Theorem 4.9 in Fasen [16]), and similar arguments as above yields (3.2) and (3.3). □

Proof of Theorem 4.1. *Step 1.* We study the extremal behavior of

$$\widetilde{\mathbf{Y}}_k^{(m)} = \sum_{j=k-m}^{k+m} \mathbf{f}(R_j, T_k - T_j) Z_j \quad \text{for } k \in \mathbb{Z}$$

and for $m > 0$ fixed. We show that $\{\widetilde{\mathbf{Y}}_k^{(m)}\}_{k\in\mathbb{Z}}$ satisfies the assumptions of Davis and Mikosch [12], Theorem 2.8 and Corollary 2.4.

We apply Lemma 2.2 to obtain

$$\lim_{n\to\infty} n\mathbb{P}(|\widetilde{\mathbf{Y}}_k^{(m)}| > a_n) = \frac{1}{\mu}\sum_{j=-m}^{m} \mathbb{E}|\mathbf{f}(R_1, T_j)|^\alpha =: \rho_m. \tag{5.6}$$

Observing that $\{\widetilde{\mathbf{Y}}_k^{(m)}\}_{k\in\mathbb{Z}}$ is $(2m+1)$-dependent and taking Lemma 2.4.2 in Leadbetter and Rootzén [23] into account, the mixing condition $\mathcal{A}(a_n\rho_m^{1/\alpha})$ (cf. Davis and Mikosch [12], p. 2052) holds for $\{\widetilde{\mathbf{Y}}_k^{(m)}\}_{k\in\mathbb{Z}}$, i. e. there exists a set of positive integers $\{r_n\}_{n\in\mathbb{N}}$ such that $r_n \to \infty$, $r_n/n \to 0$ as $n \to \infty$ and

$$\mathbb{E}\exp\left(-\sum_{j=1}^{n} f(\widetilde{\mathbf{Y}}_j^{(m)}/a_n)\right) - \left[\mathbb{E}\exp\left(-\sum_{j=1}^{r_n} f(\widetilde{\mathbf{Y}}_j^{(m)}/a_n)\right)\right]^{\lfloor n/r_n\rfloor} \stackrel{n\to\infty}{\longrightarrow} 0$$

for every bounded non negative step function f on $\overline{\mathbb{R}}^d\backslash\{\mathbf{0}\}$ with bounded support.

Also by the $(2m+1)$-dependence of $\{\widetilde{\mathbf{Y}}_k^{(m)}\}_{k\in\mathbb{Z}}$, (5.6) and $r_n = o(n)$ for $n \to \infty$, we obtain for $l > 2m+1$,

$$\mathbb{P}\left(\bigvee_{l\leq|k|\leq r_n} |\widetilde{\mathbf{Y}}_k^{(m)}| > a_n x \,\middle|\, |\widetilde{\mathbf{Y}}_0^{(m)}| > a_n x\right) \leq r_n\mathbb{P}(|\widetilde{\mathbf{Y}}_k^{(m)}| > a_n x) \stackrel{n\to\infty}{\longrightarrow} 0. \tag{5.7}$$

Define the random vectors $\mathbf{Z}^{(l)} := (Z_{-l-m}, \dots, Z_{l+m}) \in \mathbb{R}^{2(l+m)+1}$, $l \in \mathbb{N}$, and the random matrices

$$\mathbf{A}^{(l)} := \left(\ \mathbf{A}_{-l}^{(l)}, \quad \dots \quad, \mathbf{A}_{l}^{(l)}\ \right)^t \in \mathbb{R}^{(2l+1)d\times(2(l+m)+1)},$$

where $\mathbf{A}_k^{(l)} \in \mathbb{R}^{d\times(2(l+m)+1)}$ for $k=-l,\dots,l$, has entries $(\mathbf{A}_k^{(l)})_{i,j}$ in the i^{th} row and j^{th} column with value $(\mathbf{A}_k^{(l)})_{i,j} = f(R_j, T_k - T_j + t_i)$ for $j = k-m,\dots,k+m$, $i=1,\dots,d$, $k=-l,\dots,l$, and $t_d := \eta^{(1)}$. Furthermore, $(\mathbf{A}_k^{(l)})_{i,j} = 0$ for $|k-j|>m$, $j=-l-m,\dots,l+m$, $i=1,\dots,d$, and $k=-l,\dots,l$. Thus we have

