FDD and TDD Single-User Capacity Bounds

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***Abstract*—Multiple antennas increase significantly the capacity of wireless links under the assumption of perfect *channel state information* (CSI). In practice, however, in *time division duplex* (TDD) or *frequency division duplex* (FDD) systems, the available CSI is obtained with training and/or limited feedback and hence, is not perfect. In this work, we derive tight capacity bounds for the uplink and downlink of TDD/FDD systems with multiple antennas at the base station and a single-antenna user. We also present a performance comparison between the two systems based on the achievable rates and discuss further issues for a relevant comparison.**

I. Introduction

Consider a user in an isolated cell, i.e. with no intracell and intercell interference. Let us further assume a *base station* (BS) equipped with M antennas and the user with a single antenna, i.e. we have a *multiple-input single-output* (MISO) downlink and *single-input multiple-output* (SIMO) uplink. In the SIMO uplink, the BS can estimate the uplink channel with the aid of a training sequence consisting of T_{UL} pilots [1]. Hence, in a TDD or FDD uplink the base station can perform *maximum ratio combining* based on the channel estimate, incurring however in a performance degradation due to the mismatch between the true channel and the channel estimate.

Furthermore, after some calibration in a TDD system, the uplink channel estimate can be employed for the transmit beamforming in the downlink. By performing *coherent beamforming* in the downlink based on the uplink channel estimate, we experience a mismatch with the actual downlink channel due to estimation errors, similarly as in the uplink. Besides this issue, the mismatch between the beamforming and the downlink channel is further increased in the TDD downlink due to the time-varying nature of the channel.

On the other hand, in a FDD system the *transmit* CSI for the MISO downlink becomes available at the base station through *limited feedback* of B bits [2] in a three-steps process. The downlink channel is first estimated at the user by means of T_{DL} M-dimensional pilot vectors during a downlink training phase. Afterwards, the *channel direction information* (CDI) of the channel estimate is quantized with B bits and finally, the B uncoded bits corresponding to the quantized CDI estimate are fed back in the uplink from the user to the BS. With a faded- and noise-prone uplink, the B relayed bits could be received erroneously at the BS. This could lead to further increasing the mismatch in the downlink between the true channel and the available transmit CSI at the BS. The mismatch could be additionally increased due to the delay incurred in the feedback process. Thus, the transmit CSI available at the BS in the downlink is subject to estimation and quantization errors, and could be outdated and affected by erroneous feedback. Since the capacity with imperfect CSI as previously described for both systems is unknown in general, in this work we derive lower and upper bounds on the ergodic capacity with Gaussian signalling for both links in a TDD and FDD system with a single user. In addition, we present a performance comparison between the two systems based on the achievable rates and discuss further issues for a relevant comparison.

II. System Model

In both systems, each uplink/downlink time slot consists of T channel uses, where at each channel use one symbol is transmitted. The duration in seconds of one time slot is denoted by t_{s}. In addition, the uplink and downlink channels are assumed to be constant for one time slot, i.e., block fading. All transmit signals are i.i.d. zero-mean with unit-variance and the entries of all channel vectors are assumed to be i.i.d. complex Gaussian random variables with zero-mean and unit-variance. The available power is P_{UL} and P_{DL} for the uplink and downlink, respectively.

For the FDD system we consider a paired spectrum, where the uplink channel $\boldsymbol{h}_{\text{F,UL}} \in \mathbb{C}^M$ and the downlink channel $\boldsymbol{h}_{\text{F,DL}} \in \mathbb{C}^M$ are uncorrelated and considered to be flat over the respective uplink and downlink frequency band. In addition, we assume that the bandwidth of the downlink and uplink to be equal, namely W Hz, as depicted in Fig. 1.

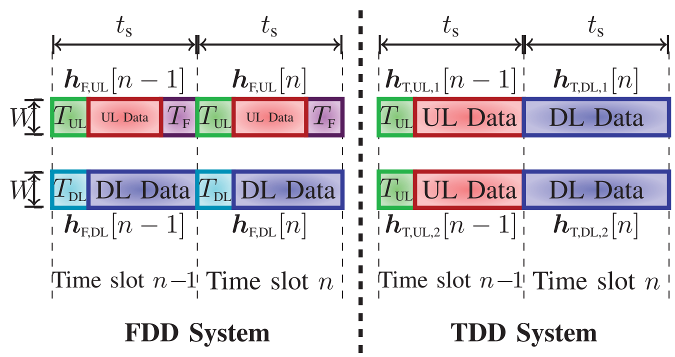

Figure 1. Consecutive Time Slots in an FDD and TDD System

With the same set of assumptions in a TDD system, each link in a TDD system employs simultaneously two frequency resource blocks, while the uplink and downlink transmissions take place over alternating time slots. The TDD uplink channels in both resource blocks are uncorrelated and denoted as $\boldsymbol{h}_{\text{T,UL},1}, \boldsymbol{h}_{\text{T,UL},2} \in \mathbb{C}^M$. Similarly $\boldsymbol{h}_{\text{T,DL},1}, \boldsymbol{h}_{\text{T,DL},2} \in \mathbb{C}^M$ denote the downlink channels. Although the available bandwidth for the TDD uplink/downlink is twice as large $(2W)$ compared to its counterpart in the FDD system, the transmission in each link of a TDD system takes place only half of the time $(\frac{t_{\text{s}}}{2})$ and thus, has the same degrees of freedom as the FDD system, i.e., Wt_{s}. Moreover, since the available power is P_{UL} and P_{DL}, the power

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on each resource blocks in the TDD uplink and downlink is $\frac{P_{\text{UL}}}{2}$ and $\frac{P_{\text{DL}}}{2}$. Hence, the TDD uplink/downlink SNR on each resource block is 3 dB smaller than the uplink/downlink SNR in the corresponding FDD system.

In order to observe the impact of the outdated CSI in the downlink, we model the time-varying nature of the channel by considering *temporally correlated block fading* between successive time slots by employing a first order Markov model as in [2], i.e., for instance the FDD and TDD downlink channel at time slot n are given by

$$\boldsymbol{h}_{\text{F,DL}}[n] = \sqrt{\alpha}\,\boldsymbol{h}_{\text{F,DL}}[n-1] + \sqrt{1-\alpha}\,\boldsymbol{g}[n-1], \quad (1)$$
$$\boldsymbol{h}_{\text{T,DL},i}[n] = \sqrt{\alpha}\,\boldsymbol{h}_{\text{T,UL},i}[n-1] + \sqrt{1-\alpha}\,\boldsymbol{g}_i[n-1], \quad (2)$$

where the elements of $\boldsymbol{g}[n-1], \boldsymbol{g}_i[n-1] \in \mathbb{C}^M$ for $i = 1, 2$ are i.i.d. zero-mean unit-variance complex Gaussian random variables and are uncorrelated with $\boldsymbol{h}_{\text{F,DL}}[n-1]$ and $\boldsymbol{h}_{\text{T,UL},i}[n-1]$ for $i = 1, 2$, respectively. Moreover, $\sqrt{\alpha}$ is the correlation coefficient, with $0 \leq \alpha < 1$. The time variations in the FDD uplink channel $\boldsymbol{h}_{\text{F,UL}}[n]$ can also be modelled similarly to (1).

