

Using Geometrical and Topological Modeling Approaches in Building Information Modeling

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ABSTRACT: This paper presents concepts for dealing with topological information in building information models. On the one hand, it shows how to derive topological relationships from "traditional" building models consisting of unconnected B-Rep bodies by means of geometric processing algorithms. On the other hand, it discusses the capabilities of topological building modeling based on relational complexes, an approach based on Algebraic Topology and the Relational Data Model. To make these complexes suitable for building modeling it was necessary to extend them by geometric properties. Finally, the paper depicts an advantageous application of cell complex-based modeling: the separation of the building model into an abstract specification of building entities (sketch), a collection of possible concrete realizations of such sketch entities (details) and a specification of the details used by sketch entities. Then a working drawing results from the spatial version of the relational operator *inner equi-join*, the so-called *fiber product*.

1 INTRODUCTION

In building and construction planning one often has to deal with information along the lines of "Which elements are connected together?" or "How are different parts separated?". In floor plan layout, for example, such connectivity information can describe the accessibility of rooms within an office or an apartment as well as describing which room lets in daylight, as this calls for connectivity between the room in question and the area outside the building. Structural analysis also requires information on the structural elements that are joined together and the joints employed in the process.

Such connectivity information is a topological property of a building, and we will later see, that it may even be called *the* topology of a building. A topological space can informally be described as a set of elements together with a specification of which element is "close to" some other elements. Then a mapping from one such space to another is called *continuous*, if the image of a point "close to" some set remains "close to" the image set and is accordingly not torn off the latter. Typical examples of such a continuous mapping are the affine transformations in the Euclidean space with its so-called natural topology. If such a continuous mapping has a continuous inverse, this mapping is called *topological isomorphism* or *homeomorphism*, for short. A typical homeomorphism is an invertible affine

transformation such as translation, rotation or shear. Note that under a homeomorphism the image of an element "not close to" some other elements is "not close to" their image either. The original space and the image space are then indistinguishable with respect to this "closeness" relation and is hence called topologically equivalent or homeomorphic.

Topological relationships between building components define the basic functionality of the building (e.g. its structural system, the connectivity of rooms) and knowledge about them can be used for a wide number of analytical tasks (evacuation simulation, building performance analysis, etc.). A dividing wall between two interior rooms, for example, has different thermal requirements than a dividing wall between an interior room and the space outside the building. So there is a good reason for topological properties to show up in building product models and, indeed, every noteworthy such model like RATAS, COMBINE or IFC deals with this kind of information (Bjoerk 1992; Dubois et al. 1995; Adachi et al. 2003). It is, however, difficult to access topological information in building product models as, generally speaking, many different approaches towards modeling topology coexist within one model.

With this heterogeneity of data types it is difficult to develop applications which have to navigate within this space of interconnected entities in order to perform thermal or structural analysis or other aspects of the building's performance (Romberg et al. 2004; van Treeck and Rank 2007). There is yet an-

other topological structure within a building product model: the data structures used to describe the geometric representation of each building element. These structures are often defined by boundary representation (b-rep) techniques using primitive geometric entities like *point*, *edge* and *face* together with connectivity information between these primitives, hence a topology for them, too.

This connectivity information, however, is only local to the specific geometric entity, which is therefore isolated from other entities in terms of b-rep connectivity. So topological relationships between building elements must either be derived from their geometric shape or stored explicitly in the building model. The realization of these diverging approaches is discussed in the following sections.

2 THE GEOMETRICAL APPROACH

What we refer to here as the “geometrical approach” involves building models which considers a building a compilation of geometric objects located in 3D space. Each of these objects has a connected and compact geometrical shape which is described using established techniques from solid modeling such as constructive solid geometry (CSG), swept representations or the particularly interesting boundary representation (b-rep) techniques (Mäntylä 1988). So far, our approach complies with how CAD systems and common product models such as IFC or CIS/2 handle building geometry today. But, whereas in product models topological relationships between geometric objects are meant to be explicitly stored, we propose to derive them from the objects’ geometry and their relative position to each other.

2.1 Specification

In (Borrmann and Rank 2008) a method is presented that makes it possible to query pairs of building elements with regard to their topological relationships. The topological relationships that are supported are *within*, *contain*, *overlap*, *touch*, *equal* and *disjoint*. Each of the predicates is formally defined by means of the 9-Intersection Model (9-IM), a model that was originally developed for 2D-GIS (Egenhofer 1991) but can easily be applied to 3D-BIM. It records the intersection between the interior, the boundary and the exterior of both objects in a matrix. Let A and B be point sets that describe a spatial object in 3D. Then the 9-IM matrix reads

$$I = \begin{pmatrix} A^\circ \cap B^\circ & A^\circ \cap \partial B & A^\circ \cap B^- \\ \partial A \cap B^\circ & \partial A \cap \partial B & \partial A \cap B^- \\ A^- \cap B^\circ & A^- \cap \partial B & A^- \cap B^- \end{pmatrix} \quad (1)$$

where A° denotes the *interior*, ∂A the *boundary* and A^- the *exterior* of A . Permitted entries of the matrix are the empty set (\emptyset), the non-empty set ($\neg\emptyset$) and the asterisk (*), the latter meaning that the respective place in the matrix is not decisive for assigning the topological predicate to a certain constellation.

