Guaranteed Cost Control over Quality-of-Service Networks

Chih-Chung Chen, Adam Molin and Sandra Hirche

Abstract— **The communication quality has a strong effect on the stability and control performance of a networked control system (NCS). To achieve superior control performance, better communication quality is desired. However, high network cost is induced. This paper aims at finding an optimal trade-off between control performance and network cost. A Quality-of-Service (QoS) communication network is introduced, where the statistical properties of the transmission delay can be controlled by adjusting its transition generator. The QoS concept enables a conjoint control of communication network and control system itself, which results in a Markovian jump linear system (MJLS) with mode-dependent delays. A delay-dependent stability condition ensuring stochastic mean-square stability is derived by using the Lyapunov-Krasovskii functional. A guaranteed cost of state evolutions is determined and the perturbation upper bound on Markov process transition generator is obtained. A performance-cost trade-off is achieved by optimizing the Markov process transition generator within the perturbation upper bound. The performance benefit of the proposed control concept is demonstrated in a numerical example.**

I. INTRODUCTION

Due to the affordability, well-developed infrastructure and widespread usage, communication networks become more and more attractive for the signal transmission in control systems, such as unmanned aerial vehicles [1], Ethernet-based car control networks [2] and teleoperation [3]. However, the use of a communication network comes at the price of non-ideal signal transmission, such as packet loss and transmission delay. Particularly, the transmission delay is well-known as a source of instability and deteriorates the control performance of closed-loop systems, see [4], [5] for a general overview on the challenges and control methodologies for networked control system (NCS). In the most NCS literatures, the communication quality, e.g. the transmission delay, is assumed to be known in advance and the controller is accordingly designed [5].

In this paper, random transmission delay modeled by Markov process is considered. Motivated by Quality-of-Service networks a conjoint control of communication network and control system is proposed. The QoS network refers to the capability of a network to provide different communication quality to different network traffic classes. Guaranteed short transmission delay leads to good control performance but needs the provision of large network resources, i.e. high network cost. Opposed to the work in [6] where the transmission delay of the communication network can be set up instantaneously, a more realizable approach is considered. It is proposed to control the probability distribution of the transmission delay by setting the transition generator of the Markov process. Furthermore, a switching remote controller is considered. The remote controller is able to monitor the transmission delay and switches synchronously with the transmission delay such that expected performance can be achieved.

As a result, a Markovian jump linear system (MJLS) with mode-dependent delay is established. The MJLS is a class of hybrid systems, whose discrete and continuous states are modeled by Markov process and corresponding linear differential equations [7]. The stability of an MJLS can be analyzed by the notion of stochastic stability introduced in [7]–[9]. MJLS with delay are widely studied in [10]–[15]. In [10], an MJLS with constant delay is considered. A delay-independent sufficient condition for stochastic mean-square stability is established. The MJLS with time-varying delay is addressed in [11]. Delay-dependent conditions for stochastic stability and stochastic stabilization are proposed in terms of linear matrix inequality (LMI). The H_2/H_∞ control is applied to MJLS with uncertain delay in [12]. In [13], the robust cost guaranteed control for MJLS with mode-dependent delay is studied. An LMI delay-dependent stability condition is proposed and the state evolutions of the closed-loop system is shown to be bounded for all admissible system uncertainties. The MJLS with mode-dependent time-varying delay is discussed in [14]. The LMI conditions for stochastic exponential mean-square stability and stabilization are proposed. A great variety of approaches can be found in [15]. The main difference of this paper among other existing literatures is that i) a delay-dependent stability condition for stochastic exponential stability is established, ii) a perturbation upper bound on Markov process transition generator is derived such that the stability condition holds and iii) a trade-off between the control performance and network cost is achieved by the optimal allocation of probability distribution of transmission delays, i.e. optimal parameter designation of Markov process transition generator. The proposed approach is numerically validated. The simulation results demonstrates the superior cost-performance benefit over the constant transmission delay network with non-switching controller.

The remainder of the paper is organized into five sections. In section 2, an MJLS with mode-dependent transmission delay is introduced. In section 3, a stochastic exponential mean square stability condition for MJLS is derived. Based on the stability condition, a perturbation upper bound on Markov process transition generator is determined. The numerical validation and performance comparison are discussed in section 4 and the summary is given in section 5.

Institute of Automatic Control Engineering, Technische Universität München, D-80290 München, Germany; `http://www.lsr.ei.tum.de`, {`chen, adam.molin`}`@tum.de`, `s.hirche@ieee.org`

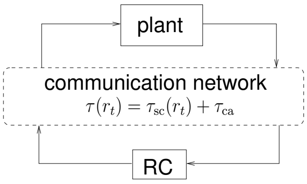

Fig. 1. Illustration of NCS interconnected by a remote controller (RC) through a communication network with round-trip transmission delay $\tau(r_t)$.