$$(\widetilde{\mathbf{Y}}_{-l}^{(m)},\dots,\widetilde{\mathbf{Y}}_{l}^{(m)})^t = \mathbf{A}^{(l)}\mathbf{Z}^{(l)} \in \mathbb{R}^{(2l+1)d}.$$

The matrix $\mathbf{A}^{(l)}$ has at most $(2m+1)$ entries in a row and $d(2m+1)$ in a column and the sequence of random matrices $(\mathbf{A}_k^{(l)})_{k=-l,\dots,l}$ is $(2m+1)$-dependent.

Since $f^+ \le \|\mathbf{A}^{(l)}\| \le (2m+1)f^+$ we can apply Lemma 2.2 and conclude that $(\widetilde{\mathbf{Y}}_{-l}^{(m)},\dots,\widetilde{\mathbf{Y}}_{l}^{(m)})$ is multivariate regularly varying of index α with spectral measure

$$\begin{aligned}&\mathbb{P}(\Theta^{(l)}\in\cdot) \qquad (5.8)\\ &\quad= \sum_{j=-l-m}^{l+m} \frac{p}{\mu\widetilde{\rho}_m}\mathbb{E}\left(|\mathbf{a}_j^{(l)}|^\alpha \mathbf{1}_{\left\{\mathbf{a}_j^{(l)}/|\mathbf{a}_j^{(l)}|\in\cdot\right\}}\right) + \frac{q}{\mu\widetilde{\rho}_m}\mathbb{E}\left(|\mathbf{a}_j^{(l)}|^\alpha \mathbf{1}_{\left\{-\mathbf{a}_j^{(l)}/|\mathbf{a}_j^{(l)}|\in\cdot\right\}}\right),\end{aligned}$$

where $\mathbf{a}_j^{(l)} = \mathbf{A}^{(l)}\mathbf{e}_j$ with the j^{th} unit vector $\mathbf{e}_j = (0,\dots,0,1,0,\dots,0)^t \in \mathbb{R}^{2(l+m)+1}$ and $\widetilde{\rho}_m := \mu^{-1}\sum_{j=-l-m}^{l+m}\mathbb{E}|\mathbf{a}_j^{(l)}|^\alpha$. Therefore by $\mathcal{A}(a_n\rho_m^{1/\alpha})$, (5.7), (5.8) and Davis and Mikosch [12], Theorem 2.8, the point processes $\sum_{k=1}^n \varepsilon_{(\widetilde{\mathbf{Y}}_k^{(m)}/a_n)}$ converge for $n\to\infty$ weakly to a point process. In the following we shall derive the explicit representation of the limit. To obtain the limit distribution we compute θ_m and $\mathcal{Q}$, where

$$\theta_m := \lim_{l\to\infty}\mathbb{E}\left(|\Theta_0^{(l)}|^\alpha - \bigvee_{j=1}^{l}|\Theta_j^{(l)}|^\alpha\right)^+ \Bigg/ \mathbb{E}|\Theta_0^{(l)}|^\alpha, \qquad (5.9)$$

$$\frac{\mathbb{E}\left(\left[|\Theta_0^{(l)}|^\alpha - \bigvee_{j=1}^{l}|\Theta_j^{(l)}|^\alpha\right]^+ \mathbf{1}\left\{\sum_{|j|\le l}\varepsilon_{(\Theta_j^{(l)})}\in\cdot\right\}\right)}{\mathbb{E}\left(|\Theta_0^{(l)}|^\alpha - \bigvee_{j=1}^{l}|\Theta_j^{(l)}|^\alpha\right)^+} \overset{w}{\Longrightarrow} \mathcal{Q}(\cdot) \qquad (5.10)$$

for $l\to\infty$, so that we can apply Corollary 2.4 of Davis and Mikosch [12].

