Unitary Precoding for MIMO Interference Networks

Andreas Dotzler,* Guido Dietl,† and Wolfgang Utschick.*

* Associate Institute for Signal Processing, Technische Universität München, 80290 Munich
E-Mail: {dotzler,utschick}@tum.de, Tel.: +49-89-289-28510, Fax: +49-89-289-28504

† DOCOMO Euro-Labs and University of Applied Sciences Landshut, E-Mail: dietl@ieee.org

***Abstract*—For MIMO interference networks, uncertainty in the spatial structure of interfering signals is a major source of performance degradation. In this work, we promote the use of linear unitary precoding, as it can be designed such that downlink transmission becomes more robust. Optimizing linear unitary precoding is a combinatorial and nonconvex problem, thus no efficient methods to compute global optimal solutions are available. Therefore, methods with reasonable complexity and acceptable performance are desired and have been investigated in research literature as well as for practical implementation. Contrary to existing work on unitary precoding, which considers receivers with a single antenna, we target scenarios with multiple receive antennas. We introduce and discuss a low-complexity method for successive user and precoder selection to enable unitary precoding for multi-antenna receivers. In addition to interference robustness, our novel method has several advantages for potential implementations, concerning the channel feedback and computation of optimal receive filters. Initial results by numerical simulations indicate that our approach has the potential to outperform existing methods.**

I. Introduction

In wireless interference networks the quality of the received signal, and therefore the data rate of the user, depends on the chosen transmit strategy, the properties of the wireless channel, noise level, and interference. The interdependence of the data rates of the users due to interference and limited resources makes it difficult to optimize the physical layer parameters, which is especially challenging in MIMO networks, as the availability of additional degrees of freedom is directly reflected in the spatial signature of the interference. The degradation of the transmission rates due to unexpected changes of the inter-cell interference is sometimes called "flash-light" effect [1]. A detailed discussion and several ideas to handle the problem can be found in [2]. Additionally, the uncertainty concerning interference results in a mismatch of presumed data rates and the rates actually achievable, this, in turn, causes impairments at the higher layers, for example the scheduler.

A potential solution is transmitter cooperation by central coordination of all transmit strategies. In general, such central approaches have high demands concerning the channel state information (CSI) available at the transmitters; usually the channels between the users and the interfering transmitters need to be known. However, it is unclear if the gains achieved do compensate for the costs to acquire cross-channel CSI in state-of-the-art deployable networks.

In this work, we focus on strategies that operate without estimation and feedback of interfering channels. Thus, interference management is limited to coordinated resource allocation in the time or frequency domain, for example via fractional reuse [3]. In turn, this demands downlink strategies that are more robust to the uncertainty in interference. In [4] two theoretic concepts are discussed. One considers the unknown interference by a worst case approximation, while the other constrains the set of allowed downlink strategies, such that interference can be better predicted. For both ideas an algorithm to compute the optimal downlink strategy with dirty paper coding (DPC) is presented. However, the high complexity of DPC prohibits practical implementation. In [2] a method termed *stabilization* suggests to restrict downlink transmission to a single user and aiming at diversity. This approach is easy to implement and robust to interference. However it sacrifices the benefits of spatial multiplexing.

These two extremes motivate to research and develop robust approaches that have a realistic chance for implementation, while keeping the so much desired abilities of adaptive MIMO transmission; meaning they allow to serve multiple users on the same resource or to provide multiple streams per user. In this work, we identify linear unitary precoding [5]–[10], also known as orthogonal beamforming or orthogonal precoding, as a potential candidate.

Unitary precoding can be implemented such that the sum transmit covariance of every transmitter is fixed and known in advance, which implies two attractive properties. First, the intra-cell interference depends only on the own channel and the own precoder and is therefore independent of the scheduled user set and their precoders. Second, for interference networks, if we assume that the channels do not change, the inter-cell interference does not change. This implies that the effect of interference can be correctly considered when deciding for the downlink strategy. Additionally, this allows for better link rate adaptation.

Unitary precoding is frequently applied in limited-feedback MIMO systems based on predefined precoder codebooks. The most popular method is *per-user unitary rate control* (PU2RC) [5], which forms the basis for the MIMO capabilities in recent releases of 3GPP-LTE.

Based on CSI at the transmitter, adaptive unitary precoding has been considered by multiple authors [6]–[10]. A global optimal algorithm for the two user case is presented in [8] and algorithms to find a local optimal solution can be found in [9], [10]. In [6], [7] low-complexity solutions for joint user and precoder selection are introduced. In Section III, we recapitulate existing work on unitary precoding in more detail. To the best of our knowledge adaptive unitary precoding has not been considered for multi-antenna receivers, which introduces some new challenges. For practical implementations a receive filter to provide a scalar input to the decoder is desired, in order to reduce complexity. The choice of the receive filter affects the maximally achievable data rate.

For our main contribution, we combine a MMSE based rate formulation, as for example used in [11], with the fixed and known interference statistics assumption of unitary precoding. Based on this formulation we introduce a successive user and precoder selection method that can be alternatively considered an extension of the work in [6] to multi-antenna receivers or an adaptation of the zero-forcing based algorithm in [12] to unitary precoding. We discuss several advantages of our method and show implications on the handling of interference, the computation of channel state feedback, and receive filter selection. Finally, we present numerical results, where our novel approach preforms at least as good as existing approaches, while having lower complexity and allowing for better link rate adaptation, therefore removing (or drastically reducing) the need for retransmissions or HARQ processes. Finally, we draw some conclusions and suggest future research directions and challenges in the field of unitary precoding in interference networks.

