An efficient computational framework for probabilistic deterioration modeling and reliability updating

Daniel Straub
Technical University Munich, Germany, straub@era.bv.tum.de

ABSTRACT: The paper presents a novel computational framework for probabilistic deterioration modeling based on dynamic Bayesian networks. The framework enables robust and efficient Bayesian updating of the deterioration model with measurements, inspections and monitoring, making it ideally suited for applications in the management of infrastructure systems, such as structural health monitoring and inspection planning. In this paper, the framework is applied to process equipment subject to CO_2 corrosion, for which inspection and monitoring data is available.

1 INTRODUCTION

In a recent paper (Straub, in press), the author has proposed a computational framework for stochastic deterioration modeling based on the dynamic Bayesian network (DBN) methodology. The proposed framework can be interpreted as a generalization of the Markov chain, which allows to efficiently include non-ergodic random variables that are the dominant source of uncertainty in most engineering models of deterioration. When the interest is in updating of the deterioration model with inspection results, monitoring data and other observations, the DBN framework has significant advantages over existing computational methods such as FORM/SORM and simulation techniques. The DBN framework is both computationally efficient and robust. In particular the latter characteristic make the framework ideally suited for application in practice for the management of deteriorating structures, because the model can be implemented in software that does not require input from an engineer specialized in reliability analysis.

In this paper, following a short summary of the DBN framework, the application of the model to corrosion in process equipment is demonstrated. Particular emphasis is set on the Bayesian updating of the deterioration model with inspection results and monitoring data. The latter have not been considered in (Straub, in press). Finally, it is outlined how the DBN model facilitates the modeling of deterioration in infrastructure systems.

2 DYNAMIC BAYESIAN NETWORKS (DBN)

Dynamic Bayesian Networks (DBN) are a special class of Bayesian networks that can represent stochastic processes (Murphy 2002; Russell and Norvig 2003). A DBN consists of a sequence of slices, each of which contains one or more BN nodes that represent random variables. The slices are connected by directed links from nodes in slice i to nodes in slice i+1. An example of a DBN is shown in Figure 1, where each slice consists of two nodes X_i and Y_i.

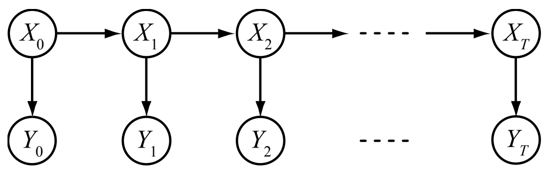

Figure 1. An example of a dynamic Bayesian network.

Like any Bayesian network, a DBN essentially is a model for a joint probability distribution. The graphical structure of the DBN represents the dependence structure among the random variables (Pearl 1988). By exploiting assumptions on (conditional) independence of random variables, the DBN enables an efficient modeling of the joint probability distribution. It is sufficient to define the conditional distributions for each variable X in the DBN given its *parents* $pa(X)$, which are the variables that have links directed towards the variable in the network. The full distribution of a DBN with discrete variables $\mathbf{X}=X_1,\ldots,X_n$ is then fully described by the local conditional probability mass functions (PMF) $p[x_i|pa(X_i)]$ through the relation

$$p(\mathbf{x}) = p(x_1, \ldots, x_n) = \prod_{i=1}^{n} p\left[x_i \middle| pa(X_i)\right] \quad (1)$$

In this paper, we consider only DBN models of discrete random variables. The interest is in computing the conditional PMF of one or several variables in the DBN given observed values of other variables. Denoting the variables of interest by $\mathbf{Y}$ and the observed variables by $\mathbf{E}$ (for evidence), we can summarize the inference problem as that of determining $p(\mathbf{y}|\mathbf{e})$. A number of exact inference algorithms are available for the efficient computation of $p(\mathbf{y}|\mathbf{e})$ (Murphy 2002).

2.1 *Markov processes*

DBN can be interpreted as a generalization of Markov process models, which have frequently been applied for the modeling of deterioration (Bogdanoff and Kozin 1985; Ishikawa et al. 1993; Rocha and Schuëller 1996; Spencer and Tang 1988). Markov deterioration processes are characterized by the fact that for a given condition at time t_1, the condition at any future time $t_2 > t_1$ is statistically independent of the condition at any past time $t_0 < t_1$. By studying the independence assumption of the DBN, it is observed that this holds also for the DBN shown in Figure 1. The joint distribution of the variables in this DBN can also be modeled by a Markov chain, yet such a model would be less efficient.

It is noted that the Markovian assumption does not hold in engineering practice, where epistemic uncertainties are prevalent (Melchers 1999; Yang 1994). Epistemic uncertainties are often time-invariant (e.g., uncertainties due to simplistic parametrical models, due to limited statistical data for empirical models, or due to incomplete knowledge of influencing parameters), thus invalidating the Markovian assumption. To overcome this shortcoming, the generic deterioration model presented in the next section corresponds to a Markov process model conditional on time-invariant random variables. As will be shown, the DBN technique enables the efficient computation of such models.