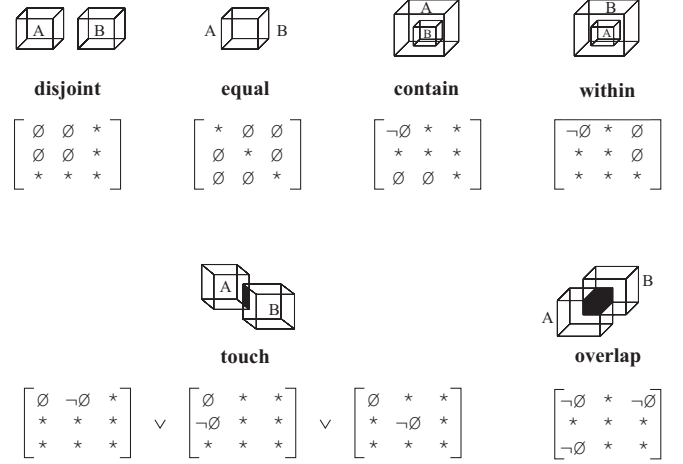


Figure 1. Some topological predicates according to the 9-intersection model.

Figure 1 shows the 9-IM matrices of all six defined topological predicates. The predicates are *mutually exclusive* and *complete* in the sense that, for any conceivable constellation, precisely one of the predicates can be applied. This allows one to offer the function *whichTopoPredicate* that returns the respective predicate for any given constellation. This function and the individual predicates are provided as operators for use in the WHERE part of a SQL query (Borrmann 2007).

By using the resulting Spatial Query Language it is possible to find all walls touching a certain slab, the heating equipment within a certain room or gas lines crossing a specific wall, for example.

2.2 Implementation

As stated above, the topological relationships between the building elements are not expected to be explicitly stored in the 3D building model. Instead, they are generated/tested on-the-fly during the processing of a spatial query on the basis of the explicit geometry of the respective building elements.

For the implementation of these topological tests we developed two approaches: the first approach is based on octree representations of the operands which are generated on demand. The resulting octrees are traversed synchronously in a breadth-first manner. On each level pairs of octants are created with one octant originating from object A and one octant from object B , both basically covering the same sector of the 3D space. Each octant provides a color combination for which specific rules can be applied. These rules may lead to filling a 9-IM matrix that is maintained to keep track of the

knowledge gained by the algorithm about the topological constellation.

In total, there are 12 positive and 9 negative rules. A positive rule can be applied when a certain color combination occurs, and a negative rule if certain color combinations do not occur over an entire level. Positive rules lead to *non-empty set* entries in the matrix, negative rules to *empty set* entries. The 9-IM matrix is successively filled by applying these rules for all octant pairs. Each time a new entry is made, the matrix is compared with the matrices of the formal definitions (Figure 1). As soon as it corresponds completely to one of the pre-defined matrices, the recursion is aborted and the algorithm returns the respective predicate. If there is any divergence between the filled matrix and the matrix of a predicate, the respective predicate is precluded. If no unequivocal decision is possible for any of the predicates, a further refinement is necessary.

Our second approach for testing topological relationships works directly on the boundary representation of the building elements. To accelerate the processing of the tests, the facets describing the objects' boundaries are hierarchically managed by means of a so-called *Axis Aligned Bounding Box* (AABB) tree (van den Bergen 1997). The algorithm performs an initial intersection test to find out whether the objects *overlap*, or not. Using the AABB tree most of the facet pairs can be excluded from detailed investigation, meaning an exact intersection test only has to be performed for a limited set of final pairs.

If the *overlap* test fails, additional tests have to be performed which are based on sending out rays from one object and counting the number of intersections of the ray and the second object. Due to the limited space available, we cannot discuss the algorithm in more detail here.

Both approaches are able to retrieve topological information from purely geometrical models. While the b-rep-based algorithm is normally considerably faster, the octree approach allows for a fuzzy treatment of topological relationships by choosing a suitable maximum refinement level. This is especially interesting as we have to be aware of impurely modeled building models where slight gaps between building elements occur without intention.

3 THE TOPOLOGICAL APPROACH

The topological approach, on the other hand, is based on the formal theory of *Algebraic Topology* which requires the digital building to be modeled according to what is known as a cw-complex, where the topology is explicitly stored and does not need to be computed.