II. System Model

We consider the downlink of a wireless network with a set of users $\mathcal{K}, K = |\mathcal{K}|$ and a set of transmitters $\mathcal{T}, T = |\mathcal{T}|$. Every user k is served by a transmitter $t(k) \in \mathcal{T}$. Without loss of generality, we assume that all transmitters have N_{tx} transmit antennas and all users have N_{rx} receive antennas. The memoryless MIMO channel from transmitter t to user k is denoted as $\boldsymbol{H}_{kt} \in \mathbb{C}^{N_{\text{rx}} \times N_{\text{tx}}}$. The transmit symbol vector of a transmitter t is $\boldsymbol{x}_t \in \mathbb{C}^{N_{\text{tx}}}$ and the receive signal $\boldsymbol{y}_k \in \mathbb{C}^{N_{\text{rx}}}$ of user k is given by

$$\boldsymbol{y}_k = \sum_{t \in \mathcal{T}} \boldsymbol{H}_{kt} \boldsymbol{x}_t + \boldsymbol{\eta}_k \tag{1}$$

where $\boldsymbol{\eta}_k$ is additive zero-mean Gaussian noise with covariance $\boldsymbol{C}_{\boldsymbol{\eta},k} = \mathrm{E}\left[\boldsymbol{\eta}_k \boldsymbol{\eta}_k^{\mathrm{H}}\right]$.

The data for a stream d intended for a user $k(d)$ is encoded in a zero-mean Gaussian signal s_d with variance $\mathrm{E}\left[|s_d|^2\right] = P_d$. A transmit beamforming filter $\boldsymbol{v}_d$, with $\boldsymbol{v}_d^{\mathrm{H}} \boldsymbol{v}_d = 1$, is used to generate the corresponding transmit symbol $\boldsymbol{v}_d s_d$. The set of data streams emitted by transmitter t is $\mathcal{D}_t$ and the resulting transmit symbol vector is $\boldsymbol{x}_t = \sum_{d \in \mathcal{D}_t} \boldsymbol{v}_d s_d$. As s_d is a zero mean Gaussian random variable, the statistic of the transmit symbol is fully described by its covariance

$$\boldsymbol{Q}_t = \mathrm{E}\left[\boldsymbol{x}_t \boldsymbol{x}_t^{\mathrm{H}}\right] = \sum_{d \in \mathcal{D}_t} P_d \boldsymbol{v}_d \boldsymbol{v}_d^{\mathrm{H}}.$$

Consider a specific data stream d' to a user $k' = k(d')$ sent by transmitter $t' = t(k')$. The receive symbol vector can be partitioned into a useful signal, an unintended signal by datastreams $\mathcal{D}_{t'} \setminus d'$ of the same transmitter (intra-cell interference), and an unintended signal from neighboring transmitters $\mathcal{T} \setminus t'$ (inter-cell interference):

$$\boldsymbol{y}_{d'} = \underbrace{\boldsymbol{H}_{k't'} \boldsymbol{v}_{d'} s_{d'}}_{\text{useful signal}} + \underbrace{\sum_{d \in \mathcal{D}_{t'} \setminus d'} \boldsymbol{H}_{k't'} \boldsymbol{v}_d s_d}_{\text{intra-cell interference}} + \underbrace{\sum_{t \in \mathcal{T} \setminus t'} \boldsymbol{H}_{k't} \boldsymbol{x}_t}_{\text{inter-cell interference}} + \boldsymbol{\eta}_{k'}.$$

The covariance of the intra-cell interference is

$$\boldsymbol{C}_{\text{intra},d'} = \sum_{d \in \mathcal{D}_{t'} \setminus d'} P_d \boldsymbol{H}_{k't'} \boldsymbol{v}_d \boldsymbol{v}_d^{\mathrm{H}} \boldsymbol{H}_{k't'}^{\mathrm{H}} \tag{2}$$

and the covariance of the inter-cell interference is

$$\boldsymbol{C}_{\text{inter},k'} = \sum_{t \in \mathcal{T} \setminus t'} \boldsymbol{H}_{k't} \boldsymbol{Q}_t \boldsymbol{H}_{k't}^{\mathrm{H}}, \tag{3}$$

which is the same for all streams to user k'.