3 THE DBN FRAMEWORK FOR STOCHASTIC DETERIORATION MODELING

3.1 *Generic deterioration model*

Consider a parametric deterioration model h that describes the extent of deterioration, d_t, as a function of time t, a set of time-invariant model parameters $\boldsymbol{\theta}$, a set of time-variant model parameters $\boldsymbol{\omega}_t = \boldsymbol{\omega}(t)$ and the initial condition d_0. For corrosion, d_t can be the dimension of the corrosion defects, e.g., the maximum depth of corrosion in a structural element, the average corrosion loss or the area of corrosion; d_t can also be a variable describing different discrete condition states, such as "no corrosion" and "initiation of corrosion"; finally, d_t can be a combination of defect dimensions and conditions. The deterioration model is written in generic form as

$$d_t = d(t) = h(t, d_0, \boldsymbol{\theta}, \boldsymbol{\omega}_1, ..., \boldsymbol{\omega}_t), \qquad t > 0 \quad (2)$$

The proposed DBN model does not replace the parametric deterioration model h. Instead, the DBN model provides a computational framework that allows accurate and efficient evaluation of d_t based on the prior stochastic model of the parameters and including all observations of the deterioration process and any of the parameters. The DBN also facilitates learning about the model parameters d_0, $\boldsymbol{\theta}$ and $\boldsymbol{\omega}_t$ based on the observations.

Hereafter, we limit ourselves to modeling deterioration as a discrete time process. Furthermore, in accordance with common deterioration models, we require that the dependence among the d_t is conditionally Markovian, i.e.,

$$\begin{aligned} & f\left(d_t \middle| d_0, \ldots, d_{t-1}, \boldsymbol{\theta}, \boldsymbol{\omega}_1, ..., \boldsymbol{\omega}_t\right) \\ & = f\left(d_t \middle| d_{t-1}, \boldsymbol{\theta}, \boldsymbol{\omega}_t\right), \qquad t = 1, 2, ..., T \end{aligned} \quad (3)$$

where f denotes the probability density function (PDF). Note that $f(d_t|d_{t-1}, \boldsymbol{\theta}, \boldsymbol{\omega}_t)$ can vary with t, i.e., the conditional Markov process is not homogenous in the general case. Additionally, we require that the time-variant model parameters $\boldsymbol{\omega}_t$ are a Markov process conditional on $\boldsymbol{\theta}$ and d_{t-1}, i.e.,

$$\begin{aligned} & f\left(\boldsymbol{\omega}_t \middle| \boldsymbol{\omega}_1, ..., \boldsymbol{\omega}_{t-1}, d_0, ..., d_{t-1}, \boldsymbol{\theta}\right) \\ & = f\left(\boldsymbol{\omega}_t \middle| \boldsymbol{\omega}_{t-1}, d_{t-1}, \boldsymbol{\theta}\right), \quad t = 1, 2, ..., T \end{aligned} \quad (4)$$

Since d_t is dependent on the time-invariant uncertain model parameters $\boldsymbol{\theta}$ for given d_{t-1}, the deterioration process is not Markovian in the unconditional case, i.e., in general it is $f(d_t|d_0, \ldots, d_{t-1}) \neq f(d_t|d_{t-1})$.

3.2 *DBN framework*

The generic deterioration model introduced above can be interpreted as a DBN. By accounting for the independence assumptions made above, the DBN model shown in Figure 2 is constructed. In this DBN, the additional vectors $\boldsymbol{\theta}_1, ..., \boldsymbol{\theta}_T$ are introduced, which are identical to the time-invariant parameters $\boldsymbol{\theta}$, i.e., they are related by the deterministic functions $\boldsymbol{\theta}_t = \boldsymbol{\theta}_{t-1}, t = 2, ..., T$ and $\boldsymbol{\theta}_1 = \boldsymbol{\theta}$. The introduction of these vectors has no effect on the computational efforts, however, it simplifies the model building process and the graphical representation of the model.

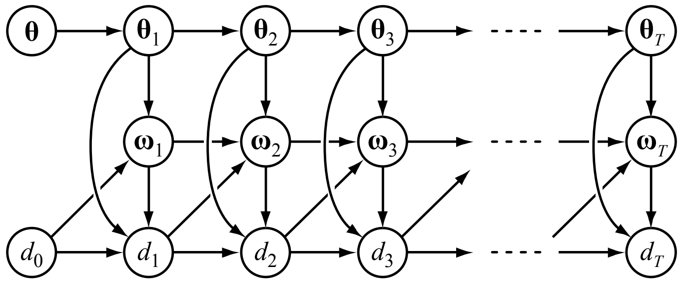

Figure 2. Generic DBN deterioration model.

The nodes in the generic DBN of Figure 2 are vectors of random variables. When the framework is applied to specific deterioration processes, it will be beneficial to replace the vectors by the individual variables of the specific models and introduce them directly in the DBN. In this way, additional independence assumptions can be encoded in the network, thus reducing the computational efforts required in evaluating the network. This will be demonstrated in the example.

Since we apply a DBN model with exclusively discrete random variables, it is necessary to discretize all continuous random variables in the model. A heuristics for discretizing the random variables is presented in (Straub, in press), which will applied in the example.