The first T_{UL} channel uses in the FDD and TDD uplink time slot are employed for uplink channel training. In the next $T_{\text{F,UL,D}}$ (for FDD) and $T_{\text{T,UL,D}}$ (for TDD) channel uses, the user transmits data to the BS. In the TDD system, $T_{\text{T,UL,D}} = T - T_{\text{UL}}$. However, in the FDD system the final T_{F} channel uses in the uplink time slot are reserved for the *uncoded* feedback of the CSI from the user to the BS, such that $T_{\text{F,UL,D}} = T - T_{\text{UL}} - T_{\text{F}}$.

Similarly as in the uplink, the first T_{DL} channel uses in the FDD downlink time slot are employed for downlink channel training. In the remaining $T_{\text{F,DL,D}} = T - T_{\text{DL}}$ channel uses of the downlink time slot, data is transmitted from the BS to the user. However, in the TDD downlink there is no need for estimating the channel vector and hence, only data is sent from the BS to the user in the downlink time slot as depicted in Fig. 1.

As a figure of merit we consider the ergodic capacity based on the imperfect CSI, which implies that coding of the data is performed over multiple time slots of the time-varying channel in each link with $\alpha < 1$.

III. Uplink Capacity Bounds

With T_{UL} pilots in the FDD uplink, the BS can obtain at time slot $n-1$ an MMSE channel estimate $\hat{\boldsymbol{h}}_{\text{F,UL}}[n-1] \in \mathbb{C}^M$ whose entries are i.i.d. zero-mean complex Gaussian random variables with variance $1-\sigma^2_{e_{\text{UL}}}(\rho_{\text{UL}})$, where [1]

$$\sigma_e^2\left(\rho_{\text{UL}}\right) = \frac{1}{1 + \rho_{\text{UL}} T_{\text{UL}}}. \quad (3)$$

where $\rho_{\text{UL}} = \frac{P_{\text{UL}}}{\sigma_{\text{n}}^2}$ with σ_{n}^2 as the variance of the AWGN and

$$\boldsymbol{h}_{\text{F,UL}}[n-1] = \hat{\boldsymbol{h}}_{\text{F,UL}}[n-1] + \boldsymbol{e}_{\text{F,UL}}[n-1], \quad (4)$$

with $\boldsymbol{e}_{\text{F,UL}}$ as the estimation error. Based on the estimate, the BS performs beamforming with $\boldsymbol{w}_{\text{F,UL}}[n-1] = \frac{\hat{\boldsymbol{h}}_{\text{F,UL}}[n-1]}{\|\hat{\boldsymbol{h}}_{\text{F,UL}}[n-1]\|_2}$ and the received signal $\boldsymbol{y}_{\text{F,UL}} \in \mathbb{C}^{T_{\text{F,UL,D}}}$ at the BS at time slot $n-1$

$$\boldsymbol{y}_{\text{F,UL}}[n-1] = \sqrt{P_{\text{UL}}}\,\|\hat{\boldsymbol{h}}_{\text{F,UL}}[n-1]\|_2\,\boldsymbol{s}_{\text{F,UL}}[n-1] + \boldsymbol{v}'_{\text{F,UL}}[n-1], \quad (5)$$

with $\boldsymbol{v}'_{\text{F,UL}}[n-1] = \sqrt{P_{\text{UL}}}\boldsymbol{w}_{\text{UL}}^{\text{H}}[n-1]\boldsymbol{e}_{\text{F,UL}}[n-1]\boldsymbol{s}_{\text{F,UL}}[n-1] + \boldsymbol{v}_{\text{F,UL}}[n-1]$, with $\boldsymbol{s}_{\text{F,UL}}[n-1] \in \mathbb{C}^{T_{\text{F,UL,D}}}$ and $\boldsymbol{v}_{\text{F,UL}}[n-1] \in \mathbb{C}^{T_{\text{F,UL,D}}}$ are the uplink transmit symbols and the zero-mean AWGN with variance σ_{n}^2.

The FDD uplink capacity $C_{\text{F,UL}}$ can be lower bounded based on the lower bound given in [3], which follows by assuming i.i.d. Gaussian signalling and that the estimation error behaves as worst case noise

$$C_{\text{F,UL}} \geq \frac{T_{\text{F,UL,D}}}{T} \mathrm{E}_{\hat{\boldsymbol{h}}_{\text{F,UL}}}\left[\log_2\left(1 + \frac{\rho_{\text{UL}}\|\hat{\boldsymbol{h}}_{\text{F,UL}}\|_2^2}{\rho_{\text{UL}}\,\sigma_e^2\left(\rho_{\text{UL}}\right)+1}\right)\right] = C_1\left(\rho_{\text{UL}}, T_{\text{F,UL,D}}\right).$$

The equality results from [4, (8.40)] and the identity $\mathrm{E}_n(z) = z^{n-1}\Gamma_{\text{inc}}(1-n,z)$, where Γ_{inc} is the incomplete gamma function and E_n is the generalized exponential integral such that

$$C_1\left(\rho_{\text{UL}}, T_{\text{F,UL,D}}\right) = \frac{T_{\text{F,UL,D}}}{T} \log_2(\mathrm{e})\mathrm{e}^{\frac{1}{\lambda(\rho_{\text{UL}})}} \sum_{k=1}^{M} \mathrm{E}_k\left(\frac{1}{\lambda(\rho_{\text{UL}})}\right), \quad (6)$$

with

$$\lambda\left(\rho_{\text{UL}}\right) = \frac{\rho_{\text{UL}}(1 - \sigma_e^2\left(\rho_{\text{UL}}\right))}{1 + \rho_{\text{UL}}\sigma_e^2\left(\rho_{\text{UL}}\right)}. \quad (7)$$

The FDD uplink capacity can be upper bounded as

$$\begin{aligned} C_{\text{F,UL}} &\leq \frac{1}{T} I\Big(\boldsymbol{s}_{\text{F,UL}}[n-1]; \boldsymbol{y}_{\text{F,UL}}[n-1] \mid \hat{\boldsymbol{h}}_{\text{F,UL}}[n-1], \boldsymbol{w}_{\text{F,UL}}^{\text{H}}[n-1]\boldsymbol{e}_{\text{F,UL}}[n-1]\Big) \\ &\leq C_2\left(\rho_{\text{UL}}, T_{\text{F,UL,D}}\right), \end{aligned}$$

where the first inequality results by assuming the receiver knows $\boldsymbol{w}_{\text{F,UL}}^{\text{H}}[n-1]\boldsymbol{e}_{\text{F,UL}}[n-1]$ besides the channel estimate. The second step follows by noting that in this case Gaussian signalling is optimum and by applying Jensen's inequality with

$$C_2\left(\rho_{\text{UL}}, T_{\text{F,UL,D}}\right) = \frac{T_{\text{F,UL,D}}}{T} \log_2\left(1 + \rho_{\text{UL}}\left((M-1)(1-\sigma_e^2(\rho_{\text{UL}}))+1\right)\right) \quad (8)$$

The TDD uplink is equivalent to the FDD uplink, with the difference that the available power on each resource block is $\frac{P_{\text{UL}}}{2}$ instead of P_{UL}. Hence, the TDD uplink capacity is lower and upper bounded by $C_1\left(\frac{\rho_{\text{UL}}}{2}, T_{\text{T,UL,D}}\right)$ and $C_2\left(\frac{\rho_{\text{UL}}}{2}, T_{\text{T,UL,D}}\right)$, by employing the FDD uplink capacity bounds from (6) and (8).

