Bayesian network as a framework for structural reliability analysis in infrastructure systems

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ABSTRACT: Structural reliability analyses are often performed for elements of complex infrastructure systems. In this paper, we demonstrate how Bayesian networks (BN) can be employed to integrate these analyses into a single model of the infrastructure system. In addition, we illustrate the capabilities of BN for Bayesian updating of the model when observations are available, such as measurements of material properties, monitoring of environmental loads or observations of past system performances. The approach is presented on an idealized example of an infrastructure system subject to natural hazard events and deterioration.

1 INTRODUCTION

We have investigated the combination of Bayesian networks (BN) with structural reliability methods (SRM) elsewhere (Straub and Der Kiureghian 2008b). The resulting concept is termed enhanced Bayesian network (eBN). The eBN facilitates the integration of the detailed modeling of a structure through SRM into the wider context of risk and decision analysis in complex civil systems, in particular for systems with evolving information, where the capabilities of the BN for Bayesian updating can be exploited. For this reason, the eBN concept is well suited for multi-scale probabilistic analysis of large infrastructure systems and, ultimately, for decision optimization, at the planning stage and in near-real-time during the operational phase. The purpose of this paper is to demonstrate this application of the eBN framework and to outline a generic framework for infrastructure risk analysis.

Other authors (Friis-Hansen 2005, Nishijima et al. 2009) have pointed out the usefulness of BN for modeling complex engineering systems due to its graphical and modular nature, which facilitates efficient and concise representation of dependences among system components. However, thus far there has not been an attempt at formalizing the combination of SRM and BN for modeling such complex systems. The present paper takes a step in this direction by applying the eBN method from Straub and Der Kiureghian (2008b) to infrastructure system risk analysis. After a presentation of general modeling principles and a generic framework for infrastructure risk analysis using BN, the capabilities and limitations of the eBN approach are illustrated by means of an example dealing with an idealized infrastructure system subject to natural hazards and deterioration.

2 THE ENHANCED BAYESIAN NETWORK METHODOLOGY

This section contains a brief introduction into the eBN methodology, following Straub and Der Kiureghian (2008b). This text assumes that the reader is familiar with both SRM and BN.

2.1 *The eBN*

We define as enhanced Bayesian networks (eBNs) a subclass of BNs that have the following properties:

a) The BN has nodes that are defined in a finite sample space (discrete nodes) and nodes that represent vectors of continuous random variables (continuous nodes).
b) The states of each discrete node that is a child of at least one continuous node are defined as domains in the outcome space of its parents, in which case the node is deterministic, or are defined by a probability mass function that is parameterized by the parent nodes, in which case the node is random (a case not considered in this paper).

To graphically distinguish the continuous nodes from the discrete nodes, we plot all continuous nodes with shaded area. On the left-hand side of Figure 1, an example of an eBN is given, in which X is a continuous node.

There exists no exact method of inference for a general BN with continuous nodes. Approximate inference methods (such as stochastic simulation or MCMC) are available, but are not considered here.

The philosophy pursued in solving the eBN is to reduce it to a BN that contains only discrete nodes. This reduced BN, referred to as rBN, can then be solved with the available algorithms for exact inference (e.g., Lauritzen and Spiegelhalter 1988), which are also implemented in a number of free and commercial software (Murphy 2001).

2.2 *Obtaining the rBN through variable elimination*

To reduce the eBN to a rBN, it is necessary to eliminate all continuous random variables in the eBN. To this end, we make use of an algorithm for variable elimination developed by Shachter (1988). We first define as *barren nodes* all random variables in the eBN without children that do not receive any evidence. In the context of the eBN, if X_i is a barren node, we can simply remove it together with the links directing to it, without changing the joint distribution of the remaining variables.

Second, we make use of theorem 2 from Shachter (1988), which states that a link from node *i* to node *j* can be reversed by adding directed links from all parents of node *i* to node *j* and from all parents of node *j* to node *i*, provided this action does not create a cyclic path in the network.

The process of eliminating a node X representing a set of continuous random variables in the eBN proceeds by first reversing all directed links from X to the children of X, $ch(X)$, until $ch(X)$ is the empty set. Then, X, together with all the links pointing to it, can simply be removed. Figure 1 shows an example of the process of obtaining the rBN from the eBN. The order of the reversing operations can be chosen freely as long as it is ensured that the resulting network is acyclic at any stage. Thus, in Figure 1, the link from X to Y_6 cannot be reversed first, since this would lead to the cycle $Y_5 \rightarrow Y_6 \rightarrow X \rightarrow Y_5$.

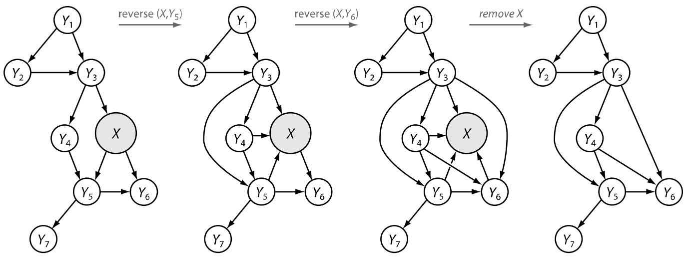

Figure 1. Illustration of an eBN and a link reversal sequence for removal of the continuous node *X*, to arrive at the rBN.

