Local linear dynamics assignment in IDA-PBC ⋆

Paul Kotyczka [a]

[a] *Institute of Automatic Control, Technische Universität München, Boltzmannstraße 15, 85748 Garching, Germany.*

Abstract

In this paper, the technique of local linear dynamics assignment is presented, which complements the powerful Interconnection and Damping Assignment Passivity Based Control (IDA-PBC) methodology. In IDA-PBC, nonlinear state feedback controllers are designed by matching the system's dynamics with a desired Port-Hamiltonian (PH) state representation. The latter consists of an energy function, which serves as the closed-loop Lyapunov function, as well as matrices, describing the virtual internal exchange and dissipation of the energy. A major difficulty in IDA-PBC is how to determine reasonable values for the large number of free design parameters. Local linear dynamics assignment offers a solution to this problem with a number of advantages: (i) Invoking the closed-loop Jacobian linearization to fix the parameter values provides transparency with respect to the resulting local dynamic behavior. (ii) An appropriate state transformation isolates the coordinates available for energy shaping. (iii) A related local linear state transformation makes the resulting system of design equations linear. (iv) Assigning a Hurwitz closed-loop Jacobian and ensuring positive semi-definiteness of the closed-loop dissipation matrix, the tedious definiteness check of the energy is omitted. The design steps are illustrated with the Ball on Wheel example.

Key words: Port-Hamiltonian systems, passivity, nonlinear control, controller parametrization, dynamics assignment.

1 Introduction

Nonlinear control literature, see e. g. Krstić et al. (1995), Khalil (1996), Sepulchre et al. (1997), provides a wide range of methods based on *Lyapunov theory*. While the nonlinear nature of the state differential equations is explicitely taken into account in Lyapunov-based techniques, also an estimate of the domain of attraction of the stabilized equilibrium is provided by the shape of the Lyapunov function. In passivity based methods like *Interconnection and Damping Assignment Passivity Based Control* (IDA-PBC), see Ortega et al. (1998), van der Schaft (2000), Ortega et al. (2002), a Lyapunov function for the closed-loop system results constructively from the controller design process. In IDA-PBC this means that first order PDEs have to be solved under positivity constraints. The so-constructed Lyapunov function has the interpretation as a new *energy function* for the closed-loop system. The class of Port-Hamiltonian (PH) systems, which arises naturally from port-based modeling and captures the internal and external energy flows and dissipation of a system, is especially suitable for physically inspired controller design, see e. g. Kugi (2001). In IDA-PBC, the original system representation is matched with a closed-loop PH system. For a predefined interconnection and dissipation structure the set of assignable energy functions is identified with the solutions of the vector-valued so-called *matching PDE*.

Different aspects of IDA-PBC have been explored in the last years, some of which shall be mentioned here: A central question is the solvability of the linear matching PDE in the case of input-affine systems, for which Cheng et al. (2005) give necessary and sufficient conditions. To circumvent the solution of the matching PDE, an alternative is proposed in Acosta and Astolfi (2009), based on solving the PDE for the elements of the gradient algebraically. The latter are integrated to define a map in state *plus* error coordinates. Then constructing a Lyapunov function for the augmented system proves closed-loop stability. In Höffner (2011) the matching problem is considered from a coordinate-free viewpoint. The problem of sampled-data implementation of IDA-PBC controllers is addressed in Tiefensee et al. (2010). An interesting class are underactuated mechanical systems. When the total energy is shaped, see Viola et al. (2007), the matching PDE for the kinetic energy becomes nonlinear and inhomogeneous. A procedure how to construct a solution of this PDE is given in Acosta et al. (2005).

⋆ This paper was not presented at any IFAC meeting. Corresponding author P. Kotyczka. Tel. +49-89-289-15663. Fax +49-89-289-15653.

Email address: kotyczka@tum.de (Paul Kotyczka).

Another problem is physical dissipation in unactuated coordinates (Gómez-Estern and van der Schaft, 2004).

Besides the stabilization of an equilibrium and the estimation of its domain of attraction, the assignment of desired transient dynamic behavior is a further important issue in nonlinear controller design. The existence of a passive output of the closed-loop PH system allows to tune the transient dynamics by *damping injection.* However, the effect of a certain output feedback gain is hardly predictable, nor can closed-loop dynamics be fixed transparently by choosing the elements of the interconnection and dissipation matrix.

The present contribution gives an answer to the question: *"How to determine the design parameters in the IDA-PBC approach such that the (local) dynamic behavior of the closed-loop system can be specified* in advance *and in a* quantifiable *way?"*

The basic idea of *local linear dynamics assignment* is to match the *linearization* of the *nonlinear* PH dynamics resulting from IDA-PBC with a *predefined* asymptotically stable linear system. The latter may result e. g. from eigenvalue assignment for the linearized original system. Both a partitioning of the state representation and a linear state transformation of the linearized system render the system of matching equations for local linear dynamics assignment *linear* in the elements of the interconnection and dissipation matrices in the equilibrium. Furthermore, the system of linear equations yields parameter values of the closed-loop energy function which can be directly realized in the *energy shaping* step of IDA-PBC. To sum up, local linear dynamics assignment provides parameter values for nonlinear IDA-PBC which guarantee prespecified local dynamics. (This neither means linear state feedback nor a feedback linearizing design.) A major advantage of the approach is – in contrast to the classical application of IDA-PBC – that from matching asymptotically stable local linear dynamics and ensuring a positive semi-definite dissipation matrix, positive definiteness of the closed-loop energy Hessian is deduced. As a consequence, the tedious definiteness check of the latter can be omitted. Local linear dynamics assignment reduces the number of remaining free design parameters in the IDA-PBC matching problem. However, to appropriately tune the nonlinear controller, in order to outperform the corresponding linear one – which is a general issue in the application of nonlinear methods – optimization over all free design parameters is required, including the prespecified properties of the closed-loop linearization. One possible criterion to assess the quality of the obtained nonlinear controllers is the extent of the domain of attraction, estimated by the shape of the energy functions.

The present paper generalizes and completes the results of the conference papers Kotyczka and Lohmann (2009), Kotyczka et al. (2010b), Kotyczka et al. (2010a), where local linear dynamics assignment has been presented for constant design matrices. Here, the technique is derived for the general case of *state-dependent* design matrices. This requires the examination of involutivity properties of a distribution of vector fields which appear in a part of the design matrix. Based thereon, important issues like the *shapeability* of the energy function, the existence of a *nonlinear transformation*, which reveals the structure of the closed-loop energy and the relation of this diffeomorphism to the *local linear transformation* used for the main result are discussed.

In Section 2, IDA-PBC for input-affine systems is summarized and the solvability condition for the vector-valued matching PDE is given. In Section 3, the shapeability of the solution of the PDE is discussed in terms of the number of characteristic coordinates. A coordinate transformation which reveals the structure of the solution, and a simplified solvability test in the case of constant coefficients are presented. The considered class of systems as well as a decomposition of the design parameters are introduced in Section 4, and the assignability of closed-loop local linear dynamics is discussed. Section 5 contains a compact derivation of the local linear dynamics assignment technique and a description of the design steps. In Section 6, the procedure is illustrated with the Ball on Wheel example while Section 7 concludes the paper with an outlook to future research.