IV. FDD Downlink Channel

In the FDD downlink, the user obtains an MMSE channel estimate $\hat{\boldsymbol{h}}_{\text{F,DL}}[n-1] \in \mathbb{C}^M$ with the aid of T_{DL} M-dimensional pilot vectors which are sent from the BS during the downlink training phase at time slot $n-1$. The entries of $\hat{\boldsymbol{h}}_{\text{F,DL}}$ are i.i.d. zero-mean complex Gaussian random variables with variance $1 - \sigma_e^2\left(\rho_{\text{DL}}\right)$ with $\rho_{\text{DL}} = \frac{P_{\text{DL}}}{M\sigma_{\text{n}}^2}$. We assume $T_{\text{DL}} \geq M$.

The channel direction information of the downlink channel $\hat{\boldsymbol{h}}_{\text{F,DL}}[n-1]$ at time slot $n-1$ is then quantized with B bits employing the *random vector quantization* (RVQ) scheme [6]. The user feeds back the index corresponding to the the beamforming vector $\boldsymbol{w}_{\text{FB}}[n]$ (to be used at time slot n due to the feedback delay of one time slot) that maximizes

$$\boldsymbol{w}_{\text{FB}}[n] = \underset{\boldsymbol{c}_i}{\operatorname{argmax}}\ |\boldsymbol{c}_i^{\text{H}}\hat{\boldsymbol{h}}_{\text{F,DL}}[n-1]|^2, \quad (9)$$

where $\boldsymbol{c}_i$, $i = 1, \ldots, 2^B$ are the i.i.d isotropically distributed unit-norm vectors in the RVQ codebook.

The downlink received signal at the user at time slot n is

$$\mathbb{C}^{T_{\text{F,DL,D}}} \ni \boldsymbol{y}_{\text{F,DL}}[n] = \sqrt{P_{\text{DL}}}\boldsymbol{w}_{\text{F,DL}}^{\text{H}}[n]\boldsymbol{h}_{\text{F,DL}}[n]\boldsymbol{s}_{\text{F,DL}}[n] + \boldsymbol{v}_{\text{F,DL}}[n], \quad (10)$$

where $\boldsymbol{s}_{\text{F,DL}}[n], \boldsymbol{v}_{\text{F,DL}}[n] \in \mathbb{C}^{T_{\text{F,DL,D}}}$ are the transmit symbols and the zero-mean AWGN with variance σ_{n}^2. The FDD downlink transmission takes place with the beamforming vector $\boldsymbol{w}_{\text{F,DL}}[n]$ which is equal to $\boldsymbol{w}_{\text{FB}}[n]$ if there were no feedback errors in the uplink at time slot $n-1$, which occurs with probability $1 - p_\epsilon$, with p_ϵ as the feedback error probability.

After error-free feedback, the equivalent downlink channel

$h_{\mathrm{DL,cf}}[n] = \boldsymbol{w}_{\mathrm{FB}}^{\mathrm{H}}[n]\boldsymbol{h}_{\mathrm{F,DL}}[n]$ at time slot n is given by

$$h_{\mathrm{DL,cf}}[n]=\sqrt{\alpha}\,\frac{\boldsymbol{w}_{\mathrm{FB}}^{\mathrm{H}}[n]\hat{\boldsymbol{h}}_{\mathrm{F,DL}}[n-1]}{\|\hat{\boldsymbol{h}}_{\mathrm{F,DL}}[n-1]\|_2}\|\hat{\boldsymbol{h}}_{\mathrm{F,DL}}[n-1]\|_2+\sqrt{\alpha}\,e_{\mathrm{DL}}+\sqrt{1-\alpha}\,g,$$

which follows from (1) and (4) and with the substitutions $e_{\mathrm{DL}} = \boldsymbol{w}_{\mathrm{FB}}^{\mathrm{H}}[n]\boldsymbol{e}_{\mathrm{F,DL}}[n-1]$ and $g=\boldsymbol{w}_{\mathrm{FB}}^{\mathrm{H}}[n]\boldsymbol{g}[n-1]$, which are independent complex Gaussian with zero mean and variance $\sigma_e^2(\rho_{\mathrm{DL}})$ and 1, respectively. From the previous expression,

$$\begin{aligned}|h_{\mathrm{DL,cf}}[n]|^2 =\;& \alpha\nu\,\|\hat{\boldsymbol{h}}_{\mathrm{F,DL}}[n-1]\|_2^2 + (1-\alpha)\,|g|^2 + \alpha\,|e_{\mathrm{DL}}|^2\\ &+ 2\sqrt{\alpha(1-\alpha)}\,\mathrm{Re}\left\{\boldsymbol{w}_{\mathrm{FB}}^{\mathrm{H}}[n]\hat{\boldsymbol{h}}_{\mathrm{F,DL}}[n-1]\,g^*+g\,e_{\mathrm{DL}}^*\right\}\\ &+ 2\alpha\,\mathrm{Re}\left\{\boldsymbol{w}_{\mathrm{FB}}^{\mathrm{H}}[n]\hat{\boldsymbol{h}}_{\mathrm{F,DL}}[n-1]\,e_{\mathrm{DL}}^*\right\}. \qquad (11)\end{aligned}$$

$$\text{with}\quad \nu = \left|\boldsymbol{w}_{\mathrm{FB}}^{\mathrm{H}}[n]\hat{\boldsymbol{h}}_{\mathrm{F,DL}}[n-1]\right|^2/\|\hat{\boldsymbol{h}}_{\mathrm{F,DL}}[n-1]\|_2^2. \qquad (12)$$

Due to the fact that $\|\hat{\boldsymbol{h}}_{\mathrm{F,DL}}\|_2^2$ is chi-squared distributed

$$\mathrm{E}\left[|h_{\mathrm{DL,cf}}[n]|^2\right] = \alpha\left(M\,\mathrm{E}\left[\nu\right]-1\right)\left(1-\sigma_e^2\left(\rho_{\mathrm{DL}}\right)\right)+1, \quad (13)$$

since g and e_{DL} are independent and from [5] we obtain

$$\mu_\nu = \mathrm{E}\left[\nu\right] = 1-2^B\mathrm{Beta}\left(2^B, \frac{M}{M-1}\right) \approx 1-2^{-\frac{B}{M-1}}. \quad (14)$$

The variance of ν is given by $\sigma_\nu^2 = \mathrm{E}\left[\nu^2\right]-\mathrm{E}^2\left[\nu\right]$, where

$$\mathrm{E}\left[\nu^2\right] = 1-2\cdot 2^B\mathrm{Beta}\left(2^B, \frac{M}{M-1}\right)+2^B\mathrm{Beta}\left(2^B, \frac{M+1}{M-1}\right).$$

A. Feedback Error Probability

The B bits are fed back *uncoded* without any error detection in the uplink to the BS with $T_{\mathrm{F}} = \frac{B}{2}$ QPSK symbols. Since no optimized labelling scheme of the feedback bits is used as in [2], one feedback symbol leads to a total feedback loss which occurs with probability p_ϵ. Assuming there was a feedback error in the uplink at time slot n–1, and with the BS unaware of the error, the received beamforming vector $\boldsymbol{w}_{\mathrm{F,DL}}[n]$ is random with respect to $\boldsymbol{h}_{\mathrm{F,DL}}[n]$, and therefore, the effective channel denoted in this case by $h_{\mathrm{DL,ef}}[n] = \boldsymbol{w}_{\mathrm{F,DL}}^{\mathrm{H}}[n]\boldsymbol{h}_{\mathrm{F,DL}}[n]$ is a zero-mean unit-variance complex Gaussian random variable.