For each node for which the incoming links are not identical in the eBN and the rBN, it is necessary to compute a probability mass function (PMF) conditioned on the parents of the variable in the rBN. More details of the procedure for computing this PMF are given in Straub and Der Kiureghian (2008b). Depending on the problem, different SRM may be used, e.g., component or system reliability methods by FORM, SORM, importance sampling, subset simulation, or Monte Carlo simulation. The fact that some of the SRM methods have unknown rates of convergence is not critical here, since the rBN can be established prior to the availability of evidence for near real-time updating.

Briefly stated, let $\mathbf{X}$ and $\mathbf{Y}$ respectively denote the set of continuous and discrete random variables in the eBN. Furthermore, let $\mathbf{Y}_C$ denote the set of discrete variables that have at least one continuous variable as parent, and $\mathbf{Y}_N$ the set of the remaining discrete variables. Let $pa'(Y_i)$ be the parents of $Y_i \in \mathbf{Y}_C$ in the eBN, which may include variables from both $\mathbf{X}$ and $\mathbf{Y}$, and $pa''(Y_i)$ be the parents of Y_i in the rBN, which can only include variables from $\mathbf{Y}$. By definition of the eBN, any state k_i of $Y_i \in \mathbf{Y}_C$ is described by a domain in the space of $\mathbf{X}$. In general, this domain may also depend on the states of the discrete variables in $pa'(Y_i)$. Thus, we denote the domain for the state k_i of Y_i as $\Omega_{\mathbf{K}_i}^{(k_i)}(\mathbf{x})$, wherein $\mathbf{K}_i$ denotes the vector of the states of the discrete variables in $pa'(Y_i)$. The conditional PMF of $\mathbf{Y}$ in the rBN is then given as a function of the probability density function (PDF) of $\mathbf{X}$, $f[\mathbf{x} \mid pa(\mathbf{X})]$, by

$$\begin{aligned} p(\mathbf{y}) &= \prod_{Y_i \in \mathbf{Y}} p\left[y_i^{(k)} \middle| pa''(Y_i)\right] \\ &= \prod_{Y_i \in \mathbf{Y}_N} p\left[y_i^{(k)} \middle| pa'(Y_i)\right] \int_{\mathbf{x} \in \Omega(\mathbf{x})} f\left[\mathbf{x} \middle| pa(\mathbf{X})\right] d\mathbf{x} \end{aligned} \tag{1}$$

with $\Omega(\mathbf{x}) = \bigcap_{Y_i \in \mathbf{Y}_C} \Omega_{\mathbf{K}_i}^{(k_i)}(\mathbf{x})$

In the second line of Equation (1) we make use of the fact that all variables in $\mathbf{Y}_N$ will have the same parents in the rBN as in the eBN.

The integration in Equation (1) can be efficiently computed using system SRM. We would need to solve such system problems for all combinations of the states of $\mathbf{Y}_C$ and $pa(\mathbf{X})$. For a large number of variables in $\mathbf{Y}_C$, the number of these computations can become prohibitive. However, this number can be reduced when the dependence structure of the specific case is taken into account. It can be shown that the number of SRM calculations required to obtain the conditional PMF of a single variable Y_i corresponds to the total number of states of $pa''(Y_i)$ times $m_i - 1$, with m_i being the number of states of Y_i. Therefore, it is crucial that the number of parents of any variable $Y_i \in \mathbf{Y}_C$ in the resulting rBN be as small as possible. In Straub and Der Kiureghian (2008b) we introduce the concept of Markov envelopes to determine the minimum number of parents of the variables in the rBN. This concept is summarized below.

2.3 *Markov envelopes*

The Markov *blanket* of a variable X includes X, the parents of X, the children of X and the parents of the children of X (e.g., Russell and Norvig 2003): $bl(X) = X \cup pa(X) \cup ch(X) \cup pa[ch(X)]$. The importance of the Markov blanket stems from the fact that for given values of the variables in $bl(X)$, the variable X is statistically independent of all other variables in the BN. Based on the Markov blanket, we identify groups of continuous variables that will become connected during the variable elimination process. Such groups, denoted by $\mathbf{X}_M$, are identified by the following procedure:

Start with a single continuous node and put it into $\mathbf{X}_M$; add all continuous nodes that are part of the Markov blanket of the first node; add all continuous nodes that are part of the Markov blankets of the additional nodes; and so on. The Markov envelope is then defined as the aggregation of all variables (discrete and continuous) that are part of the Markov blankets of all variables in $\mathbf{X}_M$: $\{\bigcup_{X_i \in \mathbf{X}_M} bl(X_i)\}$. This concept is illustrated in Figure 2. It follows from the node elimination algorithm that one discrete variable in each of the Markov envelopes will have all other discrete variables in the envelope as parents in the rBN. Therefore, the sizes of these envelopes represent a lower bound on the number of SRM computations (and on the computational complexity of the resulting rBN).