First we derive (5.9). We consider $l > 2m+1$. For $j=-m,\dots,m$ we have

$$\bigvee_{k=0}^{l}\bigvee_{i=1}^{d}|(\mathbf{A}_k^{(l)})_{i,j}|^\alpha - \bigvee_{k=1}^{l}\bigvee_{i=1}^{d}|(\mathbf{A}_k^{(l)})_{i,j}|^\alpha = \bigvee_{k=0}^{j+m}|\mathbf{f}(T_k-T_j)|^\alpha - \bigvee_{k=1}^{j+m}|\mathbf{f}(T_k-T_j)|^\alpha, \quad (5.11)$$

and for $m<|j|\le l+m$ we have

$$\bigvee_{k=0}^{l}\bigvee_{i=1}^{d}|(\mathbf{A}_k^{(l)})_{i,j}|^\alpha - \bigvee_{k=1}^{l}\bigvee_{i=1}^{d}|(\mathbf{A}_k^{(l)})_{i,j}|^\alpha = 0. \qquad (5.12)$$

Furthermore, for $j=-m,\ldots,m$,

$$\bigvee_{k=-l}^{l}\bigvee_{i=1}^{d}|(\mathbf{A}_k^{(l)})_{i,j}|^\alpha=|\mathbf{a}_j^{(l)}|^\alpha=\bigvee_{k=j-m}^{j+m}|\mathbf{f}(T_k-T_j)|^\alpha=f^{+\alpha}. \tag{5.13}$$

By taking the conditional probability under Γ_k, R_k, $k=-l-m,\ldots,l+m$ and Remark 2.3, we can calculate with deterministic variables. We apply (5.8) to compute

$$\begin{aligned}&\mathbb{E}\left(\bigvee_{k=0}^{l}|\Theta_k^{(l)}|^\alpha-\bigvee_{k=1}^{l}|\Theta_k^{(l)}|^\alpha\right)\\&\quad=\frac{1}{\mu\widetilde{\rho}_m}\sum_{j=-l-m}^{l+m}\mathbb{E}\left(|\mathbf{a}_j^{(l)}|^\alpha\left[\bigvee_{k=0}^{l}\bigvee_{i=1}^{d}\frac{|(\mathbf{A}_k^{(l)})_{i,j}|^\alpha}{|\mathbf{a}_j^{(l)}|^\alpha}-\bigvee_{k=1}^{l}\bigvee_{i=1}^{d}\frac{|(\mathbf{A}_k^{(l)})_{i,j}|^\alpha}{|\mathbf{a}_j^{(l)}|^\alpha}\right]\right).\end{aligned}$$

Taking (5.11)-(5.12) into account we receive that the right hand side is equal to

$$\begin{aligned}&\frac{1}{\mu\widetilde{\rho}_m}\left[\sum_{j=-m}^{m}\mathbb{E}\left(\bigvee_{k=0}^{j+m}|\mathbf{f}(R_j,T_k-T_j)|^\alpha\right)-\sum_{j=-m+1}^{m}\mathbb{E}\left(\bigvee_{k=1}^{j+m}|\mathbf{f}(R_j,T_k-T_j)|^\alpha\right)\right]\\&\quad=\frac{1}{\mu\widetilde{\rho}_m}\mathbb{E}\left(\bigvee_{k=-m}^{m}|\mathbf{f}(R_1,T_k)|^\alpha\right)=\frac{f^{+\alpha}}{\mu\widetilde{\rho}_m}.\end{aligned}\tag{5.14}$$

