Entropy-Based Traffic Impact Analysis and Optimization of Roadway Maintenance

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Abstract

The work presented herein addresses traffic modelling and optimization of roadway infrastructure rehabilitation for urban roadway networks and travels of approximated origins and destinations, by means of an entropy-based formulation of their vehicular movements and the impact caused by scheduled roadway closures. The perceived level of disorder (entropy metric) caused by the numerous vehicle-traveller trips in the domain under examination is subsequently utilized for the formulation of a multi-year scheduled maintenance policy in order to minimize the entropy in the system. The entropy-based analysis takes into consideration not only vehicular movements and traffic counts between multiple origins and destinations, but also annual maintenance budgets and costs, maintenance priority rules, and resource constrains. The optimization goal is to improve on maintenance schedules and minimize the impact of road closures on travellers subject to preset annual budgetary and road closure constraints. The proposed entropy-based model is shown to perform particularly well and to be an effective tool for evaluating the distribution of traffic loads and for appraising the level of disorder caused in a network. Furthermore, the entropy-based method is shown to be an excellent metric for evaluating fluctuations in traffic and/or resource assignment distributions (especially equiprobability). These properties can in turn be used for devising annual maintenance schemes and for reducing traffic loads and maintenance impacts on traffic arteries of interest. The work presented is also compared to agent-based infrastructure impact analysis (most notably Ant Colony Optimization) previously reported on by the researchers.

Keywords: infrastructure maintenance, scheduling, optimization, entropy.

1   Introduction

Roadway maintenance planning and urban traffic modelling are highly complex problems, especially in urban domains with complex street networks and high volume of unplanned vehicular movements. Roadway maintenance planning in such locales involves both the study of the street network topology but also the travel patterns (origins, destinations, paths) and the maintenance parameters (annual budgets, work prioritization, duration and cost of maintenance work). The aforementioned parameters are often difficult to evaluate and the underlying maintenance and transportation problem resolves to a stochastic (simulated) or fuzzy-estimated problem. One such possible approach to arriving at roadway maintenance and work prioritization plans involves the modelling of vehicular traffic by means of the "entropy-maximization method" (Christodoulou 2010a); a method that has previously been shown to have direct applications in transportation

planning and by which the theoretical basis for a class of forecast models on traffic demand can be based.

The work presented herein builds on previous work by Christodoulou (2010a), addressing traffic modelling by means of an entropy-based formulation of the vehicular movements in the domain studied, and work by Christodoulou (2010b) and Lukas and Borrmann (2011) on ant colony optimization and its application to path routing and maintenance scheduling. The former provides the mathematical framework of entropy and of the entropic metric utilized in measuring the impacts of vehicular movement. The latter describes the maintenance optimization problem in study and provides the theoretical framework for ant colony optimization and its applicability to path routing.

2   Problem statement

Traditionally the planning of maintenance measures in urban locales is done manually and ad-hoc. City and traffic planners typically execute maintenance measures (including planned or unplanned lane closures) primarily aiming a minimal impact on the commuters and subject to several constraints. Such constraints should be, for example, the maintenance budget, the cash flows, the network's traffic capacity, the impacts of roadway closures, the operating condition of the network, the severity and extent of the work required, the social impacts, the stakeholders and their collective influence, the prioritization of work, the tolerance to traffic delays and the availability of alternative operational pathways from one destination to another.

With the above in mind, the fundamental question to be answered in a typical roadway maintenance planning problem is twofold: (1) how can one analyze the multiple objectives and constraints and optimize/prioritize the maintenance measures, and (2) how can a reasonably good estimate be made of the traffic flow in a network, given the number of travellers, their origins and their destinations? Furthermore, as a corollary, how can this analysis be used in minimizing the impact of lane closures on traffic flow.

It should also be noted that in finding an optimal maintenance schedule one has to address not only the minimization of the impact on the traffic flow but also the minimization of the risk-of-failure of each network segment. For example, all road network bridges have to be maintained before they reach a state of collapse.

3   State of knowledge

To-date a number of studies on infrastructure assessment and roadway maintenance have been undertaken. The intent of such studies has traditionally been to assist city planners and roadway managing agencies in improving their maintenance plans and in minimizing the impacts of construction work or roadway closures on commuters.