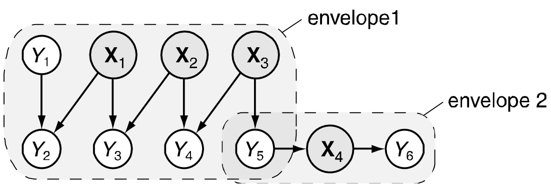

Figure 2. Illustration of Markov envelopes. X_1 and X_3 are both in $bl(X_2)$, thus the envelope contains $bl(X_1)$, $bl(X_2)$ and $bl(X_3)$, whereas $bl(X_4)$ contains no other continuous variables and forms an individual envelope.

2.4 *Including evidence*

We can distinguish two situations with respect to the availability of evidence: The situation where the evidence is already available when establishing the rBN, and the situation where the rBN is established only anticipating the evidence. In this paper we focus on the latter, since this situation occurs in near-real time decision-making. The same type of problem must be solved in preposterior decision analysis. However, we note that when the evidence is available at the time of establishing the eBN, the number of SRM calculations is significantly reduced, because every discrete random variable with known outcome is represented by a single state.

Because inference is ultimately carried out on the rBN, we require that all variables for which evidence is available, or for which an updated distribution is desired, be discrete random variables. Thus, any continuous random variable for which evidence or updating is anticipated should be replaced by an equivalent discrete random variable.

3 MODELING INFRASTRUCTURE SYSTEMS WITH THE EBN APPROACH

Complex infrastructure systems often have structural systems as elements. Examples are bridges and tunnels that are elements of a transportation system, or offshore platforms and pipelines that are elements of an oil or gas production system. The BN is well suited for such multi-scale system modeling, while also allowing modeling of non-structural elements, such as power supply or pumping units. The approach enables updating the entire system model based on observations at any scale (e.g., the monitoring of a structural component in a bridge is utilized to update the probabilistic model of the entire infrastructure system).

The eBN concept can be directly applied to the modeling of infrastructure systems. For the sake of computational feasibility, it is necessary that the size of the Markov envelopes remain small. In the following, we make observations on efficient strategies for limiting the size of these Markov envelopes in typical infrastructure systems that are distributed in space. Thereafter, we present the object-oriented BN modeling as an efficient representation of large-scale infrastructure systems.

3.1 *Markov envelopes in eBN models of infrastructure systems*

It is desirable that Markov envelopes in an eBN model be confined to the level of infrastructure system elements so that SRM can be used to solve the corresponding structural system or component problems. Because elements of infrastructure systems are often exchangeable, this approach facilitates re-using the same SRM calculations for infrastructure elements (structural systems or components) of the same type.

3.2 *Modeling common uncertain factors*

In most infrastructure systems, groups of system elements are subjected to common uncertain factors. Examples are a common hazard determining the demands on the system elements, or a common manufacturer of structural components, leading to correlation among the component capacities. The importance of these common factors (or the resulting correlation among element characteristics) has been recognized and their effect on infrastructure system reliability has been studied (e.g., Lee and Kiremidjian 2007, Straub and Der Kiureghian 2008a). As outlined in Straub et al. (2008), such common factors are even more important when considering reliability updating, since they allow learning about the entire system from observations of a single element.

If Markov envelopes are not to extend beyond individual elements of an infrastructure system, then the common uncertain factors influencing multiple elements must be modeled by discrete random variables in the eBN. Such an approach ensures that the common factors are explicitly included in the resulting rBN, reflecting the causal relations within the infrastructure system. This is illustrated in Figure 3, which shows the difference between modeling a common influencing factor by a continuous or a discrete random variable.

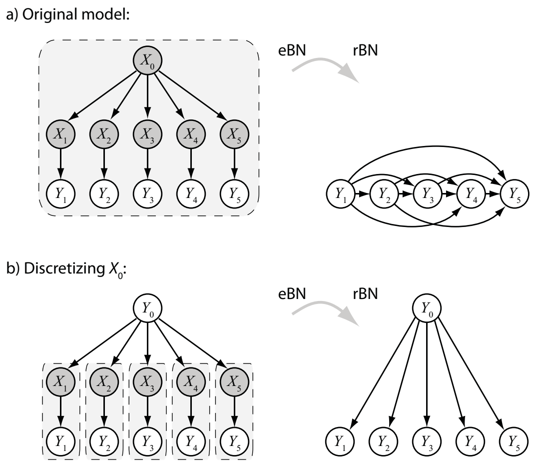

Figure 3. Illustration of the modeling of a common influencing random variable: (a) with a continuous random variable, (b) with a discrete random variable. The eBN is shown on the left side, the corresponding rBn on the right side

3.3 *Modeling infrastructure system performance*

If the system performance is defined in terms of connectivity between two or more system elements, then the eBN can be directly utilized to model system performance as a function of the element performances, which are described by binary random variables (Bensi et al. 2009). While this approach is straightforward for simple system configurations, it can become intricate for more complex systems. However, for the purpose of this paper it is sufficient to note that if the interest is in connectivity, then all the variables required to model the system performance are discrete (binary) and, therefore, are part of the rBN.

