QUANTIZATION-LOSS REDUCTION FOR SIGNAL PARAMETER ESTIMATION

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ABSTRACT

Using coarse resolution analog-to-digital conversion (ADC) offers the possibility to reduce the complexity of digital receive systems but introduces a loss in effective signal-to-noise ratio (SNR) when comparing to ideal receivers with infinite resolution ADC. Therefore, here the problem of signal parameter estimation from a coarsely quantized receive signal is considered. In order to increase the system performance, we propose to adjust the analog radio front-end to the quantization device in order to reduce the quantization-loss. By optimizing the bandwidth of the analog filter with respect to a weighted form of the Cramér-Rao lower bound (CRLB), we show that for low SNR and a 1-bit hard-limiting device it is possible to significantly reduce the quantization-loss of initially -1.96 dB. As application, joint carrier-phase and time-delay estimation for satellite-based positioning and synchronization is discussed. Simulations of the maximum-likelihood estimator (MLE) show that the optimum estimator achieves the same quantization-loss reduction as predicted by the performance bound of the optimized system.

***Index Terms*—** analog-to-digital conversion, parameter estimation, analog filtering, satellite navigation systems.

1. INTRODUCTION

Most state-of-the-art signal parameter estimation techniques assume that the receiver (or sensing device) has access to digital signals with arbitrary high precision. In practice this implies the existence of analog-to-digital conversion (ADC) with a resolution which is large enough to neglect the effect of amplitude quantization. The push toward high-speed processing and acquisition renders the assumption of having available high-resolution ADC unrealistic or even invalid. In fact, fast and high resolution ADC is expensive, power consuming and therefore not appropriate for portable devices. Coarse resolution ADC may be a cost and energy-effective solution for such applications. In particular, 1-bit ADC meets the requirements for energy efficiency and allows to use a simple analog front-end. Coarse signal quantization introduces nonlinear effects which have to be taken into account in order to obtain optimum system performance. Therefore, adapting existent methods and developing new estimation algorithms, operating on quantized data, becomes more and more important.

An early work on the subject of estimating unknown parameters based on quantized signals can be found in [1]. In [2] [3] [4] the authors studied channel parameter estimation based on a single-bit quantizer assuming uncorrelated noise. The problem of signal processing with low resolution has been considered in another line of work concerned with data transmission. It turns out that the well known reduction of low SNR channel capacity by factor $2/\pi$ (-1.96 dB) due to 1-bit quantization [5] holds also for the general MIMO case with uncorrelated noise. On the other hand it was shown by [6], that the channel capacity loss of the AWGN channel due to 1-bit quantization at the receiver can be reduced by oversampling the analog receive signal. [7] observed the possibility to improve the correlator output SNR of a quantized receiver by using high sampling frequencies and reducing the receiver bandwidth. Recently, [8] derived a lower bound on the capacity of quantized MIMO channels with noise correlations showing that the loss can be lower than with white noise.

To the best of our knowledge, estimating multiple parameters from 1-bit quantized signals with colored noise is still not covered by previous works. We provide new results on the low SNR estimation performance under 1-bit quantization and noise correlation. More precisely, we derive the Cramér-Rao lower bound (CRLB) for the estimation performance in the presence of temporally colored noise within the low SNR regime, taking into account the effects of quantization. It is observed, that the popular -1.96 dB loss [9] [10] is not valid in the presence of noise correlation. In fact, the loss can be smaller for certain favorable noise correlation. Therefore, we present a new approach aiming at optimizing the analog front-end to reduce the quantization-loss. Surprisingly, even adapting the bandwidth of the analog filter is sufficient to significantly improve the performance of 1-bit receivers. Note that such analog adjustments are highly attractive from the digital signal processing point of view as they do not require any extra computational complexity during system operation.

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2. SYSTEM MODEL

For the discussion, we consider a band-limited receive signal (one-sided bandwidth B) sampled at a rate of $1/T_s$

$$\boldsymbol{y} = \boldsymbol{s}(\boldsymbol{\theta}) + \boldsymbol{n} \in \mathbb{C}^N, \tag{1}$$

where $\boldsymbol{s}(\boldsymbol{\theta}) \in \mathbb{C}^N$ is a signal of known structure modulated through $\boldsymbol{\theta} \in \mathbb{R}^K$, the signal parameter that has to be estimated. The vector $\boldsymbol{n} \in \mathbb{C}^N$ is additive Gaussian noise with the covariance matrix entries

$$\boldsymbol{R}_{ij} = 2BN_0 \operatorname{sinc}\left(2BT_s|i-j|\right), \tag{2}$$

where N_0 is the spectral density of the noise.