Similarly, as $\bigvee_{i=1}^{d}|(\mathbf{A}_0^{(l)})_{i,j}|^\alpha=0$ for $|j|>m$, we have by (5.6),

$$\begin{aligned}\mathbb{E}|\Theta_0^{(l)}|^\alpha&=\frac{1}{\mu\widetilde{\rho}_m}\mathbb{E}\left(\sum_{j=-l-m}^{l+m}|\mathbf{a}_j^{(l)}|^\alpha\bigvee_{i=1}^{d}\frac{|(\mathbf{A}_0^{(l)})_{i,j}|^\alpha}{|\mathbf{a}_j^{(l)}|^\alpha}\right)=\frac{1}{\mu\widetilde{\rho}_m}\mathbb{E}\left(\sum_{j=-m}^{m}|\mathbf{f}(R_1,T_j)|^\alpha\right)\\&=\frac{1}{\mu\widetilde{\rho}_m}\mu\rho_m=\frac{\rho_m}{\widetilde{\rho}_m}.\end{aligned}\tag{5.15}$$

Applying (5.14)-(5.15) we obtain for the extremal index of $\{|\widetilde{\mathbf{Y}}_k^{(m)}|\}_{k\in\mathbb{Z}}$ in (5.9),

$$\theta_m=f^{+\alpha}/(\mu\rho_m).\tag{5.16}$$

We shall compute $\mathcal{Q}$ of (5.10). Following the proof of (5.14) and taking $l>2m+1$ and (5.13) into account we get for $j=-m,\ldots,m$,

$$\begin{aligned}&\mathbb{E}\left(|\mathbf{a}_j^{(l)}|^\alpha\left[\bigvee_{k=0}^{l}\bigvee_{i=1}^{d}\frac{|(\mathbf{A}_k^{(l)})_{i,j}|^\alpha}{|\mathbf{a}_j^{(l)}|^\alpha}\mathbf{1}\left\{\sum_{|k|\le l}\varepsilon_{((\mathbf{A}_k^{(l)})_{i,j}/|\mathbf{a}_j^{(l)}|)_{i=1,\ldots,d}}\in\cdot\right\}\right]\right)\\&\quad=\mathbb{E}\left(\bigvee_{k=-j}^{m}|\mathbf{f}(R_1,T_k)|^\alpha\,\mathbf{1}\left\{\sum_{|k|\le m}\varepsilon_{(\mathbf{f}(R_1,T_k)/f^+)}\in\cdot\right\}\right).\end{aligned}$$

Then analog the lines of (5.14) we obtain

$$\begin{aligned}
&\mathbb{E}\left(\left[|\Theta_0^{(l)}|^\alpha - \bigvee_{j=1}^{l} |\Theta_j^{(l)}|^\alpha\right]^+ \mathbf{1}\left\{\sum_{|j|\le l} \varepsilon_{\Theta_j^{(l)}} \in \cdot\right\}\right)\\
&= \frac{f^{+\,\alpha}}{\widetilde{\rho}_m}\left[\frac{p}{\mu}\mathbb{E}\left(\mathbf{1}\left\{\sum_{|j|\le m} \varepsilon_{(\mathbf{f}(R_1,T_j)/f^+)} \in \cdot\right\}\right) + \frac{q}{\mu}\mathbb{E}\left(\mathbf{1}\left\{\sum_{|j|\le m} \varepsilon_{(-\mathbf{f}(R_1,T_j)/f^+)} \in \cdot\right\}\right)\right]\\
&= \frac{f^{+\,\alpha}}{\mu\widetilde{\rho}_m}\mathbb{E}\left(\mathbf{1}\left\{\sum_{|j|<m} \varepsilon_{(\mathbf{f}(R_1,T_j)\chi_1/f^+)} \in \cdot\right\}\right). \qquad (5.17)
\end{aligned}$$

Hence by (5.14) and (5.17) the measure $\mathcal{Q}$ of (5.10) is defined by

$$\mathcal{Q}(\cdot) := \mathbb{P}\left(\sum_{j=-m}^{m} \varepsilon_{(\mathbf{f}(R_1,T_j)\chi_1/f^+)} \in \cdot\right).$$

Regarding (5.16) we choose

$$\widetilde{\vartheta}(dx) := \theta_m \alpha x^{-\alpha-1}\,\mathbf{1}_{(0,\infty)}(x)\,dx = \alpha f^{+\,\alpha}/(\mu\rho_m)x^{-\alpha-1}\,\mathbf{1}_{(0,\infty)}(x)\,dx.$$