If the system performance is defined in terms of system capacity, e.g., the amount of water that can be supplied to a location, then the element performances are typically expressed on a continuous scale, e.g., the amount of water that can be delivered through a particular pipe. In the eBN model, these quantities should generally be represented by discrete random variables, since otherwise the Markov envelope would encompass a large number of system elements. For an example of a pipeline, Figure 4 illustrates how the Markov envelopes can be contained to the individual system elements by discretizing the performance variable (here: discharge).

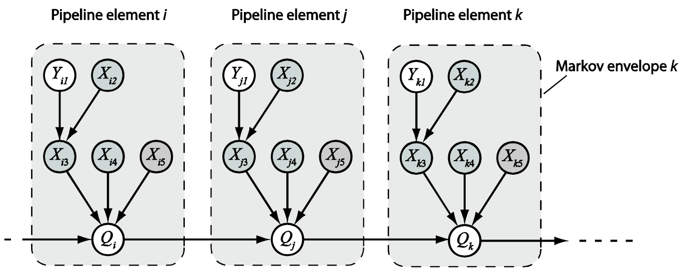

Figure 4. Modeling element and system performances in a pipeline by discrete variables. Q_i, Q_j, Q_k are the performance variables, representing discharge through the pipeline section.

3.4 *Object-oriented Bayesian networks (OOBN)*

When representing large infrastructure systems, the object-oriented Bayesian network (OOBN) methodology (Koller and Pfeffer 1997) facilitates efficient representation of the model. In an OOBN, a class is a Bayesian network in which some of the variables are defined as input and some as output. The instantiations of the classes (the objects) are embedded in a higher level BN, with which they communicate through the input and output variables (the attributes of the class). The OOBN methodology includes the general concepts of object-oriented programming, such as inheritance from a class to a subclass. However, for the purpose of the application to infrastructure analysis, it is sufficient to think of the OOBN as a BN in which sets of variables are grouped into objects, which are either a part of other, higher-level objects or directly of the top-level model. To perform inference, the OOBN is treated like a large BN. This concept, which is quite intuitive, is illustrated in Figure 5, in which rounded rectangles represent objects. As an example, the typical bridge object is connected through its input variable “spectral acceleration,” which is an output variable of the earthquake characteristics object, and its output variable “performance,” which is an input variable to the object transportation system.

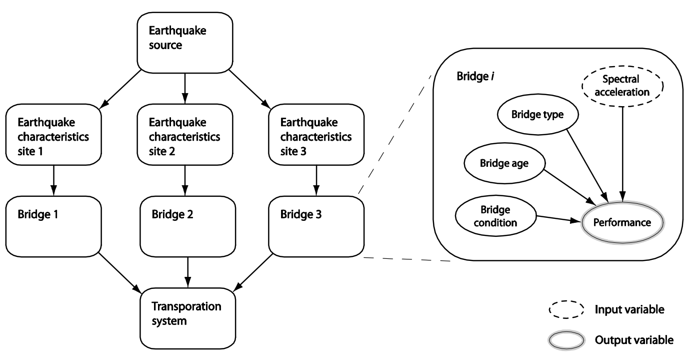

Figure 5. Illustration of the OOBN concept, considering an example of a transportation network with three bridges subject to earthquakes.

3.5 *A spatial-temporal model framework for infrastructure systems*

Based on the presented modeling principles, in the following a general eBN framework for risk analysis in infrastructure systems subject to natural hazards and deterioration is presented. The framework is summarized in Figure 6. The horizontal direction corresponds to the temporal dimension and the vertical direction to the spatial dimension of the problem. All nodes are objects (thus their rounded rectangular shape), i.e., they generally represent lower-level BNs.