2.1. Quantized Receivers

For quantized receivers, we define the signal

$$\boldsymbol{q} = Q(\boldsymbol{y}), \tag{3}$$

where for simplicity $Q(\cdot)$ is an element-wise operation, individually mapping each element $\boldsymbol{y}_n \in \mathbb{C}$ onto one point out of a finite set $\mathcal{Q} \subset \mathbb{C}$. By the orthogonality principle [13, p.177] the output $\boldsymbol{q}$ of a general quantizer can be approximated [8]

$$\boldsymbol{q} = \boldsymbol{G}\boldsymbol{y} + \boldsymbol{e} = \boldsymbol{G}\boldsymbol{s}(\boldsymbol{\theta}) + \boldsymbol{G}\boldsymbol{n} + \boldsymbol{e} = \boldsymbol{s}'(\boldsymbol{\theta}) + \boldsymbol{n}', \tag{4}$$

where

$$\boldsymbol{G} = \mathrm{E}\left[\boldsymbol{q}\boldsymbol{y}^H\right]\mathrm{E}\left[\boldsymbol{y}\boldsymbol{y}^H\right]^{-1} = \boldsymbol{R}_{\boldsymbol{q}\boldsymbol{y}}\boldsymbol{R}_{\boldsymbol{y}\boldsymbol{y}}^{-1}, \tag{5}$$

and for scenarios with low SNR

$$\boldsymbol{R}' = \mathrm{E}\left[\boldsymbol{n}'\boldsymbol{n}'^H\right] \approx \mathrm{E}\left[\boldsymbol{q}\boldsymbol{q}^H\right] = \boldsymbol{R}_{\boldsymbol{q}\boldsymbol{q}}. \tag{6}$$

While (6) just specifies the second moment of the effective noise probability density function, it can be shown that for the discussed signal model, multivariate additive Gaussian noise minimizes Fisher information (FI). Therefore, a nearly optimum estimation is attained based on the distribution

$$p(\boldsymbol{q};\boldsymbol{\theta}) = \frac{1}{\pi^N \det \boldsymbol{R}'} \exp\left[-(\boldsymbol{q} - \boldsymbol{s}'(\boldsymbol{\theta}))^H \boldsymbol{R}'^{-1}(\boldsymbol{q} - \boldsymbol{s}'(\boldsymbol{\theta}))\right].$$

3. SIGNAL PARAMETER ESTIMATION

In the following the goal is to calculate an estimate $\hat{\boldsymbol{\theta}}(\boldsymbol{q})$ of the signal parameter $\boldsymbol{\theta}$ based on the quantized observation vector $\boldsymbol{q}$. We restrict the discussion to unbiased estimators, define the conditional mean square error (MSE) matrix

$$\boldsymbol{R}_{\epsilon\epsilon}(\boldsymbol{\theta}) = \mathrm{E}_{\boldsymbol{q}|\boldsymbol{\theta}}\left[(\hat{\boldsymbol{\theta}}(\boldsymbol{q}) - \boldsymbol{\theta})(\hat{\boldsymbol{\theta}}(\boldsymbol{q}) - \boldsymbol{\theta})^T\right], \tag{7}$$

and use the weighted sum of the individual estimation errors

$$\mathrm{MSEW}(\boldsymbol{\theta}) = \operatorname{tr}\left(\boldsymbol{W}\boldsymbol{R}_{\epsilon\epsilon}(\boldsymbol{\theta})\right) \tag{8}$$

as figure-of-merit where $\boldsymbol{W}$ is a positive semi-definite matrix. Without direct access to $\boldsymbol{R}_{\epsilon\epsilon}(\boldsymbol{\theta})$ optimization of the receive system can be carried out with respect to a lower bound

$$\mathrm{MSEW}(\boldsymbol{\theta}) \geq \operatorname{tr}\left(\boldsymbol{W}\boldsymbol{F}_q^{-1}(\boldsymbol{\theta})\right). \tag{9}$$