In order to decode the stream, a receive filter $\boldsymbol{u}_{d'}^*$ is used to create a scalar symbol $y_{d'} = \boldsymbol{u}_{d'}^{\mathrm{H}} \boldsymbol{y}_{d'}$. Under the assumption that each stream is filtered and decoded independently, the achievable data rate of the stream can be upper bounded by the mutual information $r_{d'}$ between the two Gaussian signals $y_{d'}$ and $s_{d'}$:

$$r_{d'} = \log_2 \left(1 + \frac{P_{d'} |\boldsymbol{u}_{d'}^{\mathrm{H}} \boldsymbol{H}_{k't'} \boldsymbol{v}_{d'}|^2}{\boldsymbol{u}_{d'}^{\mathrm{H}} \left(\boldsymbol{C}_{\text{intra},d'} + \boldsymbol{C}_{\text{inter},k'} + \boldsymbol{C}_{\boldsymbol{\eta},k'} \right) \boldsymbol{u}_{d'}} \right). \tag{4}$$

III. Unitary Precoding – Existing Work

Collecting the precoders of datastreams $\mathcal{D}_t$ as the columns of a matrix, the precoding matrix of transmitter t is $\boldsymbol{V}_t = \left[\boldsymbol{v}_1, \ldots, \boldsymbol{v}_{|\mathcal{D}_t|}\right]$. For unitary precoding, the precoders are selected such that $\boldsymbol{V}_t$ is unitary. If we assume that every transmitter sends the maximal number of N_{tx} datastreams and equally allocates the available power P among the datastreams, that is $P_d = \frac{P}{N_{\text{tx}}}\ \forall d \in \mathcal{D}_t$, the transmit covariances are given by

$$\boldsymbol{Q}_t = \sum_{d \in \mathcal{D}_t} P_d \boldsymbol{v}_d \boldsymbol{v}_d^{\mathrm{H}} = \frac{P}{N_{\text{tx}}} \boldsymbol{V}_t \boldsymbol{V}_t^{\mathrm{H}} = \frac{P}{N_{\text{tx}}} \boldsymbol{I} \quad \forall t \in \mathcal{T}. \tag{5}$$

This implies two attractive properties of unitary precoding. First, the intra-cell interference covariance (2) only depends on the own precoder $\boldsymbol{v}_d$:

$$\begin{aligned} \boldsymbol{C}_{\text{intra},d'} &= \sum_{d \in \mathcal{D}_{t'} \setminus d'} P_d \boldsymbol{H}_{k't'} \boldsymbol{v}_d \boldsymbol{v}_d^{\mathrm{H}} \boldsymbol{H}_{k't'}^{\mathrm{H}} \\ &= \sum_{d \in \mathcal{D}_{t'}} \frac{P}{N_{\text{tx}}} \boldsymbol{H}_{k't'} \boldsymbol{v}_d \boldsymbol{v}_d^{\mathrm{H}} \boldsymbol{H}_{k't'}^{\mathrm{H}} - \frac{P}{N_{\text{tx}}} \boldsymbol{H}_{k't'} \boldsymbol{v}_{d'} \boldsymbol{v}_{d'}^{\mathrm{H}} \boldsymbol{H}_{k't'}^{\mathrm{H}} \\ &= \frac{P}{N_{\text{tx}}} \boldsymbol{H}_{k't'} \boldsymbol{H}_{k't'}^{\mathrm{H}} - \frac{P}{N_{\text{tx}}} \boldsymbol{H}_{k't'} \boldsymbol{v}_{d'} \boldsymbol{v}_{d'}^{\mathrm{H}} \boldsymbol{H}_{k't'}^{\mathrm{H}}. \end{aligned} \tag{6}$$

Second, the inter-cell interference covariance (3) is independent of the exact precoding and user schedule of the interfering transmitters

$$\begin{aligned}\boldsymbol{C}_{\text{inter},k'} &= \sum_{t\in\mathcal{T}\setminus t'} \boldsymbol{H}_{k't}\boldsymbol{Q}_t\boldsymbol{H}_{k't}^{\mathrm{H}} \\ &= \sum_{t\in\mathcal{T}\setminus t'} \frac{P}{N_{\text{tx}}}\boldsymbol{H}_{k't}\boldsymbol{H}_{k't}^{\mathrm{H}}.\end{aligned} \tag{7}$$

This means a correct estimate of the maximal achievable data rate (4) and the corresponding SINR is available when selecting the precoder for a datastream. In case any of the transmitters sends less then N_{tx} data streams, but still allocates power $\frac{P}{N_{\text{tx}}}$ to every stream, using (6) and (7) the interference power is over estimated and the true mutual information is strictly larger. This allows for a robust estimate of the achievable data rate (or SINR), which is an important metric for user scheduling and downlink resource allocation. Under the assumptions made, the data rate of every user depends only on its own precoder and is independent of the other users scheduled, thus allowing for efficient and low-complexity user selection schemes. In the following section we recapitulate related work in this field.

A. Codebook Based Unitary Precoding – PU2RC

Unitary precoding is frequently applied in limited-feedback MIMO systems based on predefined precoder codebooks. The most popular method is PU2RC [5], that forms the basis for MIMO capabilities in recent releases of 3GPP-LTE. Codebook based MIMO utilizes a codebook of $Q = 2^B$ precoding vectors, where B is the number of feedback bits available. The activated precoders $\boldsymbol{v}_1, \ldots, \boldsymbol{v}_{|\mathcal{D}_t|}$ of a transmitter t are restricted to the columns of one of the $\frac{2^B}{N_{\text{tx}}}$ unitary matrices, that constitute the codebook. After estimating the channel, every user evaluates his performance, for example the expected SINR, for all Q possible precoders and decides for his preferred codebook entry. The preferred codebook entry together with an indicator for the expected performance are reported to the serving transmitter. PU2RC allows for a simple user scheduler at the transmitter. As interference of the other users is already (correctly) considered by the user, the transmitter can freely combine users that reported an entry of the same precoding matrix, as long as they selected a different codebook entry. Thus, the PU2RC concept elegantly combines limited feedback precoding decisions and user scheduling.