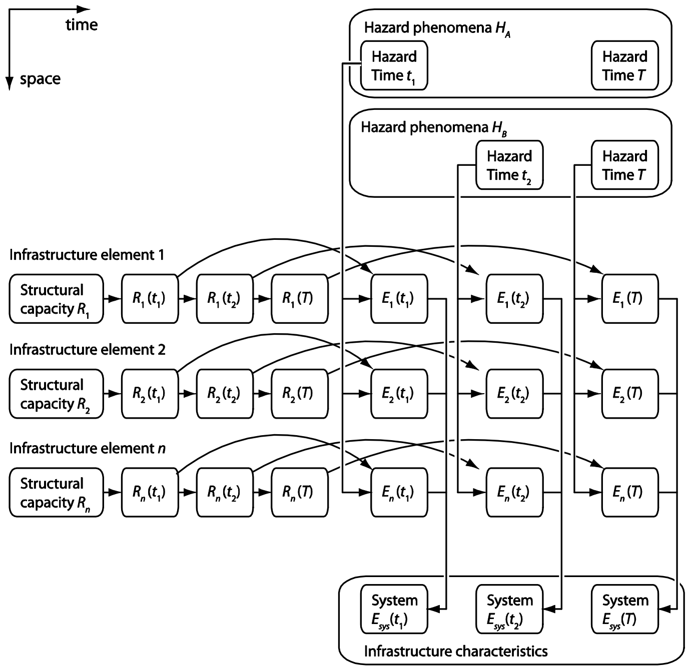

Figure 6. A spatial-temporal eBN model framework for infrastructure systems.

The framework includes objects that represent relevant natural hazards. These have as children instantiations of the hazard classes, each of which models a particular instance of a hazard at time t_i (e.g., a particular windstorm event). Next, the framework includes objects R_j that represent the time-invariant characteristics of the infrastructure elements, such as bridges and tunnels. If the element capacities are deteriorating with time, then there will be a number of objects $R_j(t_i)$ as a function of time, representing the capacities at times t_i. Next, the framework includes objects of the element performances $E_j(t_i)$ at all times t_i at which a hazard occurred (in the past) or for which a future hazard is considered. The element performances include the capacity of the elements at time t_i and the specific characteristics of the hazard at the location of the infrastructure element. Finally, the framework includes a system object that determines the infrastructure system performance $E_{sys}(t_i)$ based on the element performances at times t_i. The time instances that are included in the modeling are all points in time at which observations were made (here t_1 and t_2) and one representative future time T. It is, of course, possible to include additional time steps, but for most applications it is convenient to consider only one future time step and perform repeated evaluations of the eBN for different T.

As a general principle, in modeling an infrastructure system following the above framework, the modeler should aim at confining Markov envelopes to the individual objects shown in Figure 6. In special cases, with limited number of objects, the principle can be violated, but it is believed that most infrastructure systems can be represented by adhering to this principle.

The specific modeling within each of the objects in the framework is not a trivial matter (see Bensi et al. 2009). The final goal should be to derive general rules or guidelines on such modeling. However, this paper will focus on the illustration of the framework by means of an example.

4 EXAMPLE APPLICATION

For illustrating the application of the eBN framework for modeling infrastructure systems, we consider a system of five structures that fulfill a crucial service, such as hospitals. These structures are susceptible to failure due to extreme natural hazard events. To simplify the presentation, we assume that prior to any observation the structures have similar characteristics and are represented by identical probabilistic models. We define the infrastructure system failure as the event of failure of two or more structures during a hazard event.

Each structural system i is a one-bay elasto-plastic frame under vertical load V_i and horizontal load H_i, as shown in Figure 7. This structural system has been investigated by many authors and has been modeled using the eBN approach in Straub & Der Kiureghian (2008b). The performance of each structure is modeled by the binary variable E_i, with $E_i = 0$ being failure and $E_i = 1$ being survival of the structure.

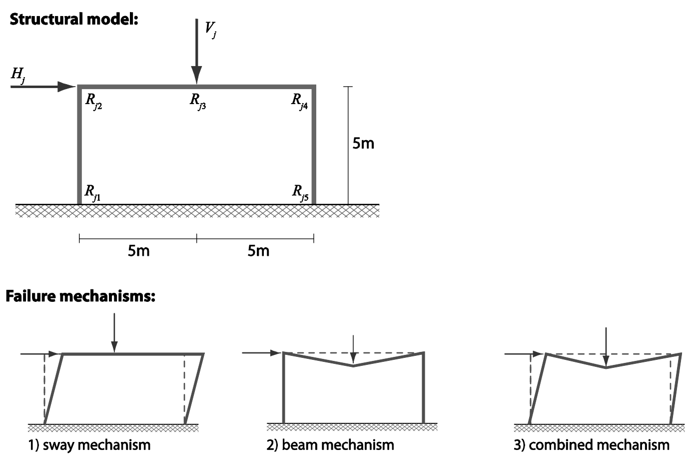

Figure 7. Typical structural system and its failure modes.

The horizontal load $H_j(t_i)$ is the maximum environmental load (e.g., earthquake or wind load) acting on the structure during a hazard event at time t_i. We use the variable $H(t_i)$ to denote the regional hazard characteristics, and we model the local force $H_j(t_i)$ conditional on $H(t_i)$, assuming statistical independence among the various $H_j(t_i)$ for given $H(t_i)$. To illustrate the effect of epistemic (model or statistical) uncertainty, we model the standard deviation of the local hazard $H_j(t_i)$, σ_{Hj}, as a random variable. The gravity load V_j is assumed to be constant with time, and statistically independent from location to location.