Notation. In this paper $\lambda_{\max}(M)$ and $\lambda_{\min}(M)$ denote the maximal and the minimal eigenvalues of matrix M, whereas M^T and $||M||$ denote the transpose and induced Euclidean norm of matrix (or vector) M, respectively. The symbol $*$ denotes the transpose of the blocks outside the main diagonal block in symmetric matrices. $\mathbb{E}$ stands for mathematical expectation and $\mathbf{P}$ for probability. $\{r_t, t \geq 0\}$ denotes a Markov process governing the mode switching in the finite set $\mathcal{S} := \{1, \ldots, N\}$ having the transition generator $\mathcal{A} = (\alpha_{i,j}),\ i, j \in \mathcal{S},\ \alpha_{i,j} > 0,\ i \neq j,\ \alpha_{i,i} = -\sum_{i \neq j} \alpha_{i,j}$. The Markov process transition probability can be defined as

$$\mathbf{P}_{i,j}(r_{t+\delta} = j | r_t = i) = \begin{cases} \alpha_{i,j}\delta + o(\delta^2),\ i \neq j \\ 1 + \alpha_{i,i}\delta + o(\delta^2),\ i = j, \end{cases}$$

where $\lim_{\delta \to 0} o(\delta^2)/\delta = 0$.

II. Problem Statement

Consider an LTI system as the plant

$$\dot{x}(t) = Ax(t) + Bu(t), \tag{1}$$

where $x \in \mathbb{R}^n$ is the state and $u \in \mathbb{R}^m$ is the control input; A and B are constant matrices with appropriate dimensions and (A, B) is controllable. The plant is interconnected by a remote controller (RC) through a communication network, see Fig. 1. The sensor-to-controller delay $\tau_{\text{sc}}(r_t)$ is random and modeled by a Markov process r_t, while the controller-to-actuator τ_{ca} delay is assumed to be constant (realizable through buffering technique). The round-trip transmission delay resulting from the communication network is $\tau(r_t) = \tau_{\text{sc}}(r_t) + \tau_{\text{ca}}$, $r_t \in \mathcal{S} := \{1, \ldots, N\}$. The transition rate of delay from $\tau(r_t = i)$ to $\tau(r_t = j),\ i,\ j \in \mathcal{S}$, is determined by a transition generator $\mathcal{A} = (\alpha_{i,j} + \Delta\alpha_{i,j})$, where $\Delta\alpha_{i,j}$ denotes the uncertainties satisfying i) $\sum_{j=1}^{N} \Delta\alpha_{i,j} = 0$ and; ii) $\Delta\alpha_{i,i} < -\alpha_{i,i},\ i = j$ and $\Delta\alpha_{i,j} > -\alpha_{i,j},\ i \neq j$.

Assume the remote controller is able to monitor the transmission delay and switches the feedback gains synchronously with the transmission delay $\tau(r_t)$. Therefore, the control laws for the remote controllers are given as

$$u(t) = K(r_t)x(t - \tau(r_t))$$

and the closed-loop system becomes

$$\dot{x}(t) = Ax(t) + BK(r_t)x(t - \tau(r_t)). \tag{2}$$

The closed-loop system (2) is an MJLS system with mode-dependent delay. For the sake of simplicity, $K(r_t)$ is written as K_i and $\tau(r_t) = \tau_i$ for each $r_t = i \in \mathcal{S}$.

Introduce a $\gamma > 0$, which is related to the convergence rate of $\mathbb{E}\{||x(t)||^2\}$ in (2) and consider a new variable $z(t) = e^{\gamma t}x(t)$. Substitute $z(t)$ into (3), it yields

$$\dot{z}(t) = \hat{A}z(t) + \hat{A}_{1i}z(t - \tau_i), \tag{3}$$

where $\hat{A} = A + \gamma I,\ \hat{A}_{1i} = e^{\gamma\tau_i}BK_i$. Using the Newton-Leibnitz formula

$$z(t - \tau_i) = z(t) - \int_{t-\tau_i}^{t} \dot{z}(s)ds,$$

and substituting it into (3), set $\xi^T(t) = [z^T(t)\ \ \dot{z}^T(t)]$. System (3) has the descriptor form

$$E\dot{\xi}(t) = (\bar{A} + \bar{A}_{1i})\xi(t) - \bar{A}_{2i}\int_{t-\tau_i}^{t} \xi(s)ds, \tag{4}$$

where

$$E = \begin{bmatrix} I & 0 \\ 0 & 0 \end{bmatrix}, \qquad \bar{A} = \begin{bmatrix} 0 & I \\ A + \gamma I & -I \end{bmatrix},$$
$$\bar{A}_{1i} = \begin{bmatrix} 0 & 0 \\ e^{\gamma\tau_i}BK_i & 0 \end{bmatrix},\ \bar{A}_{2i} = \begin{bmatrix} 0 & 0 \\ 0 & e^{\gamma\tau_i}BK_i \end{bmatrix}.$$

Consider a set of positive matrices $R_i,\ i \in \mathcal{S}$, a cost function can be defined as

$$J = \mathbb{E}\Big\{ \int_0^\infty z^T(t)R_iz(t)dt \Big\}. \tag{5}$$

Associated to the cost function (5), the cost guaranteed control is defined as follows.