Notation: $\nabla H(x)$ denotes the column vector of partial derivatives of $H(x)$ (gradient), while $\frac{\partial H(x)}{\partial x}$ is the corresponding row vector (Jacobian). For convenience, definitions of matrix valued functions like $F : \mathbb{R}^n \to \mathbb{R}^{n\times n}$ are denoted $F(x) \in \mathbb{R}^{n\times n}$. All statements regarding the local properties of vector fields, distributions, etc., like regularity or involutivity, are assumed to be valid in a sufficiently large neighborhood of the equilibrium x^*.

2 IDA-PBC

The IDA-PBC approach is briefly reviewed and the solvability condition for the matching PDE is given.

2.1 General approach

Given an input-affine system

$$\dot{x} = f(x) + G(x)u \tag{1}$$

with $x(t) \in \mathcal{X} \subseteq \mathbb{R}^n$, $u(t) \in \mathcal{U} \subseteq \mathbb{R}^m$, the IDA-PBC problem is to find a state feedback $u = \beta(x) + v$ to transform (1) into the *Port-Hamiltonian* (PH) system

$$\dot{x} = (J_d(x) - R_d(x))\nabla H_d(x) + G(x)v, \tag{2}$$

where $H_d : \mathbb{R}^n \to \mathbb{R}$ is a positive definite *storage* or *energy function* for the closed-loop system with

$$x^* = \arg\min_x H_d(x) \tag{3}$$

a strict minimum at the desired equilibrium. The interconnection and dissipation matrices $J_d(x) = -J_d^T(x)$ and $R_d(x) = R_d^T(x) \geq 0$ of dimension $n \times n$ describe energy exchange and dissipation in the closed-loop system. The collocated passive output $y = G^T(x)\nabla H_d(x)$ and the new input v obey the energy balance equation

$$\dot{H}_d(x) = -(\nabla H_d(x))^T R_d(x) \nabla H_d(x) + y^T v \leq y^T v,$$

showing passivity of the closed-loop PH system. Comparing Eqs. (1) and (2) yields a *matching equation* which must be met by all design parameters, i. e. the *design matrix* $F_d(x) := J_d(x) - R_d(x)$, the energy $H_d(x)$ as well as the corresponding control law. The fact that the number m of inputs is in general lower than the number n of state equations imposes restrictions on the achievable closed-loop dynamics, which are expressed by the projected matching equation or *matching PDE*

$$G^\perp(x) F_d(x) \nabla H_d(x) = G^\perp(x) f(x). \tag{4}$$

Herein, multiplication with the full rank left hand annihilator $G^\perp(x) \in \mathbb{R}^{(n-m)\times n}$, with $G^\perp(x)G(x) = 0$, eliminates the inputs u and v from the matching equation. The vector-valued matching PDE represents a set of $n - m$ scalar first order linear PDEs. A solution of (4) – if it exists – can be written as the sum

$$H_d(x) := \Psi(x) + \Phi(\xi(x)).$$

The *particular solution* $\Psi(x)$ solves (4) while the *homogeneous solution* $\Phi(\cdot)$ is an arbitrary smooth function of the solutions $\xi_i(x)$, $i = 1, \dots, n_\xi$, of the homogeneous PDE $G^\perp(x)F_d(x)\nabla\xi_i(x) = 0$. In the energy shaping step, $\Phi(\xi)$ is chosen to satisfy the minimum condition (3). If furthermore $R_d(x) \geq 0$ holds in a sufficiently large neighborhood of x^*, the control law (for $v \equiv 0$)

$$u = [G^T(x)G(x)]^{-1} G^T(x)[F_d(x)\nabla H_d(x) - f(x)]$$

renders the equilibrium x^* stable with $H_d(x)$ a Lyapunov function which satisfies $\dot{H}_d(x) \leq 0$ for $x \neq x^*$. Asymptotic stability can be proven using LaSalle's invariance principle, see e. g. Khalil (1996). An estimate of the domain of attraction of x^* is the region enclosed by the largest bounded and connected level set of $H_d(x)$ around x^* where $R_d(x) \geq 0$ holds. The minimum condition (3) for the closed-loop energy $H_d(x)$ is commonly verified examining the gradient and the Hessian matrix in x^*:

$$\nabla H_d(x)\big|_{x^*} = 0, \qquad Q_d := \left.\frac{\partial^2 H_d(x)}{\partial x^2}\right|_{x^*} > 0. \tag{5}$$

The first (second) order coefficients of the Taylor series expansion of the freely adjustable homogeneous solution $\Phi(\xi)$ will be called *first (second) order design parameters* of the energy function.

2.2 Solvability of the matching PDE

The vector-valued matching PDE (4) has the form

$$W(x)\nabla H_d(x) = s(x), \tag{6}$$

where the matrix $W(x) = [w_1(x) \dots w_{n-m}(x)]^T$ contains the coefficient vectors $w_i(x) \in \mathbb{R}^n$ and the vector $s(x) = [s_1(x) \dots s_{n-m}(x)]^T$ the forcing terms $s_i(x) \in \mathbb{R}$, $i = 1, \dots, n-m$, of each scalar PDE. A necessary and sufficient condition for the solvability of these PDEs has been given in Cheng et al. (2005):

Theorem 1 *The PDE* (6) *admits a solution* $H_d(x)$ *if and only if the involutive closures* $\mathrm{inv}\Delta(x)$ *and* $\mathrm{inv}\Delta'(x)$ *of the regular distributions*

$$\Delta(x) = \mathrm{span}\,\{w_1(x), \dots, w_{n-m}(x)\} \quad \textit{and} \tag{7}$$

$$\Delta'(x) = \mathrm{span}\left\{ \begin{bmatrix} w_1(x) \\ s_1(x) \end{bmatrix}, \dots, \begin{bmatrix} w_{n-m}(x) \\ s_{n-m}(x) \end{bmatrix} \right\}$$

have equal dimension $d \leq n$.

3 Properties of the matching PDE

In this section, the notion of characteristic coordinates is explained and their number for a given PDE is discussed. A coordinate transformation to allocate the effect of energy shaping is introduced and a simplified solvability test is presented for PDEs with constant coefficients.

3.1 Characteristic coordinates

Definition 2 Characteristic coordinates $\xi_i(x)$ *are independent solutions of the homogeneous PDE*

$$W(x)\nabla\xi_i(x) = 0, \tag{8}$$

summarized in the vector $\xi(x) = [\xi_1(x), \dots, \xi_{n_\xi}(x)]^T$. *Equivalently,* $\xi_i(x)$ *are quantities which remain constant along the solutions of the* system of characteristic ODEs $\dot{x} = w_j(x)$, $j = 1, \dots, n-m$, *associated to* (8).

The properties of the distribution spanned by the column vector fields of $W^T(x)$ are essential for the *number* of characteristic coordinates, which in IDA-PBC are available for shaping the closed-loop energy by $\Phi(\cdot)$. The number of independent characteristic coordinates n_ξ follows from the application of Frobenius' Theorem.

Proposition 3 *For a vector-valued PDE* (6) *there exist* $n_\xi = n - d$ *characteristic coordinates, where* d *is the dimension of the involutive closure* $\mathrm{inv}\Delta$ *according to* (7).