3.3 *Including observations in the model*

Observations, such as measurements, monitoring data or inspection results, can be included in the model. If a variable of the model is observed directly, then the variable is instantiated in the DBN. Typically, however, the observation is indirect or associated with a measurement uncertainty. In this case, a specific variable representing the observation must be introduced in the DBN as a child of the observed variable. Figure 3 exemplarily shows the modeling of observations of the extent of damage, obtained from inspections of the component. The node $\mathbf{Z}_t$ is fully defined by $p(\mathbf{z}_t \mid d_t)$, which is the likelihood function of the observation and corresponds to common inspection models such as PoD (Probability of Detection), see also Straub (submitted). When establishing the DBN model, variables $\mathbf{Z}_t$ should be included in the DBN for all potential future observations. Whenever an observation is available, the corresponding variable can be instantiated without modifying the DBN. This is one of the key advantages of the DBN framework: Because the evaluation of the DBN with exact inference algorithms is computationally robust, once the DBN is established, all types of evidence can be included automatically. This allows including the model in software that can be run without the support of an expert in structural reliability, with applications, e.g., in health monitoring and inspection planning.

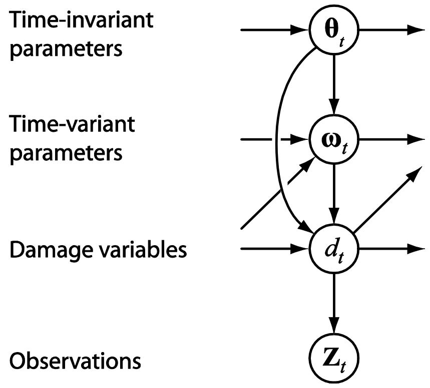

Figure 3. Including an inspection event (observation of the damage variables) in the DBN.

3.4 *Inference*

The purpose of the DBN model is to facilitate the computation of the conditional probability distribution of D_t and other variables of interest, given observations $\mathbf{e}$ from inspection and monitoring (e.g., the observation of an inspection result $Z_t = e_t$ in accordance with Figure 3). An efficient exact algorithm to compute this conditional probability distribution was proposed in (Straub, in press) and is here employed. The algorithm is an adopted version of the "forward-backward" algorithm. The main constituents of this algorithm are

– the forward operation (for time t), which computes $p(\boldsymbol{\theta}_t, \boldsymbol{\omega}_t, d_t \mid \mathbf{e}_1, ..., \mathbf{e}_t)$ by means of a recursive algorithm given in the appendix of (Straub, in press);
– the backward operation (for time t), which computes $p(\mathbf{e}_{t+1}, ..., \mathbf{e}_T \mid \boldsymbol{\theta}_t, \boldsymbol{\omega}_t, d_t)$ by means of a recursive algorithm given in the appendix of (Straub, in press).

When the interest is in computing the reliability at time t given the evidence up to time t, the forward operation is performed. This case is known as "filtering". When the interest is in computing the reliability at a future time T conditional on evidence up to the present t, which is the case known as "prediction", the forward operation is performed for time T, whereby the whereby the likelihood functions for $\mathbf{e}_{t+1}, ..., \mathbf{e}_T$ are set equal to one: $p(\mathbf{e}_i \mid d_i) = 1$, $i = t+1, \ldots, T$ (i.e., no evidence is entered for these variables). Finally, for the case where the interest is in computing the probability over the state at a past time t given evidence up to time T, which is the case known as "smoothing", both the forward operation and the backward operation are carried out for time t. We then obtain

$$p\left(\boldsymbol{\theta}_t, \boldsymbol{\omega}_t, d_t \mid \mathbf{e}_0, ..., \mathbf{e}_T\right) \propto \\ p\left(\boldsymbol{\theta}_t, \boldsymbol{\omega}_t, d_t \mid \mathbf{e}_0, ..., \mathbf{e}_t\right) p\left(\mathbf{e}_{t+1}, ..., \mathbf{e}_T \mid \boldsymbol{\theta}_t, \boldsymbol{\omega}_t, d_t\right) \quad (5)$$

Equation (5) is an application of Bayes' rule, whereby $p(\boldsymbol{\theta}_t, \boldsymbol{\omega}_t, d_t \mid \mathbf{e}_1, ..., \mathbf{e}_t)$ is the prior probability. The likelihood function is $p(\mathbf{e}_{t+1}, ..., \mathbf{e}_T \mid \boldsymbol{\theta}_t, \boldsymbol{\omega}_t, d_t)$, the result of the backward operation, because of independence of $\mathbf{e}_{t+1},...,\mathbf{e}_T$ from $\mathbf{e}_1,...,\mathbf{e}_t$ for given $\boldsymbol{\theta}_t, \boldsymbol{\omega}_t, d_t$, as prescribed by the DBN structure.

Let the number of states of $d_t, \boldsymbol{\omega}_t, \boldsymbol{\theta}_t$ be $m_d, m_{\boldsymbol{\omega}}, m_{\boldsymbol{\theta}}$, respectively. As demonstrated in (Straub, in press), in the general case the computation time for filtering is $O[(m_d^2 m_{\boldsymbol{\omega}} + m_d m_{\boldsymbol{\omega}}^2) m_{\boldsymbol{\theta}} t]$, whereas for predicting and smoothing it is $O[(m_d^2 m_{\boldsymbol{\omega}} + m_d m_{\boldsymbol{\omega}}^2) m_{\boldsymbol{\theta}} T]$. By exploiting the independence assumptions among the variables in $\boldsymbol{\theta}_t$ and in $\boldsymbol{\omega}_t$, the computation time can be further reduced. This will be shown for the presented example of CO_2 corrosion in the following.