Definition 1: Consider the closed-loop system (3). If there exists a positive scalar $\bar{J}$ such that the cost function (5) satisfies $J \leq \bar{J}$, then $\bar{J}$ is said to be a guaranteed cost for the closed-loop system (3).

Before the main result is introduced, the following definition and lemmas have to be given.

Definition 2: System (3) is stochastic exponential mean square stable if for any initial condition $x(t_0, r_{t_0})$, there exist positive constants b, and ρ such that for all $t \geq t_0$

$$\mathbb{E}\big\{||x(t)||^2 | x(t_0, r_{t_0})\big\} \leq b||x(t_0, r_{t_0})||^2 e^{-\rho(t-t_0)}.$$

Lemma 1: [15] Let X and Y be real constant matrices with appropriate dimensions. Then

$$X^TY + Y^TX \leq \varepsilon X^TX + \frac{1}{\varepsilon}Y^TY$$

holds for any $\varepsilon > 0$.

Lemma 2: Let X, Y be positive definite matrices and a, b be scalars satisfying $a > 0$ and $a > b$. Then

$$\lambda_{\max}(aX + bY) \leq \lambda_{\max}(aX + aY).$$

Proof: It is noted that

$$(aX + bY)^T(aX + bY) \leq (aX + aY)^T(aX + aY).$$

Pre- and post-multiply the above inequality by the normalized eigenvector v^T and v, which corresponds to the maximal eigenvalue, i.e. $\lambda_{\max}(aX + bY)$. It becomes

$$\begin{aligned} \lambda_{\max}^2(aX + bY) &= v^T(aX + bY)^T(aX + bY)v \\ &\leq v^T(aX + aY)^T(aX + aY)v. \end{aligned} \tag{6}$$

According to the definition of second order induced norm (Euclidean norm) of matrix, it has

$$\begin{aligned}\lambda_{\max}^2(aX+aY) &= ||aX+aY||^2\\ &= \max_{\|v\|_2=1} v^T(aX+aY)^T(aX+aY)v\end{aligned} \tag{7}$$

Combine (6) and (7), it yields

$$\lambda_{\max}(aX+bY) \le \lambda_{\max}(aX+aY)$$

■

III. Main Result

The objective of this section is to derive a stability condition for NCS with mode-dependent delay and a perturbation upper bound on Markov process transition generator. In Theorem 1, the Markov transition generator uncertainties $\Delta\alpha_{i,j}$ are assumed to be zero. A delay-dependent stability condition is derived by using the Lyapunov-Krasovskii functional and a guaranteed cost for the cost function (5) is found. Based on the stability condition in Theorem 1, a perturbation upper bound $\Delta\alpha_{i,j}$ on the Markov process transition generator is determined in Theorem 2 such that the stability condition and guaranteed cost in Theorem 1 are still valid.

Theorem 1: For the closed-loop system (3) with a given scalar $\gamma > 0$, matrix $R_i > 0$, if there exist $P_i > 0$ and $M > 0$ such that the following matrix inequality holds for all $r_t = i \in \mathcal{S}$

$$\begin{bmatrix}\Omega_{1i} & \tau_i P_i^T\bar{A}_{2i}\\ * & -\tau_i M\end{bmatrix} < 0, \tag{8}$$

where $\bar{\alpha}_i = |\alpha_{i,i}|$ and

$$\begin{aligned}&\bar{\tau} = \max_{i\in\mathcal{S}}\{\tau_i\}, \quad \underline{\tau} = \min_{i\in\mathcal{S}}\{\tau_i\}, \quad \hat{\tau} = \frac{1}{2}(\bar{\tau}^2 - \underline{\tau}^2),\\ &\Omega_{1i} = (\bar{A}+\bar{A}_{1i})^T P_i + P_i^T(\bar{A}+\bar{A}_{1i}) + \bar{\alpha}_i\hat{\tau}M\\ &\quad + \sum_{j=1}^N \alpha_{i,j}EP_j + \tau_i M + R_i,\end{aligned}$$

then the system is stochastic exponential mean square stable and the cost function (5) is bounded by

$$\begin{aligned}J &\le \bar{J}(P_{r_0}, M, \bar{\alpha}_{r_0})\\ &= \xi^T(0)EP_{r_0}\xi(0) + \int_{-\tau_{r_0}}^0\int_\theta^0 \xi^T(s)M\xi(s)dsd\theta\\ &\quad + \bar{\alpha}_{r_0}\int_{-\bar{\tau}}^{-\underline{\tau}}\int_\theta^0 \xi^T(s)M\xi(s)(s-\theta)dsd\theta.\end{aligned} \tag{9}$$