The initial element capacities (hinge plastic moment capacities) $R_{jk,}(t_0)$ within a structure are equicorrelated, but statistically independent among structures. In the model we include the possibility of performing measurements of the element capacities at time t_0. A measurement M_{jk} will result in the true element capacity $R_{jk,}(t_0)$ plus an additive measurement error ε_{jk}:

$$M_{jk} = R_{jk}(t_0) + \varepsilon_{jk} \tag{1}$$

The resulting probabilistic model is summarized in Table 1.

Table 1. Probabilistic model (excluding deterioration).

Variable	Distribution	Mean	St.Dev.	Correlation
$R_{jk}(t_0)$ i,j=1,...,5 [kNm]	Joint lognormal	150	30	$\rho_{R_{j,k}R_{j,l}} = 0.3, k \neq l$
V_j [kN]	Gamma	60	12	-
$H_j(t_i)$ [kN]	Lognormal	$H(t_i)$	σ_{Hj}	-
$H(t_i)$ [kN]	Weibull	30	15	-
σ_{Hj} [kN]	Lognormal	10	6	-
ε_k [kNm]	Normal	0	10	-

The eBN modeling of an individual structure, including measurements of element capacities, is presented in Straub & Der Kiureghian (2008b). The performance of the structure at time t_0 is described by three limit state functions corresponding to the failure mechanisms:

$$\begin{aligned} g_{j1}(t_0) &= r_{j1}(t_0) + r_{j2}(t_0) + r_{j4}(t_0) + r_{j5}(t_0) - 5h_j(t_0) \\ g_{j2}(t_0) &= r_{j2}(t_0) + 2r_{j3}(t_0) + r_{j4}(t_0) - 5v_j \\ g_{j3}(t_0) &= r_{j1}(t_0) + 2r_{j3}(t_0) + 2r_{j4}(t_0) + r_{j5}(t_0) - 5h_j(t_0) - 5v_j \end{aligned} \tag{3}$$

The structure fails if any of the mechanisms occur. For a given horizontal loading h_j, the probability of failure of structure j at time zero (which is computed with SRM) is thus

$$\begin{aligned} &\Pr\left[F_j(t_0) \mid H_j(t_0) = h_j\right] = \\ &\Pr\left[\{g_1(t_0) \leq 0\} \cup \{g_2(t_0) \leq 0\} \cup \{g_3(t_0) \leq 0\} \mid H_j(t_0) = h_j\right] \end{aligned} \tag{4}$$

We introduce the variable $R_j(t_0)$ to denote the initial capacity of the structure with respect to the horizontal load h_j, and note that the conditional cumulative distribution function (CDF) of $R_j(t_0)$ (the fragility function) is

$$F_{R_j(t_0)}(r) = \Pr[F_j(t_0) \mid H_j(t_0) = r] \tag{5}$$

Next, we consider the deterioration of the structures. Deterioration is modeled by factors $D_j(t_{i-1}, t_i)$, such that the capacity of the structure at time t_i is

$$R_j(t_i) = R_j(t_{i-1}) \cdot D_j(t_{i-1}, t_i) \tag{6}$$

Simplifying, $D_j(t_{i-1}, t_i)$ is a Beta distributed random variable with mean $1 - 0.01(t_i - t_{i-1})$ and standard deviation $0.03(t_i - t_{i-1})^{0.5}$, defined in the range zero to one. It is possible to implement a more sophisticated deterioration model, e.g., as in Straub (2009). However, as discussed later, there are certain fundamental limitations to the deterioration modeling in the context of infrastructure system analysis by BN.

The eBN model of the infrastructure system is summarized in Figures 8 and 9. To enhance readability, the temporal and spatial dimensions of the model are shown separately. Note that we have adhered to the principle of containing the Markov envelopes within the objects of the eBN. It is assumed that measurements will be performed on R_{j4}. For that reason R_{j4} is modeled as a discrete random variable. In this example, SRM calculations are performed only to determine the initial capacity of each structure.

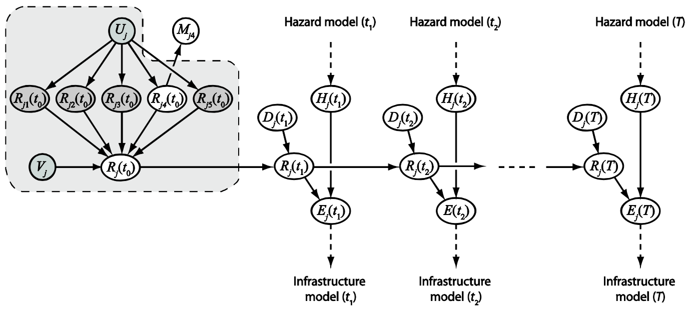

Figure 8. Model of structure j in the infrastructure eBN (temporal dimension).