4 DBN MODELING OF CO_2 CORROSION

4.1 *Deterioration model*

The example from (Straub and Faber 2007) is studied, considering CO_2 corrosion in a pressurized pipe, a common deterioration mechanism in oil ad gas process plants. CO_2 corrosion is modeled by a parametric model originally developed in (DeWaard and Milliams 1975, DeWaard et al. 1991), which predicts a linear corrosion rate R as a function of number of influential parameters. For illustrational purposes, a simple version of the model is utilized, with the corrosion rate defined as a function of operating pressure P_o, operating temperature T_o and the partial pressure of CO_2. Other influencing parameters, such as the pH value or the flow rate are not explicitly accounted for.

CO2 corrosion leads to spatially distributed defects (pits). The DeWaard-Milliams model is a worst-case model, and it can be assumed that the corrosion rate R relates to the maximum defect size in a given area of the considered process equipment. To account for the conservatism of this model, a model correction factor X_M is included, following (Sydberger 1995).

The environmental conditions vary with time. Accordingly, P_o and T_o are modeled as random processes. As a consequence, the corrosion rate is also varying with time, and the maximum defect depth at time T is determined as

$$D(t) = D_0 + \int_0^T R(t)\,dt \tag{6}$$

D_0 is the initial defect depth. The corrosion rate $R(t)$ at any time t is calculated as a function of $T_o(t)$, the CO_2 fugacity $f_{CO_2}(t)$ and a model uncertainty X_M:

$$R(t) = X_M \cdot 10^{\left[5.8 - 1710/T_o(t) + 0.67 \cdot \log_{10} f_{CO_2}(t)\right]} \tag{7}$$

Here, the temperature $T_o(t)$ is expressed in [K] and the CO_2 fugacity $f_{CO_2}(t)$ is calculated as

$$f_{CO_2}(t) = P_{CO_2}(t) \cdot 10^{P_o(t)\left[0.0031 - 1.4/T_o(t)\right]} \tag{8}$$

where $P_o(t)$ is expressed in [bar] and $P_{CO_2}(t)$ is the partial pressure of CO_2. $P_{CO_2}(t)$ is a function of $P_o(t)$ and the fraction of CO_2 in the gas phase, n_{CO_2}:

$$P_{CO_2}(t) = n_{CO_2} P_o(t) \tag{9}$$

4.2 *Failure criterion*

The considered failure mode is leakage, i.e., the failure event occurs when the largest corrosion depth $D(t)$ exceeds the wall thickness of the pipe, W. The corresponding limit state function is

$$g_{F(t)} = w - D(t) \tag{10}$$

The extension of the model to the bursting failure mode is straightforward (Ahammed and Melchers 1996), and corresponds to replacing w in Equation (10) with the critical defect size at which failure occurs. Note, however, that in most practical applications the uncertainty related to the failure mechanism is smaller than the uncertainty related to the corrosion process.

4.3 *Stochastic modeling*

The operating pressure $P_o(t)$ and temperature $T_o(t)$ are random processes. To include model uncertainty, the mean values of these processes, M_P and M_T, are modeled by random variables. This reflects the fact that these values depend on the operation of the process system and are not known with certainty at the time of design. For given values of M_P and M_T, the operating pressure $P_o(t)$ and the operating temperature $T_o(t)$ are modeled as Poisson square wave processes (Madsen 1986). Such a process has different intervals, whose starting points are generated by a Poisson process with intensity ν. In each interval, the value of the process is redefined, and is here Normal distributed with mean values M_P and M_T and standard deviations σ_P and σ_T. It is assumed that the underlying Poisson process is identical for $P_o(t)$ and $T_o(t)$, because both will change at times when the operational conditions are changing. In addition, it is assumed that $P_o(t)$ and $T_o(t)$ at any time t are correlated by a correlation factor ρ_{PT}. The covariance functions of these processes are derived in (Straub and Faber 2007). For the DBN modeling, however, these are not required. It is sufficient to note that the processes have the Markov property.

The model uncertainty X_M is Weibull distributed with mean 0.4 and coefficient of variation 0.8 in accordance with (Sydberger 1995). All random variables are summarized in Table 1.

Table 1. Stochastic model

Variable	Distribution	Mean	St.Dev.	Correlation
W [mm]	Deterministic	24	-	-
D_0 [mm]	Deterministic	0.0	-	-
X_M [-]	Weibull	0.4	0.32	-
n_{CO2} [-]	Deterministic	0.01	-	-
P_O [bar]	Normal	M_P	σ_P=15	ρ_{PT} = 0.8
T_O [K]	Normal	M_T	σ_T=15	ρ_{PT} = 0.8
M_P [bar]	Normal	303	7	-
M_T [K]	Normal	100	5	-
ν [yr^{-1}]	Deterministic	4	-	-