B. Adaptive Unitary Precoding – Single-Antenna Receivers

Assuming (full) channel knowledge at the transmitter, adaptive precoding techniques promises further gains. Methods for linear precoding are often based on zero-forcing (ZF-precoding), as this is the optimal strategy for an isolated cell and zero noise [13]. Finding the optimal ZF-precoding downlink strategy is a combinatorial problem, and there exist several low-complexity solutions [12], [13]. For single antenna receivers ZF-precoding reduces to a user selection, as the precoders can be easily computed in case the user set is given. For multi-antenna receivers, one possible strategy is the *Linear Successive Allocation* (LISA) algorithm introduced in [12], where the authors also provide an overview on existing work on ZF-precoding.

Although less frequently considered, advanced downlink strategies with for adaptive unitary precoding are available [6]–[10]. In the following, we summarize existing literature on adaptive unitary precoding that, to the best of our knowledge, is only available for single-antenna receivers. For scalar receive symbols the noise power is $\sigma^2_{\eta,k'}$, intra-cell interference power (6) is

$$\sigma^2_{\text{intra},d'} = \frac{P}{N_{\text{tx}}}\left\|\boldsymbol{H}_{k't'}\right\|_2^2 - \frac{P}{N_{\text{tx}}}|\boldsymbol{H}_{k't'}\boldsymbol{v}_d|^2$$

and inter-cell interference power (7) is

$$\sigma^2_{\text{inter},k'} = \sum_{t\in\mathcal{T}\setminus t'} \frac{P}{N_{\text{tx}}}\left\|\boldsymbol{H}_{k't}\right\|_2^2.$$

By introducing $\rho_k = \frac{|\boldsymbol{H}_{k't'}\boldsymbol{v}_k|}{\|\boldsymbol{H}_{k't'}\|_2}$ the rate expression (4) can be written as

$$r_{d'} = \log_2\left(1 + \frac{\frac{P}{N_{\text{tx}}}\left\|\boldsymbol{H}_{k't'}\right\|_2^2\rho_k^2}{\sigma^2_{\eta,k'} + \sigma^2_{\text{inter},k'} + \frac{P}{N_{\text{tx}}}\left\|\boldsymbol{H}_{k't'}\right\|_2^2(1-\rho_k^2)}\right), \tag{8}$$

which is the version most commonly used in existing literature.

For a fixed user set, optimization of the precoding matrix under a unitary constraint is, unfortunately, a nonconvex problem. A global optimal algorithm is only known for the two user case [8]. Algorithms to find a local optimal solution can be found in [9], [10]. Both elegantly tackle the optimization of unitary precoding in case the user set is known. However, if the number of users is larger than the number of available transmit antennas the user set has to be determined by an additional user selection scheme, which motivates algorithms that find the user set and precoders jointly [6], [7].

In [7] the first user's precoder is selected to match the users channel. Based on this precoder an orthonormal basis is created (Gram-Schmidt), which is used as the precoding matrix. Now, the remaining users are iteratively allocated to the $N_{\text{tx}} - 1$ determined precoders. The performance can be improved by trying all users as the first user.

In [6] the precoding matrix is created on the fly while performing a successive user and precoder selection. In every iteration, for every user the best precoder that is orthogonal to the already fixed precoders is determined. This can be assured by a projection matrix that itself is iteratively updated. The next user is selected as the one that leads to the highest increase in performance. Similar ideas for successive user allocation are also used for ZF-precoding [12], [13].

IV. Adaptive Unitary Precoding – Multi-Antenna Receivers

To enable adaptive unitary precoding for multi-antenna receivers we use an MMSE based rate expression to formulate the downlink problem, similar to the formulation used in [11]. For multi-antenna receivers we assume that a receive filter is used to obtain a scalar symbol. In this case, the data rate of the user (4) depends on the choice of the receive filter.

For the data stream d' with fixed precoder $\boldsymbol{v}_{d'}$, the filter that minimizes the mean squared error between s_d and y_d (MMSE filter) is given by

$$\boldsymbol{u}_{\mathrm{MMSE},d'}^{\mathrm{H}} = \boldsymbol{v}_{d'}^{\mathrm{H}} \boldsymbol{H}_{k't'} \boldsymbol{C}_{\boldsymbol{y}_{k'}}^{-1} \tag{9}$$

where $\boldsymbol{C}_{\boldsymbol{y}_{k'}}$ is the covariance of the receive symbol vector (1) of user k'. Now, for general precoding strategies the covariance depends on the precoder $\boldsymbol{v}_{d'}$. For unitary precoding and under assumption (5) the covariance of the receive symbol vector is given by

$$\boldsymbol{C}_{\boldsymbol{y}_{k'}} = \boldsymbol{C}_{\boldsymbol{\eta},k'} + \sum_{t\in\mathcal{T}} \frac{P}{N_{\text{tx}}} \boldsymbol{H}_{k't} \boldsymbol{H}_{k't}^{\mathrm{H}},$$

which does not depend on $\boldsymbol{v}_{d'}$.