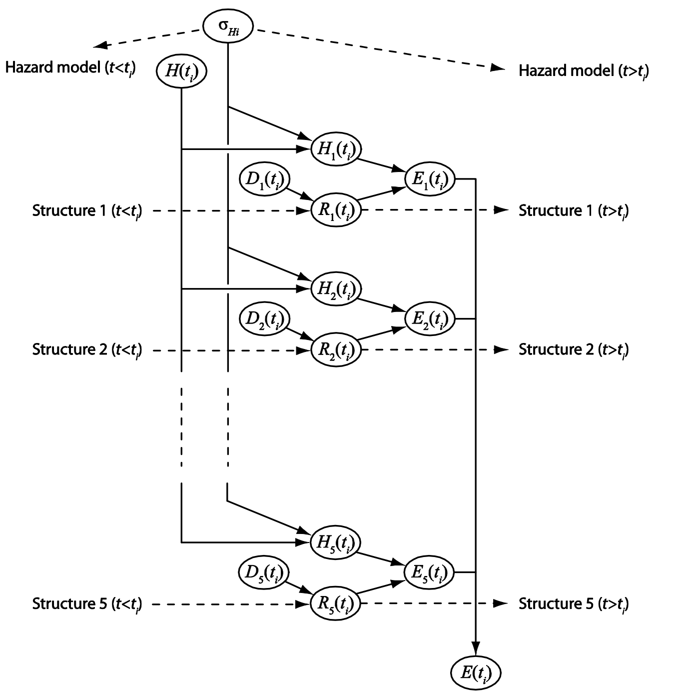

Figure 9. eBN model of the infrastructure at an instance of time (spatial dimension).

The infrastructure performance $E(t_i)$ is represented by a single node in the eBN. In the terminology of Bensi et al. (2009), this is the "naïve" modeling approach. Given the small number of structures (infrastructure elements) in this example, this approach is reasonable. In the general case, a different approach following Bensi et al. (2009) would be appropriate.

4.1 *Computations*

The SRM computations in the example are performed by FORM analysis. The resulting rBN is computed using the free BN software Genie available from the Decision Systems Laboratory of the University of Pittsburgh. Figure 10 shows the reliability of the infrastructure system as a function of time for the case where no evidence is available (the design situation). Without evidence, Monte Carlo simulation (MCS) can be employed to validate the results obtained with the eBN approach, as shown in Figure 10.

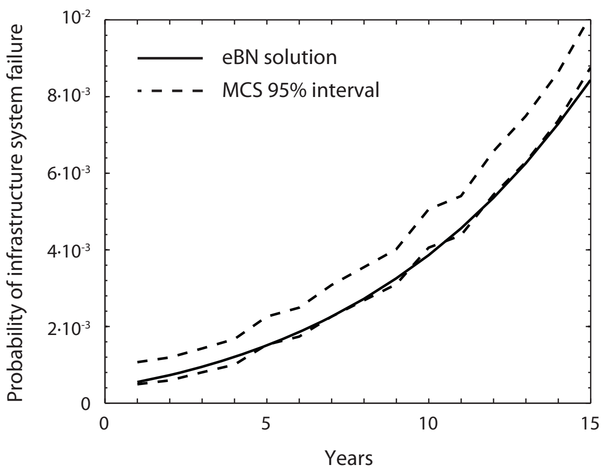

Figure 10. Probability of failure of the infrastructure system as a function of time – no evidence case.

The eBN facilitates updating the reliability of the system for any evidence. Exemplarily, we consider five evidence cases, assuming that the infrastructure system has been subjected to two hazard events in the past. The evidence cases are summarized in Table 2. Note that we include only observations of survival of structures. If failures were observed, it would be necessary to modify the eBN to account for the disposition of the failed structure, e.g., rebuilt or repaired. Such modifications are straightforward, but are not included due to page limitation.

4.2 *Results*

Generally, observing likely events has little influence on the probabilistic model. In the case of the considered example, observing that a structure has not failed alters the reliability only marginally. Note that this is not necessarily true for systems with little uncertainty in the loading because of an implicit proof-loading effect (Nishijima and Faber 2008). Therefore, we neglect such evidence, even though in cases 2-5 it is known that the infrastructure system survived in years 1-4 (for t_1 = 5 years) and 6-9 (for t_2 = 10 years).

Table 2. Evidence cases for the numerical investigation.

Observed variable	Case 1	Case 2	Case 3	Case 4	Case 5
M_{14} [kNm]	100	-	-	-	100
M_{24} [kNm]	120	-	-	-	120
M_{34} [kNm]	160	-	-	-	160
M_{44} [kNm]	90	-	-	-	90
M_{54} [kNm]	120	-	-	-	120
$H(t_1)$ [kN]	-	100	100	>50	100
$H(t_2)$ [kN]	-	80	80	>50	80
$\boldsymbol{E}_1(t_1)$	-	-	1	1	1
$\boldsymbol{E}_1(t_2)$	-	-	1	1	1
$\boldsymbol{E}_2(t_1)$	-	-	1	1	1
$\boldsymbol{E}_2(t_2)$	-	-	1	1	1
$\boldsymbol{E}_3(t_1)$	-	-	1	1	1
$\boldsymbol{E}_3(t_2)$	-	-	1	1	1
$\boldsymbol{E}_4(t_1)$	-	-	1	1	1
$\boldsymbol{E}_4(t_2)$	-	-	1	1	1
$\boldsymbol{E}_5(t_1)$	-	-	1	1	1
$\boldsymbol{E}_5(t_2)$	-	-	1	1	1
$E(t_1)$	-	1	1	1	1
$E(t_2)$	-	1	1	1	1