4.4 *Model of inspection and monitoring*

During an inspection, the deepest identified defect in the pipe element is measured and recorded. It is, however, not guaranteed that the identified defect is the actual deepest defect in the element, which might be missed at the inspection. The inspection model developed in (Straub, submitted) accounts for this possibility of missing the largest defect, and is applied here to represent inspection results. The model gives the likelihood of measuring a defect size d_{mt} given that the true largest defect size is d_t:

$$L(d_t) = f_{D_{mt}}(d_{mt} | d_t)$$
$$= PoD(d_t) f_\varepsilon(d_t - d_{mt}) \qquad (11)$$
$$+ \frac{1 - PoD(d_t)}{F_{D_t}(d_t)} \int_{-\infty}^{d_t - d_{mt}} f_{D_t}(d_{mt} + \varepsilon) f_\varepsilon(\varepsilon) d\varepsilon$$

where PoD is the Probability of Detection function, ε is the additive measurement error and $f_\varepsilon(\cdot)$ its corresponding PDF, $F_{D_t}(\cdot)$ is the cumulative probability distribution (CDF) of D_t, and $f_{D_t}(\cdot)$ the corresponding PDF. The PoD function is taken from (Straub, submitted) as

$$PoD(d_t) = \frac{\exp[-1.07 + 2.57 \ln(d_t)]}{1 + \exp[-1.07 + 2.57 \ln(d_t)]} \qquad (12)$$

The measurement error is Normal distributed with zero mean and standard deviation $\sigma_\varepsilon = 1\text{mm}$.

In the DBN, the inspections are included by the nodes D_{mt}, whose conditional PMF is defined by the discretized version of the likelihood function (11). Whenever an inspection result d_{mt} is available, the corresponding node in the DBN is instantiated.

Monitoring data is considered for the variables $P_o(t)$ and $T_o(t)$. It is assumed that these can be monitored exactly, i.e., the observed values correspond to the true values. Therefore, monitoring data is included in the DBN by directly instantiating the respective variables of the DBN deterioration model.

4.5 *DBN model*

The resulting DBN model for the example is shown in Figure 4. Because the model has discrete time steps, we write all variables with index t, e.g., $R(t) = R_t$.

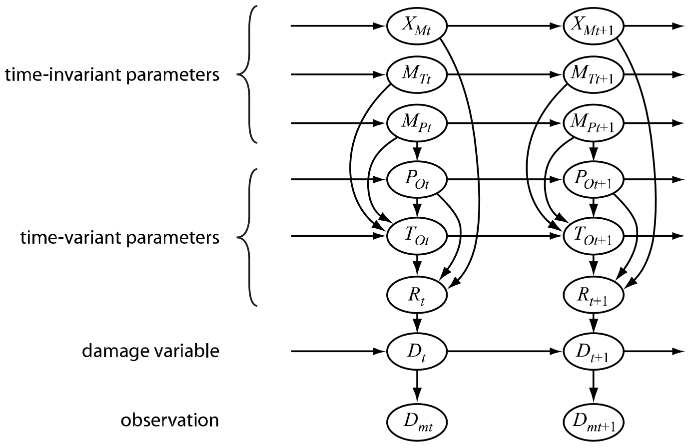

Figure 4. The DBN model for CO2 corrosion.

With the inference algorithm from (Straub, in press), the computational efforts for performing inference on this network are $O[(m_{P_O}^2 m_{T_O} m_R m_D + m_{P_O} m_{T_O}^2 m_R m_D + m_{P_O} m_{T_O} m_R^2 m_D + m_{P_O} m_{T_O} m_R m_D^2) m_{X_M} m_{M_T} m_{M_P} t]$ for the filtering case, where m are the number of discrete states of the respective random variables.

The discretization scheme, selected following the heuristics suggested in (Straub, in press), is summarized in Table 2.

Table 2. Stochastic model

Variable	# of states	Interval boundaries
X_M [-]	16	0, exp{ln(0.1):[ln(4.0/0.1)]/14:ln(4.0)}, ∞
P_O [bar]	9	0, 34:(116-34)/7:116, ∞
T_O [K]	14	0, 243:10:363, , ∞
M_P [bar]	7	86:4:114
M_T [K]	10	288:3:318
R_t [mmyr^{-1}]	21	0, exp{ln(0.1):[ln(5.0/0.1)]/19:ln(5.0)}, ∞
D_t [mm]	10	0, exp{ln(0.1):[ln(24/0.1)]/98:ln(24)}, ∞
Z_t [mm]	145	0, exp{ln(0.1):[ln(24/0.1)]/98:ln(24)}, ∞

In the general case, the resulting computation times are in the order of 1-2 CPU hours on a standard PC with a 2GHz processor. If monitoring data is available for $T_O(t)$ and $P_O(t)$, these can be reduced down to a few CPU seconds. These CPU times are large, not least in comparison to the ones reported in (Straub, in press) for other deterioration models (which are in the order of a few CPU seconds in all cases). To some extent this is because the discretization schemes shown here are not optimized, yet the main reason is that the number of random variables in a time slice is higher in the CO_2 model presented here. It is possible to reduce this number, and therefore the computation time, by a strategy outlined in the discussion section later. It is noted that the computation times reported here are not necessarily critical, since the cost of computer time is negligible. The fact that the computations are robust and do not require the intervention of the engineer will be the crucial factor in many applications.

5 NUMERICAL RESULTS

5.1 *Unconditional case*

Figure 5 summarizes the reliability for the pipe element prior to including inspection and monitoring results. For this case, Monte Carlo simulation (MCS) can be performed, which is used to verify the outcomes of the DBN model. As observed in Figure 5, the results obtained with the DBN are in reasonable agreement with the MCS results.