Similar as for the payload phase, the BS can perform receive beamformng based on the uplink channel estimate for the feedback detection. The SNR during the feedback phase at time slot $n-1$ is approximated with $\overline{\gamma}_{\mathrm{F}}[n-1] = \frac{P_{\mathrm{UL}}\|\hat{\boldsymbol{h}}_{\mathrm{F,UL}}[n-1]\|_2^2}{\sigma_{\mathrm{n}}^2+P_{\mathrm{UL}}\sigma_e^2(\rho_{\mathrm{UL}})}$, such that the average feedback error probability is given as

$$p_\epsilon = \mathrm{E}\left[1-\left(1-p_{\mathrm{b}}\left(\overline{\gamma}_{\mathrm{F}}[n-1]\right)\right)^B\right] \approx 1-\left(1-\mathrm{E}[p_{\mathrm{b}}\left(\overline{\gamma}_{\mathrm{F}}[n-1]\right)]\right)^B, \quad (15)$$

where the expectation is taken over the channel estimate $\hat{\boldsymbol{h}}_{\mathrm{F,UL}}[n-1]$ and p_{b} is the bit error probability with QPSK and maximum ratio combining given by [8, (7.20)].

V. Downlink Capacity Bounds

We now focus on bounds on the ergodic capacity $C_{\mathrm{F,DL}}$ for the FDD downlink with limited feedback beamforming. An upper bound to FDD downlink capacity is given by

$$\begin{aligned}C_{\mathrm{F,DL}} \le\;& \frac{T_{\mathrm{F,DL,D}}}{T}p_\epsilon\,\mathrm{E}_{h_{\mathrm{DL,ef}}}\left[\log_2\left(1+\frac{P_{\mathrm{DL}}}{\sigma_{\mathrm{n}}^2}|h_{\mathrm{DL,ef}}[n]|^2\right)\right]\\ &+\frac{T_{\mathrm{F,DL,D}}}{T}(1-p_\epsilon)\,\mathrm{E}_{h_{\mathrm{DL,cf}}}\left[\log_2\left(1+\frac{P_{\mathrm{DL}}}{\sigma_{\mathrm{n}}^2}|h_{\mathrm{DL,cf}}[n]|^2\right)\right]\\ \le\;& \frac{T_{\mathrm{F,DL,D}}}{T}\Bigg(p_\epsilon\;\log_2(\mathrm{e})\,\mathrm{e}^{\frac{\sigma_{\mathrm{n}}^2}{P_{\mathrm{DL}}}}\;\mathrm{E}_1\left(\frac{\sigma_{\mathrm{n}}^2}{P_{\mathrm{DL}}}\right)\\ &+(1-p_\epsilon)\log_2\left(1+\frac{P_{\mathrm{DL}}}{\sigma_{\mathrm{n}}^2}\left(\alpha(M\mathrm{E}\left[\nu\right]-1)\left(1-\sigma_e^2(\rho_{\mathrm{DL}})\right)+1\right)\right)\Bigg), \quad (16)\end{aligned}$$

where the first step follows by assuming a genie informs the user of the effective downlink channel, i.e., either $h_{\mathrm{DL,cf}}[n]$ or $h_{\mathrm{DL,ef}}[n]$, such that Gaussian signalling is optimum. The first term in the final expression results similarly as (6) and the second one from Jensen's inequality and (13).

Denote $C_{\mathrm{F,DL}} = \frac{1}{T}I_{\mathrm{F,DL}}$ with the mutual information $I_{\mathrm{F,DL}} = I\left(\boldsymbol{s}_{\mathrm{F,DL}}[n];\boldsymbol{y}_{\mathrm{F,DL}}[n]\right)$. Let the binary random variable θ indicate a feedback error event: $\theta[n-1]=1$ in case of a feedback error at time slot $n-1$ and $\theta[n-1]=0$ otherwise. Using the short-hand notation $I_{(A;B|C)} = I\left(A;B\mid C\right)$ we can derive a lower bound to $I\left(\boldsymbol{s}_{\mathrm{F,DL}}[n];\boldsymbol{y}_{\mathrm{F,DL}}[n]\right)$ in the FDD downlink

$$\begin{aligned}I_{\mathrm{F,DL}}=\;&I_{(\boldsymbol{s}_{\mathrm{F,DL}}[n];\boldsymbol{y}_{\mathrm{F,DL}}[n])}+I_{(\boldsymbol{s}_{\mathrm{F,DL}}[n],h_{\mathrm{DL}}[n];\boldsymbol{y}_{\mathrm{F,DL}}[n])}-I_{(\boldsymbol{s}_{\mathrm{F,DL}}[n],h_{\mathrm{DL}}[n];\boldsymbol{y}_{\mathrm{F,DL}}[n])}\\ \overset{(a)}{=}\;&I_{(h_{\mathrm{DL}}[n];\boldsymbol{y}_{\mathrm{F,DL}}[n])}+I_{(\boldsymbol{s}_{\mathrm{F,DL}}[n];\boldsymbol{y}_{\mathrm{F,DL}}[n]|h_{\mathrm{DL}}[n])}-I_{(h_{\mathrm{DL}}[n];\boldsymbol{y}_{\mathrm{F,DL}}[n]|\boldsymbol{s}_{\mathrm{F,DL}}[n])}\\ \overset{(b)}{\ge}\;&I_{(\boldsymbol{s}_{\mathrm{F,DL}}[n];\boldsymbol{y}_{\mathrm{F,DL}}[n]|h_{\mathrm{DL}}[n])}-I_{(h_{\mathrm{DL}}[n];\boldsymbol{y}_{\mathrm{F,DL}}[n]|\boldsymbol{s}_{\mathrm{F,DL}}[n])}\\ &-I_{(\boldsymbol{s}_{\mathrm{F,DL}}[n],\theta[n-1];\boldsymbol{y}_{\mathrm{F,DL}}[n]|h_{\mathrm{DL}}[n])}+I_{(\boldsymbol{s}_{\mathrm{F,DL}}[n],\theta[n-1];\boldsymbol{y}_{\mathrm{F,DL}}[n]|h_{\mathrm{DL}}[n])}\\ &-I_{(h_{\mathrm{DL}}[n],\theta[n-1];\boldsymbol{y}_{\mathrm{F,DL}}[n]|\boldsymbol{s}_{\mathrm{F,DL}}[n])}+I_{(h_{\mathrm{DL}}[n],\theta[n-1];\boldsymbol{y}_{\mathrm{F,DL}}[n]|\boldsymbol{s}_{\mathrm{F,DL}}[n])}\\ \overset{(c)}{=}\;&I_{(\theta[n-1];\boldsymbol{y}_{\mathrm{F,DL}}[n]|h_{\mathrm{DL}}[n])}+I_{(\boldsymbol{s}_{\mathrm{F,DL}}[n];\boldsymbol{y}_{\mathrm{F,DL}}[n]|h_{\mathrm{DL}}[n],\theta[n-1])}\\ &-I_{(\theta[n-1];\boldsymbol{y}_{\mathrm{F,DL}}[n]|\boldsymbol{s}_{\mathrm{F,DL}}[n])}-I_{(h_{\mathrm{DL}}[n];\boldsymbol{y}_{\mathrm{F,DL}}[n]|\boldsymbol{s}_{\mathrm{F,DL}}[n],\theta[n-1])}\\ \overset{(d)}{\ge}\;&I_{(\boldsymbol{s}_{\mathrm{F,DL}}[n];\boldsymbol{y}_{\mathrm{F,DL}}[n]|h_{\mathrm{DL}}[n],\theta[n-1])}-I_{(h_{\mathrm{DL}}[n];\boldsymbol{y}_{\mathrm{F,DL}}[n]|\boldsymbol{s}_{\mathrm{F,DL}}[n],\theta[n-1])}\\ &-I_{(\theta[n-1];\boldsymbol{y}_{\mathrm{F,DL}}[n]|\boldsymbol{s}_{\mathrm{F,DL}}[n])}\\ \overset{(e)}{=}\;&I_{(\boldsymbol{s}_{\mathrm{F,DL}}[n];\boldsymbol{y}_{\mathrm{F,DL}}[n]|h_{\mathrm{DL}}[n],\theta[n-1])}-I_{(h_{\mathrm{DL}}[n];\boldsymbol{y}_{\mathrm{F,DL}}[n]|\boldsymbol{s}_{\mathrm{F,DL}}[n],\theta[n-1])}\\ &-h\left(\theta[n-1]\mid\boldsymbol{s}_{\mathrm{F,DL}}[n]\right)+h\left(\theta[n-1]\mid\boldsymbol{s}_{\mathrm{F,DL}}[n],\boldsymbol{y}_{\mathrm{F,DL}}[n]\right)\\ \overset{(f)}{\ge}\;&I_{(\boldsymbol{s}_{\mathrm{F,DL}}[n];\boldsymbol{y}_{\mathrm{F,DL}}[n]|h_{\mathrm{DL}}[n],\theta[n-1])}-I_{(h_{\mathrm{DL}}[n];\boldsymbol{y}_{\mathrm{F,DL}}[n]|\boldsymbol{s}_{\mathrm{F,DL}}[n],\theta[n-1])}\\ &-h_{\mathrm{b}}(p_\epsilon), \qquad (17)\end{aligned}$$