3.1 Related work

Using the Algebraic Topology for modeling buildings has already been proposed by a number of researchers, e.g. in (Huhnt and Gielsdorf 2006; Clemen and Gielsdorf 2008). However, the potential of this formal approach with respect to concrete applications has not been fully developed so far. In the context of 3D Geographic Information Systems (GIS), simplicial complexes have been employed to model soil layers, for example (Breunig et al. 1994; Pilouk 1996; Shi et al. 2003). Simplexes are a generalization of closed, filled triangles and are often used instead of cells to set up a so-called simplicial complex. These are, however, very restricted and demand a costly partitioning of building elements into tetrahedra. We therefore propose using cells, the more general version of a topological primitive.

3.2 Cell complexes

We start with the observation that a building resides in the Euclidean space, in such a way that the latter is somehow divided into a finite number of parts by the former. The difference with the topological approach is that these parts need not be compact and that they lead to a complete *partitioning* of the Euclidean space: every point in space belongs to exactly one unique building part, a room or the exterior space. Note that hierarchically aggregated spatial objects cannot be expressed with only one such partitioning.

This partitioning immediately leads to a topology for the building elements, which is called *final topology* or *quotient topology*. This is the topology of a building mentioned above.

3.2.1 Architectural Complexes.

From a theoretical point of view, the statement that the Euclidean space is finitely partitioned by architecture is still too weak. We say that every building part can also be further subdivided into a finite number of cells which set up a special case of a topological space with interesting algebraic properties. It is the so-called cw-complex—a composition of cells frequently used in *Algebraic Topology* (Hatcher 2002).

An *n-cell* (cell of dimension n) is a topological space which is homeomorphic to the interior of an n -dimensional cube. For the special case of $n=0$ we say a 0-cell is a space with exactly one point.

Low dimensional n -cells are often called *topological primitives* in volume modeling. A 0-cell is also called *vertex*, “1-cell” is a synonym for “edge”, 2-cells are “faces” and 3-cells are also called “volumes”. Figure 2 shows the canonical cells found in a closed cube. It is important, however, that a cell's boundary is never part of the cell.

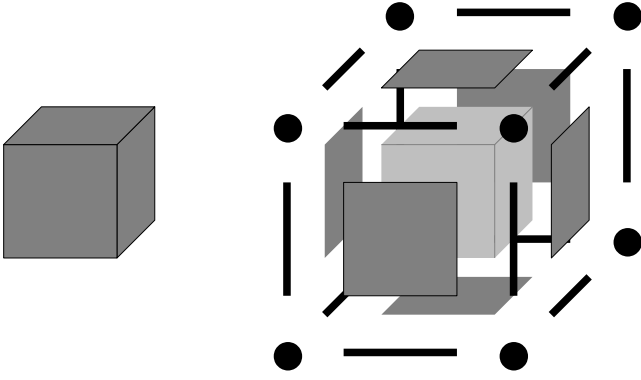


Figure 2. The canonical cellular decomposition of a closed 3d-cube

Now these cells can be combined to form a topological space which is called *cw-complex*. This is a topological space partitioned into a set C of cells where the boundary of each cell is a union of a finite number of cells in C of strictly lower dimension than n . We always assume C be finite.

Then for some integer number i , the set $X_i \subseteq X$, the union of $C_i \subseteq C$ of cells of at most dimension i is called i -skeleton of X . This is also a *cw-complex* with partitioning C_i . Each $i+1$ -cell is also said to be *attached* to the i -skeleton. Note that i may even be negative, in which case $X_i = \emptyset$.

The 0-skeleton is simply the discrete vertex set (or point cloud) and the 1-skeleton is a graph, where the edges are attached to the vertices. Attaching faces to the 1-skeleton can be done by either filling loops or by creating cavities and gives rise to the 2-skeleton (a sponge-like structure) which is then completed to form the 3-skeleton by filling cavities with volumes. Theoretically, attaching a volume could also create hyper-cavities, but we will refrain from that. Note that, if one neglects the rooms of a building, it also displays a “sponge-like structure”. Note also that a single b-rep volume model is likewise a *cw-complex*.

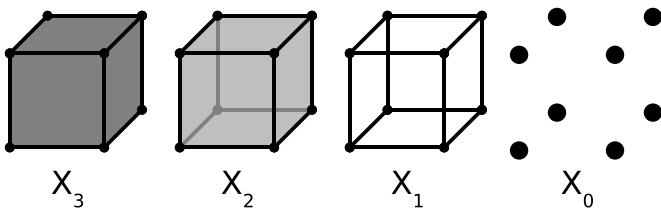


Figure 3. The skeletons of the canonical *cw-complex* of a three-dimensional closed cube, beginning with X_3 on the left and ending with X_0 to the right. Note that $X_n = X_3$ for all $n \geq 3$ and $X_n = \emptyset$ if i is a negative integer.