Proof: Suppose $r_t = i \in \mathcal{S}$ and denote $\xi(t) = z(t+s)$, $-\max_{i\in\mathcal{S}}\{\tau_i\} \le s \le 0$. Consider a Lyapunov candidate as the following

$$V(\xi(t),i) = V_1(\xi(t),i) + V_2(\xi(t),i) + V_3(\xi(t),i), \tag{10}$$

where

$$\begin{aligned}&V_1(\xi(t),i) = \xi^T(t)EP_i\xi(t),\\ &V_2(\xi(t),i) = \int_{-\tau_i}^0\int_{t+\theta}^t \xi^T(s)M\xi(s)dsd\theta,\\ &V_3(\xi(t),i) = \bar{\alpha}_i\int_{-\bar{\tau}}^{-\underline{\tau}}\int_{t+\theta}^t \xi^T(s)M\xi(s)(s-t-\theta)dsd\theta.\end{aligned}$$

Define $\bar{\alpha}_i = |\alpha_{i,i}|$, then

$$\begin{aligned}\mathcal{L}V_1(\xi(t),i) &= \dot{\xi}^T(t)EP_i\xi(t) + \xi^T(t)P_i^T E\dot{\xi}(t)\\ &\quad + \sum_{j=1}^N \alpha_{i,j}\xi^T(t)EP_j\xi(t)\\ &= \xi^T(t)\Big[\big(\bar{A}+\bar{A}_{1i}\big)^T P_i + P_i^T\big(\bar{A}+\bar{A}_{1i}\big)\\ &\quad + \sum_{j=1}^N \alpha_{i,j}EP_j\Big]\xi(t)\\ &\quad - 2\xi^T(t)P_i^T\bar{A}_{2i}\int_{t-\tau_i}^t \xi(s)ds.\end{aligned}$$

According to lemma 1, $\mathcal{L}V_1(\xi(t),i)$ becomes

$$\begin{aligned}\mathcal{L}V_1(\xi(t),i) &\le \xi^T(t)\Big[\big(\bar{A}+\bar{A}_{1i}\big)^T P_i + P_i^T\big(\bar{A}+\bar{A}_{1i}\big)\\ &\quad + \tau_i P_i^T\bar{A}_{2i}M^{-1}\bar{A}_{2i}^T P_i\\ &\quad + \sum_{j=1}^N \alpha_{i,j}EP_j\Big]\xi(t)\\ &\quad + \int_{t-\tau_i}^t \xi^T(s)M\xi(s)ds.\end{aligned}$$

Similarly to [14], it has

$$\begin{aligned}\mathcal{L}V_2(\xi(t),i) &\le \tau_i\xi^T(t)M\xi(t) - \int_{t-\tau_i}^t \xi^T(s)M\xi(s)ds\\ &\quad + \bar{\alpha}_i\int_{-\bar{\tau}}^{-\underline{\tau}}\int_{t+\theta}^t \xi^T(s)M\xi(s)dsd\theta.\\ \mathcal{L}V_3(\xi(t),i) &= \frac{1}{2}\bar{\alpha}_i(\bar{\tau}^2-\underline{\tau}^2)\xi^T(t)M\xi(t)\\ &\quad - \bar{\alpha}_i\int_{-\bar{\tau}}^{-\underline{\tau}}\int_{t+\theta}^t \xi^T(s)M\xi(s)dsd\theta.\end{aligned}$$

It yields

$$\begin{aligned}\mathcal{L}V(\xi(t),i) &= \xi^T(t)\Big[\big(\bar{A}+\bar{A}_{1i}\big)^T P_i + P_i^T\big(\bar{A}+\bar{A}_{1i}\big)\\ &\quad + \tau_i P_i^T\bar{A}_{2i}M^{-1}\bar{A}_{2i}^T P_i + (\bar{\alpha}_i\hat{\tau}+\tau_i)M\\ &\quad + \sum_{j=1}^N \alpha_{i,j}EP_i\Big]\xi(t)\\ &= \xi^T(s)\Theta_i\xi(s),\end{aligned} \tag{11}$$

where $\hat{\tau} = \frac{1}{2}(\bar{\tau}^2 - \underline{\tau}^2)$.