The vehicle routing problem (VRP) has been researched extensively over the years and numerous methodologies have been proposed (Samanta and RoY, 2005) ranging from mathematical models, to numerical models, and most recently artificial intelligence techniques. Examples of such research work are given below. An algorithm calculating time-dependent shortest paths from all network nodes to a given destination node can be found in Ziliaskopoulos and Mahmassani (1993), while a multimodal trip distribution function and a methodology for measuring accessibility and effectiveness of road networks by use of the impedance curves in their proposed model can be found in Levinson and Kumar (1994). Heuristic algorithms for solving general transit network design problems were subsequently introduced by Baaj and Mahmassani (1995) and Braca et al. (1997). The work by Li et al. (2002) looks at a combined model for time-dependent trip distribution and traffic assignment, assuming known time-dependent departure rates from origins and overall arrival rates at destinations. The model then seeks the estimation of the origin-destination matrix according to the observed

entropy value and the subsequent minimization of the total system travel time. A mixed-integer multiple-commodity network flow model formulating bus movements and passenger flows at various time intervals was also examined and presented in Yan and Chen (2002). The model manages the interrelationships between passenger trip demands and bus trip supplies to produce the best bus routes and timetables for the given network. Notable are also the various artificial intelligence techniques applied to VRP, such as genetic algorithms (Chien et al., 2001; Baker and Ayechew, 2003; Fan and Machemehl, 2006; Thangiah and Nygard, 2005), interactive meshing, neural networks and evolutionary algorithms (Creput and Koukam, 2007).

In terms of entropic measures and their application to VRP, notable are the general models in Chandler et al. (1958), examining traffic dynamics and liking driving as an action done on the verge of instability, as well as the works in Wang et al. (2006) and Wilson (1970a, 1970b) on the concept of entropy and on equilibrium distributions derived from kinetic energy and from entropy maximization as tools that may allow the prediction of the consequences of specific policy decisions. A method for estimating the optimal distribution of cars in a traffic network based on a variant of the maximum-entropy method was presented in Das et al. (2000) and a general multi-objective transportation problem with an additional entropy objective function was presented in Samanta and Roy (2005). A generalized transport planning model based on entropy maximization, for both symmetric and asymmetric traffic flow was also presented in Agrawal et al. (2005), whereas an entropy approach to describing inhabitant trip distributions and developing a model based on origin moments was described in Wang et al. (2006).

4 The use of an entropic metric for traffic modelling and roadway maintenance planning

Entropy is generally thought of as a metric of a system's state of disorder (the higher a system's entropy is, the more disordered the system is) and generally systems tend to move toward higher entropy values, at which system stabilization may be sought. As a metric, entropy's relation to disorder may have a direct application to traffic flow accounting since it allows a traffic planner to utilize the entropy metric to approximate the vehicular movement in the network at a state close traffic chaos.

In general terms, entropy is defined as the uncertainty associated with traffic movements in the space boundaries under consideration and is related to the probability distribution of traveller-trips within the system by origin and destination. In mathematical terms, the fundamental general equation for entropy is Eq. (1),

$$H_x = P_x \ln\left(\frac{1}{P_x}\right) \tag{1}$$

and the basic assumptions and equations for the transportation problem are as follows.

If we assume that the network in examination consists of n_o origins and n_d destinations, and we define P_{ij} as the probability that a number of vehicles originates from location i destined to location j during the time interval of interest, then by use of Eq. (1) the total system vehicular entropy becomes

$$H_T = -\sum_{j=1}^{n_d}\sum_{i=1}^{n_o} P_{ij} \ln\left(\frac{1}{P_{ij}}\right) \tag{2}$$

where, P_x is the probability of occurrence of event x, which in the absence of a probability distribution function can be assumed to be the statistical probability for event x. In the case of traffic modelling, P_{ij} can be expressed in terms of the traffic loads (T_{ij}) between locations i and j as a percentage of the

total traffic (T) in the system. Since a closed system is assumed, the entropy-maximization traffic modelling problem can be expressed by,

$$\max\left\{H_T = \sum_{j=1}^{n_d}\sum_{i=1}^{n_o}\frac{T_{ij}}{T}\ln\left(\frac{T_{ij}}{T}\right)\right\}$$