By plugging (9) into (4) followed by some calculus the rate expression can be formulated as

$$r_{d'} = -\log_2\left(1 - \frac{P}{N_{\text{tx}}} \boldsymbol{v}_{d'}^{\mathrm{H}} \boldsymbol{H}_{k't'}^{\mathrm{H}} \boldsymbol{C}_{\boldsymbol{y}_{k'}}^{-1} \boldsymbol{H}_{k't'} \boldsymbol{v}_{d'}\right). \tag{10}$$

Note that (9) also is the filter that maximizes (4), which in the context of interference networks is known as the interference rejection combiner (IRC), which can be shown to be a scaled version of (9).

Based on (10) we are able to construct a method for unitary linear precoding in the downlink. Every transmitter $t \in \mathcal{T}$ has to determine its N_{tx} datastreams $\mathcal{D}_t \subseteq \mathcal{D}$ by selecting precoders and allocating them to the assigned users $\mathcal{K}_t$. Our idea is to perform a successive stream selection, where in every iteration we add a new data stream by selecting the user and a precoder that maximally increases performance. In this work, performance is measured by a weighted sum of the users data rates. The data rate of a user is the sum of all streams $\mathcal{D}_k$ assigned to the user, that is $r_k = \sum_{d\in\mathcal{D}_k} r_d$. The user weights are $\boldsymbol{w} = [w_1, \ldots, w_K]$.

When a new data stream is added, it needs to be assured that all precoders are mutually orthogonal. As a consequence, if $d-1$ datastreams with precoders $\boldsymbol{V}_{d-1} = [\boldsymbol{v}_1, \ldots, \boldsymbol{v}_{d-1}]$ have already been determined, a condition for the newly added precoder is $\boldsymbol{V}_{d-1}^{\mathrm{H}} \boldsymbol{v}_d = \boldsymbol{0}$. To decide for the next data stream we compute the increase of performance.

In a first step, for ever user we compute the best performing precoder, which is found as

$$\boldsymbol{v}_k \in \underset{\boldsymbol{v}^{\mathrm{H}}\boldsymbol{v}=1}{\operatorname{argmax}} \left\{ \boldsymbol{v}^{\mathrm{H}} \boldsymbol{H}_{kt}^{\mathrm{H}} \boldsymbol{C}_{\boldsymbol{y}_k}^{-1} \boldsymbol{H}_{kt} \boldsymbol{v} : \boldsymbol{V}_{d-1}^{\mathrm{H}} \boldsymbol{v} = \boldsymbol{0} \right\}.$$

The orthogonality constraint can be considered by including a projection matrix

$$\boldsymbol{v}_k \in \underset{\boldsymbol{v}^{\mathrm{H}}\boldsymbol{v}=1}{\operatorname{argmax}} \left\{ \boldsymbol{v}^{\mathrm{H}} \boldsymbol{P}^{(d),\mathrm{H}} \boldsymbol{H}_{kt}^{\mathrm{H}} \boldsymbol{C}_{\boldsymbol{y}_k}^{-1} \boldsymbol{H}_{kt} \boldsymbol{P}^{(d)} \boldsymbol{v} \right\},$$

where

$$\boldsymbol{P}^{(d)} = \boldsymbol{I} - \boldsymbol{V}_{d-1} (\boldsymbol{V}_{d-1}^{\mathrm{H}} \boldsymbol{V}_{d-1})^{-1} \boldsymbol{V}_{d-1}^{\mathrm{H}} = \boldsymbol{I} - \boldsymbol{V}_{d-1} \boldsymbol{V}_{d-1}^{\mathrm{H}}.$$

The preferred precoding vector of user k is the normalized eigenvector that corresponds to the strongest eigenvalue of $\boldsymbol{P}^{(d),\mathrm{H}} \boldsymbol{H}_{kt}^{\mathrm{H}} \boldsymbol{C}_{\boldsymbol{y}_k}^{-1} \boldsymbol{H}_{kt} \boldsymbol{P}^{(d)}$.

Second, after computing the potential data rate of the new stream for every user

$$r_k = -\log_2\left(1 - \frac{P}{N_{\text{tx}}} \boldsymbol{v}_k^{\mathrm{H}} \boldsymbol{P}^{(d),\mathrm{H}} \boldsymbol{H}_{kt}^{\mathrm{H}} \boldsymbol{C}_{\boldsymbol{y}_k}^{-1} \boldsymbol{H}_{kt} \boldsymbol{P}^{(d)} \boldsymbol{v}_k\right),$$

the user that leads to the highest increase in weighted-sum rate

$$k(d) \in \underset{k\in\mathcal{K}_t}{\operatorname{argmax}} \{w_k r_k\},$$

is selected and $\boldsymbol{v}_d = \boldsymbol{v}_{k(d)}$.