Since $\max_{\theta\in[-2\bar{\tau},0]}\{||\xi(t+\theta)||\} \le \varphi||\xi(t)||$ for some $\varphi > 0$ [16], the following can be established

$$\begin{aligned}V(\xi(t),i) &\le \Big[\lambda_{\max}(EP_i + \varphi\lambda_{\max}(M)\Big]||\xi(t)||^2\\ &\le \bar{\Lambda}_i||\xi(t)||^2,\end{aligned} \tag{12}$$

where

$$\begin{aligned}\varphi &= \frac{1}{2}\bar{\tau}^2 + \frac{1}{6}(\bar{\tau}^3 - \underline{\tau}^3)\bar{\alpha}_i\\ \bar{\Lambda}_i &= \lambda_{\max}(EP_i + \varphi\lambda_{\max}(M)).\end{aligned}$$

Combining (11) and (12) yields

$$\frac{\mathcal{L}V(\xi(t),i)}{V(\xi(t),i)} \le -\min_{i\in\mathcal{S}}\left\{\frac{\lambda_{\min}(-\Theta_i)}{\bar{\Lambda}_i}\right\} \triangleq -\rho_0$$

and

$$\mathbb{E}\big\{\mathcal{L}V(\xi(t),i)\big\} \le -\rho_0\mathbb{E}\big\{V(\xi(t),i)\big\}. \tag{13}$$

By applying Dynkin's formula into (13) it becomes

$$\begin{aligned}&\mathbb{E}\big\{V(\xi(t),r_t)\big\} - \mathbb{E}\big\{V(\xi(0),r_0)\big\}\\ &= \mathbb{E}\bigg\{\int_0^t \mathcal{L}V(\xi(s),r_s)ds\bigg\}\\ &\le -\rho_0\int_0^t \mathbb{E}\big\{\mathcal{L}V(\xi(s),r_s)ds\big\}.\end{aligned} \tag{14}$$

Using the Gronwall-Bellman lemma, (14) results in

$$\mathbb{E}\big\{V(\xi,r_t)\big\} \le e^{-\rho_0 t}\mathbb{E}\big\{V(\xi(0),r_0)\big\}.$$

Since

$$\begin{aligned}V(\Xi(t),i) &\ge \Big[\lambda_{\min}(EP_i + \varphi\lambda_{\min}(M)\Big]||\xi(t)||^2\\ &= \underline{\Lambda}_i||\xi(t)||^2,\end{aligned}$$

it is established that

$$\mathbb{E}\big\{||\xi(t)||^2\big\} \le e^{-\rho_0 t}\frac{\mathbb{E}\big\{V(\xi(0),r_0)\big\}}{\min_{i\in\mathcal{S}}\big\{\underline{\Lambda}_i\big\}}. \tag{15}$$

Equation (15) provides the proof for stochastic exponential mean square stability.

Due to the fact $z(t) = [I\ 0]\xi(t)$, consider Dynkin's formula and (11), the cost function (5) becomes

$$\begin{aligned}J &= \mathbb{E}\bigg\{\int_0^T \xi^T(t)\begin{bmatrix}I\\0\end{bmatrix}R_i\begin{bmatrix}I\ 0\end{bmatrix}\xi(t)dt\bigg\}\\ &= \mathbb{E}\bigg\{\int_0^T \Big[\xi^T(t)\begin{bmatrix}I\\0\end{bmatrix}R_i\begin{bmatrix}I\ 0\end{bmatrix}\xi(t) + \mathcal{L}V(\xi(t),i)\Big]dt\bigg\}\\ &\quad - \mathbb{E}\bigg\{\int_0^T \mathcal{L}V(\xi(t),i)dt\bigg\}\\ &= \mathbb{E}\bigg\{\int_0^T \Big[\xi^T(t)\begin{bmatrix}I\\0\end{bmatrix}R_i\begin{bmatrix}I\ 0\end{bmatrix}\xi(t) + \mathcal{L}V(\xi(t),i)\Big]dt\bigg\}\\ &\quad - \mathbb{E}\big\{V(\xi(T),r_T)\big\} + \mathbb{E}\big\{V(\xi(0),r_0)\big\}\\ &\le \mathbb{E}\bigg\{\int_0^T \xi^T(t)\bar{\Theta}_i\xi(t)dt + V(\xi(0),r_0)\bigg\},\end{aligned} \tag{16}$$

where $\bar{\Theta}_i = \Theta_i + \begin{bmatrix}I\\0\end{bmatrix}R_i\begin{bmatrix}I\ 0\end{bmatrix}$. By the requirement $\bar{\Theta}_i < 0$, it results in (8) and concludes

$$\begin{aligned}J &= \mathbb{E}\bigg\{\int_0^T \xi^T(t)ER_iE\xi(t)dt\bigg\} \le V(\xi(0),r_0)\\ &= \bar{J}(P_{r_0},M,\bar{\alpha}_{r_0}).\end{aligned} \tag{17}$$

Apply Schur complement to $\bar{\Theta}_i$, it results in (8) and completes the proof. ■

Remark 1: According to [14], inequality (15) can be rewritten as

$$\mathbb{E}||x(t)||^2 \le e^{-(\rho_0+2\gamma)t}\frac{\mathbb{E}V(\Xi(0),r_0)}{\min_{i\in\mathcal{S}}\big\{\underline{\Lambda}_i\big\}}. \tag{18}$$

Therefore, the given γ in Theorem 1 ensures the decay rate of trajectory $\mathbb{E}\{||x(t)||^2\}$ and determines the control performance.