Taking (5.6) into account, we apply Davis and Mikosch [12], Theorem 2.8 and Corollary 2.4, and obtain for $n \to \infty$,

$$\sum_{k=1}^{n} \varepsilon_{\left(\rho_m^{1/\alpha}\widetilde{\mathbf{Y}}_k^{(m)}/a_n\right)} \overset{w}{\Longrightarrow} \sum_{k=1}^{\infty}\sum_{j=-m}^{m} \varepsilon_{(\mathbf{f}(R_k,T_{k,j})\chi_k\widetilde{P}_k/f^+)},$$

where $\sum_{k=1}^{\infty} \varepsilon_{\widetilde{P}_k}$ is a PRM($\widetilde{\vartheta}$) in $M_P(\overline{\mathbb{R}}^d\backslash\{\mathbf{0}\})$. By Hsing [18], Lemma 4.1.2, the convergence of the sequence of point processes $\kappa_n((0,1]\times\cdot)$ is equivalent to the convergence of κ_n, if the so called $\Delta(a_n)$ condition is satisfied, which is similar to condition $\mathcal{A}(a_n)$. Note, that by the $(2m+1)$-dependence of $\{\widetilde{\mathbf{Y}}_k^{(m)}\}_{k\in\mathbb{Z}}$ the $\Delta(a_n)$ condition holds. This implies, replacing $\widetilde{\vartheta}$ by ϑ, and $\{\widetilde{P}_k\}_{k\in\mathbb{N}}$ by $\{P_k\}_{k\in\mathbb{N}}$, respectively, that for $n\to\infty$,

$$\sum_{k=1}^{\infty} \varepsilon_{\left(k/(n\mu),\widetilde{\mathbf{Y}}_k^{(m)}/a_n\right)} \overset{w}{\Longrightarrow} \sum_{k=1}^{\infty}\sum_{j=-m}^{m} \varepsilon_{(s_k,(\mathbf{f}(R_k,T_{k,j})\chi_k)P_k)}. \qquad (5.18)$$

Step 2. For fixed $m>0$ we study the extremal behavior of

$$\mathbf{Y}_k^{(m)} = \sum_{j=k-m}^{k+m} \mathbf{f}(R_j,\Gamma_k-\Gamma_j)Z_j \quad \text{for } k\in\mathbb{Z}.$$

Note, that $\mathbf{f}(R_j, T_k - T_j) = \mathbf{f}(R_j, \Gamma_{k+1} - \Gamma_{j+1})$ for $k, j \in \mathbb{N}_0$ by (4.8). Then also $\{\widetilde{\mathbf{Y}}_k^{(m)}\}_{k\geq m} \stackrel{d}{=} \{\mathbf{Y}_{k+1}^{(m)}\}_{k\geq m}$, although $\{\mathbf{Y}_{k+1}^{(m)}\}_{k\in\mathbb{Z}}$ is not stationary. Thus, the asymptotic point process behavior of $\{\widetilde{\mathbf{Y}}_k^{(m)}\}_{k\in\mathbb{N}}$ and $\{\mathbf{Y}_k^{(m)}\}_{k\in\mathbb{N}}$ is the same. Regarding (5.18) we obtain for $n \to \infty$,

$$\sum_{k=1}^{\infty} \varepsilon_{\left(k/(n\mu), \mathbf{Y}_k^{(m)}/a_n\right)} \stackrel{w}{\Longrightarrow} \sum_{k=1}^{\infty} \sum_{j=-m}^{m} \varepsilon_{(s_k, \mathbf{f}(R_k, T_{k,j})\chi_k P_k)}. \tag{5.19}$$

Step 3. We study the extremal behavior of $\{\mathbf{Y}_k\}_{k\in\mathbb{Z}}$ given by

$$\mathbf{Y}_k = \sum_{j=-\infty}^{\infty} \mathbf{f}(R_j, \Gamma_k - \Gamma_j) Z_j.$$