PROOF. Denote $w_{1,\ldots,n-m}$ the vector fields of $\Delta_0 := \Delta$ according to (7) and $w_{n-m+1,\ldots,d}$ the Lie-brackets which are added one after another to construct the distributions $\Delta_{1,\ldots,d-(n-m)}$ where $\Delta_{d-(n-m)} = \mathrm{inv}\Delta$. According to Frobenius' Theorem, see e. g. Isidori (1995), the annihilator of the *involutive* d-dimensional distribution $\mathrm{inv}\Delta$ is spanned by exactly $n-d$ differentials, i. e. there exist $n-d$ functions ξ_i, $i = 1, \ldots, n-d$, such that

$$\frac{\partial \xi_i(x)}{\partial x} w_j(x) = 0, \quad j = 1, \ldots, d.$$

The functions ξ_i remain constant along solutions of the characteristic ODEs $\dot{x} = w_j(x)$, $j = 1, \ldots, n-m$, and hence are characteristic coordinates. If there were more, i. e. $n-d+k$ characteristic coordinates, Frobenius' Theorem would require $\Delta_{d-(n-m)-k}$ to be involutive, which contradicts the fact that $\mathrm{inv}\Delta = \Delta_{d-(n-m)}$. $\square$

Consequently, if the distribution (7) is involutive and the PDE (6) is solvable according to Theorem 1, the solution can be shaped with the maximum number of m characteristic coordinates. This *maximum shapeability* is favorable in the energy shaping step of IDA-PBC.

3.2 Transformation of the matching PDE

To allocate the effect of energy shaping by the homogeneous solution $\Phi(\cdot)$ to a subset of coordinates, a transformation of the (matching) PDE of type (6) is useful.

Proposition 4 *Given the vector-valued PDE* (6)*, with* $W(x)$ *of full rank* $n-m$*. Assume the vector fields* $w_j(x)$*,* $j = 1, \ldots, n-m$*, span an involutive distribution. Then* (6) *can be transformed into*

$$\begin{bmatrix} 0 & \bar{W}_\eta(\chi) \end{bmatrix} \nabla \bar{H}_d(\chi) = \bar{s}(\chi) \tag{9}$$

by a diffeomorphism $\chi = \tau(x)$ *with* $\chi = [\xi^T \quad \eta^T]^T$*,* $\xi \in \mathbb{R}^m$*,* $\eta \in \mathbb{R}^{n-m}$*, a nonsingular matrix* $\bar{W}_\eta(\chi) \in \mathbb{R}^{(n-m)\times(n-m)}$ *and* $\bar{s}(\chi) = s \circ \tau^{-1}(\chi)$*. The solution of* (9) *takes the form*

$$\bar{H}_d(\xi, \eta) = \bar{\Psi}(\xi, \eta) + \bar{\Phi}(\xi),$$

with a particular solution $\bar{\Psi}(\xi, \eta)$ *and a solution* $\bar{\Phi}(\xi)$ *of the homogeneous PDE. The requirement* $\nabla \bar{H}_d(\chi)\big|_{\chi^*} = 0$ *can be achieved if* $s(x^*) = 0$.

PROOF. The condition to transform the (transposed) row vector fields of the principal part in (6) is

$$\begin{bmatrix} 0_{m\times(n-m)} \\ \bar{w}_{\eta,j}(\chi) \end{bmatrix} = \begin{bmatrix} \frac{\partial \xi(x)}{\partial x} \\ \frac{\partial \eta(x)}{\partial x} \end{bmatrix} w_j \circ \tau^{-1}(\chi), \quad j = 1, \ldots, n-m, \tag{10}$$

with $\bar{w}_{\eta,j}(\chi)$ the column vectors of $\bar{W}_\eta^T(\chi)$. Under the involutivity assumption, there exist m independent functions $\xi_i(x)$, $i = 1, \ldots, m$, such that the first row of (10) holds with $\frac{\partial \xi(x)}{\partial x}$ of maximum rank m. The functions $\eta_i(x)$, $i = 1, \ldots, n-m$, can be defined arbitrarily such that $\frac{\partial \tau(x)}{\partial x}$ is nonsingular and consequently $\bar{W}_\eta(\chi)$ is of full rank. From the Inverse Function Theorem (Isidori, 1995) it follows that $\tau(x)$ locally is a diffeomorphism. The sole dependency of $\bar{\Phi}(\cdot)$ on ξ follows from the structure of the homogeneous PDE (replace $\bar{s}(\chi)$ by 0 in (9)). With $s(x^*) = \bar{s}(\chi^*) = 0$ also $\nabla_\eta \bar{H}_d(\chi)\big|_{\chi^*} = 0$ holds, and $\nabla_\xi \bar{H}_d(\chi)\big|_{\chi^*} = 0$ can be achieved by the first order parameters of the free function $\bar{\Phi}(\xi)$. $\square$

3.3 Simplified solvability test

Using the above transformation, solvability of (6) is easy to verify *with respect to the elements* of a *constant* matrix W (Kotyczka and Lohmann, 2009):

Proposition 5 *The PDE* (6) *with* $W = const.$ *admits a solution if and only if for all* $i, j = 1, \ldots, n-m$

$$L_{w_i} s_j(x) - L_{w_j} s_i(x) = 0, \tag{11}$$

where $L_{w_i} s_j(x)$ *denotes the Lie-derivative of* $s_j(x)$ *along the constant vector field* w_i.

PROOF. The statement can be proven either by application of Theorem 1 or by *Poincaré's Lemma*, see e. g. Lévine (2009). First, transform (6) with $W = const.$ into $\nabla_\eta \bar{H}_d(\xi, \eta) = \bar{s}(\chi)$ by the regular linear coordinate change $x = [* \quad W^T]\chi$. Solvability now is equivalent to the question whether the 1-form $\omega = \sum_{j=1}^{n-m} \bar{s}_j(\xi, \eta) d\eta_j$ is exact on the considered subset of $\mathbb{R}^{n-m}$ (ξ here only plays the role of a parameter). Necessary and sufficient condition on a star-shaped region is

$$\frac{\partial \bar{s}_j(\xi, \eta)}{\partial \eta_i} - \frac{\partial \bar{s}_i(\xi, \eta)}{\partial \eta_j} = 0, \quad i, j = 1, \ldots, n-m,$$

which corresponds to the interchangeability of the second order partial derivatives of the solution. The back-transformation yields (11). $\square$

4 Problem structure and assumptions

In order to obtain a systematic procedure for the parameter choice in IDA-PBC, only input-affine systems with a special structure are considered. Correspondingly, the target dynamics is restricted to a structured representation. In this section, furthermore, the assignability of local linear dynamics is briefly discussed, before the assumptions for the main result are formulated.