Remark 2: For constant transmission delay, i.e. $\tau_i = \tau$ and $\alpha_{i,j} = 0$, Theorem 1 is applicable to systems with constant delay.

Theorem 2: Consider an NCS in (3) satisfying the matrix inequality (8) in Theorem 1. Let the Markov process transition generator be perturbed by $\Delta\alpha_{i,j}$, for $i,j\in\mathcal{S}$, satisfying $\sum_{j=1}^N \Delta\alpha_{i,j} = 0$ and

$$\begin{cases}\Delta\alpha_{i,j} > -\alpha_{i,j},\ i\ne j,\\ \Delta\alpha_{i,i} < -\alpha_{i,i},\ i = j.\end{cases}$$

If the perturbations on the Markov process transition generator satisfy

$$\Delta\bar{\alpha}_i \le \bar{\chi}_i, \tag{19}$$

where

$$\begin{aligned}\Delta\bar{\alpha}_i &= |\Delta\alpha_{i,i}| = \sum_{j\ne 1}^N \Delta\alpha_{i,j},\\ \bar{\chi}_i &= \lambda_{\max}^{-1}\bigg(\hat{\tau}M + \sum_{j\ne i}^N P_j - P_i\bigg)\lambda_{\min}(-\bar{\Theta}_i),\\ \bar{\Theta}_i &= \big(\bar{A} + \bar{A}_{1i}\big)^T P_i + P_i^T\big(\bar{A} + \bar{A}_{1i}\big) + \sum_{j=1}^N \alpha_{i,j}P_i\\ &\quad + \tau_i P_i^T\bar{A}_{2i}M^{-1}\bar{A}_{2i}^T P_i + (\bar{\alpha}_i\hat{\tau} + \tau_i)M + R_i,\end{aligned}$$

then the system is also stochastic exponential mean square stable and the guaranteed cost of cost function (5) is bounded by

$$\begin{aligned}J &\le \bar{J}(P_{r_0},M,\bar{\alpha}_{r_0})\\ &= \xi^T(0)EP_{r_0}\xi(0) + \int_{-\tau_{r_0}}^0\int_\theta^0 \xi^T(s)M\xi(s)dsd\theta\\ &\quad + (\bar{\alpha}_{r_0} + \Delta\alpha_{i,j})\int_{-\bar{\tau}}^{-\underline{\tau}}\int_\theta^0 \xi^T(s)M\xi(s)(s-\theta)dsd\theta.\end{aligned} \tag{20}$$

Proof: The perturbed closed-loop system is exponential mean-square stable, if the matrix inequality in (8) is satisfied, i.e.

$$\Delta\bar{\alpha}_i\hat{\tau}M + \sum_{j=1}^N \Delta\alpha_{i,j}P_j + \bar{\Theta}_i < 0.$$

where $\bar{\Theta}_i < 0$ is determined in Theorem 1 as

$$\begin{aligned}\bar{\Theta}_i &= \big(\bar{A} + \bar{A}_{1i}\big)^T P_i + P_i^T\big(\bar{A} + \bar{A}_{1i}\big) + \sum_{j=1}^N \alpha_{i,j}P_i\\ &\quad + \tau_i P_i^T\bar{A}_{2i}M^{-1}\bar{A}_{2i}^T P_i + (\bar{\alpha}_i\hat{\tau} + \tau_i)M + R_i.\end{aligned}$$

Note that $\Delta\bar{\alpha}_i = |\Delta\alpha_{i,i}| = \sum_{j\ne 1}^N \Delta\alpha_{i,j}$, it has

$$\lambda_{\max}\bigg(\sum_{j=1}^N \Delta\alpha_{i,j}P_j\bigg) \le \lambda_{\max}\bigg(\Delta\alpha_{i,i}P_i + \Delta\bar{\alpha}_i\sum_{j\ne i}^N P_j\bigg).$$

Since Choose a $\Delta\bar{\alpha}_i$ such that the following inequality is satisfied

$$\Delta\bar{\alpha}_i\hat{\tau}\lambda_{\max}(M) + \Delta\bar{\alpha}_i\lambda_{\max}\left(\sum_{j\neq i}^{N} P_j - P_i\right) \leq \lambda_{\min}(-\Theta_i).$$

Therefore, the upper bound of $\Delta\bar{\alpha}_i$ can be determined by

$$\Delta\bar{\alpha}_i \leq \bar{\chi}_i = \lambda_{\max}^{-1}\left(\hat{\tau}M + \sum_{j\neq i}^{N} P_j - P_i\right)\lambda_{\min}(-\Theta_i).$$

Substituting the perturbed Markov process transition generator $\mathcal{A} = (\alpha_{i,j} + \Delta\alpha_{i,j})$ into the proof in Theorem 1 yields the guaranteed cost in (20). ■

IV. Quality of Service Control

Quality of service (QoS) refers to the capability of a network to provide different communication quality to different network traffic. Therefore, it allows the adjustment of the communication network, such as IPv6. Guaranteed high communication quality, e.g. short transmission delay, requires larger bandwidth and induces high network cost. Due to the limited network resources, it is desirable to allocate the resources to each application to certain level of control performance. Inspired by the QoS concept, a conjoint control of communication network and control system is proposed.