The matrix $\boldsymbol{F}_q(\boldsymbol{\theta})$ is the so called FI matrix [11] with the property [12, p. 3]

$$\boldsymbol{R}_{\epsilon\epsilon}(\boldsymbol{\theta}) \succeq \boldsymbol{F}_q^{-1}(\boldsymbol{\theta}) \tag{10}$$

for a receiver using q bits of resolution. For the considered signal model (4) the entries of $\boldsymbol{F}_q(\boldsymbol{\theta})$ are given by

$$\boldsymbol{F}_{q,ij}(\boldsymbol{\theta}) = 2 \cdot \mathrm{Re}\left\{\frac{\partial \boldsymbol{s}'^H(\boldsymbol{\theta})}{\partial \theta_i}\boldsymbol{R}'^{-1}\frac{\partial \boldsymbol{s}'(\boldsymbol{\theta})}{\partial \theta_j}\right\}. \tag{11}$$

4. 1-BIT HARD-LIMITING RECEIVER

For a hard-limiting ADC with 1-bit resolution the element-wise quantization operation $Q(\cdot)$ is defined by

$$Q(x) = \operatorname{sign}\left(\mathrm{Re}\left\{x\right\}\right) + \mathrm{j}\operatorname{sign}\left(\mathrm{Im}\left\{x\right\}\right). \tag{12}$$

Fortunately, for this quantizer a low SNR gain-noise model [8] can be specified using the arcsine-law [13, p. 438]

$$\begin{aligned}\boldsymbol{R}' &= \frac{2}{\pi}\arcsin\left(\frac{1}{2BN_0}\boldsymbol{R}\right)\\ \boldsymbol{G} &= \sqrt{\frac{2}{\pi}}\sqrt{\frac{1}{2BN_0}}\boldsymbol{I}.\end{aligned} \tag{13}$$

If the noise covariance $\boldsymbol{R}$ is a scaled identity matrix

$$\begin{aligned}\boldsymbol{F}_{\text{1-bit},ij}(\boldsymbol{\theta}) &= \frac{1}{BN_0}\mathrm{Re}\left\{\frac{\partial \boldsymbol{s}^H(\boldsymbol{\theta})}{\partial \theta_i}\left(\arcsin\left(\boldsymbol{I}\right)\right)^{-1}\frac{\partial \boldsymbol{s}(\boldsymbol{\theta})}{\partial \theta_j}\right\}\\ &= \frac{2}{\pi}\boldsymbol{F}_{\infty,ij}(\boldsymbol{\theta}).\end{aligned} \tag{14}$$

This verifies the established result [9] [10] that 1-bit hard-limiting quantization at low SNR leads to a loss of

$$10\log\left(\frac{2}{\pi}\right) = -1.96 \text{ dB} \tag{15}$$

in system performance when comparing to an ideal receive system with ADC of infinite resolution.

5. SATELLITE-BASED POSITIONING

In order to visualize the possible performance improvements, we use a GNSS signal parameter estimation scenario. Here the analog receive signal is modeled by

$$y(t) = e^{j\phi}\sqrt{C}x(t-\tau) + n(t), \tag{16}$$

with ϕ being a carrier-phase shift, C the carrier power and τ a time-delay. As satellite signal GPS L1 C/A (satellite 1) [14] is used. For observation periods smaller than 20 ms [15]

$$x(t) = \sum_{k=-\infty}^{+\infty} b_k h(t - kT_c), \tag{17}$$

with $b_k \in \{-1, 1\}$ being the k-th symbol of a periodic binary sequence $\boldsymbol{b}$ with 1023 elements and chip-duration of $T_c = 977.52$ ns such that $x(t)$ repeats after $T_0 = 1$ ms. The pulse

$$h(t) = \frac{1}{\pi\sqrt{T_c}}\left[\mathrm{Si}\left(2\pi B\Big(t + \frac{T_c}{2}\Big)\right) - \mathrm{Si}\left(2\pi B\Big(t - \frac{T_c}{2}\Big)\right)\right]$$

is the band-limited version of a rectangular pulse filtered with an ideal low-pass filter with bandwidth B where by definition

$$\mathrm{Si}(t) = \int_0^t \frac{\sin(x)}{x} dx. \tag{18}$$

A discussion regarding quantized positioning with advanced BOC pulse forms can be found in [16].