where $h_{\mathrm{DL}}[n] = \boldsymbol{w}_{\mathrm{F,DL}}^{\mathrm{H}}[n]\boldsymbol{h}_{\mathrm{F,DL}}[n]$ is the effective downlink channel which as mentioned before is either equal to $h_{\mathrm{DL,cf}}[n]$ after correct feedback or to $h_{\mathrm{DL,ef}}[n]$ after erroneous feedback. Step (a) follows from applying the two definitions of the chain rule $I_{(A,B;C)} = I_{(B;C)} + I_{(A;C|B)} = I_{(A;C)} + I_{(B;C|A)}$ on the two identical terms. Inequality (b) results from the non-negativity of the mutual information. Step (c) arises similar to (a) from the chain rule on both pairs of identical terms, while step (d) follows from the non-negativity of the mutual information. Step (e) results from the definiton of the mutual information and step (f) arises from the non-negativity of the entropy and from $h\left(\theta[n-1]\mid\boldsymbol{s}_{\mathrm{F,DL}}[n]\right) \ge h\left(\theta[n-1]\right)$ which is the binary entropy function $h_{\mathrm{b}}(p_\epsilon)$ with probability p_e for the error event $\theta[n-1]=1$.

The first two terms in (17) can be computed from

$$\begin{aligned}I_{(\boldsymbol{s}_{\mathrm{F,DL}}[n];\boldsymbol{y}_{\mathrm{F,DL}}[n]|h_{\mathrm{DL}}[n],\theta[n-1])} &= (1-p_\epsilon)I_{(\boldsymbol{s}_{\mathrm{F,DL}}[n];\boldsymbol{y}_{\mathrm{F,DL}}[n]|h_{\mathrm{DL,cf}}[n])}\\ &\quad+p_\epsilon\,I_{(\boldsymbol{s}_{\mathrm{F,DL}}[n];\boldsymbol{y}_{\mathrm{F,DL}}[n]|h_{\mathrm{DL,ef}}[n])} \qquad (18)\\ I_{(h_{\mathrm{DL}}[n];\boldsymbol{y}_{\mathrm{F,DL}}[n]|\boldsymbol{s}_{\mathrm{F,DL}}[n],\theta[n-1])} &= (1-p_\epsilon)I_{(h_{\mathrm{DL,cf}}[n];\boldsymbol{y}_{\mathrm{F,DL}}[n]|\boldsymbol{s}_{\mathrm{F,DL}}[n])}\\ &\quad+p_\epsilon\,I_{(h_{\mathrm{DL,ef}}[n];\boldsymbol{y}_{\mathrm{F,DL}}[n]|\boldsymbol{s}_{\mathrm{F,DL}}[n])}. \qquad (19)\end{aligned}$$

The first term in (18) represents the mutual information given the effective channel after correct feedback. This mutual information can be lower bounded as

$$\begin{aligned}I_{(\boldsymbol{s}_{\mathrm{F,DL}}[n];\boldsymbol{y}_{\mathrm{F,DL}}[n]|h_{\mathrm{DL,cf}}[n])} &\overset{(a)}{\ge} T_{\mathrm{F,DL,D}}\,\mathrm{E}\left[\log_2\left(1+\frac{P_{\mathrm{DL}}|h_{\mathrm{DL,cf}}[n]|^2}{\sigma_{\mathrm{n}}^2}\right)\right]\\ &\overset{(b)}{\ge} T_{\mathrm{F,DL,D}}\,\mathrm{E}_{\nu,\|\hat{\boldsymbol{h}}_{\mathrm{F,DL}}\|_2^2}\left[\log_2\left(1+\frac{P_{\mathrm{DL}}\alpha\nu\,\|\hat{\boldsymbol{h}}_{\mathrm{F,DL}}[n-1]\|_2^2}{\sigma_{\mathrm{n}}^2}\right)\right]\end{aligned}$$

$$\stackrel{(c)}{\geq} T_{\text{F,DL,D}}\left(1-\frac{\sigma_\nu}{2\mu_\nu}\right)\mathrm{E}_{\|\hat{\boldsymbol{h}}_{\text{F,DL}}\|_2^2}\left[\log_2\left(1+\frac{P_{\text{DL}}\alpha\mu_\nu\,\|\hat{\boldsymbol{h}}_{\text{F,DL}}\|_2^2}{\sigma_{\text{n}}^2}\right)\right] \quad (20)$$

$$\stackrel{(d)}{=} T_{\text{F,DL,D}}\left(1-\frac{\sigma_\nu}{2\mu_\nu}\right)\log_2(\mathrm{e})\,\mathrm{e}^{1/\chi}\sum_{k=1}^{M}\mathrm{E}_k(1/\chi), \quad (21)$$

where (a) results from assuming Gaussian signalling. The inequality in step (b) results from the fact, which was verified numerically, that $x = |h_{\text{DL,cf}}[n]|^2$ given in (11) stochastically dominates [7] the random variable $y = \alpha\nu\,\|\hat{\boldsymbol{h}}_{\text{F,DL}}[n-1]\|_2^2$. For step (c), we employ the fact that ν, defined in (12), and $\|\hat{\boldsymbol{h}}_{\text{F,DL}}[n-1]\|_2^2$ are independent and we make use of a lower bound on concave functions given in [9, (15)] where the standard deviation $d_\nu \leq \sigma_\nu$. Finally, the last result (21) follows similar to the derivation of (6) with

$$\chi = \frac{P_{\text{DL}}\alpha\mu_\nu(1-\sigma_e^2(\rho_{\text{DL}}))}{\sigma_{\text{n}}^2}. \quad (22)$$