4.1 Class of systems

In the considered class of systems, "actuated" coordinates are distinguished from "unactuated" states:

Definition 6 *In input-affine systems of the form*

$$\begin{bmatrix} \dot{x}_\alpha \\ \dot{x}_\nu \end{bmatrix} = \begin{bmatrix} f_\alpha(x) \\ f_\nu(x) \end{bmatrix} + \begin{bmatrix} G_\alpha(x) \\ 0 \end{bmatrix} u, \tag{12}$$

where $G_\alpha(x) \in \mathbb{R}^{m \times m}$ is a nonsingular square matrix on $\mathcal{X}$, the upper part $x_\alpha \in \mathbb{R}^m$ of the state vector $x \in \mathcal{X} \subseteq \mathbb{R}^n$ contains the actuated *coordinates, while $x_\nu \in \mathbb{R}^{n-m}$ denotes the* unactuated *coordinates.*

The IDA-PBC matching equation now turns into

$$\begin{bmatrix} f_\alpha(x) \\ f_\nu(x) \end{bmatrix} + \begin{bmatrix} G_\alpha(x) \\ 0 \end{bmatrix} u \overset{!}{=} \begin{bmatrix} F_\alpha(x) \\ F_\nu(x) \end{bmatrix} \nabla H_d(x), \tag{13}$$

where the design matrix $F_d(x)$ has been partitioned into $F_\alpha(x) \in \mathbb{R}^{m \times n}$ and $F_\nu(x) \in \mathbb{R}^{(n-m) \times n}$. The submatrices

$$F_\alpha(x) = \begin{bmatrix} \alpha_1^T(x) \\ \vdots \\ \alpha_m^T(x) \end{bmatrix}, \quad F_\nu(x) = \begin{bmatrix} \nu_1^T(x) \\ \vdots \\ \nu_{n-m}^T(x) \end{bmatrix}$$

are composed of the row vectors

$$\begin{aligned} \alpha_i^T(x) &= \begin{bmatrix} \alpha_{i1}(x) & \dots & \alpha_{in}(x) \end{bmatrix}, \quad i = 1, \dots, m, \\ \nu_j^T(x) &= \begin{bmatrix} \nu_{j1}(x) & \dots & \nu_{jn}(x) \end{bmatrix}, \quad j = 1, \dots, n-m, \end{aligned}$$

containing the scalar design parameters. The simplest possible left hand annihilator

$$G^\perp = \begin{bmatrix} 0_{(n-m) \times m} & I_{n-m} \end{bmatrix} \tag{14}$$

is used such that the second row of (13)

$$f_\nu(x) = F_\nu(x) \nabla H_d(x) \tag{15}$$

directly represents a matching PDE, which *only* depends on the elements of the lower submatrix $F_\nu(x)$. When (3) has been achieved with $R_d(x) \geq 0$, the stabilizing control law follows from the first row of (13):

$$u = G_\alpha^{-1}(x)[F_\alpha(x) \nabla H_d(x) - f_\alpha(x)].$$

4.2 Assignable local linear dynamics

The linearization of (12) at an admissible equilibrium (x^*, u^*) has the form

$$\begin{bmatrix} \Delta\dot{x}_\alpha \\ \Delta\dot{x}_\nu \end{bmatrix} = \begin{bmatrix} A_\alpha \\ A_\nu \end{bmatrix} \begin{bmatrix} \Delta x_\alpha \\ \Delta x_\nu \end{bmatrix} + \begin{bmatrix} B_\alpha \\ 0 \end{bmatrix} \Delta u, \tag{16}$$

with $A_\alpha \in \mathbb{R}^{m \times n}$, $A_\nu \in \mathbb{R}^{(n-m) \times n}$, the nonsingular matrix $B_\alpha \in \mathbb{R}^{m \times m}$. The Δ-quantities are the deviations of state and input from the equilibrium. If (16) is controllable, there exists a linear state feedback $\Delta u = K \Delta x$ such that desired closed-loop linearized dynamics

$$\Delta\dot{x} = A_d \Delta x \quad \Leftrightarrow \quad \begin{bmatrix} \Delta\dot{x}_\alpha \\ \Delta\dot{x}_\nu \end{bmatrix} = \begin{bmatrix} A_{d,\alpha} \\ A_\nu \end{bmatrix} \begin{bmatrix} \Delta x_\alpha \\ \Delta x_\nu \end{bmatrix}, \tag{17}$$

can be achieved with *arbitrary* eigenvalues of the Hurwitz matrix $A_d = A + BK$. The linear eigenvalue assignment problem will be an intermediate step to determine the design parameters for nonlinear IDA-PBC.

Remark 7 *It is well known that stabilizability of the linearization is only sufficient for stabilizability of* (12)*, see Nijmeijer and van der Schaft (1990), Bacciotti (1988) for details. To concentrate on the core question of transparent dynamics assignment in the nonlinear IDA-PBC design process, the* critical *cases (e. g. uncontrollable eigenvalues of A on the imaginary axis) are excluded here.*

4.3 Assumptions

Before stating the main result of the paper, the assumptions are summarized:

Assumption 8 *The state equations of the considered system have the form* (12) *with sufficiently smooth vector fields on the right hand side and the linearization* (16) *in the desired equilibrium (x^*, u^*) is controllable.*

Note that every input-affine system with input matrix of full rank m, whose column vector fields span an involutive distribution, can be transformed into the above representation. The smoothness assumption guarantees that the solution of the matching PDE (4), if it exists, is smooth, and hence, the Hessian matrix Q_d according to (5) is defined. This allows to employ the linearization arguments in the derivation of the main result.

Assumption 9 *The matching PDE* (15) *is solvable with a full rank matrix $F_\nu(x)$, whose smooth (row) vector fields $\nu_j^T(x)$, $j = 1, \ldots, n-m$, span an involutive distribution.*

This assumption ensures that a full rank matrix A_d can be assigned for the *linearized* closed-loop system and there exists the maximum number of m characteristic coordinates for the energy shaping step of IDA-PBC. The row vectors of every *constant* matrix F_ν span involutive distributions and solvability of the PDE is easily tested by condition (11) in this case. To construct a *state-dependent* matrix $F_\nu(x)$ such that Assumption 9 holds, is complicated in general. However, it can be shown that the row vector fields of matrices $F_\nu(x)$, as constructed in Subsection 5.5, span involutive distributions.

5 Main result

The procedure described in this section provides values of the IDA-PBC design parameters such that

- desired dynamics of the linearization is achieved while
- the energy $H_d(x)$ serves as a Lyapunov function to prove stability and determine an estimate for the domain of attraction of the closed-loop equilibrium x^*.

5.1 Matching of local linear dynamics

The basic idea is to match the linearization of the closed-loop system with a desired matrix A_d according to (17):

$$\Delta\dot{x} = \frac{\partial}{\partial x}\left(F_d(x)\nabla H_d(x)\right)\bigg|_{x^*} \overset{!}{=} A_d \Delta x. \tag{18}$$

Under Assumption 9, the row vector fields of $F_\nu(x)$ span an involutive distribution. Proposition 4 then states that $\nabla H_d(x)|_{x^*} = 0$ can be achieved (by the first order parameters of the homogeneous solution) if $f_\nu(x^*) = 0$, i. e. x^* is an admissible equilibrium. Hence (18) leads to

$$F_d(x^*)Q_d\Delta x \overset{!}{=} A_d\Delta x \qquad \text{with} \tag{19}$$

$$Q_d := \frac{\partial^2}{\partial x^2}H_d(x)\bigg|_{x=x^*} = \frac{\partial^2}{\partial \Delta x^2}\Delta H_d(\Delta x)\bigg|_{\Delta x=0}$$

the Hessian of the closed-loop energy in the desired equilibrium. $\Delta H_d(\Delta x) := H_d(x^*) + \frac{1}{2}\Delta x^T Q_d \Delta x$ denotes the quadratic approximation of $H_d(x)$ at x^*. If (19) can be solved for the free IDA-PBC parameters "hidden" in $F_d(x^*)$ and Q_d, clearly regularity of both matrices follows if A_d is Hurwitz.

Proposition 10 *If* (19) *with A_d a Hurwitz matrix is solved such that*

$$R_d(x^*) = -\frac{1}{2}(F_d(x^*) + F_d^T(x^*)) \geq 0, \tag{20}$$

then Q_d is positive definite.