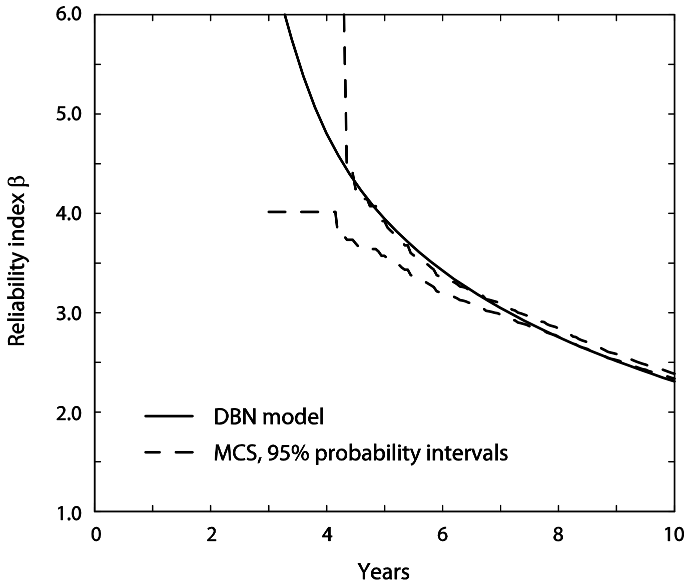

Figure 5. The reliability index for the case without evidence.

5.2 *Including inspection results*

Two series of inspection outcomes are considered, with measured defect sizes as summarized in the following with the corresponding inspection time in brackets:

1) 3mm(2yr); 3mm(3yr); 4mm(4yr); 5mm(6yr); 6mm(8yr).
2) 13mm(5yr); 18mm(7yr).

Figure 6 presents the reliability index updated with these inspection results, as evaluated with the DBN. The results presented here correspond to filtering. i.e., at each time t, the reliability index in Figure 6 includes all evidence available up to time t. This is a common case encountered in the integrity management of engineering systems.

5.3 *Including monitoring data*

Next, we consider the situation when the values of the influencing parameters T_O and P_O are recorded. It is noted that such data is readily available in the operation of process systems, yet an integrity management procedure is necessary for storing the data for the purpose of planning inspections and repairs. The two considered monitoring data sets are summarized in Figure 7. The resulting reliability indexes are presented in Figure 8.

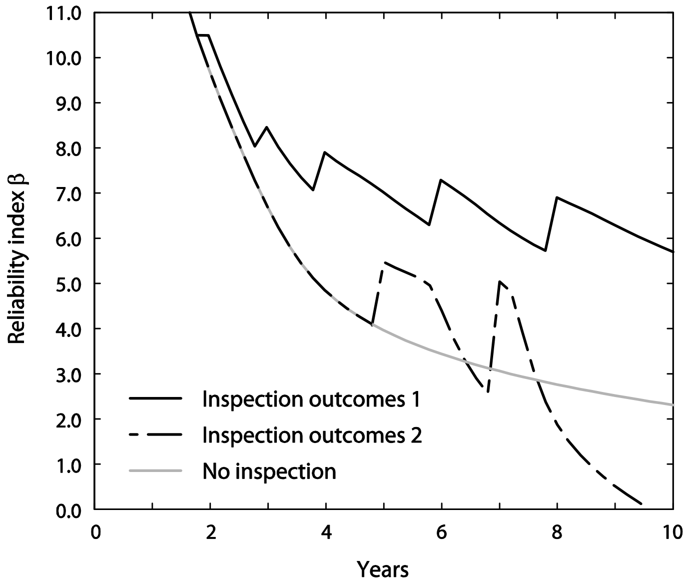

Figure 6. The reliability index for the case with inspection results.

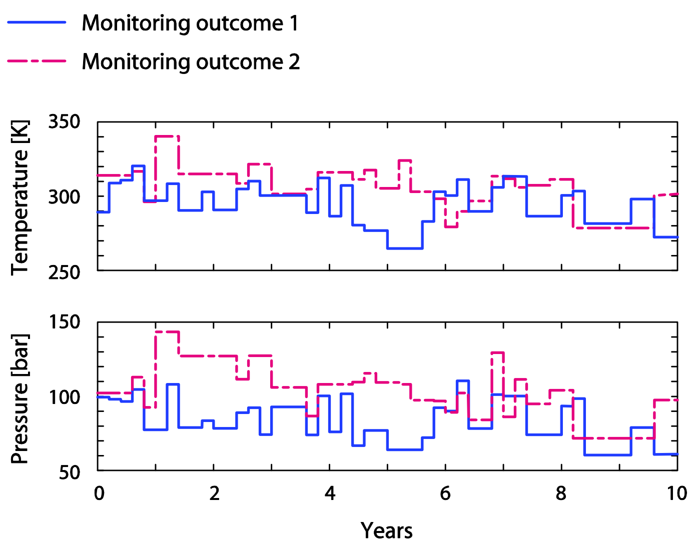

Figure 7. The monitoring data for the updating example.

