Corrigendum to Baltrunas, Daley and Klüppelberg "Tail behaviour of the busy period of a GI/GI/1 queue with subexponential service times" [Stochastic Process. Appl. 111 (2004) 237-258.]

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Abstract

The purpose of this note is to correct an error in [1], and to give a more detailed argument to a formula whose validity has been questioned over the years. These details close a gap in the proof of Theorem 4.1 as originally stated, the validity of which is hereby strengthened.

All details refer to [1]. On p. 250, line -3, the moment generating function of the truncated random variable $V = UI_{\{U<y\}}$ is given, but a term was omitted in error. The line should read, correctly, for given $y > -\mu$

$$\widetilde{f}(s) = \int_{-\infty}^{y} \mathrm{e}^{sv} \mathrm{d}P(V \le v) + P(U > y) = E[\mathrm{e}^{sU} \mid U \le y]P(U \le y) + P(U > y), \quad s \in \mathbb{R}.$$

We follow the missing term $P(U > y)$ through the argument.

After eq. (4.11) we decompose $\widetilde{f}(s)$ for $sy > 1$ into

$$\widetilde{f}(s) = J_0 + J_1 + P(U > y).$$

In the following we find $J_0 = 1 + O(1)s^2$ and $J_1 = O(1)s^2$. From Lemma 3.6(b) we know that the moment index $\kappa \ge \beta$, which in turn is greater than 2 by Condition B(ii). Consequently, $E[X^2] < \infty$. Since $U = X - Y - \mu$, this implies that U has finite second moment. Invoking Markov's inequality, we obtain that $P(U > y) \le P(U > 1/s) \le s^2 E[U^2] = O(1)s^2$. Substituting this into (4.11) yields (4.14) as in the paper.

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The second point of concern is the last line of p. 252. We give here a more detailed calculation showing that it holds for given y for sufficiently small $s > 0$. Since the i.i.d. random variables V_k^s have finite variance, Chebychev's inequality gives

$$P\left(\sum_{k=1}^{n} V_k^s > t\right) = P\left(\sum_{k=1}^{n} V_k^s - nE[V^s] > t - nE[V^s]\right) \le \frac{n\mathrm{Var}(V^s)}{(t - nE[V^s])^2} \le \frac{nE[(V^s)^2]}{(t - nE[V^s])^2},$$

and the right-hand side above is bounded by $(n/t^2)E[(V^s)^2]$, as asserted on the last line of p. 252, provided that $E[V^s] < 0$. To show that for large enough y we have $E[V^s] < 0$ for all sufficiently small $s > 0$, observe that, no matter what finite $y > 0$ is given, we have $E[V^s]\big|_{s=0+} = E[UI_{\{U<y\}}] < 0$, and $E[V^s]$ has a finite derivative in s for all small enough $s > 0$. This implies that $E[V^s] < 0$ for all sufficiently small $s > 0$, as required for our argument. This has shown in fact that the probability above is bounded by $(n/t^2)\mathrm{Var}(V^s)$, which is tighter than what we claimed originally.

References

[1] Baltrunas, A., Daley, D.J. and Klüppelberg, C. (2004) Tail behaviour of the busy period of GI/G/1 queue with subexponential service times. *Stoch. Proc. Appl.* **111**(2), 237-258.