We have to attend to the non-stationarity of the sequences $\{\mathbf{Y}_k\}_{k\in\mathbb{Z}}$, $\{\mathbf{Y}_k^{(m)}\}_{k\in\mathbb{Z}}$ and $\{\mathbf{Y}_k - \mathbf{Y}_k^{(m)}\}_{k\in\mathbb{Z}}$. With (4.8) we have

$$\begin{aligned}\mathbb{P}\left(\bigvee_{k=1}^{n} |\mathbf{Y}_k - \mathbf{Y}_k^{(m)}| > a_n x\right) &\leq \sum_{i=1}^{d} \sum_{k=1}^{n} \mathbb{P}\left(\left|\sum_{|k-j|>m} f(R_j, \Gamma_k - \Gamma_j + t_i) Z_j\right| > a_n x\right) \\ \leq \sum_{i=1}^{d} n\mathbb{P}\left(\left|\sum_{|j|\geq m} f(R_j, \Gamma_j + t_i) Z_j\right| > a_n x/2\right) &+ \sum_{k=1}^{n} \mathbb{P}(|f(R_1, \Gamma_k + t_i) Z_1| > a_n x/2).\end{aligned} \tag{5.20}$$

Since $\sum_{k=1}^{\infty} \mathbb{E}|f(R_1, \Gamma_k + t_i)|^\delta = \mu \int_{\mathbb{R}_+} \int_{\mathbb{R}_+} |f(r,s)|^\delta \, ds \, \pi(dr) < \infty$ we have by Example 2.5 *(b)* that the last term of (5.20) tends to 0. For the first term of (5.20) we have by a simple generalization of Proposition 3.3,

$$n\mathbb{P}\left(\left|\sum_{|j|\geq m} f(R_j, \Gamma_j + t_i) Z_j\right| > a_n x/2\right) \stackrel{n\to\infty}{\longrightarrow} \frac{(x/2)^{-\alpha}}{\mu} \sum_{|j|\geq m} \mathbb{E}|f(R_j, \Gamma_j + t_i)|^\alpha \stackrel{m\to\infty}{\longrightarrow} 0.$$

Thus, $\lim_{m\to\infty} \lim_{n\to\infty} \mathbb{P}\left(\bigvee_{k=1}^{n} |\mathbf{Y}_k - \mathbf{Y}_k^{(m)}| > a_n x\right) = 0$. Then following the proof of Resnick [30], Proposition 4.2.7, along the lines, and regarding Step 2, gives for $n \to \infty$,

$$\sum_{k=1}^{\infty} \varepsilon_{(k/(n\mu), \mathbf{Y}_k/a_n)} \stackrel{w}{\Longrightarrow} \kappa. \tag{5.21}$$

Step 4. The point process behavior of κ_n.
We need to invoke the decomposition (4.3) in $\mathbf{Y}(t) = \mathbf{Y}_1(t) + \mathbf{Y}_2(t)$ for $t \in \mathbb{R}$. Then

$$\mathbf{Y}(\Gamma_k) = \mathbf{Y}_k + \mathbf{Y}_2(\Gamma_k) \quad \text{for } k \in \mathbb{Z}. \tag{5.22}$$

Similarly as (5.20) we have

$$\begin{aligned}\mathbb{P}(|\mathbf{Y}(\Gamma_k)| > a_n x) \;\; &\leq \;\; \sum_{i=1}^{d} \mathbb{P}\left(\left|\sum_{\substack{j=-\infty\\ j\neq k}}^{\infty} f(R_j, T_j + t_i) Z_j\right| > a_n x/2\right) \\ &\quad + \mathbb{P}(|\mathbf{Y}_2(\Gamma_k)| > a_n x/2). \end{aligned} \tag{5.23}$$