(3)

subject to,

$$T_{ij} \geq 0 \quad \forall i,j$$

$$\sum_{j=1}^{n_o}\frac{T_{ij}}{T} = \frac{O_i}{T} \quad \forall i$$

$$\sum_{j=1}^{n_d}\frac{T_{ij}}{T} = \frac{D_j}{T} \quad \forall j$$

(4)

with O_i being the total volume of traffic originating from region i, and D_j being the total volume of traffic arriving at region j. As shown in Agrawal and el. (2005) and Christodoulou (2010), the entropy equation can be refined by considering a subdivision of the network into smaller regions of interest, the existence of various traffic flow paths in the network and the vehicular movements in-between the regions. The revised equations (Christodoulou, 2010) become,

$$\max\left\{H_k = -\sum_{i\in a_k}\sum_{j\in b_k}\left[\frac{T_{ij}}{T}\ln\left(\frac{T_{ij}}{T}\right)\right]\right\}$$

(5)

subject to,

$$\frac{T_{ij}}{T} \geq 0 \quad \forall\left(i\in a_k \text{ and } j\in b_k\right)$$

$$\sum_{j\in b_k}\frac{T_{ij}}{T} = U_i^{'} \quad \forall i\in a_k$$

$$\sum_{i\in a_k}\frac{T_{ij}}{T} = V_j^{'} \quad \forall j\in b_k$$

(6)

$$\sum_{i\in a_k}\sum_{j\in b_k}\frac{T_{ij}}{T} = 1$$

The parameters in Eq. (5)-(6) are defined as follows: i and j are the regions of interest; T_{ij} is the traffic between regions i and j; T is the total traffic flow generated in the system; K is the total number of blocks in the system; U_i is the probability of traffic originating from region i; V_j is the probability of traffic destined to region j; U_i is the probability of traffic originating from region i of some block k and destined to various regions of the same block; V_j is the probability of traffic destined to region j of some block k and originated from various regions of the same block; and a_k and b_k are the set of origins and destinations respectively that belong to block k.

5   Case study network

5.1   General problem definition

Given a roadway network consisting of n_r road segments of variable unidirectional or bidirectional traffic flows and capacities, n_j junction nodes, n_o origins and n_d destinations, a multi-year maintenance time horizon n_t, annual maintenance budgets b_r and additional network parameters as shown in Table 1, optimize the roadway maintenance schedule so as to minimize the impacts to traffic flow while also satisfying all budgetary and operational constraints set on the road network.

5.2   Model network

Figure 1 shows the topology of the case-study roadway network. The network, which was reported upon by Lukas and Borrmann (2011), consists of 103 streets in need of maintenance in the next 15 years. Of the 103 streets, a maximum of only 10 streets can be maintained each year. Maintenance reduces the vehicular capacity of a street by 50% and maintenance schedules shall be developed for the next 5 years. The maintenance costs per street, the annual budget constraints, the street capacities and the number of travellers per origin and destination are also given (the parameter notation and the values utilized in the analysis are shown in Table 1).
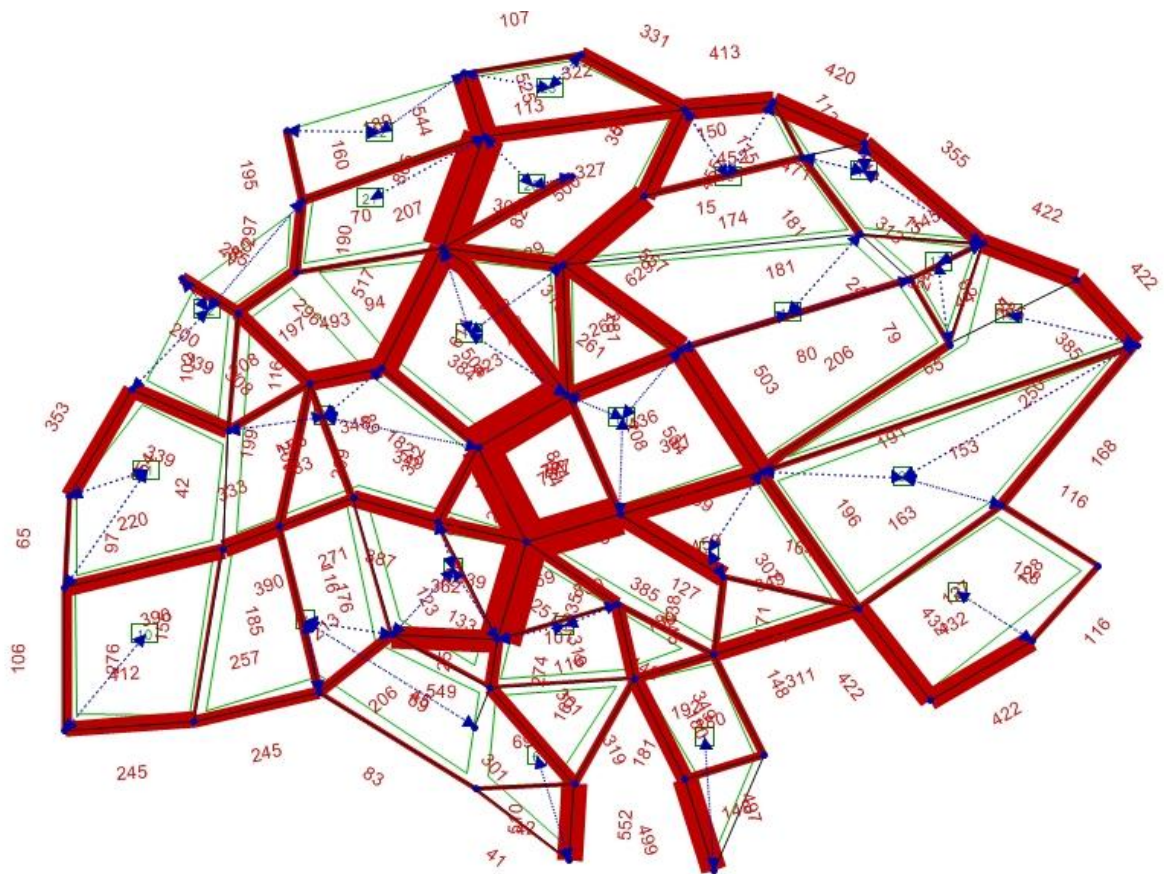

