Ruin probabilities in the presence of heavy–tails and interest rates

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Abstract

We study the infinite time ruin probability for the classical Cramér-Lundberg model, where the company also receives interest on its reserve. We consider the large claims case, where the claim size distribution F has a regularly varying tail. Hence our results apply for instance to Pareto, loggamma, certain Benktander and stable claim size distributions. We prove that for a positive force of interest δ the ruin probability $\psi_\delta(u) \sim \kappa_\delta(1 - F(u))$ as the initial risk reserve $u \to \infty$. This is quantitatively different from the non-interest model, where $\psi(u) \sim \kappa \int_u^\infty (1 - F(y))dy$.

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1 Introduction

We consider the classical Cramér-Lundberg model as follows:

(i) The claim sizes $(X_k)_{k\in\mathbb{N}}$ are positive iid rv's having common distribution function F and finite mean $\mu = EX_1 < \infty$.

(ii) The claims happen at random time points $0 < T_1 < T_2 < \dots$. The claim arrival process $(N(t))_{t\ge 0}$ defined by

$$N(t) = \begin{cases} card\,\{k \ge 1 : T_k \le t\} & t > 0\,, \\ 0 & t = 0\,, \end{cases}$$

is a homogeneous Poisson process with intensity $\lambda > 0$.

(iii) Finally, $(N(t))_{t\ge 0}$ and $(X_k)_{k\in\mathbb{N}}$ are assumed to be independent processes.

The classical risk process $(U(t))_{t\ge 0}$ is defined as

$$U(t) = u + ct - S(t)\,, \quad t \ge 0\,,$$

with large initial capital u, total claim amount $S(t) = \sum_{n=1}^{N(t)} X_n$ and premium rate $c > 0$. Hence $U(t)$ describes the risk reserve at time $t \ge 0$.

In addition to the linear premium income the company also receives interest on its reserves and we assume a constant force of interest $\delta > 0$. (Notice that $\delta > 0$ can also model the difference of the force of interest and the force of inflation.) The risk reserve is then

$$U_\delta(t) = ue^{\delta t} + c\int_0^t e^{\delta v}\,dv - \int_0^t e^{\delta(t-v)}dS(v)\,, \quad t \ge 0\,.$$

The *ruin probability in infinite time* is defined as

$$\psi_\delta(u) = P\left(\inf_{0\le t<\infty} U_\delta(t) < 0 \mid U_\delta(0) = u\right)\,, \quad u \ge 0\,. \tag{1.1}$$

Our main result in this context is Corollary 2.4 where we prove for claimsize distribution functions with regularly varying tail that for $\delta > 0$

$$\psi_\delta(u) \sim \kappa_\delta\,(1 - F(u))\,, \quad u \to \infty\,,$$

for some explicit positive constant κ_δ. ($\sim$ means that the quotient of lefthand side and righthand side tends to 1 as $u \to \infty$.)

This paper can be considered as a heavy-tailed counterpart of a recent paper by Sundt and Teugels (1995). They study the behaviour of $\psi_\delta(u)$ under a Cramér condition. They derive bounds for $\psi_\delta(u)$ in the case of exponentially fast decreasing tails of the claim size distribution function F. Thus their work follows the tradition of Gerber (1971), Delbaen and Haezendonck (1987), Boogaert and Crijns (1987), Boogaert and Haezendonck (1987), Embrechts and Dassios (1989), Embrechts and Schmidli (1994) and Willmot (1989).

The proofs in Sundt and Teugels (1995) are based on the integral equation (1.2) which has been derived by martingale methods by Delbaen and Haezendonck (1987). This equation also serves as the starting point of our investigation. Furthermore, we write $\psi(u) = \psi_0(u)$ for the ruin probability in the classical case.