3.2.2 Complex Based Modeling

At the most detailed level, the building elements are cells which are basically the same objects used in b-rep volume modeling. Only the 3-cell as a “primi-

tive” is missing in some of these models which mostly identify a volume with its boundary instead of providing it as a separate primitive.

We *attach* volumes to faces instead of such identification and will therefore model the connectivity information in a different way. Later the notion of a cell will be relaxed and walls or slabs can be considered faces by simply disregarding their relatively small thickness.

We will now proceed to describe a relational schema for an algebraic complex. It is a well-known fact that each *cw-complex* has an associated algebraic complex of this kind.

3.3 Relational Complexes

Cw-complexes are often defined by specifying an inductive procedure of starting with the 0-skeleton X_0 and then attaching $i+1$ -cells to the i -skeleton until the complex is finished (Hatcher 2002, p. 5).

We will repeat this inductive procedure with an example volume element, a closed unit cube of dimension 3, and, in parallel, give a relational representation of the associated algebraic complex to illustrate the notion of a *relational complex* (Paul 2007). We only present here what we call the dynamic version of a relational complex, where all cells are stored in one single table together with an attribute to indicate the dimension.

We start with our specimen cube's vertices and establish the 0-skeleton as a relation with the schema

`CELLS(id: \mathbb{Z} , dim: \mathbb{Z})`

with primary key {id, dim}. The coordinates of these cells set up another relation of schema

`COORD(id: \mathbb{Z} , dim: {0} \rightarrow x, y, z: \mathbb{R})`

We indicate the primary key by an arrow pointing from the primary key attributes to all other attributes. The attribute `COORD.dim` is constantly the integer 0 and only used to establish a reference to `CELLS`. We call each entry in `COORD` a *point* and each entry in `CELLS` with `dim=0` a *vertex*. Each point must reference a vertex.

So by simply storing the eight vertices and points in the corresponding relations we obtain the 0-skeleton of our specimen cube.

Now we have to paste edges to vertices. This will be done by adding records with a `dim`-value of 1 to the cells table and calling these records *edges* or 1-cells. To be able to attach these cells to vertices we finally define our last additional relational schema

`BD(a, dimA, b, dimB \rightarrow alpha: \mathbb{Z})`

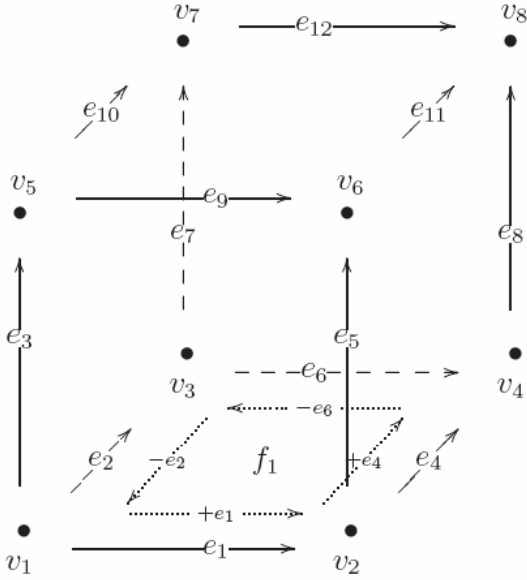


Figure 4: The 1-skeleton of a cube consists of vertices and edges. A label v_i stands for cells tuple $(i,0)$, e_i for $(i,1)$ and f_i for $(1,2)$.

where $(a, \dim A)$ and $(b, \dim B)$ are references to CELLS. We will call this relation the boundary relation. We also insist on $\dim A > \dim B$ for all entries in the boundary relation and, in addition, if these dimension attributes differ by more than one, the *alpha* value must be zero.

Note that BD defines a partial integer matrix D , which maps some pairs (e, v) of cells to an integer $D(e, v)$ and is undefined for others. To obtain a total matrix, all undefined entries can be regarded as zero entries. Each edge e of our cube running from vertex a to vertex b will now be stored as one record $(e, 1)$ in the cells relation and two additional records $(e, 1, a, 0, -1)$ and $(e, 1, b, 0, +1)$ in the boundary relation, indicating that edge e (of dimension 1) starts at vertex a (of dimension 0) and ends in b . The orientation of each edge can be arbitrarily chosen and, in our example, we simply let an edge run from lower id to higher id. This produces a 1-skeleton, displayed in figure 4 with the following boundary relation (some entries have been omitted):

CELL		BD				
id	dim	a	dimA	b	dimB	alpha
1	0					
2	0	1	1	1	0	-1
		1	1	2	0	+1
	...	2	1	1	0	-1
8	0	2	1	3	0	+1
1	1					
2	1					
		12	1	7	0	-1
	...	12	1	8	0	+1
12	1					

This is simply the well-known incidence matrix of a graph, an approach which at first glance seems a verbose and inefficient way to store a directed

graph. Note, however, that firstly the usual way of storing such a graph as a relation with schema

EDGE (edgeId \rightarrow origin, target)

only changes the storage complexity by a constant factor and, secondly, the advantages of our approach will become evident in the following section.