Figure 1. Case-study traffic network (adopted from Lukas and Borrmann, 2001)

Table 1. Parameters of model network (Figure 1).

Parameter	Notation	Value	Comments
MaintenanceTimeHorizon	t	5 years	Constraint
MaintenanceMaxAnnualBudget	b_t	3.0 million	Constraint
MaintenanceMaxRoadsClosedPerAnnum	m_t	10	Constraint
RoadNodes	n_j	61	
TrafficOriginatingFromNode	o_j	Variable	Stochastically distributed
TrafficDestinedToNode	d_j	Variable	Stochastically distributed
RoadSegments	n_r	103	
RoadHourlyCapacity	h_r	Variable	
RoadLength	l_r	Variable	
RoadDeadlineToRepair	r_r	Variable	
RoadCostToRepair	c_r	Variable	
RoadVehicularCapacity	p_r	Variable	
RoadTrafficFlowDirection	f_r	Variable	
Areas	n_a	23	
AreaOriginatingFrom	g_a	Variable	Stochastically distributed
AreaDestinedTo	s_a	Variable	Stochastically distributed
Travellers	n_v	12700	
NumberOfPossiblePaths	n_p	855	

6   Solution methodology

Solution of the case-study problem entails the conversion of the network topology into a directed graph with arcs representing the road segments and with nodes representing the road junctions. The arcs are directional (to indicate traffic flows) and have vehicular capacities, while each enclosed area (neighbourhood) has a number of residing travellers whose origin (starting node) and destination (ending node) are stochastically distributed. The network topology is completed with a nodal connectivity matrix and a path list, so as to provide the means to simulate traffic movements and to calculate the entropy caused by path, road segment and node.