The survival probability is then

$$\overline{\psi}_\delta(u) = 1 - \psi_\delta(u)\,, \quad u \geq 0\,.$$

Similarly we write

$$\overline{F}(u) = 1 - F(u)\,, \qquad \overline{F}_I(u) = \frac{1}{\mu}\int_u^\infty \overline{F}(v)\,dv\,, \quad u \geq 0\,,$$

for the tail of the claim size distribution and of the integrated tail distribution function of F respectively. The basic integral equation is given as (see Sundt and Teugels)

$$\overline{\psi}_\delta(u) = \frac{c}{c+\delta u}\overline{\psi}_\delta(0) + \frac{1}{c+\delta u}\int_0^u \overline{\psi}_\delta(u-y)(\delta + \lambda\overline{F}(y))dy\,, \quad u \geq 0\,. \tag{1.2}$$

Defining

$$G_\delta(u) = \frac{\overline{\psi}_\delta(u) - \overline{\psi}_\delta(0)}{1 - \overline{\psi}_\delta(0)}\,, \quad u \geq 0\,, \tag{1.3}$$

we find that $G_\delta(u)$ is a distribution function with tail

$$1 - G_\delta(u) = \psi_\delta(u)/\psi_\delta(0)\,, \quad u \geq 0\,,$$

hence it is up to a constant the ruin probability. The above equation (1.2) transforms into

$$cG_\delta(u) + \delta\int_0^u v\,dG_\delta(v) = K_\delta F_I(u) + \rho\,G_\delta * F_I\,, \tag{1.4}$$

where $K_\delta = \rho\overline{\psi}_\delta(0)/\psi_\delta(0)$, or

$$\psi_\delta(0) = \frac{\rho}{K_\delta + \rho} \tag{1.5}$$

and $\rho = \lambda\mu$.

The natural class of heavy–tailed claim size distribution functions in risk theory is the class of subexponential distributions. We recall that H with $H(0) = 0$ is a *subexponential distribution function* if

$$\lim_{x\to\infty} \frac{\overline{H}^{n*}(x)}{\overline{H}(x)} = n$$

holds for all $n \in I\!N$ where H^{n*} denotes the n-fold convolution of H. For an overview of their properties and their importance in risk theory see Embrechts, Klüppelberg and Mikosch (1996).

In the classical case with zero interest, $c > \rho$ and subexponential F_I the following asymptotic relation has been derived by Embrechts and Veraverbeke (1982)

$$\psi(u) \sim \frac{\kappa}{\mu} \int_u^\infty \overline{F}(v)dv = \kappa\, \overline{F}_I(u)\,, \quad u \to \infty\,, \tag{1.6}$$

where $\kappa = \rho/(c-\rho)$.

The main purpose of this paper is to develop a similar result for the case with interest rates. Unfortunately, our methods, which are based on Abel-Tauber-theorems are restricted to the subclass of df's with regularly varying tails and are not extendable to the whole class of subexponential claim size distributions.

We prove that for $\delta > 0$ (1.6) is no longer valid. Instead we obtain for $\delta > 0$

$$\psi_\delta(u) \sim \kappa_\delta \overline{F}(u)\,, \quad u \to \infty\,, \tag{1.7}$$

where κ_δ is some positive constant. The result, although unexpected, becomes clear from equation (2.11): for $\delta = 0$ the second term on the lhs vanishes, leaving an equation which relates the asymptotics of the tails of G_δ (hence of ψ_δ) and F_I. For $\delta > 0$, however, the second term on the lefthand side gives the leading term of the asymptotic result, yielding (1.7). By quite different methods, Asmussen (1996) obtained recently analogous results for the whole class of subexponential claimsize distributions.

2 Ruin estimates in the case of heavy–tails and interest rates

The key to the approximation (1.6) is the Pollacek–Khinchine formula

$$\overline{\psi}(u) = \left(1 - \frac{\rho}{c}\right) \sum_{n=0}^{\infty} \left(\frac{\rho}{c}\right)^n F_I^{n*}(u)\,, \tag{2.8}$$

where $\rho = \lambda\mu$. Taking tails on both sides and dividing by $\overline{F}_I(u)$ yields for $c > \rho$

$$\frac{\psi(u)}{\overline{F}_I(u)} = \left(1 - \frac{\rho}{c}\right) \sum_{n=0}^{\infty} \left(\frac{\rho}{c}\right)^n \frac{\overline{F}_I^{n*}(u)}{\overline{F}_I(u)}\,.$$

Then the definition of subexponential df's and an application of Lebesgue's dominated convergence theorem imply convergence of the quotient to $\kappa = \rho/(c-\rho)$ giving the result (1.6).

In the presence of interest rates an explicit Pollacek–Khinchine formula as (2.8) can not be obtained. But on the level of Laplace transforms a similar representation is possible.