The GNSS signal spectrum can be written

$$X(\omega) = B(\omega)H(\omega), \tag{19}$$

with $B(\omega)$ being the discrete spectrum of the satellite sequence $\boldsymbol{b}$ and $H(\omega)$ the Fourier transform of the pulse $h(t)$. The sampled and quantized receive signal is

$$\boldsymbol{q} = \boldsymbol{s}'(\boldsymbol{\theta}) + \boldsymbol{n}' = \sqrt{\frac{2}{\pi}}\sqrt{\frac{C}{2BN_0}} e^{j\phi} \boldsymbol{x}(\tau) + \boldsymbol{n}', \tag{20}$$

with $\boldsymbol{\theta} = \begin{bmatrix}\phi & \tau\end{bmatrix}^T$ and $\boldsymbol{x}_k(\tau) = x(kT_s - \tau)$.

5.1. FI - Formulations and Asymptotic Analysis

For the considered estimation problem the diagonal elements of the FI matrix $\boldsymbol{F}_{\text{1-bit}}(\boldsymbol{\theta})$ are

$$\begin{aligned}
\boldsymbol{F}_{\text{1-bit},11} &= \frac{C}{BN_0} \cdot \boldsymbol{x}^H(\tau) \arcsin\left(\frac{1}{2BN_0}\boldsymbol{R}\right)^{-1} \boldsymbol{x}(\tau) \\
\boldsymbol{F}_{\text{1-bit},22} &= \frac{C}{BN_0} \cdot \frac{\partial \boldsymbol{x}^H(\tau)}{\partial \tau} \arcsin\left(\frac{1}{2BN_0}\boldsymbol{R}\right)^{-1} \frac{\partial \boldsymbol{x}(\tau)}{\partial \tau},
\end{aligned} \tag{21}$$

and the off-diagonal elements $\boldsymbol{F}_{\text{1-bit},12}$ and $\boldsymbol{F}_{\text{1-bit},21}$ vanish. Under the assumption that the band-limited signal $x(t)$ is periodic the sampled signal vector $\boldsymbol{x}(\tau)$ can be written

$$\boldsymbol{x}(\tau) = \boldsymbol{D}\boldsymbol{T}(\tau)\tilde{\boldsymbol{x}}, \tag{22}$$

where $\boldsymbol{D} \in \mathbb{C}^{N\times N}$ is a modified DFT matrix

$$\boldsymbol{D}_{ni} = \frac{1}{\sqrt{N}} e^{\mathrm{j}\, 2\pi \frac{(n-(N/2+1))(i-(N/2+1))}{N}}, \tag{23}$$

$\boldsymbol{T}(\tau) \in \mathbb{C}^{N\times N}$ a diagonal matrix with entries

$$\boldsymbol{T}_{nn}(\tau) = e^{-\mathrm{j}(n-(N/2+1))\omega_0 \tau}, \tag{24}$$

and $\tilde{\boldsymbol{x}}$ the vector with the Fourier coefficients

$$\tilde{\boldsymbol{x}} = \left[X\Big(-\frac{N}{2}\omega_0\Big), \ldots, X\Big(\Big(\frac{N}{2} - 1\Big)\omega_0\Big)\right]^T, \tag{25}$$

with $\omega_0 = \frac{2\pi}{T_0}$. For the signal derivative $\frac{\partial \boldsymbol{x}(\tau)}{\partial \tau}$ such an approach allows to write

$$\frac{\partial \boldsymbol{x}(\tau)}{\partial \tau} = \boldsymbol{D}\boldsymbol{T}(\tau)\boldsymbol{\Omega}\tilde{\boldsymbol{x}}, \tag{26}$$

with $\boldsymbol{\Omega} \in \mathbb{C}^{N\times N}$ being a diagonal matrix with entries

$$\boldsymbol{\Omega}_{nn} = -\mathrm{j}\,\omega_0(n - (N/2+1)). \tag{27}$$

Further, if N is sufficiently large any covariance matrix $\boldsymbol{R}$ has approximately the structure of a circulant Toeplitz matrix and is therefore diagonalized

$$\boldsymbol{R} \approx \boldsymbol{D}\boldsymbol{\Lambda}\boldsymbol{D}^H, \tag{28}$$

such that $\boldsymbol{\Lambda} \in \mathbb{R}^{N\times N}$ forms a diagonal matrix with entries

$$\boldsymbol{\Lambda}_{nn} = \frac{1}{T_0}\psi\Big((n - (N/2+1))\omega_0\Big), \tag{29}$$