Note that our method allows for multiple datastreams per user, but assumes that these streams are filtered and decoded separately. Further, the term $\boldsymbol{M}_k = \boldsymbol{H}_{kt}^{\mathrm{H}} \boldsymbol{C}_{\boldsymbol{y}_k}^{-1} \boldsymbol{H}_{kt}$ is a constant that is specific for every user and can be precomputed in advance, for instance at the user, see Section IV-B. As all precoders are mutually orthogonal, the projection matrix itself can be iteratively updated

$$\boldsymbol{P}^{(d+1)} = \boldsymbol{P}^{(d)} - \boldsymbol{v}_d \boldsymbol{v}_d^{\mathrm{H}}.$$

In the following, we relate our novel approach to existing work. First, it is straight-forward to verify that for single antenna receivers (10) is identical to (8) and therefore our approach can be considered a generalization of the method in [6] to multiple receive antennas. Second, a successive user allocation with linear ZF-precoding for MIMO systems is presented in [12], called *Linear Successive Allocation* (LISA). So our novel approach can alternatively be seen as a modification of the LISA algorithm to unitary precoding. As the authors in [6] did not coin a name for their algorithm, we name the presented method Unitary-LISA, which we summarize in Algorithm 1. In the following we investigate in more detail the implications on system design and point out several advantages of Unitary-LISA.

A. Robustness to Interference

One idea to obtain robust downlink strategies is to remove (or reduce) the uncertainty in interference by selecting a transmission strategy such that the sum transmit covariance is constant. A theoretic analysis can be found in [4], where a constraint $\boldsymbol{Q}_t \preceq \frac{P}{N_{\text{tx}}} \boldsymbol{I}$ is imposed on the sum transmit covariance. However, the presented optimal algorithm assumes DPC and is therefore not relevant for practical implementation.

Under the assumptions made, unitary precoding automatically fulfills the constraint $\boldsymbol{Q}_t \preceq \frac{P}{N_{\text{tx}}} \boldsymbol{I}$ and Unitary-LISA provides an algorithm with low-complexity that might have a realistic chance for implementation.

This advantage of a constraint sum transmit covariance is reflected in the covariance of the receive symbol $\boldsymbol{C}_{\boldsymbol{y}_k}$ that only depends on channels. As channel change much slower than the user schedules, the covariance $\boldsymbol{C}_{\boldsymbol{y}_k}$ can be well estimated. Further, channel realization are correlated over time and frequency. This is reflected in $\boldsymbol{C}_{\boldsymbol{y}_k}$ and these correlations (if known) can be used for better estimation.

for $k \in \mathcal{K}$ **do**

$\quad \boldsymbol{M}_k \leftarrow \boldsymbol{H}_{kt}^{\mathrm{H}} \boldsymbol{C}_{\boldsymbol{y}_k}^{-1} \boldsymbol{H}_{kt}$

end

$\boldsymbol{P}^{(d)} \leftarrow \boldsymbol{I}$

for $d = 1 \ldots, N_{tx}$ **do**

$\quad$ **for** $k \in \mathcal{K}_t$ **do**

$$\boldsymbol{v}_k \in \underset{\boldsymbol{v}^{\mathrm{H}}\boldsymbol{v}=1}{\operatorname{argmax}} \left\{ \boldsymbol{v}^{\mathrm{H}} \boldsymbol{P}^{(d),\mathrm{H}} \boldsymbol{M}_k \boldsymbol{P}^{(d)} \boldsymbol{v} \right\}$$

$$\boldsymbol{r}_k \leftarrow -\log_2 \left(1 - \boldsymbol{v}_k^{\mathrm{H}} \boldsymbol{P}^{(d),\mathrm{H}} \boldsymbol{M}_k \boldsymbol{P}^{(d)} \boldsymbol{v}_k \right)$$

$\quad$ **end**

$$k(d) \in \underset{k \in \mathcal{K}_t}{\operatorname{argmax}} \{ w_k r_k \}$$

$$\boldsymbol{v}_d \leftarrow \boldsymbol{v}_{k(d)}$$

$$\boldsymbol{P}^{(d+1)} \leftarrow \boldsymbol{P}^{(d)} - \boldsymbol{v}_d \boldsymbol{v}_d^{\mathrm{H}}$$

end

Algorithm 1: Unitary-LISA

B. Channel Feedback

Having a good estimate of $\boldsymbol{C}_{\boldsymbol{y}_k}$ enables us to compute $\boldsymbol{M}_k = \boldsymbol{H}_{kt}^{\mathrm{H}} \boldsymbol{C}_{\boldsymbol{y}_k}^{-1} \boldsymbol{H}_{kt}$ in case the users channel is known. Both $\boldsymbol{C}_{\boldsymbol{y}_k}$ and $\boldsymbol{H}_{kt}$ are estimated at the user. We therefore suggest to base the channel feedback for Unitary-LISA on $\boldsymbol{M}_k$. As the value of $\boldsymbol{M}_k$ is correlated over time and frequency one could consider advanced feedback mechanisms, for example based on tracking the eigenmodes of $\boldsymbol{M}_k$. Further, Unitary-LISA can be robust to inter-cell interference without estimation or feedback of interfering channels.

C. Receive Filter

The optimal receive filter (9) for a datastream requires knowledge of the covariance $\boldsymbol{C}_{\boldsymbol{y}'_k}$, which can be easily estimated by using the sample covariance as a sufficient statistic. Further, computing the receive filter requires knowledge of the effective channel $\boldsymbol{v}_{d'} \boldsymbol{H}_{k't'}$, which can be estimated by sending known training symbols that are precoded with $\boldsymbol{v}_{d'}$. In LTE these are called user specific reference signals and were introduced to support non-codebook based precoding [14].