The control structure is illustrated in Fig. 2. The remote controller (RC) adjusts the communication quality, i.e. the probability distribution of transmission delay, such that good control performance and efficient usage of network resources are achieved. In order to increase the control performance, the feedback gain of the remote controller is synchronously switched with the transmission delays [14].

The goal of the conjoint control of network and control system is to balance the guaranteed control performance versus network cost. Assume that the switching feedback gains K_i, $i \in \mathcal{S}$, are found by Theorem 1. Then, a cost function incorporating the trade-off between network resources and control performance can be formulated in the following

$$J(\mathcal{A}_{\text{QoS}}) = \lim_{T\to\infty} \mathbb{E}\left\{\bar{J}(P_{r_0}, M, \bar{\nu}_{r_0}) + \frac{1}{T}\int_0^T C(r_t)dt\right\}. \quad (21)$$

The first term in (21) refers to the expected guaranteed control performance depending on the distribution of the initial Markov process r_0. The second term refers to the network cost associated with the probability distribution of transmission delays. The remaining degree of freedom in the design procedure is the choice of transition generator $\mathcal{A}_{\text{QoS}}$ bounded by (19). The resulting optimization problem is given by

$$\min_{\mathcal{A}_{\text{QoS}}\in\mathbb{A}} J(\mathcal{A}_{\text{QoS}}), \quad (22)$$

where $\mathbb{A} = \{\nu_{i,j}, i,j \in \mathcal{S}\}$ is the set of admissible transition generators satisfying $\sum_{j=1}^{N} \nu_{i,j} = 0$ and

$$\alpha_{i,i} \geq \nu_{i,i} \geq \alpha_{i,i} + \bar{\chi}_i.$$

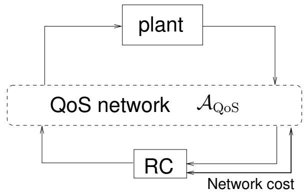

Fig. 2. Networked control systems with QoS communication network.

The perturbation $\bar{\chi}_i$ is determined by Theorem 2. The optimal transition generator $\mathcal{A}^*_{\text{QoS}}$ can be computed off-line and set for the QoS network when the control action starts.

The benefit of QoS control is studied in the following simulation example. A comparison between the QoS control approach and non-switching approach is performed with respect to control performance and network cost.

A. *Example: NCS with QoS communication network*

Consider an QoS communication network having two different traffic classes, i.e. transmission delay $\tau_1 = 80$ ms and $\tau_2 = 150$ ms. The associated network cost is given by $C_1 = 1.7$ and $C_2 = 1$, meaning higher cost for shorter transmission delay. The transition generator, i.e. $\mathcal{A} = (\alpha_{i,j})$, $i,j \in \mathcal{S} = \{1,2\}$, is considered as an QoS parameter of network and controlled by the remote controller to affect the final distribution of transmission delays.

Given an LTI plant (1) with values $A = -1.7$, $B = 0.5$, set $\gamma = 1.8$, $R_1 = R_2 = 10^{-5}$ and transition generator

$$\mathcal{A} = \begin{bmatrix} -1 & 1 \\ 1 & -1 \end{bmatrix}.$$

Solving Theorem 1 by `Penbmi` solver in Matlab, the feasible feedback gains in the closed-loop system (3) are $K_1 = -2.2$ and $K_2 = -1.1$ with the corresponding positive definite matrices

$$P_1 = 10^{-3}\begin{bmatrix} 0.213 & 0.075 \\ 0.075 & 0.134 \end{bmatrix},$$
$$P_2 = 10^{-3}\begin{bmatrix} 0.266 & 0.109 \\ 0.109 & 0.212 \end{bmatrix},$$
$$M = 10^{-3}\begin{bmatrix} 0.510 & -0.017 \\ -0.017 & 0.644 \end{bmatrix}.$$

The perturbations of transition generator, $\Delta\alpha_{1,1} = -\Delta\alpha_{1,2}$ and $\Delta\alpha_{2,2} = -\Delta\alpha_{2,1}$, is determined by (19) and have the values

$$0 \leq \Delta\alpha_{1,2} \leq 0.58, \quad 0 \leq \Delta\alpha_{2,1} \leq 4.03.$$