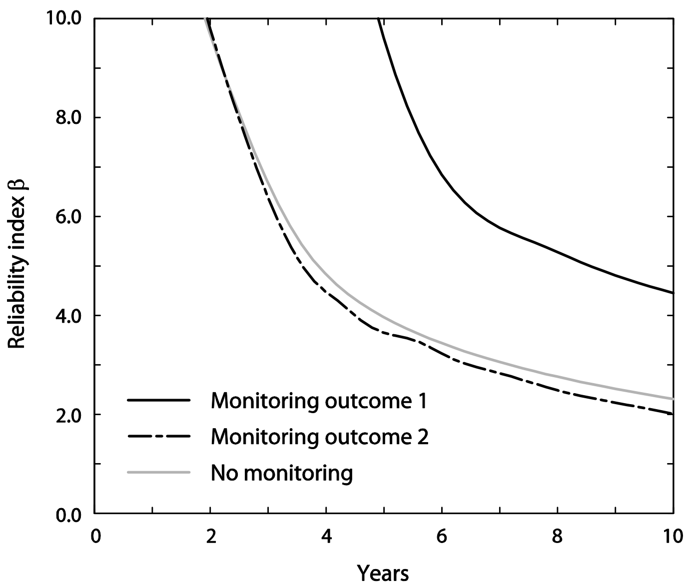

Figure 8. The reliability index for the case with monitoring data.

From Figure 8 it is observed that the monitoring data can have a significant influence on the reliability. This is particularly the case for data set 1 with lower observed values of pressure and temperature (the mean value of the T_{Ot} series is 293.5K, the one of the P_{Ot} series is 92.9). Because gathering the data will be relatively cheap in many instances, such monitoring of influencing variables can represent an efficient way to reduce the risk due to deterioration.

In general, monitoring data will be available in combination with inspection results. Updating of the model with such combined information is straightforward with the DBN model, an exemplarily result is presented in Figure 9, combining the monitoring data set 1 with the inspection outcomes 2 from above. It is interesting to note that including the monitoring data in addition to the inspection results increases the reliability. In the case without inspection results, the same monitoring data leads to a decrease in reliability (Figure 8). Without inspection results, the effect of observing mean values of pressure and temperature that are larger than expected is stronger than the effect of the reduction in uncertainty. Combined with inspection results, which lead to higher reliability indexes, the reduction in uncertainty has the stronger effect.

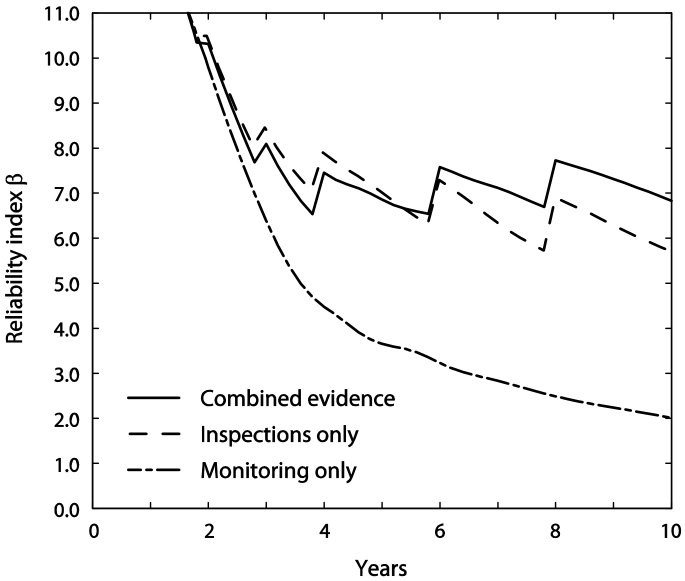

Figure 9. The reliability index for the case with monitoring data set 1 and inspection results 2.

Besides computing the reliability, the DBN readily computes the updated distribution of any variable in the model. Exemplarily, the posterior distribution of X_M given the monitoring data set 1 and the inspection outcomes 1 is shown in Figure 10. This inference problem belongs to the smoothing class. Since in this case the inspections indicate large defect sizes, the posterior PDF of X_M is shifted towards larger values. Furthermore, it is observed that the uncertainty on the value of X_M for this component has been significantly reduced.

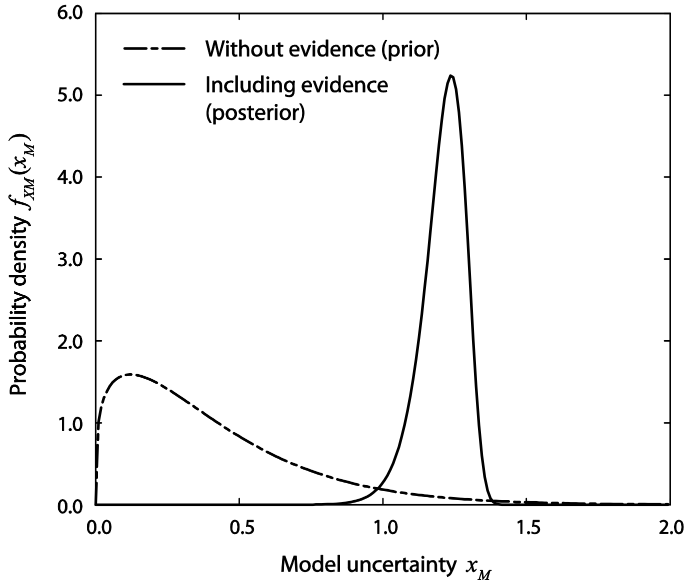

Figure 10. The PDF of X_M, updated with monitoring data set 1 and inspection results 1.