The second term in (18) represents the mutual information given the effective channel $h_{\text{DL,ef}}[n]$ after erroneous feedback. This mutual information is given by [4, (8.40)]

$$I_{(\boldsymbol{s}_{\text{F,DL}}[n];\boldsymbol{y}_{\text{F,DL}}[n]|h_{\text{DL,ef}}[n])} = T_{\text{F,DL,D}}\log_2(\mathrm{e})\mathrm{e}^{\frac{\sigma_{\text{n}}^2}{P_{\text{DL}}}}\mathrm{E}_1(\sigma_{\text{n}}^2/P_{\text{DL}}). \quad (23)$$

The second term in (17) can be viewed as a penalty for not really knowing the effective downlink channel $h_{\text{DL}}[n]$. The first term in (19) can be *upper* bounded as follows

$$I_{(h_{\text{DL,cf}}[n];\boldsymbol{y}_{\text{F,DL}}[n]|\boldsymbol{s}_{\text{F,DL}}[n])} \leq \log_2(\mathrm{e})\mathrm{e}^{\frac{\sigma_{\text{n}}^2}{P_{\text{DL}}\mathrm{E}[|h_{\text{DL,cf}}[n]|^2]}}\sum_{k=1}^{T_{\text{F,DL,D}}}\mathrm{E}_k\left(\frac{\sigma_{\text{n}}^2}{P_{\text{DL}}\mathrm{E}\left[|h_{\text{DL,cf}}[n]|^2\right]}\right), \quad (24)$$

which results by assuming $h_{\text{DL,cf}}[n]$ to be Gaussian distributed which maximizes the entropy given the variance of the effective channel, which is upper bounded by $\mathrm{E}\left[|h_{\text{DL,cf}}[n]|^2\right]$ given in (13). In (24), we have also employed [4, (8.40)].

By employing [4, (8.40)] we can also compute in closed form the second term in (19)

$$I_{(h_{\text{DL,ef}}[n];\boldsymbol{y}_{\text{F,DL}}[n]|\boldsymbol{s}_{\text{F,DL}}[n])} = \log_2(\mathrm{e})\mathrm{e}^{\sigma_{\text{n}}^2/P_{\text{DL}}}\sum_{k=1}^{T_{\text{F,DL,D}}}\mathrm{E}_k\left(\frac{\sigma_{\text{n}}^2}{P_{\text{DL}}}\right), \quad (25)$$

since $\boldsymbol{s}_{\text{F,DL}}[n]$ is Gaussian distributed with unit variance.

A lower bound to the FDD downlink capacity under limited feedback beamforming and Gaussian signalling is given by plugging (21) and (23) in (18), and (24) and (25) in (19), and afterwards substituting the resulting expressions for (18) and (19) in (17) and finally with $C_{\text{F,DL}} = \frac{1}{T}I_{\text{F,DL}}$, we obtain

$$\begin{aligned} C_{\text{F,DL}} \geq{}& \frac{(1-p_\epsilon)\log_2(\mathrm{e})}{T}\left(T_{\text{F,DL,D}}\left(1-\frac{\sigma_\nu}{2\mu_\nu}\right)\mathrm{e}^{1/\chi}\sum_{k=1}^{M}\mathrm{E}_k\left(1/\chi\right)\right.\\ &\left.-\,\mathrm{e}^{\frac{\sigma_{\text{n}}^2}{P_{\text{DL}}\mathrm{E}[|h_{\text{DL,cf}}[n]|^2]}}\sum_{k=1}^{T_{\text{F,DL,D}}}\mathrm{E}_k\left(\frac{\sigma_{\text{n}}^2}{P_{\text{DL}}\mathrm{E}\left[|h_{\text{DL,cf}}[n]|^2\right]}\right)\right)-\frac{h_{\text{b}}(p_\epsilon)}{T}\\ &+\frac{p_\epsilon}{T}\log_2(\mathrm{e})\,\mathrm{e}^{\frac{\sigma_{\text{n}}^2}{P_{\text{DL}}}}\left(T_{\text{F,DL,D}}\,\mathrm{E}_1\left(\frac{\sigma_{\text{n}}^2}{P_{\text{DL}}}\right)-\sum_{k=1}^{T_{\text{F,DL,D}}}\mathrm{E}_k\left(\frac{\sigma_{\text{n}}^2}{P_{\text{DL}}}\right)\right). \quad (26)\end{aligned}$$

An upper bound on the downlink capacity $C_{\text{T,DL}}$ in a TDD system can be derived similarly to the upper bound for the FDD case given in (16), by ignoring the effects of the quantization errors, feedback errors, by recalling that there is no training phase and that the available power on each resource block is $\frac{P_{\text{DL}}}{2}$ instead of P_{DL}:

$$C_{\text{T,DL}} \leq \log_2\left(1+\frac{P_{\text{DL}}}{2\sigma_{\text{n}}^2}\left(\alpha(M-1)\left(1-\sigma_e^2\left(\frac{\rho_{\text{DL}}}{2}\right)\right)+1\right)\right). \quad (27)$$

Following partially the derivation of the lower bound on the FDD downlink capacity, $C_{\text{T,DL}}$ can be lower bounded as

$$\begin{aligned} C_{\text{T,DL}} \geq{}& \log_2(\mathrm{e})\,\mathrm{e}^{\frac{2\sigma_{\text{n}}^2}{\alpha P_{\text{DL}}\left(1-\sigma_e^2\left(\frac{\rho_{\text{DL}}}{2}\right)\right)}}\sum_{k=1}^{M}\mathrm{E}_k\left(\frac{2\sigma_{\text{n}}^2}{\alpha P_{\text{DL}}\left(1-\sigma_e^2\left(\frac{\rho_{\text{DL}}}{2}\right)\right)}\right)\\ &-\frac{1}{T}\log_2(\mathrm{e})\,\mathrm{e}^{2\sigma_{\text{n}}^2/P_{\text{DL}}}\sum_{k=1}^{T}\mathrm{E}_k(2\sigma_{\text{n}}^2/P_{\text{DL}}). \quad (28)\end{aligned}$$

VI. Performance Analysis

In this section we provide some simulation results in order to assess the capacity bounds. Let us consider the following scenario: $T = 200$, $M = 8$, $B = 30$, $T_{\text{UL}} = T_{\text{DL}} = 10$ and $\alpha = 0.9994$[1]. In addition, we assume $P_{\text{UL}} = \frac{P_{\text{DL}}}{M}$, i.e., the user has $\frac{1}{M}$-th the available power of the base station, thus introducing a scaling of the available uplink power with the number of antennas. Hence, with $T_{\text{UL}} = T_{\text{DL}}$ the estimation error in the uplink and downlink of a given system are equal, for instance in an FDD system $\sigma_e^2(\rho_{\text{UL}}) = \sigma_e^2(\rho_{\text{DL}})$. With the described scenario, we depict in Fig. 2, the uplink and downlink capacity bounds for both systems as a function of the SNR parameterized by the FDD uplink and downlink SNR.