On the one hand, by Proposition 3.3,

$$\begin{aligned}&\mathbb{P}\left(\left|\sum_{\substack{j=-\infty\\ j\neq k}}^{\infty} f(R_j, T_j + t_i) Z_j\right| > a_n x/2\right) \qquad (5.24)\\ &\leq \mathbb{P}\left(\left|\sum_{j=-\infty}^{\infty} f(R_j, \Gamma_j + t_i) Z_j\right| > a_n x/6\right) + 2\mathbb{P}\left(f^{+}|Z_1| > a_n x/6\right) = O(1/n)\end{aligned}$$

for $n \to \infty$. On the other hand, the Lévy measure of Y_2 has bounded support, so that by Sato [34], Theorem 26.1, and $a_n \in \mathcal{R}_{1/\alpha}$, we have

$$\mathbb{P}(|\mathbf{Y}_2(\Gamma_k)| > a_n x/2) \leq d\mathbb{P}(|Y_2(0)| > a_n x/2) = o(1/n) \quad \text{ for } n \to \infty. \tag{5.25}$$

Regarding (5.23)-(5.25) there exists a r.v. W such that

$$\mathbb{P}(|\mathbf{Y}(\Gamma_k)| > a_n x) \leq \mathbb{P}(W > a_n x) = O(1/n) \quad \text{ for } n \to \infty.$$

Thus, by (5.22), (5.25), Lemma 2.4 and Example 2.5 *(a)* the point process behavior of the sequence $\{\mathbf{Y}(\Gamma_k)\}_{k\in\mathbb{Z}}$ is the same as that of $\{\mathbf{Y}_k\}_{k\in\mathbb{Z}}$. Furthermore, we can shift the time scale. This completes the proof. □

Proof of Theorem 4.4. *(a)* Define the disjoint intervals

$$I_k = \left[\eta^{(1)} + \Gamma_k, \eta^{(1)} + \Gamma_{k+1}\right) \quad \text{ for } k \in \mathbb{N}. \tag{5.26}$$

Let $\widetilde{Y}(t) := \sum_{j=-\infty}^{\infty} \widetilde{f}(R_j, t - \Gamma_j) Z_j^{+}$ for $t \in \mathbb{R}$ be a mixed MA process, which is by Proposition 3.1 stationary and i. d. Define $\widetilde{\mathbf{Y}}(t) = (\widetilde{Y}(t+t_1), \ldots, \widetilde{Y}(t+t_{d-1}), \widetilde{Y}(t+\eta^{(1)}))$. Applying Theorem 4.1 to $\widetilde{\mathbf{Y}}$ yields for $T \to \infty$,

$$\widetilde{\kappa}_T := \sum_{k=1}^{\infty} \varepsilon_{\left(\Gamma_k/T, \widetilde{\mathbf{Y}}(\Gamma_k)/a_T\right)} \stackrel{w}{\Longrightarrow} \sum_{k=1}^{\infty} \sum_{j=-\infty}^{\infty} \varepsilon_{(s_k, \widetilde{\mathbf{f}}(R_k, T_{k,j}) P_k \chi_k^{+})} =: \widetilde{\kappa}. \tag{5.27}$$

Moreover, define

$$\overline{\mathbf{Y}}_k := \widetilde{\mathbf{Y}}(\Gamma_k) + \sup_{s\in I_k} \mathbf{Y}_2(s), \tag{5.28}$$

where we understand $\sup_{s\in I_k}\mathbf{Y}_2(s)$ computing the supremum coordinatenwise. Then

$$\mathbf{Y}(t) \leq \overline{\mathbf{Y}}_k \quad \text{for } t \in I_k, \tag{5.29}$$

again coordinatenwise. Keep in mind that

$$\mathbb{P}(|\overline{\mathbf{Y}}_k| > a_n x) \leq \mathbb{P}(|\widetilde{\mathbf{Y}}(\Gamma_k)| > a_n x/2) + \mathbb{P}(\sup_{s\in I_k}|\mathbf{Y}_2(s)| > a_n x/2). \tag{5.30}$$