We will now paste faces to the 1-skeleton to achieve the 2-skeleton—the shell covering the cube's volume. Starting with the bottom face f_1 of our example in figure 4, we store it as a 2-dimensional cell—a record $(1,2)$ consisting of cell number 1 and dimension 2. Then we observe that this cell is bounded by the edges e_1 , e_4 , e_6 and e_2 , the orientation of which is, as we remember, arbitrarily chosen. As with the edges, we define an arbitrary orientation of the face, say counter-clockwise when seen from above, and then “correct” the orientation of the edges in relation to the face so that they are all pointing in a counter-clockwise direction. This is achieved by attaching a positive or negative sign to each edge indicating whether this edge has the desired orientation of the faces or whether it is pointing in the opposite direction. This produces *signed edges* $+e_1$, $+e_4$, $-e_6$ and $-e_2$. We say the origin of a negative edge $-e$ is the target of the positive edge $+e$ and vice versa. The meaning of this sign is similar to the Orientation-attribute of an Ifc-OrientedEdge: An Orientation value of true corresponds to the plus sign $+1$ and false to the minus sign -1 . In our case, however, these signs are relative to the bounded face and therefore *not* a property of the edge itself but rather a property of the edge-face association. We accordingly get the following additional records for face 1:

		BD				
CELL		a	dimA	b	dimB	alpha
id	dim					
	...					
1	2	1	2	1	1	+1
		1	2	4	1	+1
		1	2	6	1	-1
		1	2	2	1	-1

Note that these boundary entries are sometimes called *half edges*.

In total we get six more entries in the cells table—one for each face—and twenty-four additional entries in the boundary table. The reader can verify that, if the boundary table is considered a sparse cells \times cells-matrix where the rows are indexed by $(a, \dim A)$ -pairs, the column index is $(b, \dim B)$ and the entries are the associated alpha values, then it returns a sparse zero matrix if multiplied by itself.

When the 2-skeleton is finished, we still do not have a valid volume model. Firstly, the volume itself is still missing; we only have an empty shell so far.

Secondly, the faces do not have a consistent orientation, some are oriented counter-clockwise when seen from the inside to the outside and others then have clockwise orientation. Now we want each face seen from “outside” the cube to have a counter-clockwise orientation. We will say a face *points to* a side if, seen from this side, its boundary has a counter-clockwise orientation. Assuming the faces' orientations have been chosen in such a way that the horizontal faces point upwards, the frontal faces point to the rear and the sagittal faces to the right, then some faces have to be turned over (flipped). This flipping is also relative to the volume object so we obtain an entry (1,3) in the cells table, meaning that we have cell number 1 of dimension 3 and six more entries in the boundary table, thus completing the relational complex:

		BD				
		a	dimA	b	dimB	alpha
CELL		...				
id	dim	1	3	1	2	-1
...		1	3	2	2	-1
1	3	1	3	3	2	-1
		1	3	4	2	+1
		1	3	5	2	+1
		1	3	6	2	+1

A boundary relation must always satisfy the fundamental property of homological algebra that, as a matrix, the product with itself must give a zero matrix, hence the boundary of the boundary is zero, which algebraically expresses the spatial property that each cell is circumscribed by its boundary. Each edge is referenced twice by a volume: once in its original orientation and once flipped to face in the opposite direction, so the sum of the products of the alpha entries has to be zero. The same is true for a vertex, which has an incoming edge and an outgoing edge with respect to a face.

This approach has many interesting properties:

- The data model is very easy and based on the relational model.
- Each “topological primitive” is of one common data type CELL and the dimension of such a cell is only indicated by an attribute.
- The data model can hence be used for spatial instances of arbitrary dimension.
- The key values of the boundary relation define the topology for the primitives.
- Common cells can always be reused.
- Every b-rep volume model can be represented as a relational complex.
- The asymptotical storage complexity is not worse than other geometric modeling approaches.

One property might be considered a drawback. As this model is based on homology, it cannot store a specific sequence in which edges encircle a face as some volume models do. Translating a volume model from IfcTopologyResource into a relational complex “abelianizes” every IfcEdgeLoop element (Hatcher 2002, p.99), hence every permutation of edges from such a loop is considered equal to the original sequence. Such a loss of information, however, is only of theoretical interest and we (still) see no practical relevance in it.

3.4 Relational Operators

Basing this model on the relational model offers an interesting perspective: If one takes a relational view of topological spaces, one might, conversely, ask how relational operators can be given a topological meaning. Indeed, except for the outer join, every operator in relational algebra has a counterpart in point set topology or such topological version of an operator in question can be easily constructed.