6 DISCUSSION

The DBN framework enables robust probabilistic inference when information on the deterioration process becomes available. Unlike for the deterioration processes investigated in (Straub, in press), the example presented here still has a main drawback in that the CPU times required for performing inference are in the order of one hour on a standard PC with a 2GHz processor unless monitoring data on some of the variables is available. For applications in which this computational effort becomes critical, a modified version of the model must be utilized. The model as developed and presented here is the straightforward implementation of the general framework. If CPU time needs to be reduced by orders of magnitude, simplifications must be made. For the presented example of CO_2 corrosion, it has been shown in (Straub and Faber 2007) that replacing the random process models of T_O and P_O with time-invariant, equivalent values of T_O and P_O changes the results only slightly as long as these values are not directly observed. Therefore, a strategy for making the DBN modeling of CO_2 corrosion more efficient is to separate the model: For those time slices at which T_O and P_O are observed, the model as presented here is employed. For all times with no observations, e.g., for all future times T, the model can be simplified significantly because, with T_O and P_O represented by time-invariant random variables, R becomes time-invariant, and it is sufficient to have R_T and D_T in these time slices, instead of all random variables. A similar strategy is presented in (Straub, in press), where monitoring evidence is not considered.

Extension of the presented DBN framework to the modeling of entire engineering systems is possible. To this end, the DBN model of each element is defined conditional on common influencing variables. Such a joint model is a Bayesian network, yet no longer a DBN.

7 CONCLUSIONS

A computational framework for deterioration modeling is presented and investigated by means of an example considering CO_2 corrosion in process equipment. It has been shown that the framework allows including inspection results and monitoring data through Bayesian updating in a robust manner, i.e., the computations can be performed automatically. Such robust computations have a strong potential for applications in the asset integrity management of engineering systems.

REFERENCES

Ahammed, M., and Melchers, R. E. (1996). "Reliability Estimation of Pressurised Pipelines Subject to Localised Corrosion Defects." *International Journal of Pressure Vessels and Piping*, **69**, p. 267-272.

Bogdanoff, J. L., and Kozin, F. (1985). *Probabilistic Models of Cumulative Damage*, John Wiley & Sons, New York.

DeWaard, C., and Milliams, D. E. (1975). "Carbonic Acid Corrosion of Steel." *Corrosion*, **31**(5), p. 177-181.

DeWaard, C., Lotz, U., and Milliams, D. E. (1991). "Predictive Model for CO2 Corrosion Engineering in Wet Natural Gas Pipelined." *Corrosion*, **47**(12), p. 976-985.

Ishikawa, H., Tsurui, A., Tanaka, H., and Ishikawa, H. (1993). "Reliability assessment of structures based upon probabilistic fracture mechanics." *Probabilistic Engineering Mechanics*, **8**(1), p. 43-56.

Madsen, H. O., Krenk, S., and Lind, N. C. (1986). *Methods of Structural Safety*, Pentice-Hall, Inc., Englewood Cliffs, New Jersey 07632, New Jersey.

Melchers, R. E. (1999). "Corrosion Uncertainty Modelling for Steel Structures." *Journal of Constructional Steel Research*, **52**, p. 3–19.

Murphy, K. P. (2002). *Dynamic Bayesian networks: representation, inference and learning*. PhD thesis, University of California, Berkeley.

Pearl, J. (1988). *Probabilistic reasoning in intelligent systems : networks of plausible inference*, Morgan Kaufmann Publishers, San Mateo, Calif.

Rocha, M. M., and Schuëller, G. I. (1996). "A probabilistic criterion for evaluating the goodness of fatigue crack growth models." *Engineering Fracture Mechanics*, **53**(5), p. 707-731.

Russell, S. J., and Norvig, P. (2003). *Artificial intelligence: a modern approach*, Prentice-Hall, Englewood Cliffs, N.J.

Spencer, B. F., and Tang, J. (1988). "A Markov Process Model for Fatigue Crack Growth." *Journal of Engineering Mechanics, Trans. ASCE*, **114**(12), p. 2134-2157.

Straub, D. (in press). "Stochastic Modeling of Deterioration Processes through Dynamic Bayesian Networks." *Journal of Engineering Mechanics, Trans. ASCE*, in press.

Straub D. (submitted). "Quantitative modeling of inspection quality for spatially distributed defects." Submitted to *Structural Safety*.

Straub, D., and Faber, M. H. (2007). "Temporal Variability in Corrosion Modeling and Reliability Updating." *Journal of Offshore Mechanics and Arctic Engineering, Trans. ASME*, **129**(4), p. 265-272.

Sydberger, T., Edwards, J. D., and Mørk, K. J. "A Probabilistic Approach to Prediction of CO2-Corrosion and its Application to Life Cycle Cost Analysis of Oil and Gas Equipment." Proc. *Corrosion95*, paper #65.

Yang, J. N. "Application of reliability methods to fatigue, quality assurance and maintenance." *Proceedings of the 6th International Conference of Structural Safety & Reliability* Balkema Rotterdam, p. 3-18.