On the one hand, we obtain

$$\widetilde{\mathbf{Y}}(\Gamma_k) = \sum_{j=-\infty}^{\infty}\widetilde{\mathbf{f}}(R_j,\Gamma_k-\Gamma_j)Z_j^+ \stackrel{d}{=} \sum_{\substack{j=-\infty\\ j\neq k}}^{\infty}\widetilde{\mathbf{f}}(R_j,T_j)Z_j^+ \leq \sum_{j=-\infty}^{\infty}\widetilde{\mathbf{f}}(R_j,T_j)Z_j^+, \tag{5.31}$$

where by Proposition 3.3 and the independence of $\sum_{\substack{j=-\infty\\ j\neq 0}}^{\infty}\widetilde{\mathbf{f}}(R_j,T_j)Z_j^+$ and $\widetilde{\mathbf{f}}(R_1,0)Z_1^+$,

$$\lim_{n\to\infty} n\mathbb{P}\left(\left|\sum_{j=-\infty}^{\infty}\widetilde{\mathbf{f}}(R_j,T_j)Z_j^+\right| > a_n x\right) = p\mu^{-1}x^{-\alpha}\sum_{j=-\infty}^{\infty}\mathbb{E}\widetilde{\mathbf{f}}(R_1,T_j)^{\alpha}. \tag{5.32}$$

On the other hand, the Lévy measure of Y_2 has bounded support. Using Markov's inequality, Braverman and Samorodnitsky [7], Lemma 2.1, $a_n \in \mathcal{R}_{1/\alpha}$ and the independence of I_k and Y_2 yields

$$\mathbb{P}(\sup_{s\in I_k}|\mathbf{Y}_2(s)| > a_n x/2) \;\;\leq\;\; d(1/\mu+1)\mathrm{e}^{-a_n x/2}\mathbb{E}\exp(\sup_{0\leq s\leq 1}|Y_2(s)|) = o(1/n) \tag{5.33}$$

for $n\to\infty$. Regarding (5.30)-(5.33), we obtain that there exists a r.v. W such that

$$\mathbb{P}(|\overline{\mathbf{Y}}_k| > a_n x) \leq \mathbb{P}(W > a_n x) = O(1/n) \quad \text{for } n\to\infty.$$

Thus, by (5.28), (5.33), Lemma 2.4 and Example 2.5 *(a)* the point process behavior of the sequence $\{\overline{\mathbf{Y}}_k\}_{k\in\mathbb{Z}}$ is the same as that of $\{\widetilde{\mathbf{Y}}(\Gamma_k)\}_{k\in\mathbb{Z}}$. Furthermore, we can shift the time scale. This together with (5.27) yields for $T\to\infty$,

$$\widetilde{\kappa}_T := \sum_{k=1}^{\infty}\varepsilon_{\left(\Gamma_k/T,\overline{\mathbf{Y}}_k/a_{\lfloor T\rfloor}\right)} \stackrel{w}{\Longrightarrow} \widetilde{\kappa}.$$

Taking (5.29) into account, we obtain on the one hand for $I=[0,1)\times\mathbb{R}_+^d\setminus(0,x]^d$,

$$\lim_{T\to\infty}\mathbb{P}(a_{\lfloor T\rfloor}^{-1}M(T)\leq x) \geq \mathbb{P}(\widetilde{\kappa}(I)=0) = \exp\left(-{f^+}^{\alpha}px^{-\alpha}\right). \tag{5.34}$$

On the other hand, Theorem 4.1 applied to $\mathbf{Y}$ yields

$$\lim_{T\to\infty}\mathbb{P}(a_{\lfloor T\rfloor}^{-1}M(T)\leq x) \leq \mathbb{P}(\kappa(I)=0) = \exp\left(-{f^+}^{\alpha}px^{-\alpha}\right). \tag{5.35}$$

The result follows from (5.34) and (5.35).

(b) The proof follows a long the lines of Fasen [16], Theorem 5.8, where the normalizing constants for subexponential Lévy driven MA processes in MDA(Λ) are calculated. The only difference is that the point process results for regularly varying processes are applied here; in particular Theorem 4.1 and the results for discrete-time MA processes (see Example 4.6). □

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