The following table gives some example relational operators and their counterparts in point set topology.

relational algebra	point set topology
selection	subspace
rename	homeomorphism
Cartesian product	product space
inner equi join	fiber product
disjoint union	topological sum

We will later present a topological version of the Cartesian product of two complexes, hence a counterpart of Cartesian product in *algebraic* topology.

3.5 Some Geometric Properties

In topology, the geometry of cells and cw-complexes is ignored. So it is necessary to extend the data model by geometric properties in order to make it suitable for geometric modeling.

If we first consider solely planar objects with straight edges and plane surfaces, then it is sufficient to store the location of the vertices in the COORD table because all other cells' geometries are then defined by linear interpolation. This assumption is too restrictive for practical purposes, but the appropriate extension of the model presented here can be discussed independently. We will now describe some basic geometric properties. In (Breunig et al. 1994) a similar approach was presented for simplicial complexes.

Measures. Length, surface area, volume and maybe even their higher dimensional analogous are important properties which must be derived from the given model. The surface of a planar polygonal surface f

can be computed by first triangulating the polygon and then summing up the surfaces of each such triangle. This triangulation, however, need not cover the surface exactly. It is also possible to fix an arbitrary point p in the supporting hyperplane and then for each edge (a,b) in the polygon's boundary compute the cross product

$$\overrightarrow{pa} \times \overrightarrow{pb}.$$

The sum of all such cross products is a vector, where its direction gives the face's orientation and its length is twice the area of the face if all edges (a,b) are aligned with the orientation of the face itself. The reference point p may even lie anywhere so the area of a face f is simply

$$\text{area}(f) = \left\| \sum_{(a,b) \in d(f)} \vec{a} \times \vec{b} \right\| \quad (2)$$

where $d(f)$ is the set of all edges in the boundary of f . This formula, however, does not take into account that the orientations of the edges are arbitrary. But then we can use the signs from our boundary relation and hence the sum in the above formula can be replaced by:

$$\sum_{(f,e,\beta) \in \text{BD}} \sum_{\{(e,a,-1), (e,b,+1)\} \subseteq \text{BD}} \beta \cdot \frac{\vec{a} \times \vec{b}}{2}. \quad (3)$$

The cross product $a \times b$ is first computed for the boundary points a, b of each edge e bounding f . This is then multiplied by β , the flipping of the orientation of e relative to face f .

A similar approach enables us to compute the volume of a 3-cell in our complex. We triangulate the surface (boundary) of the volume object and compute the volume of each cone (pyramid) atop such a triangle with an appropriate sign indicating whether the cone's volume is attached to the outside or the inside of the triangle. A natural choice for the cone's apex is the origin in \mathbb{R}^3 .

$$\text{vol}(v) = \left\| \sum_{(a,b,c) \in \text{surface}(v)} \frac{1}{3} \cdot \begin{vmatrix} a_x & a_y & a_z \\ b_x & b_y & b_z \\ c_x & c_y & c_z \end{vmatrix} \right\| \quad (4)$$

where $\text{surface}(v)$ is a triangulation of v 's boundary such that all triangles (a,b,c) have the same orientation when viewed from the outside. Note that there is a close relationship between the cross product used to compute the area and the determinant used here to compute the volume.

he triangulation of a face, say f_1 from our example, can simply be done by fixing one boundary point p of f_1 and then for each boundary edge e of f_1 using the triangle (p,a,b) , where e runs from a to b . If p happens to be equal to a or b the triangle's volume is zero, and it can be left to the discretion of the

implementor and his complexity considerations whether such volume is computed and added or whether it is discarded in advance. Hence for each face f we choose one arbitrary vertex v_f and compute the triangles surface(v) as:

$$\{ (a,b,v) \mid \text{BD}(v,f,\alpha_{vf}), \text{BD}(f,e,\alpha_{fe}), \\ \text{BD}(e,a,-\alpha_{vf} \cdot \alpha_{fe}), \\ \text{BD}(e,b,+\alpha_{vf} \cdot \alpha_{fe}) \}$$

Note that most of the predicate in the above set builder expression is an inner equi join of copies of BD, hence 'surface' can also be generated by an SQL query. The arbitrarily chosen v_f can be the one with the minimal identifier so it can be chosen by the min-operator with a 'group by' clause. The flipping of orientation is recognized by simply multiplying the corresponding alpha-value.

3.6 Application: From Sketch to Working Drawing

We have now presented a concept for storing an arbitrary cell complex. A building, however, is commonly conceived as a union of building elements like doors, walls or columns which can be treated as if they were cells. A door, for example, somehow resembles a 2-cell combining two volumes if we disregard its relatively shallow thickness. The relational model presented above has the advantage that it is not restricted to cells and can store arbitrary complexes. It clearly makes sense to store building elements and the spatial relationships among them as spatial entities and not the cells they are made of because, firstly, this represents the semantics of a building and, secondly, many such elements have repeated patterns, and the explicit storage of the cell decomposition of, say, a frequently used joint would lead to redundancies.