Unfortunately, there exists no representation of subexponential distributions in terms of Laplace transforms. But for the subclass of df's with regularly varying tails, Abel and Tauberian theorems can be applied and yield approximations for $\psi_\delta(u)$ as $u \to \infty$.

Definition 2.1 *A measurable function $f : I\!R^+ \to I\!R^+$ is called* regularly varying with index *a if*

$$\lim_{x\to\infty} \frac{f(xt)}{f(x)} = t^a\,, \qquad \textit{for all } t > 0\,,$$

holds for some $a \in I\!R$. We then write $f \in RV(a)$. If $a = 0$, we say that f is slowly varying.

Obviously we have for $f \in RV(a)$ that

$$f(x) = x^a\,\ell(x) \quad \text{with some slowly varying function } \ell\,.$$

For further properties of regularly varying functions we refer to the monograph by Bingham, Goldie and Teugels (1987). For the claim size distribution F we shall require the representation

$$\overline{F}(x) = x^{-\alpha}\ell(x)\,, \quad x > 0\,, \tag{2.9}$$

with some slowly varying function ℓ and some $\alpha > 1$. The condition $\alpha > 1$ ensures a finite mean μ. Then we get for the integrated tail distribution

$$1 - F_I(x) \sim x^{-(\alpha-1)}\ell(x)/(\mu(\alpha-1))\,.$$

In our approach we use the following integral transforms.

Definition 2.2 *For any df H such that $H(0) = 0$ and for any $p \ge 0$ we define the L_p-*transform

$$L_pH(s) = \int_0^\infty e^{-st}(st)^p dH(t)\,. \tag{2.10}$$

Remarks

(i) L_0H is the usual Laplace transform of H.

(ii) L_p is an integral transform with kernel $K(t) = e^{-t}t^p$.

For any df H such that $H(0) = 0$ we define for $x \geq 0$

$$H_p(x) = \int_0^x t^p \, dH(t) \,,$$

then H_p is a measure defining function on $(0,\infty)$ with $H_p(\infty) \leq \infty$. If $\overline{H}(x) = x^{-\alpha}\ell(x)$ where $p > \alpha$ and ℓ is slowly varying, then $H_p(\infty) = \infty$.

Taking L_p-transforms in equation (1.4) we obtain

$$cL_pG_\delta(s) + \delta\int_0^\infty e^{-sx}(sx)^p x dG_\delta(x) = K_\delta L_pF_I(s) + \rho L_p(G_\delta * F_I)(s) \,.$$

Equivalently,

$$cL_pG_\delta(s) + \frac{\delta}{s}L_{p+1}G_\delta(s) = K_\delta L_pF_I(s) + \rho L_p(G_\delta * F_I)(s) \,. \tag{2.11}$$

We shall use equation (2.11) in order to prove our main result. In the case without interest, i.e. $\delta = 0$ the second term on the lefthand side vanishes and the analysis changes.

Theorem 2.3 *Assume that the claim size distribution function F is regularly varying with representation*

$$\overline{F}(x) \sim x^{-\alpha}\ell(x) \,, \quad x \to \infty \,,$$

for $\alpha > 1$ and some slowly varying function $\ell(x)$. Then

$$\overline{G}_\delta(x) \sim \frac{K_\delta + \rho}{\delta\mu\alpha}x^{-\alpha}\ell(x) \,, \quad x \to \infty \,.$$

The following Corollary is an immediate consequence of (1.3) and (1.5).

Corollary 2.4 *Assume the conditions of Theorem 2.3 hold. Then*

$$\psi_\delta(u) \sim \frac{\rho}{\alpha\mu\delta}u^{-\alpha}\ell(u) = \frac{\lambda}{\alpha\,\delta}\overline{F}(u) \,, \quad u \to \infty \,,$$

where u is the initial risk reserve, δ the force of interest and $\rho = \lambda\mu$.

This result is different from (1.6), the interest earned by the insurance company reduces the order of the tail by one power of u.