PROOF. The solution of (19) implies $\text{rank}(Q_d) = n$. Solving (19) for $F_d(x^*)$ and substituting in (20), one gets

$$A_d Q_d^{-1} + Q_d^{-1}A_d^T = -2R_d(x^*),$$

a Lyapunov equation with at least a solution $Q_d^{-1} \geq 0$ if $R_d(x^*) \geq 0$, see Boyd (2005). Regularity of Q_d implies regularity of Q_d^{-1}. Hence $Q_d^{-1} > 0$ and finally $Q_d > 0$. □

When assigning a Hurwitz matrix A_d, it is sufficient to check $R_d(x^*) \geq 0$ to deduce positive definiteness of $H_d(x)$. The argumentation is related to Proposition 1 in Prajna et al. (2002) that for any (asymptotically) stable linear system a PH representation can be derived.

5.2 Local linear coordinate transformation

The linear transformation of the deviation coordinates

$$\Delta x = F_d^T(x^*)\Delta z \tag{21}$$

turns the local linear matching equation (19) into

$$\underbrace{F_d(x^*)Q_dF_d^T(x^*)}_{=:\tilde{Q}_d}\Delta z = A_dF_d^T(x^*)\Delta z, \tag{22}$$

where $\tilde{Q}_d$ describes the quadratic approximation of the closed-loop energy in new *local* deviation coordinates: $\Delta\tilde{H}_d(\Delta z) := H_d(x^*) + \frac{1}{2}\Delta z^T\tilde{Q}_d\Delta z$. By partitioning A_d and $F_d(x^*)$ into their actuated and unactuated parts, one obtains

$$\begin{bmatrix} \tilde{Q}_{\alpha\alpha} & \tilde{Q}_{\alpha\nu} \\ \tilde{Q}_{\nu\alpha} & \tilde{Q}_{\nu\nu} \end{bmatrix} = \begin{bmatrix} A_{d,\alpha}F_\alpha^T(x^*) & A_{d,\alpha}F_\nu^T(x^*) \\ A_\nu F_\alpha^T(x^*) & A_\nu F_\nu^T(x^*) \end{bmatrix}, \tag{23}$$

since (22) has to hold for any Δz. The submatrices

$$\tilde{Q}_{ij} := \frac{\partial^2\Delta\tilde{H}_d(\Delta z)}{\partial\Delta z_i\Delta z_j}, \quad i,j \in \{\alpha, \nu\},$$

denote the second order partial derivatives of the closed-loop energy in the directions of the subvectors of

$$\Delta z = \begin{bmatrix}\Delta z_\alpha^T & \Delta z_\nu^T\end{bmatrix}^T, \qquad \Delta z_\alpha \in \mathbb{R}^m, \;\; \Delta z_\nu \in \mathbb{R}^{n-m},$$

required to fulfill (22). Now it will be clarified how the subset Δz_α of local coordinates is related to the characteristic coordinates of the PDE (15). The solution of (15) using computer algebra provides a set of characteristic coordinates $\xi(x) \in \mathbb{R}^m$. Additional coordinate functions

$\eta(x) \in \mathbb{R}^{n-m}$ can be defined to complete a diffeomorphism $\chi = \tau(x)$. To assign local linear dynamics, a matrix $\tilde{Q}_d$ which satisfies (22) must be realized by shaping the homogeneous solution $\Phi(\xi)$. The following proposition relates the submatrix $\tilde{Q}_{\alpha\alpha}$ (defined with respect to Δz_α) to $\Phi(\xi)$.

Proposition 11 *Given a set of characteristic coordinates $\xi \in \mathbb{R}^m$. Then an arbitrary matrix*

$$\tilde{Q}_{\alpha\alpha} = \frac{\partial^2 \Delta \tilde{H}_d(\Delta z)}{\partial \Delta z_\alpha^2} \tag{24}$$

can be achieved by appropriate choice of the homogeneous solution $\Phi(\xi)$ of (15).

PROOF. First, it is shown that the deviation coordinates $\Delta\xi$, expressed through Δz, only depend on Δz_α. The characteristic coordinates $\xi(x)$ satisfy $\frac{\partial \xi(x)}{\partial x} F_\nu^T(x) = 0$. The local approximation of $\xi(x)$ is $\Delta\xi = \left.\frac{\partial \xi(x)}{\partial x}\right|_{x^*} \Delta x$, thus $\frac{\partial \Delta\xi}{\partial \Delta x} F_\nu^T(x^*) = 0$ holds. With $F_\nu(x) = [\,0 \;\; I\,] F_d(x)$ and (21) it follows that

$$0 = \frac{\partial \Delta\xi}{\partial \Delta x}\frac{\partial \Delta x}{\partial \Delta z_\nu} = \frac{\partial \Delta\xi}{\partial \Delta z_\nu}.$$

In a similar way, it is shown that $\frac{\partial \Delta z_\alpha}{\partial \Delta \eta} = 0$, i. e. a one-to-one correspondence $\bar{Q}_{\xi\xi} = C^T \tilde{Q}_{\alpha\alpha} C$, $C \in \mathbb{R}^{m\times m}$, can be established between (24) and

$$\bar{Q}_{\xi\xi} := \frac{\partial^2 \Delta \bar{H}_d(\Delta\xi, \Delta\eta)}{\partial \Delta\xi^2},$$

which can be assigned *arbitrarily* by $\Phi(\xi)$. □

In the case $\left.\frac{\partial \xi(x)}{\partial x}\right|_{x^*} = \frac{\partial \Delta z_\alpha}{\partial \Delta x}$, the identity $\tilde{Q}_{\alpha\alpha} = \bar{Q}_{\xi\xi}$ holds. Otherwise, the matrix C has to be determined.

Now the effect of $\Phi(\xi)$ is *exclusively* allocated to the submatrix $\tilde{Q}_{\alpha\alpha}$. Note that $\tilde{Q}_{\alpha\alpha}$ is the sum of the Hessian matrices with respect to Δz_α of the particular solution $\Psi(\cdot)$ (fixed) and the homogeneous part $\Phi(\cdot)$ (assignable):

$$\tilde{Q}_{\alpha\alpha} = \tilde{Q}_{\alpha\alpha}^{\Psi} + \tilde{Q}_{\alpha\alpha}^{\Phi}.$$

If $H_d(x)$ is indeed a solution of the matching PDE, the second order partial derivatives interchange. This symmetry requirement on $\tilde{Q}_d$ according to (23) provides necessary conditions for the values of $F_d(x)$ in x^*.