As optimal receive filters are correctly assumed when the transmission strategy is decided, link rate adaptation can be performed without uncertainty, which reduces outages, retransmissions and HARQ processes and therefore improves performance.

V. Simulation Results

To evaluate the performance of the proposed algorithm, we provide numerical simulations using a realistic channel. The spatial signatures of the channels are created by the software IMTAphy [15] using the default parameters for the urban macro-cell scenario. The antenna configuration is $N_{\text{tx}} = N_{\text{rx}} = 4$. We generated channels for a single time-slot, thus assuming zero-mobility and a channel that is constant over time. Corresponding to a bandwith of 10 MHz for uplink and downlink. We consider 50 physical resource blocks (PRBs) for the downlink and for every PRB we compute a single channel matrix in the frequency domain, which is assumed to be representative. We use a scenario with sectorized sites and 3×19 cells. The 570 users, an average of 10 users per cell, are associated to their serving transmitter taking a 1 dB handover margin into account. We consider a proportional fair scheduler that for every time-slot successively computes the weighted sum-rate for all 50 PRBs and updates the weights afterwards. The performance of the users is evaluated by averaging over 1000 time-slots, thus considering 50000 PRBs, each with different weights and therefore different downlink strategies.

As a reference for ZF-precoding we use the LISA algorithm in [12]. For link rate adaptation we used the methods described [2]: *genie* and *gambling*. In a first step, every transmitter decides for its downlink strategy, where we assumed that the inter-cell interference is given by (7). For the *LISA - Gambling* method, the stream is encoded with a rate as presumed by the outcome of the LISA algorithm. In case the real mutual information is smaller than expected, an outage occurs and the throughput for this resource block is zero. The performance could be enhanced by several methods, for example by coding over multiple resource blocks, using a more conservative rate allocation, or by HARQ. Such methods have not been considered in our simulations. Instead, we include a result named *LISA - Genie*, where, as for *LISA - Gambling*, the transmission strategies are computed based on a prediction of the interference. Afterwards, we compute the true mutual information and assume the data streams are coded to this rate and are transmitted error-free. We therefore consider the performance of *LISA - Genie* as an upper bound on what could be achieved by LISA, when more advanced methods for link rate adaptation and/or HARQ are used.

For unitary precoding we include PU2RC with the codebook of LTE, which consists of 16 unitary matrices. Using the rate expression (10) it is possible to extend the existing algorithms to multiple antennas. We further modify them to support weighted sum-rate, enforce an equal power allocation of $\frac{P}{N_{\text{tx}}}$ per stream, and remove a potential stopping criterion that would allocate less than N_{tx} streams. The modified version of algorithm B in [7] is labeled as *Gram-Schmidt*. As discussed in Section IV, the Unitary-LISA algorithm can be considered an extension of the algorithm in [6] to multiple receive antennas.

For the local optimal solution we do not expect a difference in performance of [10] and [9] and therefore implemented the method in [9]. As the local optimal algorithms do not provide a method for user selection and because trying all user combinations is infeasible, we use a simple scheme where we select those N_{tx} users that have the highest weighted rate in case in case they can freely select their precoder.

Figure 1 shows the empirical cumulative distribution of

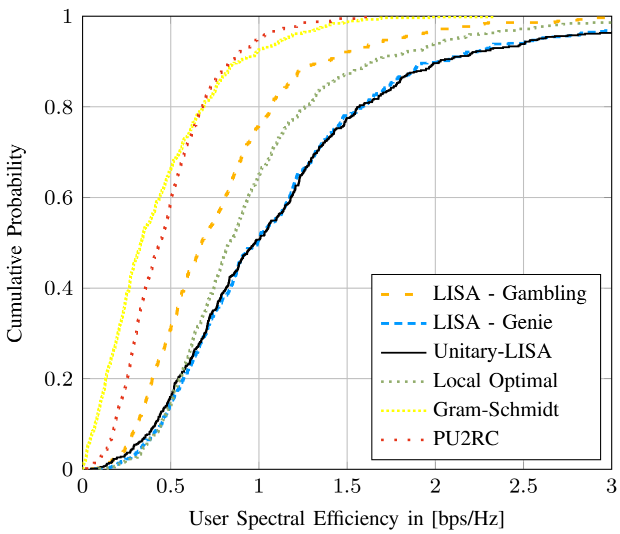

Fig. 1. Empirical Cumulative Distribution of the User Data Rates

the average user data rates. Although one would expect the best performance by the optimization based approach (*Local Optimal*), this approach is outperformed by *LISA - Genie* and *Unitary-LISA*, which certainly results from the suboptimal user selection. This statement is supported by simulations with N_{tx} users per transmitter that have been performed, but are not included in this work. For this scenario, the local optimal approach slightly outperforms all other algorithms.

Further, we observe that *PU2RC* and *Gram-Schmidt* are not competitive. However, one has to consider that *PU2RC* is based on limited feedback contrary to the perfect CSI assumption of the adaptive approaches, which makes it an unfair comparison. See Section VI for further remarks on limited feedback.