where $\psi(\omega)$ is the power-spectral density of the random Gaussian signal with covariance matrix $\boldsymbol{R}$. For the information measure related to the phase-rotation ϕ

$$\begin{aligned}
\boldsymbol{F}_{\text{1-bit},11} &= \frac{C}{BN_0} \cdot \boldsymbol{x}^H(\tau) \arcsin\left(\frac{1}{2BN_0}\boldsymbol{R}\right)^{-1} \boldsymbol{x}(\tau) \\
&\approx 2C \cdot \tilde{\boldsymbol{x}}^H \boldsymbol{\Lambda}_{\text{1-bit}}^{-1} \tilde{\boldsymbol{x}} \\
&= \frac{2C}{T_0} \sum_{n=-\frac{N}{2}}^{\frac{N}{2}-1} \frac{|X(n\omega_0)|^2}{\psi_{\text{1-bit}}(n\omega_0)} = F_{\text{1-bit},\phi}(B, T_s),
\end{aligned} \tag{30}$$

with

$$\psi_{\text{1-bit}}(\omega) = 2BN_0T_s \sum_{k=-\infty}^{k=\infty} \arcsin\left(\mathrm{sinc}\left(2BkT_s\right)\right) e^{-\mathrm{j}\,\omega k T_s}. \tag{31}$$

Letting the time of one signal period go to infinity, $T_0 \to \infty$ the FI asymptotically becomes

$$\bar{F}_{\text{1-bit},\phi} \approx \frac{C}{\pi} \int_{-2\pi B}^{2\pi B} \frac{|X(\omega)|^2}{\psi_{\text{1-bit}}(\omega)} \mathrm{d}\omega. \tag{32}$$

For the FI measure of the time-delay τ

$$\begin{aligned}
\boldsymbol{F}_{\text{1-bit},22} &= \frac{C}{BN_0} \cdot \frac{\partial \boldsymbol{x}^H(\tau)}{\partial \tau} \arcsin\left(\frac{1}{2BN_0}\boldsymbol{R}\right)^{-1} \frac{\partial \boldsymbol{x}(\tau)}{\partial \tau} \\
&\approx 2C \cdot \tilde{\boldsymbol{x}}^H \boldsymbol{\Omega}^H \boldsymbol{\Lambda}_{\text{1-bit}}^{-1} \boldsymbol{\Omega} \tilde{\boldsymbol{x}} \\
&= \frac{2C}{T_0} \sum_{n=-\frac{N}{2}}^{\frac{N}{2}-1} \frac{(n\omega_0)^2 |X(n\omega_0)|^2}{\psi_{\text{1-bit}}(n\omega_0)} = F_{\text{1-bit},\tau}(B, T_s),
\end{aligned} \tag{33}$$

with the asymptotic form

$$\bar{F}_{\text{1-bit},\tau} \approx \frac{C}{\pi} \int_{-2\pi B}^{2\pi B} \frac{\omega^2 |X(\omega)|^2}{\psi_{\text{1-bit}}(\omega)} \mathrm{d}\omega. \tag{34}$$

5.2. System Optimization - Bandwidth and Sampling

For system optimization we choose $N = 2046$, $B = \frac{1}{T_c}$, $T_s = \frac{T_c}{2}$ and use an ideal system with infinite resolution as reference in order to formulate a normalized version of (9)

$$\text{NMSEW}(\rho,\mu) \geq \alpha \cdot \text{NMSE}_\phi(\rho,\mu) + (1-\alpha) \cdot \text{NMSE}_\tau(\rho,\mu), \tag{35}$$

with

$$\text{NMSE}_{\phi/\tau}(\rho,\mu) = \frac{F_{\infty,\phi/\tau}(B, T_s)}{F_{\text{1-bit},\phi/\tau}(\rho B, \frac{T_s}{\mu})}, \tag{36}$$

where the bandwidth fraction satisfies $0 \leq \rho \leq 1$ and the oversampling factor $\mu \geq 1$. Note, that with infinite resolution

$$F_{\infty,\phi/\tau}(\rho B, \frac{T_s}{\mu}) \leq F_{\infty,\phi/\tau}(B, T_s) \tag{37}$$

for $\rho \leq 1$, $\mu \geq 1$ which does not hold for the 1-bit case. In fact, Fig. 1 visualizes the individual quantization-loss

$$\chi_{\phi/\tau}(\rho,\mu) = -10 \log \text{NMSE}_{\phi/\tau}(\rho,\mu) \tag{38}$$

with respect to the bandwidth ρ. Without oversampling, i.e.