    The network, mapped in a typical spreadsheet, is solved by means of a commercially available Monte-Carlo simulation program (*Oracle Crystal Ball*[TM]). One million simulation runs per solution stage (i.e. for each annual maintenance cycle) are executed, simulating at each simulation run the following: the vehicular load by origin and destination node, the path chosen by each commuter and the roads closed for maintenance (<=10 per annum). The entropy values for each traversed path are computed by use of Eq. 5-6 and then distributed equally among the road segments on the path. The total system entropy, the total annual maintenance cost and the total number of road segments maintained per annum are then maximized (subject to the predefined budget and road closures constraints) and the solution refined until converge to an optimal solution is obtained (Fig. 2). To avoid a computationally-demanding exhaustive enumeration and to accelerate convergence (Fig. 2), the simulation utilizes a neurofuzzy modeller (*Oracle Crystal Ball Decision Optimizer*[TM]) which cuts the computation time to about 30 minutes per optimization stage (i.e. for each annum).
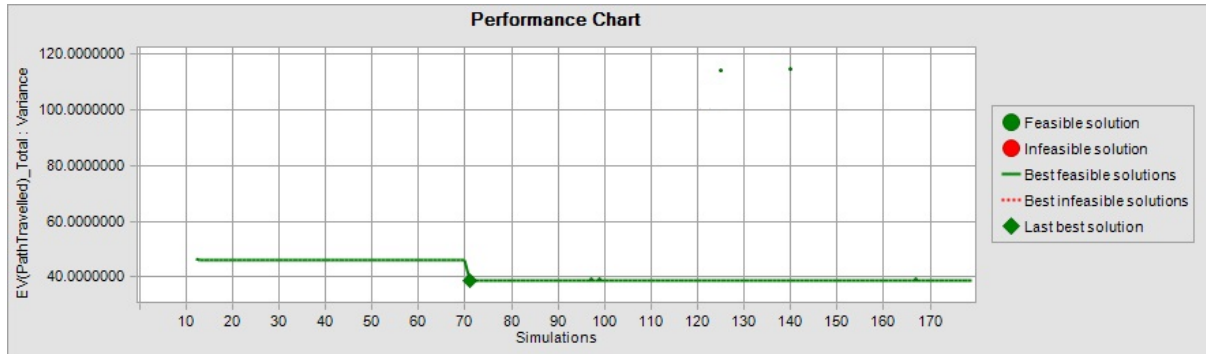
Figure 2.

7 Results

The aforementioned entropy-maximization approach to the roadway maintenance case-study problem results in the maintenance plan shown below (Table 2). The solution satisfies all objectives and constraints set in the definition of the problem and compares favourably with the solutions obtained by Lukas and Borrmann (2011) by use of ant colony optimization (ACO).

Table 2. Obtained 4-year maintenance plan

Road	Repair By (year)	Repair On (year)	Cost to Repair	Road	Repair By (year)	Repair On (year)	Cost to Repair
3	2	1	175,413	6	4	3	186,502
7	1	1	100,724	9	3	3	153,449
38	1	1	165,837	15	4	3	143,094
44	1	1	305,387	33	3	3	239,770
53	2	1	149,668	43	5	3	398,524
58	1	1	288,728	45	4	3	144,784
61	1	1	395,741	54	3	3	358,998
84	2	1	399,980	68	3	3	110,824
89	1	1	332,759	76	4	3	108,887
91	1	1	176,272	95	4	3	176,630
8	3	2	374,924	16	4	4	271,226
12	2	2	260,778	17	5	4	356,477
21	3	2	229,205	28	6	4	127,274
47	2	2	306,319	31	8	4	348,431
66	2	2	181,367	35	6	4	112,406
71	4	2	134,330	48	7	4	149,803
77	4	2	399,287	56	5	4	292,245
80	4	2	352,658	57	6	4	186,596
81	2	2	225,730	73	8	4	108,377
90	6	2	397,694	92	5	4	329,473

8 Conclusion

The proposed entropy-based model is shown to perform particularly well and to be an effective tool for evaluating the distribution of traffic loads and for appraising the level of disorder caused in a network. Furthermore, the entropy-based method is shown to be an excellent metric for evaluating

fluctuations in traffic and/or resource assignment distributions (especially equiprobability). These properties can in turn be used for devising annual maintenance schemes and for reducing traffic loads and maintenance impacts on traffic arteries of interest. As in the case of ACO, the entropy-maximization method provides good solutions to the underlying NP-hard maintenance planning problem by use of an intelligent path traversal algorithm. Unlike ACO or other heuristic techniques, though, the entropy-maximization method is guaranteed to converge to a good solution without falling into local minima. It should also be noted that the entropy method does not utilize any traffic simulator (such as VISUM) to account for the traffic impacts in the network, since the computation relies only on the origin and destination of each traveller (Eq. 5-6). This attribute reduces the computation time significantly compared to agent-based infrastructure impact analysis methods.

Ongoing and future work on the subject matter aims the incorporation of additional optimization goals and constraints in the analysis (beyond the traffic impact, maintenance deadlines and budget constraints) and increased computational efficiency of the algorithms used.

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