To avoid such redundancies a topological version of relational decomposition and inner equi joins is needed and might also set up what can be called a "topological design theory". Such topological inner joins are well known and are called fiber products. They will be illustrated here. This replaces cells by composite objects which resemble architectural elements in a natural way.

A Simple Example. Figure 5 gives an introductory example. On the left-hand side of the figure there is a two-dimensional complex representing a (simple) floor plan. On the right-hand side we have another two-dimensional complex, which resembles this floor plan in a less detailed view where each wall and wall joint has collapsed to a cell. We will call the left-hand side complex a *working drawing* and the right-hand side complex a *sketch* to make the intention clear. So there is an obvious continuous mapping from the working drawing onto the sketch.

The illustration between working drawing and sketch shows the parts of the working drawing which have collapsed to one common cell by this mapping, its so-called fibers. One observes that, in this example, each such fiber representing a wall is essentially the extrusion of the wall's profile along the cell that represents the wall in the sketch. Such extrusion is a special case of product space and, in fact, each fiber in the working drawing is essentially the Cartesian product of a cell in the sketch with some cw-complex, and this complex repeatedly occurs as a factor. We accordingly have a set of product spaces (a generalization of extrusion) where one factor is some subset from the sketch and the other factor is a complex, which we may call the *detail* or the *profile* of the sketch element.

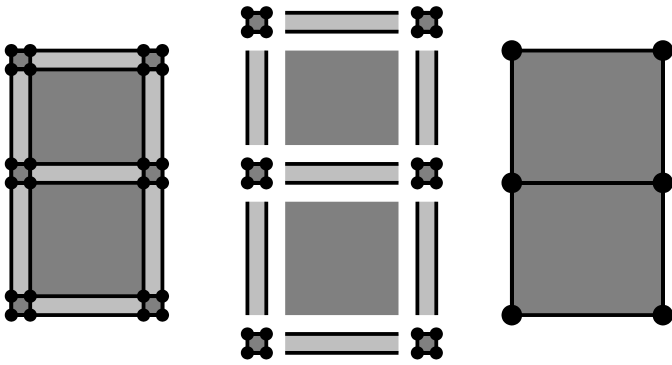


Figure 5: Two complexes representing a simplified floor plan and the fibers of the obvious mapping between these complexes drawn in the middle.

Formalization of the Example. Each detail can now be considered a subcomplex of a collection of details. If such subcomplex is given a unique name, then this designation is a mapping from the cells of the collection of details to the set of identifiers where each detail is a fiber of this mapping which we might call “belongsTo”.

We can now assign a detail to a sketch element by assigning each cell such a detail identifier, thus producing another mapping, say “uses”, from the sketch to the set of detail identifiers. Figure 6 shows these two mappings.

If we now take a detail identifier then, on the one hand, there is the set of cells in the details collection which belong to the identifier and, on the other hand, the set of cells in the sketch which use this detail, so we have two fibers.

Our working drawing consists of the union of products of such pairs of fibers or, in short, the fiber product of the two mappings “belongsTo” and “uses”. If a cell d belongs to a detail i which is also used by a cell s from the sketch, then the pair (d,s) belongs to the fiber product. Then the obvious mapping from the working drawing to the sketch is sim-

ply the projection of the pair (d,s) to the second component s .

Now we need a topology for the cells of the working drawing. We will proceed to show how the boundary matrix of the Cartesian product of two relational complexes detail (D,M) and sketch (S,N) can be computed, according to the EILENBERG-ZILBER-theorem (Eilenberg and Zilber 1953).

First, of course, the dimension of a pair (d,s) in our Cartesian product is simply the sum of the dimensions of both components, hence

$$\dim(d,s) = \dim(d) + \dim(s) \quad (5)$$

To obtain the boundary of a cell, we take the partial boundary matrix M for the complex which consists of the details and a partial boundary matrix N for our sketch. Then we define the partial product matrix $P=M \otimes N$: First set

$$\sigma(d) = (-1)^{\dim(d)} \quad (6)$$

to get a negative sign iff the dimension of a cell is odd and then set

$$P_{(d,s),(e,t)} := \begin{cases} \text{undefined} & : (d,s) = (e,t) \\ M_{d,e} & : s = t \\ \sigma(d)M_{s,t} & : d = e \\ \text{undefined} & : \text{otherwise} \end{cases} \quad (7)$$

The reader may verify that the matrix product $P \cdot P$ then returns a partial zero matrix.

Then our relational product complex is

$$(D, M) \times (S, N) := (D \times S, M \otimes N), \quad (8)$$

a generalisation of the extrusion of an m -dimensional profile (D,M) along an n -dimensional axis (S,N) to obtain the $n+m$ -dimensional product complex as the “extruded object”.