Notice that the result of Corollary 2.4 is independent of the finite mean μ. Actually, the result holds true for all claim size distributions with regularly varying tail $\overline{F}(u) = u^{-\alpha}\ell(u)$ for $\alpha > 0$. Observe that $E(ln_+ X) < \infty$ is a necessary and sufficient condition for ruin not to occur almost surely (see e.g. Asmussen (1987), Chapter 13). From an insurance applications point of view the result for infinite mean is of minor interest. So, we only prove Corollary 2.4, i.e. the case $\alpha > 1$

in detail below. However, a minor change in this proof enables us to deal with the cases $\alpha < 1$ (here the mean $\mu = \infty$) and $\alpha = 1$ (μ can be finite or infinite) as well. We will now indicate what has to be changed: whereas in the case $\mu < \infty$ the tail of $\int_u^\infty \overline{F}(y)dy$ determines the asymptotic behaviour of ψ_δ in equation (1.2), for $\mu = \infty$ the function $\widetilde{F}(u) = \int_0^u \overline{F}(y)dy$ is overtaking this role. Nevertheless, the arguments go through similarly for $\alpha < 1$, when we replace $\overline{F_I}$ by $\widetilde{F}$. A careful study of the arguments for $\alpha = 1$, and μ either finite or infinite, shows that the result remains true also in this case. For the sake of completeness we formulate the general result.

Proposition 2.5 *Assume that F is a df concentrated on $[0,\infty)$ with representation*

$$\overline{F}(u) = u^{-\alpha}\ell(u)\,,$$

for $\alpha > 0$ and some slowly varying function ℓ. Let ψ_δ be defined by (1.1). Then

$$\psi_\delta(u) \sim \frac{\lambda}{\alpha\,\delta} u^{-\alpha}\ell(u) = \frac{\lambda}{\alpha\,\delta}\overline{F}(u)\,, \quad u \to \infty\,.$$

3 Proof of Theorem 2.3

The following is a version of Karamata's theorem in terms of Stieltjes integrals formulated e.g. in Bingham, Goldie and Teugels (1987), Prop. 1.6.4.

Lemma 3.1 *Let F be a df such that $\overline{F}(x) = x^{-\alpha}\ell(x)$ for $\alpha > 0$ and $p > \alpha$. Define F_p by $F_p(x) = \int_0^x t^p\, dF(t)$. Then*

$$F_p(x) \sim \frac{\alpha}{p-\alpha} x^{p-\alpha}\ell(x)\,, \quad x \to \infty\,.$$

Conversely this asymptotic equality implies that $\overline{F}(x) \sim x^{-\alpha}\ell(x)$.

Proposition 3.2 *(a) Let F be a distribution function and $p > \alpha > 0$. Then the following two assertions are equivalent*

(i) $\overline{F}(x) \sim x^{-\alpha}\ell(x)\,, \quad x \to \infty\,.$

(ii) $L_pF(s) \sim \alpha\Gamma(p-\alpha)s^\alpha\ell(1/s)\,, \quad s \to 0+\,.$

In particular,

$$L_pF(1/x) \sim \alpha\Gamma(p-\alpha)\overline{F}(x)\,, \quad x \to \infty\,.$$

(b) If $p < \alpha$ and $F \in RV(-\alpha)$ then $L_pF(s) \sim s^p \int_0^\infty t^p\, dF(t)\,, \quad s \to 0+\,.$

Proof (a) Obviously we have by the definition of F_p that

$$L_pF(s) = s^p L_0 F_p(s)\,.$$

Hence L_pF is the Laplace transform of the measure generating function F_p. We apply Karamata's Abel-Tauber-theorem (Feller (1971), Theorem 1, Section XIII.5) giving

$$F_p(x) \sim \frac{\alpha}{p-\alpha} x^{p-\alpha}\ell(x)\,,$$

if and only if

$$L_0F_p(s) \sim \alpha\Gamma(p-\alpha)s^{-(p-\alpha)}\ell(1/s)\,, \quad s\to 0+\,.$$

In particular,

$$\frac{L_0F_p(1/x)}{F_p(x)} \to \Gamma(p-\alpha+1)\,, \quad x\to\infty\,.$$

This together with Lemma 3.1 implies the result. It is simply a reformulation of this in terms of $\overline{F}$ and L_pF. (b) is obvious. □