The optimization problem (22) is solved numerically by the `fmincon` algorithm from the Matlab optimization toolbox. With the initial condition $x(t_0) = 10$, $t_0 \in [-\tau_2,\ 0]$, the cost function (22) is minimized by the transition generator

$$\mathcal{A}^*_{\text{QoS}} = \begin{bmatrix} -1.58 & 1.58 \\ 1 & -1 \end{bmatrix}$$

with $J = 1.30$. The simulation is performed 1000 times with uniformly distributed initial probability distribution of transmission delay for a time horizon of $T = 3$ s. A sample path and probability distribution of the transmission delay is shown in Fig. 3 (a) and (b). Note that the final probability is determined by transition generator and has the value 39% for $\tau_1 = 80\,$ms and 61% for $\tau_2 = 150\,$ms. For comparison, two communication networks are investigated. In the proposed QoS communication network, the transition generator, i.e. the probability distribution of transmission delays, is designated such that the control performance and network cost are optimized. Furthermore, the remote controller is synchronously switched with the transmission delays. The benchmark communication network has constant transmission delay and the system is equipped with non-switching remote controller. The remote controller has stronger feedback gain K_1 for shorter transmission delay τ_1, and weaker feedback gain K_2 for longer transmission delay τ_2. The evolution of mean trajectory $\bar{x}(t)$ is shown in Fig. 3 (c) for comparison. The mean trajectory converges exponentially towards a ball around the origin of radius $||x(t)|| = 0.05$ after $t_{0.05} = 1.94\,$s, close to the non-switching controller with shorter transmission delay (+12%), see Table. However, the network cost is 23.6% less than non-switched with shorter transmission delay. Clearly, the trade-off between control performance and network cost is achieved.

TABLE I

Control performance and network cost.

	$t_{0.005}$ [s]	Network cost [unit]
QoS network	1.94	1.27
K_1 with delay τ_1	1.71	1.7
K_2 with delay τ_2	2.26	1

Open problems that will be addressed in the future research includes: i) the implementation of delay-dependent switched controller over QoS communication network using time-stamping technique. This requires precise synchronization between sensors and controller; ii) the optimal Markov process transition generator is solved off-line. An implementable online optimization algorithm is desirable.

V. CONCLUSIONS

Inspired by Quality-of-Service network, this paper introduces a novel control approach toward a conjoint control of communication network, i.e. the distribution probability of transmission delays, and control system. The control approach is based on Markovian jump linear system with mode-dependent delay. A sufficient condition ensuring stochastic exponential mean-square stability is derived by using Lyapunov-Krasovskii functional. Accordingly, a perturbation upper bound on Markov process transition generator is determined by the stability condition. An optimal trade-off between the control performance and network resource usage is achieved by optimal allocation of distribution probability for transmission delays. The numerical example shows superior performance benefit of the proposed control approach over the traditional control design approach.

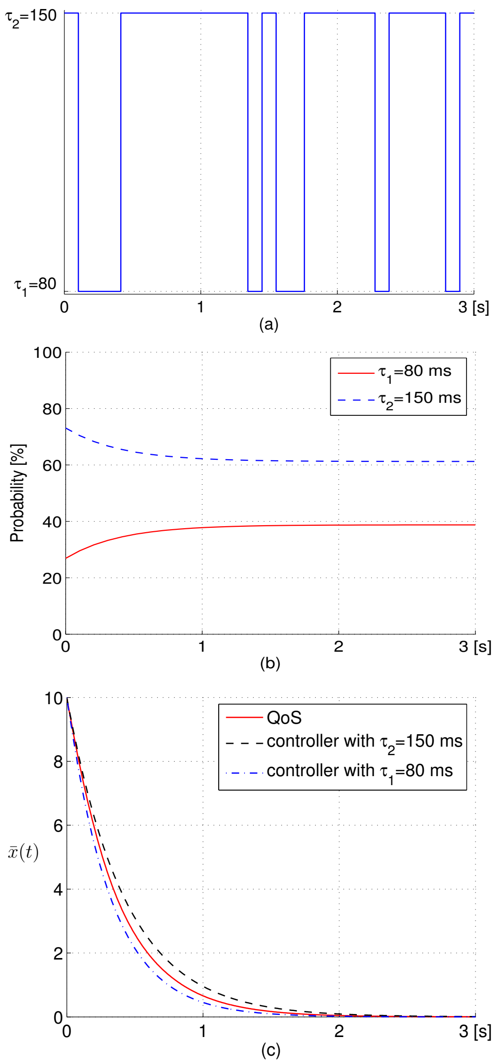

Fig. 3. A sample path of the transmission delay (a), probability distribution with initial distribution of 27% for τ_1 and 73% for τ_2 (b) and the mean state trajectory of switching controller with QoS network (solid line), non-switching controller with shorter delay network (dash-dotted line) and non-switching controller with longer delay network (dashed line) (c).

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