If (D,M) is the collection of all details for building elements instead of just one detail and (S,N) the

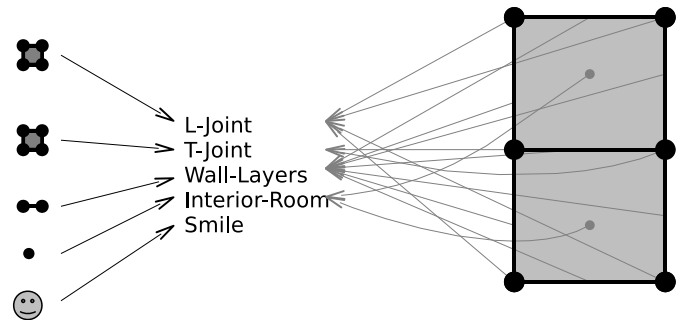


Figure 6: The floor plan sketch, the collection of details and the two mappings involved: “belongsTo” from left to middle and “uses” from right to middle.

entire sketch of the building, then the resulting product space gives a huge, complicated space where each detail is applied to each building element, where most of them do not make sense. However, it contains the products of fibers we are interested in as subspaces. It consists of all pairs (s,d) where the detail identifiers of s and d are the same. This produces an inner equi join

$$D[\text{Join}]S \subseteq D \times S \quad (9)$$

and so we can also restrict the huge product matrix $M \otimes N$ by removing all the rows and all the columns from $M \otimes N$ which are not in $D[\text{Join}]S$ and end up with the much smaller matrix $M \otimes N|_{D[\text{Join}]S}$. We accordingly obtain the pair

$$(D[\text{Join}]S, M \otimes N|_{D[\text{Join}]S}) \quad (10)$$

which may or may not be a relational complex of our desired working drawing. Hence, additional consistency rules need to be defined to guarantee that this, indeed, is the desired result.

First, the resulting working drawing should also be a complex which amounts to testing whether $D[\text{Join}]S$ is closed in $D \times S$.

Second, as we have an inner join, we must take care that no building element gets lost in the result. Therefore the mapping “belongsTo” must map detail cells *onto* the identifiers (i.e. be surjective) and each sketch element must use such a detail. It is *not* possible to simply use a right outer join instead (Paul 2008, p.186).

Third, the connectivity information within the sketch may get lost in the working drawing. We suppose that it is possible to define a topology on the details telling us if and how they can be connected to each other. The use of these details must then be consistent with this connectivity information. The assignment of identifiers, i.e. the map “belongsTo”, carries this topology over to the identifiers set such that “belongsTo” becomes continuous. This “image” of a topology is also called a *final topology*. The desired consistency in the use of details is then nothing else than continuity of the mapping “uses”. This continuity of the mappings involved is also the formal prerequisite that a union of products of fibers may be called fiber product.

Fourth, the projection from our working drawing back to the sketch must be isomorphic to the original sketch. An important consistency rule to assure this is topological monotony of the mapping “belongsTo”, i.e. each connected set of detail identifiers must have a connected pre-image.

Building Models as Assemblies of Cell Complexes. The above concept separates a building model into three parts: a sketch of building elements, a specification of possible details for these sketch elements and a specification determining which of the possible

details is chosen for each element. To obtain a valid building model, additional topological consistency rules for the references between these elements must be observed. This separation is done in a similar way to the decomposition in relational database design.

4 COMPARISON & CONCLUSIONS

By “geometrical model” we refer to the traditional approach of developing product models by means of combining isolated geometric objects. For building models of this type, topological relationships can be computed using the algorithms presented in Section 2. In the topological approach these spatial relationships and the geometric objects can be integrated into one entity, a complex. This integration can be done at several levels of detail which are connected by continuous mappings.

The topological model is intended as a formal front-end for spatial data modeling and, similar to the relational model, it is based on a mathematical theory. Apart from this formality we see the following advantages:

- A relational complex is an extremely simple data model and so it promises that many different programming tasks which involve spatial navigation and analysis can be accomplished in a similar manner. It is a well-known fact that the simplicity of the relational model is an important factor towards improved quality of databases (Abiteboul et al. 1995, p.28)
- The spatial structure and other semantic properties are kept strictly apart and many semantics can be dynamically modeled, keeping the static data model simple. An application can navigate across a joining edge without knowing if this joint is an `IfcConnectionGeometry` path between two walls or a door frame and, if it needs to know, can look it up in some standardized extension of the buildings parts library used.

An overall advantage may lie in its formal foundations: It is strictly based on the underlying mathematical theory. So every engineering question which might be of a topological nature can, in principle, be found by analysing this model; every adequate topologist's tool can be applied to the modeled object and every other (finite) topological data model can be translated from and to a relational complex with all its topological properties unchanged. Indeed, the initial motivation to define this model was the wish to have a common reference model—a formal basis with which all other spatial modeling approaches can be compared.

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