5.3 System of linear equations

Putting the above considerations together, the main theorem of the paper can be formulated:

Proposition 12 *Given the IDA-PBC problem in* (13)*, with the simplest left hand annihilator* (14) *to extract the matching PDE* (15)*. Let the linearization be controllable in the desired equilibrium (x^*, u^*) (Assumption 8). Assume the matching PDE to be solvable with $F_\nu(x)$ of full rank $n-m$ and its row vectors $\nu_j^T(x)$, $j = 1, \dots, n-m$, to span an involutive distribution (Assumption 9).*

Choose the free design parameters $\tilde{Q}_{\alpha\alpha}^{\Phi}$, which can be arbitrarily assigned by the homogeneous solution $\Phi(\xi)$ of (15)*, and the values of the elements of $F_d(x^*)$ from the solution of the linear system of equations*

$$0 = A_{d,\alpha} F_\alpha^T(x^*) - F_\alpha(x^*) A_{d,\alpha}^T \tag{25a}$$

$$0 = A_{d,\alpha} F_\nu^T(x^*) - F_\alpha(x^*) A_\nu^T \tag{25b}$$

$$\tilde{Q}_{\alpha\alpha}^{\Phi} = A_{d,\alpha} F_\alpha^T(x^*) - \tilde{Q}_{\alpha\alpha}^{\Psi} \tag{25c}$$

for a given Hurwitz matrix A_d such that (20) *holds. In addition, ensure $\nabla H_d(x)|_{x^*} = 0$ by the choice of first order parameters of $\Phi(\xi)$.*

The result is a parametrization of IDA-PBC such that the linear approximation of the asymptotically stable closed-loop PH dynamics is (17)*, and the energy function $H_d(x)$ has a minimum in the desired equilibrium x^*.*

PROOF. The involutivity assumption ensures the existence of exactly m characteristic coordinates. Consequently, $\tilde{Q}_{\alpha\alpha}^{\Phi}$ as required by (25c), which is part of the northwestern subequation of (23), can be realized by second order parameters in $\Phi(\xi)$. Equations (25a), (25b) express the symmetry requirement of the northwestern and off-diagonal submatrices of $\tilde{Q}_d$. Symmetry of the southeastern submatrix is automatically given when the matching PDE is solvable. In the SISO case, Eq. (25a) is trivially true. The solution of the system of equations (25) is equivalent to solving the matching equation for local linear dynamics (19) such that Q_d is symmetric. Applying Proposition 10 completes the proof. □

The proposition provides a method how to determine the values of the elements $\alpha_{ik}(x)$, $\nu_{jk}(x)$ of the design matrix in the equilibrium x^*, as well as the first and second order parameters of the homogeneous solution $\Phi(\xi)$. The question *how* these functions have to be like to ensure solvability of the matching PDE, maximum shapeability of the energy function etc. is, however, not addressed. The simplest case, which is a reasonable starting point, is to take $F_d = const$. Then (a) solvability of the matching PDE (15) is easy to check via condition (11), (b) the distribution spanned by the constant vector fields ν_j, $j = 1, \dots, n-m$, is always involutive, and (c) if (25) is solved such that $R_d \geq 0$, then this property holds globally. Note that it depends on the coordinate choice if interconnection and dissipation structures are represented by constant matrices or not (Höffner, 2011).

The matrix equations (25) represent $m \cdot n$ scalar linear equations, which is the number of free elements in $A_{d,\alpha}$. On the other hand, there are $(m^2+m)/2$ free parameters in $\tilde{Q}_{\alpha\alpha}$ and n^2 entries in $F_d(x^*)$ (under the constraint $R_d(x^*) \geq 0$). Although the design freedom is reduced by the solvability condition for the matching PDE, the system of equations is usually underdetermined, which allows to use the remaining design freedom e. g. to maximize the estimated domain of attraction of x^*, see Kotyczka et al. (2010b).

5.4 Design procedure with constant design matrices

The design steps to apply local linear dynamics assignment with constant design matrices are described below.

Step 1 Define the desired dynamics of the closed-loop linearization (17).

Step 2 (1st preconditioning of F_d) Establish algebraic relations between the elements of F_ν such that solvability of the matching PDE is ensured by condition (11).

Step 3 (2nd preconditioning of F_d) Establish relations between the elements of F_d such that $R_d \geq 0$ is possible. This includes inequalities, but also setting those off-diagonal elements of R_d zero which correspond to zeros on the diagonal. Furthermore, certain choices of elements may simplify the definiteness check of R_d.

Step 4 Represent the solution of the matching PDE (15) with respect to the free parameters.

Step 5 Ensure $\nabla H_d(x)|_{x^*} = 0$ by the choice of first order parameters in $\Phi(\xi)$.

Step 6 Solve the system of linear equations (25) for the free parameters. Determine the second order parameters of $\Phi(\xi)$ from $\tilde{Q}^{\Phi}_{\alpha\alpha}$.

Step 7 Fix the remaining free parameters, e. g. to optimize the shape of the resulting closed-loop energy $H_d(x)$.

The presented approach in a sense reverts the order of design steps in the "classical" application of IDA-PBC, where energy shaping is followed by damping injection to adjust the dynamic behavior. Whereas here, closed-loop (linearized) dynamics is *predefined* and the shape of the energy is adjusted at the end of the design process.

5.5 Construction of state-dependent design matrices

A constant design matrix may be insufficient to successfully design an IDA-PBC controller, cf. the last example

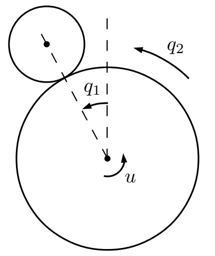

Fig. 1. Sketch of the Ball on Wheel system

in Kotyczka and Lohmann (2009), where the solvability condition restricts the design parameters in such a way that it is not possible to achieve $R_d \geq 0$ and $Q_d > 0$ at the same time. A state-dependent matrix $F_\nu(x)$ may be more suitable for the problem. Such a matrix can be systematically constructed as follows.

Take the matching PDE (15) and multiply with a nonsingular matrix $D(x) \in \mathbb{R}^{(n-m)\times(n-m)}$. This corresponds to the use of a modified left hand annihilator $G^\perp(x) = [0 \;\; D(x)]$. The *equivalent matching PDE* is

$$\underbrace{D(x)f_\nu(x)}_{\hat{f}_\nu(x)} = \underbrace{D(x)F_\nu(x)}_{\hat{F}_\nu} \nabla H_d(x). \tag{26}$$

Parametrize $F_\nu(x)$ by $F_\nu(x) = D^{-1}(x)\hat{F}_\nu$ with $\hat{F}_\nu =$ *const.* and choose $D(x)$ such that $\hat{f}_\nu(x)$ becomes a less restrictive forcing term with respect to the (simplified) solvability condition. It can be easily checked that using this construction, the involutivity Assumption 9 is satisfied. The first preconditioning in Step 2 is carried out for the elements of $\hat{F}_\nu$ in the equivalent PDE (26). Consequently, the solution of the matching PDE depends only on the constant parameters $\hat{\nu}_{jk}$. The second preconditioning in Step 3 and the solution of the system of equations in Step 6 are based on the state-dependent matrix $F_d(x)$ of the original problem, evaluated in x^*.