Lemma 3.3 *Let H be a df such that with some $p>0$ we obtain $\int_0^\infty x^p dH(t) = \infty$. Then we have*

$$\lim_{s\to 0+} \frac{sL_pH(s)}{L_{p+1}H(s)} = 0\,.$$

Proof For large $M>0$ and any $\epsilon>0$ we obtain for s sufficiently small

$$\begin{aligned}
\frac{L_{p+1}H(s)}{s} &= s^p \int_0^\infty e^{-sx}x^{p+1}dH(x) \\
&\geq s^p \int_M^\infty e^{-sx}x^{p+1}dH(x) \\
&\geq M \int_M^\infty e^{-sx}(sx)^p dH(x) \\
&= M\left(L_pH(s) - \int_0^M e^{-sx}(sx)^p dH(x)\right) \\
&\geq M\left(L_pH(s) - \epsilon\int_0^M x^p dH(x)\right)\,.
\end{aligned}$$

Hence we obtain for $0<s\leq s(M)$

$$\frac{L_{p+1}H(s)}{sL_pH(s)} \geq M\left(1-\epsilon\frac{\int_0^M x^p dH(x)}{L_pH(s)}\right) \geq \frac{M}{2}\,,$$

which yields the desired result. □

Lemma 3.4 *Let H and K be distribution functions on $(0,\infty)$ and $p\in\mathbb{N}$. Then*

$$\begin{aligned}
(i) L_p(H*K)(s) &= \sum_{\nu=0}^{p}\binom{p}{\nu} L_\nu H(s)\, L_{p-\nu}K(s) \quad \textit{for } p\in\mathbb{N}\,. \\
(ii) L_p(H*K)(s) &\leq 2^{p-1}(L_pH(s)L_0K(s) + L_0H(s)L_pK(s))\,.
\end{aligned}$$

Proof

$$\begin{aligned}
L_p(H*K)(s) &= \int_0^\infty e^{-sx}(sx)^p\, d(H*K)(x) \\
&= s\int_0^\infty \int_0^x e^{-sx}((sx)^p - p(sx)^{p-1})H(x-u)\, dK(u)\, dx \\
&= s\int_0^\infty \int_u^\infty e^{-s(x-u)}e^{-su}((sx)^p - p(sx)^{p-1})H(x-u)\, dx\, dK(u) \\
&= s\int_0^\infty \int_0^\infty e^{-sv}e^{-su}((s(v+u))^p - p(s(v+u))^{p-1})H(v)\, dv\, dK(u) \\
&= \int_0^\infty \int_0^\infty e^{-sv}e^{-su}((s(v+u))^p)\, dH(v)\, dK(u) \\
&\leq 2^{p-1}(L_pH(s)L_0K(s) + L_0H(s)L_pK(s))\,.
\end{aligned}$$

For the last inequality we used that for $u, v \geq 0$ we have $(u+v)^p \leq 2^{p-1}(u^p + v^p)$. Obiously the equality in our statement (i) follows from the calculation above as well. □

Lemma 3.5 *Assume that H and K are distribution functions on $(0,\infty)$ such that $\overline{H}(x) = x^{-\alpha}\ell(x)$ as $x \to \infty$, where $\alpha > 0$ and ℓ a slowly varying function and let $p \in I\!N$ and $p > \alpha$.*

(i) If $\lim_{s\to 0} L_pK(s)/L_pH(s) = 0$, then

$$L_p(H*K)(s) \sim L_pH(s) \sim \alpha\Gamma(p-\alpha)s^\alpha\ell(1/s)\,.$$

(ii) If $\lim_{x\to\infty} \overline{K}(x)/\overline{H}(x) = 0$, then

$$L_p(H*K)(s) \sim L_pH(s) \sim \alpha\Gamma(p-\alpha)s^\alpha\ell(1/s)\,, \quad s \to 0+\,.$$

(iii) If $\overline{K}(x) = x^{-\alpha}\ell_1(x)$ with some slowly varying function $\ell_1(x)$, then

$$L_p(H*K)(s) \sim L_pH(s) + L_pK(s) \sim \alpha\Gamma(p-\alpha)s^\alpha(\ell(1/s) + \ell_1(1/s))\,, \quad s \to 0+\,.$$

Proof (i) Observe that

$$\begin{aligned}
L_{p-\nu}K(s) &\leq s^{p-\nu}\int_0^1 t^{p-\nu}\, dK(t) + s^{p-\nu}\int_1^\infty t^p\, e^{-st}\, dK(t) \\
&\leq s^{p-\nu} + s^{-\nu}L_pK(s) = s^{-\nu}\, o\,(L_pH(s))\,.
\end{aligned}$$