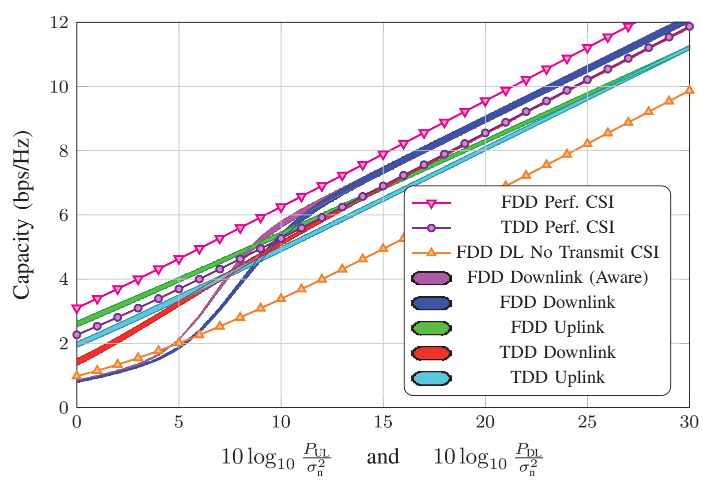

Figure 2. Capacity vs. SNR

The actual capacity with imperfect CSI for each of the links lies in the shaded region enclosed by the lower and upper bounds. As a reference we have included the capacity with perfect CSI which is the same for the uplink and downlink in a given system. An upper bound on the FDD downlink capacity without transmit CSI is also depicted. Recall that the TDD uplink/downlink SNR is 3 dB smaller than in the equivalent FDD uplink/downlink.

For low SNRs, the FDD downlink capacity with the BS being unaware of feedback errors can be smaller than the capacity when there is no transmit CSI. For a low downlink

[1]Assuming $\sqrt{\alpha} = \mathrm{J}_0(2\pi t_{\text{s}} f_{\text{c}} s/c)$, α results from considering a time slot duration $t_{\text{s}} = 1$ ms, a carrier frequency $f_{\text{c}} = 2$ GHz and a user's velocity along the line from the BS to the user of $s = 3$ km/h.

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SNR, the SNR in the corresponding uplink is assumed to be $\frac{1}{M}$-th the downlink SNR and hence, the occurrence of feedback errors in the uplink is not negligible. The FDD downlink capacity is mainly dominated by the capacity achieved after erroneous feedback and therefore, for such scenarios the performance in the FDD downlink is practically equivalent to that of random beamforming with a single user or to having only one transmit antenna. Since we assume the base station and the user to be unaware of the feedback errors, after erroneous feedback the BS sends only one stream with a random beamforming vector instead of transmitting M independent streams with equally shared power which is the optimum strategy when the base station has no transmit CSI.

However, if the BS were aware of a feedback error, it could employ, for instance, the preceding beamforming vector fed back in the previous time slot, if the precedent feedback message was received error-free[2]. The capacity bounds for this case are depicted in Fig. 2 with the additional legend (Aware). For moderate to high SNRs, there is practically no gain, since the feedback error probability for such SNRs is negligible. Nonetheless, even if the BS is aware of the feedback errors for very low SNRs, it cannot simply adopt the optimum transmit strategy for the no transmit CSI case, since the user would need to informed that the BS transmits M streams instead of only one stream. Another straightforward approach to counter the feedback errors for a given low downlink SNR is to increase the uplink SNR, which in our case would imply that the uplink SNR no longer scales as $\frac{1}{M}$-th the downlink SNR.

We now compare in Fig. 2 the TDD downlink with the FDD downlink when the BS is unaware of feedback errors. For up to an FDD downlink SNR of 9 dB, a TDD system achieves a higher capacity even though the TDD downlink SNR is 3 dB smaller. This is mainly due to the errors in transmission of the $B = 30$ feedback bits (15 QPSK symbols) in the FDD uplink. With smaller feedback error probabilities, i.e. with FDD downlink SNRs larger than 10 dB, which imply FDD uplink SNRs larger than 4 dB, the performance of the FDD downlink is for sure better than the TDD downlink.

Observe the slightly increasing absolute loss in capacity of the FDD uplink/downlink and TDD uplink with respect to the perfect CSI case, which results from the training and/or feedback overhead. We emphasize, nonetheless, that the assumed training and feedback length are not optimum at all SNRs.Due to the overhead, the slope of the FDD uplink/downlink and TDD uplink bounds are not strickly the same as that of the capacity with perfect CSI for high SNRs. However, assuming a fixed $T_{\text{DL}} \geq M$ for the FDD downlink, so that as $T \to \infty$ then $\frac{T_{\text{F,DL,D}}}{T} \to 1$ and with $\frac{P_{\text{UL}}}{\sigma_{\text{n}}^2} \to \infty$ and $\frac{P_{\text{DL}}}{\sigma_{\text{n}}^2} \to \infty$ such that we can ignore the feedback and estimation errors at high SNR, the FDD downlink capacity can be approximated using (20)

$$C_{\text{F,DL}} \approx \left(1 - \frac{\sigma_\nu}{2\mu_\nu}\right) \mathrm{E}\left[\log_2\left(1 + \frac{P_{\text{DL}}\alpha\mu_\nu \left\|\boldsymbol{h}_{\text{DL}}\right\|_2^2}{\sigma_{\text{n}}^2}\right)\right] \quad (29)$$

since the channel is estimated practically perfectly and fed back error-free. Considering $1 - \frac{\sigma_\nu}{2\mu_\nu} \approx 1$, comparing (29) with the corresponding expression for the perfect CSI case, we observe that the outdating and the quantization lead to an SNR degradation of $\alpha\mu_\nu \leq 1$.The SNR degradation suggests that in order that an FDD downlink, given the same resources (assuming the overhead, estimation errors and feedback errors are negligible) can match the performance of a TDD downlink with outdating (c.f. (27)), we would require

$$\alpha\left(1 - 2^{-\frac{B}{M-1}}\right) = \frac{\alpha}{2}, \quad (30)$$

where we used (14) and since the available power in the TDD downlink is half of that in the FDD downlink. For a given number of antennas M, an FDD downlink requires at least

$$B = (M-1) \quad \text{feedback bits}, \quad (31)$$

in order to match the equivalent TDD downlink, neglecting the effect of the overhead, estimation and feedback errors.

We now take a look at the FDD capacity bounds as a function of the number of feedback bits B. For Fig. 3, we consider the following scenario $T = 500$, $M = 8$, $T_{\text{UL}} = T_{\text{DL}} = 10$ and $\alpha = 0.9994$. The downlink SNR is $10\log_{10}\frac{P_{\text{DL}}}{\sigma_{\text{n}}^2} = 10$ dB and consequently $10\log_{10}\frac{P_{\text{UL}}}{\sigma_{\text{n}}^2} \approx 4$ dB.