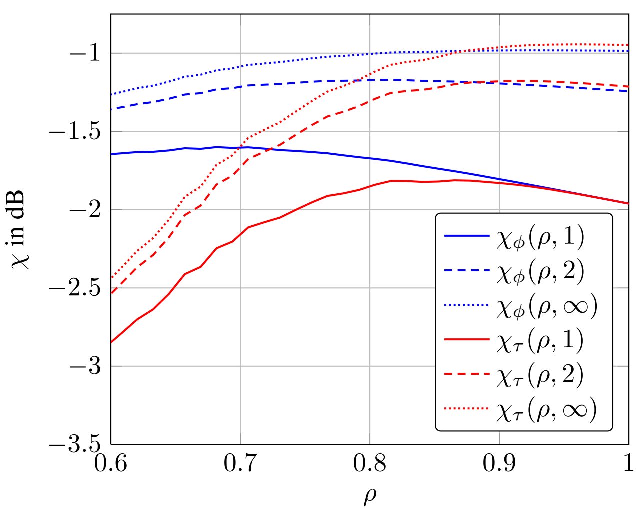

Fig. 1. 1-bit Quantization-loss versus bandwidth ρ

$\mu = 1$, the best performance (-1.6 dB) in ϕ is attained with a bandwidth fraction of $\rho = 0.68$. For τ the best configuration (-1.81 dB) is $\rho = 0.86$. With oversampling the tendency is to use the full bandwidth ($\rho = 1$). Fig. 2 shows the feasibility region for different oversampling factors $\mu \geq 1$ which is attained by plotting the values NMSE_ϕ and NMSE_τ after solving

$$\rho^\star = \arg\min_{\rho} \text{NMSEW}(\rho,\mu) \quad \text{s.t.} \quad 0 \leq \rho \leq 1 \tag{39}$$

for fixed μ and different weightings α. It can be observed that without oversampling a compromise between the two parameters has to be made. Increasing for example the performance of the carrier-phase measurement results in a performance loss for the time-delay. This is not the case with oversampling where the highest performance gain is attained by doubling the sampling rate of the receiver. Further increasing the sampling rate only gives marginal additional performance. Also the performance gains for the maximum-likelihood estimator (MLE) on different points of the feasibility regions are plotted in Fig. 2. It can be observed that the estimator achieves the same improvement through the analog front-end design as predicted by the optimized performance bound.

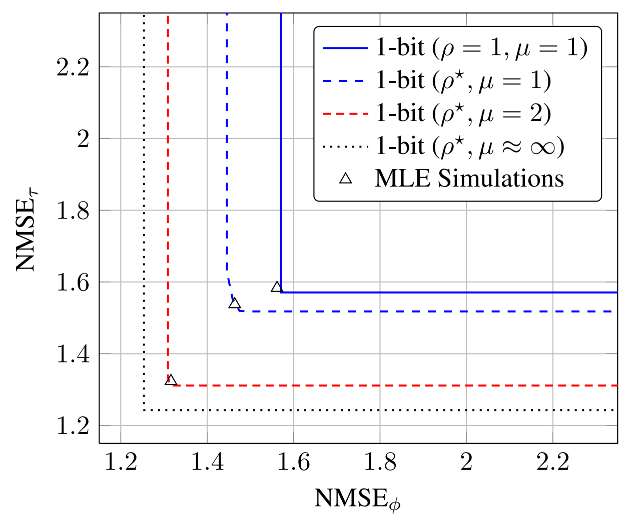

Fig. 2. Feasibility Region - Normalized MSE

6. CONCLUSION

Simple ADC allows to reduce the complexity of analog receive systems and to simplify digital signal processing. Here it was proposed to optimize the analog radio front-end with respect to a theoretic performance measure behind an quantization device. For the extreme case of 1-bit quantization it was shown that the well-known -1.96 dB performance loss is not valid in the context of low SNR signal parameter estimation if appropriate noise correlation is present. Here these correlations were produced in a controlled fashion through the analog preprocessing chain leading to significantly higher system performances. In practice these improvements are achieved by calculation of the optimum estimator.

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