Therefore and from Proposition 3.2(b) above, we obtain for $0 \leq \nu < \alpha$

$$L_{p-\nu}K(s)\, L_\nu H(s) = O(s^\nu)\, s^{-\nu}\, o(L_pH(s)) = o(L_pH(s))\,,\ s \to 0+\,.$$

For $p-1 \geq \nu > \alpha$ we find as $s \to 0+$

$$L_{p-\nu}K(s)\, L_\nu H(s) = O(s^\alpha\, \ell(1/s))\, o(1) = o(L_pH(s))\,,$$

since $L_\mu K(s) = o(1)$ as $s \to 0+$ for all $\mu \geq 1$. Using Lemma 3.4 the proof is complete if $\alpha \notin \mathbb{N}$. Since $L_p(H*K)(s)/(s^\alpha \ell(1/s))$ is equicontinuous in $\alpha \in (\alpha_0 - \varepsilon, \alpha_0]$ for $\alpha_0 \in (0,p) \cap \mathbb{N}$ for large s, the result holds for all $\alpha \in (0,p)$.

(ii) The condition implies the assumption of part (i).

(iii) This part follows from the Proposition in section VIII. 8 in Feller's book and our Proposition 3.2. (See also Lemma 2 in Embrechts and Goldie (1980) for a more general case of convolution tails.) □

Proposition 3.6 *Assume that F_I and G_δ satisfy equation (2.11). Then we have*

$$\lim_{s \to 0+} \frac{L_p G_\delta(s)}{L_p F_I(s)} = 0\,.$$

Proof Assume there exists a sequence $s_n \to 0+$ such that $\frac{L_p G_\delta(s_n)}{L_p F_I(s_n)} \geq \varepsilon_0 > 0$. Then (2.11), Lemma 3.3 and Lemma 3.4 imply for large n

$$\begin{aligned} s_n^{-1} L_{p+1} G_\delta(s_n)(1+o(1)) &= (K_\delta/\delta) L_p F_I(s_n) + (\rho/\delta) L_p(G_\delta * F_I)(s_n) \\ &\leq (K_\delta/\delta) L_p F_I(s_n) + (\rho/\delta) 2^{p-1} (L_p G_\delta(s_n) L_0 F_I(s_n) + L_0 G_\delta(s_n) L_p F_I(s_n)) \\ &\leq (K_\delta/(\varepsilon_0 \delta)) L_p G_\delta(s_n) + (\rho/\delta) 2^p (1 + \frac{1}{\varepsilon_0}) L_p G_\delta(s_n) \\ &\leq (K_\delta/(\varepsilon_0\delta) + 2^{p+1}\rho/(\delta\,\varepsilon_0)) L_p G_\delta(s_n)\,. \end{aligned}$$

This implies that the quotient

$$\frac{L_{p+1} G_\delta(s_n)}{s_n L_p G_\delta(s_n)} \not\to 0\,,$$

which contradicts Lemma 3.3. □

We can now prove our main result.

Proof of Theorem 2.3 We derive the asymptotics from equation 2.11. Choose an integer $p > \alpha$. By Lemma 3.3 we have

$$\frac{\delta}{s} L_{p+1} G_\delta(s) \sim c L_p G_\delta(s) + \frac{\delta}{s} L_{p+1} G_\delta(s) = K_\delta L_p F_I(s) + \rho L_p (G_\delta * F_I)(s)\,.$$

Then by Proposition 3.6, Lemma 3.5 and the equation $1 - F_I(x) \sim x^{-(\alpha-1)} \ell(x)/(\mu(\alpha-1))$ we obtain

$$L_{p+1} G_\delta(s) \sim \frac{K_\delta + \rho}{\delta} s L_p F_I(s) \sim \frac{K_\delta + \rho}{\delta} \frac{(\alpha-1)\Gamma(p+1-\alpha)}{\mu(\alpha-1)} s^\alpha \ell(1/s)\,.$$

Finally we conclude from Proposition 3.2 that

$$\overline{G}_\delta(x) \sim \frac{1}{\alpha} \frac{K_\delta + \rho}{\delta\mu} x^{-\alpha} \ell(x)\,.$$

□

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