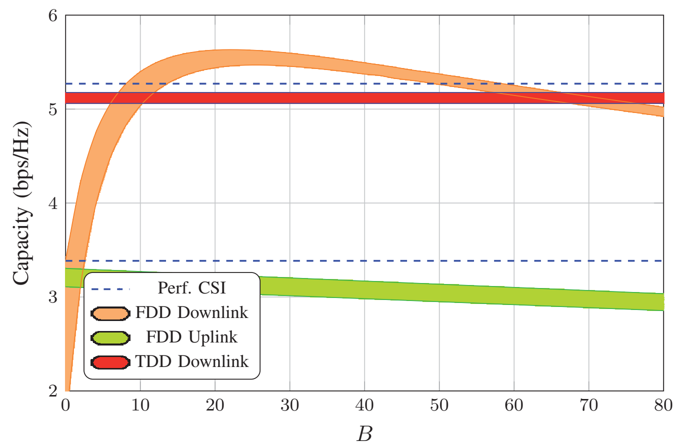

Figure 3. Capacity vs. Number of Feedback Bits B

Here we can observe the tradeoff between the quantization error (number of feedback bits) and the feedback error probability in the downlink. The maximum for both FDD downlink capacity bounds is achieved with around 22-26 bits and hence, we posit that this is also optimum for the actual FDD downlink capacity. In addition, as the number of feedback bits increase, the reduction in the quantization error, i.e. $\mu_\nu \to 1$, cannot compensate the increase in the errors in the transmission of the feedback message which consequently leads to a decrease of the downlink capacity. Note also how the achievable uplink rate decreases with increasing feedback overhead.

The result given by (31), suggests that with $M = 8$ antennas we need to set $B = 7$ in order that the FDD downlink can match the TDD downlink. In Fig. 3, however, the lower bound for the FDD downlink with $B = 7$ lies slightly below the achievable rate in the TDD downlink. This is due to the fact that in this case the factor in front of the FDD lower bound is not exactly one: $1 - \frac{\sigma_\nu}{2\mu_\nu} \approx 0.9265$. The simulation result indicates that at least $B = 10$ are required for the FDD lower bound to be larger than or equal than the TDD lower bound.

[2]The BS could assume there was a feedback error, if the current beamforming vector is not highly *correlated* (depending on the mobility scenario and the codebook size) with previously correctly received beamforming vectors.

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Let us now depict the capacity bounds for the FDD and TDD downlink as a function of the number of antennas. The paremeter setting in Fig. 4 is as follows: $T = 200$, $B = 30$, $T_{\text{UL}} = T_{\text{DL}} = 20$ and $\alpha = 0.9994$. The FDD downlink SNR is $10\log_{10}\frac{P_{\text{DL}}}{\sigma_0^2} = 13$ dB which implies the TDD downlink to be 10 dB. The performance with perfect CSI is also shown as reference. The TDD downlink capacity bounds increase logarithmically with M as a consequence of the antenna gain achieved with maximum ratio transmission based on the estimated channel. However, this is not completely possible in the FDD downlink due to the quantization errors. First, we point out that the effect of the feedback errors is negligible[3] for $M \geq 3$. With the assumed fixed B for all M, the absolute antenna gain, i.e. $M\;\mathrm{E}[\nu] \approx M(1-2^{-\frac{B}{M-1}})$ increases monotonically with M, such that the capacity also increases, albeit slowly, with M but not logarithmically as in the TDD case. Due to this reason, under the given setting a TDD downlink surely outperforms an FDD downlink for $M \geq 14$.

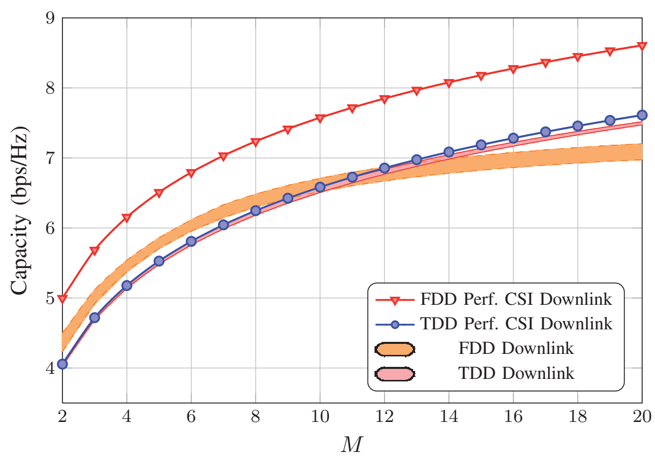

Figure 4. Capacity vs. Number of Antennas M

VII. Summary and Discussion

We have provided tight bounds for the single-user capacity in both links of FDD and TDD systems with imperfect CSI. Given a set of resources (time, frequency and transmit power) available for a two-way communication systems, an FDD system is able to perform better than a TDD system: an FDD uplink outperforms always a TDD uplink and the FDD downlink, assuming the number of feedback bits are properly chosen, could also outperform the corresponding TDD downlink. However, a comparison between the two systems is not as simple and straightforward as it may seem. For instance, the energy consumption in an FDD system is twice as large as in a TDD system, since the links in a TDD system operate only half the time. Although this might not be a major inconvience at the base station, this could imply a larger battery at the user terminal in an FDD systems as compared to a TDD system. If we limit the energy consumption instead of the available power, then the downlink SNR in both systems is the same and the TDD downlink would outperform the FDD downlink.

[3]With $M = 3$ the feedback error probability is only $p_\epsilon \approx 7\%$. Although the uplink SNR decreases with increasing M since we assume $P_{\text{UL}} = \frac{P_{\text{DL}}}{M}$, the receive antenna gain increases with M in such a way that the feedback error probability actually decreases with M, i.e. $p_\epsilon \leq 7\%$ for $M \geq 3$.

In addition, there are practical losses and implementation issues which could be taken into account. An FDD transceiver requires duplexers, which can incurr in transmit losses of 2-3 dB, and thus reduce the 3 dB SNR advantage of the FDD system over the TDD system. Nonetheless, a TDD system requires a switch to change between transmission and reception which can have losses of about 0.5-1 dB. Furthermore, a TDD system requires calibration at the BS which may insert additional losses. Hence, after considering all these aspects, we could still have a net SNR advantage of an FDD system compared to a TDD system.

In contrast to an FDD system, in a TDD system there is also the need of a time guard interval which mainly depends on the proximity of neighboring base stations. Depending on the network layout and cell sizes, this guard interval might represent a relevant overhead.

Another important aspect is the flexibility of a TDD system over an FDD system to handle asymmetric traffic between the uplink and downlink more easily. The duration of the uplink slot and the downlink slot can be modified adaptively in order to match the ratio of uplink/downlink traffic. This flexibility may however pose some problems in a multi-cell scenario. In case neighboring cells have a different UL/DL time slot partition (as a consequece of a different traffic asymmetry) or are not synchronized, a base station might be receiving at the same time when a neighboring base station is transmitting. The resulting interference can be stronger than the intercell interference resulting from the transmission of a user in a neighboring cell, since there might be a line of sight between the two base stations. One possibility to avoid this interference is through synchronization, which would, nonetheless, restrict all cells to have the same UL/DL traffic allocation. This would obviously hinder the beforementioned flexibility of a TDD system over an FDD system to dynamically allocate traffic in the uplink and downlink of each cell.

Based on the proposed arguments, a detailed comparison could also include the component losses, calibration, energy consumption and synchronization, but is out of the scope of this work. The actual decision which system is preferred depends on several factors and not just on one figure of merit.

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