6 Example: Ball on Wheel

To illustrate the design procedure, the Ball on Wheel system is considered (Fig. 1). The goal is to balance a ball (hollow sphere with radius r_b and mass m_b) on top of a wheel (radius r_w and moment of inertia J_w) by actuating a motor (torque u) in the hub of the wheel. q_1 denotes the angle between the rotation axes of wheel and ball, q_2 is the absolute rotation angle of the wheel. The system could be modeled as 2 DOF mechanical system in Hamiltonian formulation and controlled by the IDA-PBC approach for underactuated mechanical systems (Acosta et al., 2005) to stabilize the desired equilibrium $q^* = \dot{q}^* = 0$. However, disregarding the wheel angle q_2 and defining $x_1 = \dot{q}_1$, $x_2 = q_1$, and $x_3 = -\frac{7}{2}\frac{r_w+r_b}{r_w}\dot{q}_1 + \dot{q}_2$, results in the input-affine state space representation

$$\dot{x} = \begin{bmatrix} f_\alpha(x) \\ f_\nu(x) \end{bmatrix} + \begin{bmatrix} g_\alpha \\ 0 \end{bmatrix} u = \begin{bmatrix} a \sin x_2 \\ x_1 \\ -b \sin x_2 \end{bmatrix} + \begin{bmatrix} c \\ 0 \\ 0 \end{bmatrix} u,$$

with $a = (5J_w + 2m_b r_w^2)g/d$, $b = 5g/(2r_w)$, $c = 2r_w/d$, $d = (r_w + r_b)(7J_w + 2m_b r_w^2)$, and g the gravitational acceleration. The first ansatz for the closed-loop Port-Hamiltonian dynamics is

$$\dot{x} = \begin{bmatrix} F_\alpha \\ F_\nu \end{bmatrix} \nabla H_d(x) = \begin{bmatrix} \alpha_{11} & \alpha_{12} & \alpha_{13} \\ \nu_{11} & \nu_{12} & \nu_{13} \\ \nu_{21} & \nu_{22} & \nu_{23} \end{bmatrix} \nabla H_d(x),$$

with a constant design matrix F_d and the energy $H_d(x)$ to be determined such that it takes a minimum in $x^* = 0$.

Step 1 A linear state feedback $u_{\text{lin}} = Kx$ is designed for the linearized system, assigning three closed-loop eigenvalues in -10. The matrix A_d takes the form

$$A_d = \begin{bmatrix} a_{d,11} & a_{d,12} & a_{d,13} \\ 1 & 0 & 0 \\ 0 & -b & 0 \end{bmatrix}.$$

Step 2 The first preconditioning of the design matrix with respect to the solvability of the matching PDE $f_\nu(x) = F_\nu \nabla H_d(x)$ yields

$$b \cos x_2 \nu_{12} + \nu_{21} = 0 \qquad \Rightarrow \qquad \nu_{12} = \nu_{21} = 0.$$

The parametrized dissipation matrix at this stage is

$$R_d = \begin{bmatrix} -\alpha_{11} & -\frac{1}{2}(\alpha_{12} + \nu_{11}) & -\frac{1}{2}\alpha_{13} \\ -\frac{1}{2}(\alpha_{12} + \nu_{11}) & 0 & -\frac{1}{2}(\nu_{13} + \nu_{22}) \\ -\frac{1}{2}\alpha_{13} & -\frac{1}{2}(\nu_{13} + \nu_{22}) & -\nu_{23} \end{bmatrix}.$$

Step 3 To allow R_d to be positive semidefinite, $\nu_{22} = -\nu_{13}$ and $\nu_{11} = -\alpha_{12}$ are indispensable. Setting for simplicity $\alpha_{13} = 0$ results in $R_d = \text{diag}\{-\alpha_{11}, 0, -\nu_{23}\}$, which is positive semidefinite when at least one of the inequalities $\alpha_{11} \le 0$ and $\nu_{23} \le 0$ holds strictly.

Step 4 The matching PDE after this second preconditioning of F_d is

$$\begin{bmatrix} x_1 \\ -b \sin x_2 \end{bmatrix} = \begin{bmatrix} -\alpha_{12} & 0 & \nu_{13} \\ 0 & -\nu_{13} & \nu_{23} \end{bmatrix} \nabla H_d(x)$$

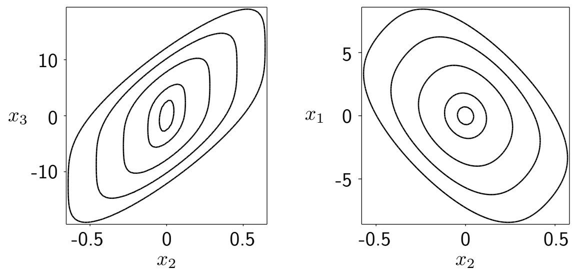

Fig. 2. Sections of the contour surfaces of $H_d(x)$ in x-coordinates for $x_1 = 0$ and $x_3 = 0$

and has the particular solution

$$\Psi(x) = -\frac{1}{2\alpha_{12}} x_1^2 - \frac{b}{\nu_{13}} \cos x_2.$$

The freely adjustable homogeneous solution is set up as

$$\Phi(\xi) = \frac{1}{2}\mu_2 \xi^2 + \frac{1}{4}\mu_4 \xi^4,$$

with $\xi = z_\alpha = e_1^T F_d^{-T} x$ the characteristic coordinate resulting from the linear coordinate transformation (21), which holds globally in the case of constant F_d.

Step 5 The given choice of $\Phi(\xi)$ ensures $\nabla H_d(x)|_{x^*} = 0$.

Step 6 The system of equations (25) becomes

$$\begin{bmatrix} 0 \\ 0 \end{bmatrix} = \begin{bmatrix} -\alpha_{12} & 0 & \nu_{13} \\ 0 & -\nu_{13} & \nu_{23} \end{bmatrix} \begin{bmatrix} a_{d,11} \\ a_{d,12} \\ a_{d,13} \end{bmatrix} - \begin{bmatrix} 1 & 0 & 0 \\ 0 & -b & 0 \end{bmatrix} \begin{bmatrix} \alpha_{11} \\ \alpha_{12} \\ 0 \end{bmatrix},$$

$$\mu_2 = \left.\frac{\partial^2 \tilde{\Phi}}{\partial z_\alpha^2}\right|_{z^*} = a_{d,11}\alpha_{11} + a_{d,12}\alpha_{12} - \left.\frac{\partial^2 \tilde{\Psi}}{\partial z_\alpha^2}\right|_{z^*},$$

where

$$\left.\frac{\partial^2 \tilde{\Psi}}{\partial z_\alpha^2}\right|_{z^*} = -\frac{\alpha_{11}^2}{\alpha_{12}} + b\frac{\alpha_{12}^2}{\nu_{13}},$$

and can be solved besides for μ_2, e.g. for α_{12} and ν_{13}. The parameters α_{11}, ν_{23}, as well as μ_4 remain free.

Step 7 The values $\alpha_{11} = \nu_{23} = -1$ are chosen arbitrarily and the weighting $\mu_4 = 4$ is supposed to render the nonlinear IDA-PBC control law

$$u_{\text{IDA}} = \frac{1}{c}\Big(-\frac{\alpha_{11}}{\alpha_{12}} x_1 - (a - \frac{\alpha_{12}}{\nu_{13}} b) \sin x_2 + \\ + (\mu_2 z_\alpha + \mu_4 z_\alpha^3) \circ z_\alpha(x)\Big)$$

more aggressive with respect to larger deviations from the equilibrium, compared to its purely linear counterpart $u_{\text{lin}} = Kx$. In Fig. 2, sections of the closed contour surfaces of the energy are displayed. Figure 3 shows transients of the simulated nonlinear system (with plant parameters from Ho et al. (2009)) under both control laws, as well as the normal force between ball and wheel, which remains positive along the transients. For $\mu_4 = 0$ (not displayed) the plots under u_{IDA} and u_{lin} virtually coincide. The choice $\mu_4 = 4$ does not alter local linear dynamics, but provides a stronger reaction and faster transients.

Note that the free parameter values in the example, as well as the function $\Phi(\cdot)$ are chosen arbitrarily in order to illustrate the design procedure in a compact manner. The optimization according to Step 7 is omitted and the reader is referred to Kloiber and Kotyczka (2012) for recent results on this topic.