Finally, we can see that *Unitary-LISA* clearly outperforms *LISA - Gambling* and closely matches the performance of *LISA - Genie*, that we consider an upper bound. Further, Unitary-LISA has lower complexity and the advantage of better link rate adaptation, therefore removing (or drastically reducing) the need for retransmissions or HARQ processes.

VI. Conclusions and Future Work

The main contribution of our work is the Unitary-LISA algorithm, which enables linear unitary precoding for multi-antenna receivers. Further, we illustrated that unitary precoding reduces the uncertainty in interference, therefore making downlink transmission more robust. Initial results by numerical simulations indicate that Unitary-LISA has the potential to outperform existing methods and should therefore be considered an alternative to ZF-precoding for MIMO interference networks.

For completeness, the numerical simulations included PU2RC, which it is based on limited feedback, contrary to the perfect CSI assumption of the adaptive approaches. It is unclear if the adaptive approaches are still superior in case realistic feedback (and the resulting overhead) is considered. In this work, this has been left aside by considering static channels.

This creates two interesting directions of research in the field of unitary precoding. First one should consider adaptive unitary precoding for limited and quantized channel feedback and, second, one should consider time variant channels. This means to develop and evaluate new methods for estimation and prediction of channels and realistic channel feedback in future work on the topic.

References

[1] A. Osseiran and A. Logothetis, “Closed loop transmit diversity in WCDMA HS-DSCH,” in *Proc. IEEE 61st Vehicular Technology Conference, VTC 2005-Spring*, vol. 1, 2005, pp. 349 – 353.

[2] M. T. Ivrlac and J. A. Nossek, “Intercell-Interference in the Gaussian MISO broadcast channel,” in *IEEE Global Telecommunications Conference, 2007. GLOBECOM ’07*, Nov. 2007, pp. 3195–3199.

[3] A. Dotzler, W. Utschick, and G. Dietl, “Fractional reuse partitioning for MIMO networks,” in *2010 IEEE Global Telecommunications Conference (GLOBECOM 2010)*. IEEE, Dec. 2010, pp. 1–5.

[4] A. Dotzler and W. Utschick, “Interference robustness for MIMO interference networks,” in *SPAWC (invited paper)*, 2012.

[5] H. Kim, C. S. Park, and K. B. Lee, “On the performance of multiuser MIMO systems in WCDMA / HSDPA : Beamforming , feedback and user diversity,” *IEICE Transactions on Communications*, vol. E89-B, no. 8, pp. 2161–2169, 2006.

[6] J. Duplicy, D. P. Palomar, and L. Vandendorpe, “Adaptive orthogonal beamforming for the MIMO broadcast channel,” in *2nd IEEE International Workshop on Computational Advances in Multi-Sensor Adaptive Processing, 2007. CAMPSAP 2007*. IEEE, Dec. 2007, pp. 77–80.

[7] R. de Francisco, M. Kountouris, D. T. Slock, and D. Gesbert, “Orthogonal linear beamforming in MIMO broadcast channels,” in *IEEE Wireless Communications and Networking Conference, 2007.WCNC 2007*. IEEE, Mar. 2007, pp. 1210–1215.

[8] A. Jalali and D. J. Love, “Closed-form expression for optimal two-user MIMO unitary precoding,” *IEEE Communications Letters*, vol. 13, no. 4, pp. 251–253, Apr. 2009.

[9] S. Wagner, S. Sesia, and D. Slock, “On unitary beamforming for MIMO broadcast channels,” in *2010 IEEE International Conference on Communications (ICC)*. IEEE, May 2010, pp. 1–5.

[10] R. de Francisco and D. T. Slock, “An optimized unitary beamforming technique for MIMO broadcast channels,” *IEEE Transactions on Wireless Communications*, vol. 9, no. 3, pp. 990–1000, Mar. 2010.

[11] A. J. Tenenbaum and R. S. Adve, “Linear processing and sum throughput in the multiuser MIMO downlink,” *IEEE Transactions on Wireless Communications*, vol. 8, no. 5, pp. 2652–2661, May 2009.

[12] C. Guthy, W. Utschick, and G. Dietl, “Low-Complexity linear Zero-Forcing for the MIMO broadcast channel,” *IEEE Journal of Selected Topics in Signal Processing, Special Issue on Managing Complexity in Multiuser MIMO Systems*, vol. 3, no. 6, pp. 1106–1117, 2009.

[13] T. Yoo and A. Goldsmith, “On the optimality of multiantenna broadcast scheduling using zero-forcing beamforming,” *IEEE Journal on Selected Areas in Communications*, vol. 24, no. 3, pp. 528– 541, Mar. 2006.

[14] S. Sesia, I. Toufik, and M. Baker, *LTE, The UMTS Long Term Evolution: From Theory to Practice*. Wiley, Apr. 2009.

[15] J. Ellenbeck, O. Mushtaq, F. Sheikh, and Q. Safiullah Shahrukh, “Calibrating an efficient C++ implementation of the ITU-R M.2135 channel model for use in system-level simulations,” in *International Workshop on Propagation and Channel Modeling for Next-Generation Wireless Networks*, Mar. 2011. [Online]. Available: http://www.lkn.ei.tum.de/personen/jan/imtaphy/iwpcm2011.pdf