t_1 = 5yr; t_2 = 10yr

Figure 11 summarizes the results for the different evidence cases. In case 1, which includes the measurement of structural element capacities at time t_0, the posterior probability of failure is increased due to the low measured capacities. The effect is quite pronounced, due to the fact that the element capacities within a structure are positively correlated, so that by measuring one element, information is gained also on the other elements in the structure. In cases 2-4, which include observations of performances of the structures and the infrastructure, the posterior probability of failure is decreased since no failure is observed. The more detailed the available information is, the larger the effect on the posterior model. Case 3 includes the most information. In contrast, case 2 assumes that instead of observing all structures individually, only the entire infrastructure performance is observed. Case 4 assumes that only limited information on the hazard is available: Instead of an exact value of the hazard intensity, it is only known that the hazard event was a strong one. Finally, case 5 demonstrates that it is possible to combine information on the capacity, the hazard and the system performance in a single analysis.

5 DISCUSSION

The presented eBN approach is a powerful tool for reliability analysis of individual structures in the context of infrastructure system analysis and for facilitating learning of the models with observations. However, it is important to be aware of the limitations of the methodology, which may not be obvious to the reader with limited experience with BNs.

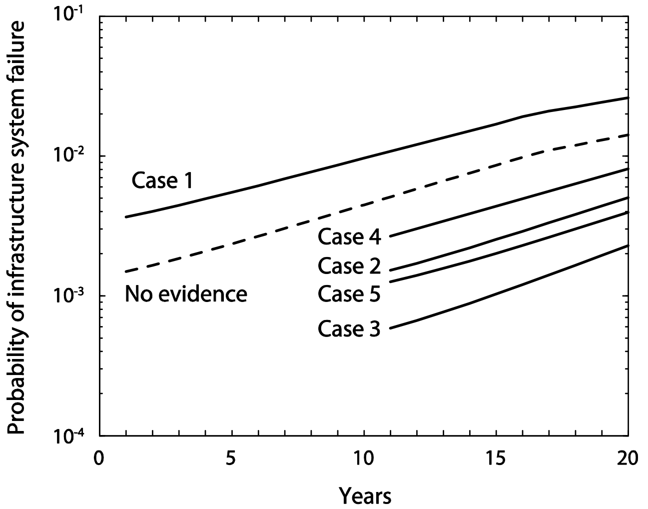

Figure 11. Probability of failure of the infrastructure system as a function of time for the different evidence cases.

The two main limitations of the eBN methodology are: (a) the need for limiting the number of discrete random variables within the Markov envelopes, and (b) the requirement of a rBN with a manageable computational complexity. These are discussed separately in the following.

We have presented strategies to limit the size of the Markov envelopes. These strategies, however, limit our flexibility in modeling. In particular, in order to contain the Markov envelopes to single time steps, we can have only a limited number of variables that are connected from one time step to another. For this reason, deterioration must be modeled at the level of the structural system, e.g., through a reduction in the overall capacity $R_j(t)$, as in the presented example, rather than at the level of structural elements, which is more realistic. With this modeling, it is not clear how measurements of component characteristics at a time *t* can be included in the infrastructure model.

An additional limitation is the need to represent the capacity of the structural system by a few variables only (by one variable $R_j(t)$ in the considered example). Such a model is straightforward as long as there is only one dominant time-varying load, but if there are several dominant load cases, then the capacity of a structure with respect to one load case will be dependent of the capacity with respect to another load case. It is yet to be studied how this dependence can be included in the eBN model.

The computational complexity of the resulting rBN is dependent on the type and amount of available evidence, since, in the general case, solving the rBN requires the computation of the joint probability of all evidence. This limits the number of observations of the infrastructure performance with unknown or uncertain hazard (such as in example case 4). In this case, the computational complexity increases exponentially with the number of observations. A second limitation is on the number of infrastructure elements that can be considered. If evidence on past infrastructure system performance is available, the algorithm for solving the rBN must manipulate the joint distribution of the performance of all elements at one time step. This limits the applicability of the model to around 15-20 infrastructure elements. For the presented example, these limitations were not observed as only two past events and five infrastructure elements were considered.

6 CONCLUSIONS

A framework for combining structural reliability methods with Bayesian network for infrastructure risk analysis is presented. Structural reliability methods allow computation of failure probabilities for structural components or systems defined by continuous random variables. The BN methodology facilitates updating of the model with evidence, such as results of inspections or observations of hazards and system performances. The applicability of the framework has been demonstrated on an example and its limitations have been discussed.

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