7 Conclusions

The technique of local linear dynamics assignment for the parametrization of IDA-PBC has been described. By solving a linear system of equations for the design parameters, predefined dynamic behavior of the closed-loop linearization can be achieved. Furthermore, the definiteness check of the closed-loop energy Hessian matrix can be omitted. Local linear dynamics assignment is one way to reduce systematically the extensive freedom in the choice of the IDA-PBC design parameters.

The method is applicable to set-point control (Kotyczka et al., 2010a) and also to design passivity based tracking controllers (Kotyczka et al., 2010b), where the peculiarities of time-varying systems have to be accounted for. The idea can also be transferred to the stabilization of underactuated mechanical systems (Kotyczka, 2011). The results presented in this paper, together with an optimization with respect to the domain of attraction (Kloiber and Kotyczka, 2012), are first steps to enhance the applicability of IDA-PBC and to use it e. g. for automated nonlinear controller design. This however, remains a challenging task due to the large number of available design quantities in IDA-PBC.

An interesting issue, which gives rise to future work in the context of the presented approach, is dynamics assignment when passivity based methods are applied to systems which have been partially feedback linearized – this is especially relevant in the control of underactuated mechanical systems. Furthermore, it could be worthwhile examining how the idea to use the linearization for dynamics assignment can be exploited for certain classes of infinite-dimensional (discretized) systems.

Acknowledgements

The support of the head of institute, Boris Lohmann, and the helpful remarks of Joachim Deutscher on the manuscript are cordially acknowledged, as well as the valuable comments of the reviewers.

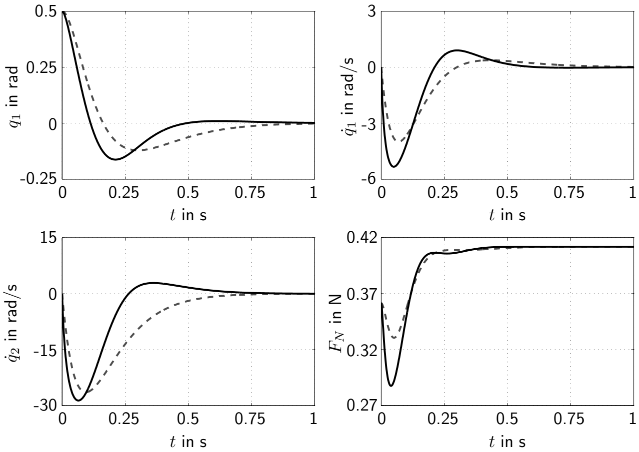

Fig. 3. Transients of q_1, $\dot{q}_1$, $\dot{q}_2$ and the normal force F_N under both IDA-PBC (solid) and the linear control law (dashed)

References

Acosta, J. A. and Astolfi, A. (2009). On the PDEs arising in IDA-PBC. In *Proceedings of the Conference on Decision and Control / Chinese Control Conference, Shanghai*, pages 2132–2137.

Acosta, J. A., Ortega, R., Astolfi, A., and Mahindrakar, A. D. (2005). Interconnection and damping assignment passivity-based control of mechanical systems with underactuation degree one. *IEEE Transactions on Automatic Control*, 50(12):1936–1955.

Bacciotti, A. (1988). The local stabilizability problem for nonlinear systems. *IMA Journal of Mathematical Control and Information*, 5:27–39.

Boyd, S. (2005). Linear dynamical systems. Lecture Notes, Stanford University, www.stanford.edu/class/ee363/notes/lq-lyap-notes.pdf.

Cheng, D., Astolfi, A., and Ortega, R. (2005). On feedback equivalence to port controlled Hamiltonian systems. *Systems & Control Letters*, 54:911–917.

Gómez-Estern, F. and van der Schaft, A. J. (2004). Physical damping in IDA-PBC controlled underactuated mechanical systems. *European Journal of Control*, 10:451–468.

Höffner, K. (2011). *Geometric Aspects of Interconnection and Damping Assignment – Passivity-Based Control.* PhD thesis, Queen's University, Kingston.

Ho, M.-T., Tu, Y.-W., and Lin, H.-S. (2009). Controlling a ball and wheel system using full-state-feedback linearization. *IEEE Control Systems Magazine*, pages 93–101.

Isidori, A. (1995). *Nonlinear Control Systems.* Springer-Verlag, London.

Khalil, H. K. (1996). *Nonlinear Systems.* Prentice-Hall.

Kloiber, T. and Kotyczka, P. (2012). Estimating and enlarging the domain of attraction in IDA-PBC. In *Proceedings of the IEEE Conference on Desision and Control, Maui.*

Kotyczka, P. (2011). Local linear dynamics assignment in IDA-PBC for underactuated mechanical systems.

In *Proceedings of the IEEE Conference on Decision and Control / European Control Conference, Orlando*, pages 6534–6539.

Kotyczka, P., Koch, G., Pellegrini, E., and Lohmann, B. (2010a). Transparent parametrization of nonlinear IDA-PBC for a hydraulic actuator. In *Proceedings of the IFAC Symposium on Nonlinear Control Systems, Bologna*, pages 1122–1127.

Kotyczka, P. and Lohmann, B. (2009). Parametrization of IDA-PBC by assignment of local linear dynamics. In *Proceedings of the European Control Conference, Budapest*, pages 4721–4726.

Kotyczka, P., Volf, A., and Lohmann, B. (2010b). Passivity based trajectory tracking control with predefined local linear error dynamics. In *Proceedings of the American Control Conference, Baltimore*, pages 3429–3434.

Krstić, M., Kokotović, P. V., and Kanellakopoulos, I. (1995). *Nonlinear and Adaptive Control Design*. John Wiley & Sons, New York.

Kugi, A. (2001). *Non-Linear Control Based on Physical Models: Electrical, Hydraulic, and Mechanical Systems*. Springer-Verlag, New York.

Lévine, J. (2009). *Analysis and Control of Nonlinear Systems*. Springer-Verlag.

Nijmeijer, H. and van der Schaft, A. J. (1990). *Nonlinear dynamical control systems*. Springer-Verlag, New York.

Ortega, R., Loría, A., Nicklasson, P. J., and Sira-Ramírez, H. (1998). *Passivity-based Control of Euler-Lagrange Systems*. Springer-Verlag, London.

Ortega, R., van der Schaft, A. J., Maschke, B., and Escobar, G. (2002). Interconnection and damping assignment passivity-based control of port-controlled Hamiltonian systems. *Automatica*, 338:585–596.

Prajna, S., van der Schaft, A. J., and Meinsma, G. (2002). An LMI approach to stabilization of linear port-controlled Hamiltonian systems. *Systems & Control Letters*, 45:371–385.

Sepulchre, R., Janković, M., and Kokotović, P. V. (1997). *Constructive Nonlinear Control*. Springer-Verlag, London.

Tiefensee, F., Monaco, S., and Normand-Cyrot, D. (2010). IDA-PBC under sampling for port-controlled Hamiltonian systems. In *Proceedings of the American Control Conference, Baltimore*, pages 1811–1816.

van der Schaft, A. J. (2000). *L2-Gain and Passivity Techniques in Nonlinear Control*. Springer-Verlag, London.

Viola, G., Ortega, R., Banavar, R., Acosta, J. A., and Astolfi, A. (2007). Total energy shaping control of mechanical systems: Simplifying the matching equations via coordinate changes. *IEEE Transactions on Automatic Control*, 52(6):1093–1099.