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***Abstract*—Optimization of transmit strategies with linear transceivers in multiple-input multiple-output (MIMO) broadcast channels generally leads to nonconvex problems, which cannot be solved efficiently in a globally optimal manner. Instead, it is necessary to resort to suboptimal algorithms. In this paper, we evaluate the application of a gradient descent algorithm for the optimization of the energy efficiency in such a system. Since the quality of the obtained locally optimal solutions depends on the initialization, a successive stream allocation is introduced and combined with the gradient algorithm. Comparison with a globally optimal reference algorithm for the special case of single receive antennas shows that the obtained solutions are close to the global optimum. For the MIMO case, the energy per bit achievable with dirty paper coding, which is a lower bound for the case of linear transceivers, is used as benchmark, and good performance of the gradient-based methods is shown for MIMO systems as well.**

I. Introduction

In the last few years, energy efficiency has become an important design criterion for wireless communication systems (e.g., [1]–[3]). Of course, an important task in this context is to develop circuits with reduced energy consumption. However, even for a given value of the energy consumption of the circuit elements (modeled as a constant in this paper), the energy efficiency can be improved by adapting the transmit power such that it fits best to the channel conditions and the circuit power (e.g., [4]). In this paper, we consider this optimization for a multiple-input multiple-output (MIMO) broadcast channel. Therefore, the following literature review concentrates on works considering energy efficiency of broadcast channels [5]–[12]. Publications on the energy efficiency of point-to-point transmission (e.g., in multicarrier settings [4], [13], [14] or multiantenna systems [15]) are not discussed in detail.

From an information theoretic point of view, the optimal transmit strategy in MIMO broadcast channels is an interference precompensation technique called dirty paper coding (DPC) [16], [17]. This technique is also needed as an ingredient for the optimal solution to the energy efficiency problem in MIMO broadcast channels, and algorithms based on this technique were proposed in [5]–[7]. However, even approximate DPC as in [18] has prohibitive complexity for online implementation since it involves vector quantization, and additional problems arise, e.g., due to imperfect channel state information at the transmitter. Also an approximate implementation with Tomlinson-Harashima precoding (THP) has drawbacks such as the shaping, power, and modulo loss (e.g., [19]). Thus, MIMO techniques with linear transceivers—i.e., the case where nonlinear operations (encoding, detection, ...) are only applied to single data streams while all operations that involve more than one data stream have to be linear (e.g., [20])—have been intensively studied by researchers and widely applied in practical systems.

In the context of energy efficiency optimization, broadcast channels with linear transceivers were considered for the special case of single-antenna receivers in [8]–[11]. The works [8], [9] concentrated mainly on the question of user selection in systems with more users than transmit antennas. In [10], [11], the combination of fairness and energy efficiency was the main topic of the investigations, which is not considered in this paper. In other works on energy efficiency, the case of imperfect channel state information was studied (e.g., [12], [21]), which is outside of the focus of our work as well.

The aim of our work is to propose a good suboptimal solution for the energy efficiency optimization with linear transceivers in broadcast channels with multiple antennas at the transmitter and receivers. In the literature on MIMO broadcast channels with linear transceivers, efficient suboptimal algorithms have been proposed, amongst others, for the sum rate maximization problem (e.g., [22]–[27]). These methods are based on alternating optimization of transmit and receive filters [22], [23], successive stream allocation and zero-forcing [24], [25], or on gradient algorithms [26], [27]. In this work, we focus on the application of gradient-based methods [26], [27] to the problem of energy efficiency optimization (Section III). Applying other methods known from sum rate maximization to the energy efficiency problem is left open as a topic for future research.

Since optimization problems in MIMO broadcast channels with linear transceivers are generally nonconvex, the above-mentioned approaches cannot be expected to always converge to the global optimum. To find good local optima, a gradient algorithm was combined with a successive stream allocation in [27]. A significant improvement due to this modification compared to a pure gradient method as in [26] was observed for the sum rate maximization in [27]. In Section IV, we introduce the combination of a gradient method and successive allocation for the energy efficiency optimization.

Since the obtained solutions are not globally optimal, numerical simulations have to be performed to evaluate how good the obtained solutions are on average and to study the influence of the successive allocation on the quality of the obtained solutions. As we need a benchmark for these simulations, we discuss how the energy efficiency problem with linear transceivers can be solved in a globally optimal manner for the special case of single-antenna receivers (Section V). Even though the proposed method based on monotonic optimization has prohibitive complexity for application in a real system, it is very helpful for judging the quality of the solutions obtained by the gradient method.

For systems with arbitrary numbers of receive antennas, we do not have such a globally optimal reference algorithm for linear transceivers. Therefore, we resort to a comparison with the globally optimal dirty paper coding solution, which provides a lower bound on the energy per bit achievable with linear transceivers. Although this bound is not tight in general, it gives an indication how good the linear solutions are.

II. System Model and Problem Formulation

We consider the downlink of a communication system that consists of a base station equipped with M antennas and K user terminals, where the kth terminal is equipped with N_k antennas. We assume frequency flat channels $\boldsymbol{H}_{k,\mathrm{DL}} \in \mathbb{C}^{N_k \times M}$ and additive circularly symmetric Gaussian noise $\boldsymbol{\eta}_k \sim \mathcal{CN}(\mathbf{0}, \boldsymbol{C}_k)$. Due to the uplink-downlink duality for MIMO broadcast channels with linear transceivers [28], we can perform the optimization in the dual uplink with channel matrices $\boldsymbol{H}_k = \boldsymbol{H}_{k,\mathrm{DL}}^{\mathrm{H}} \boldsymbol{C}_k^{-\frac{1}{2}} \in \mathbb{C}^{M \times N_k}$, where the same per-user rates can be achieved with the same sum transmit power as in the downlink. The solution can be transformed back to the downlink afterwards as described in [28].

In the dual uplink, data transmission with linear transceivers can be described by

$$\boldsymbol{y} = \sum_{k=1}^{K} \boldsymbol{H}_k \boldsymbol{T}_k \boldsymbol{x}_k + \boldsymbol{\eta} \tag{1}$$

where $\boldsymbol{T}_k \in \mathbb{C}^{N_k \times S_k}$ are beamforming matrices, $\boldsymbol{x}_k \sim \mathcal{CN}(\mathbf{0}, \mathbf{I}_{S_k})$ are circularly symmetric Gaussian data symbols, and $\boldsymbol{\eta} \sim \mathcal{CN}(\mathbf{0}, \mathbf{I}_M)$. The number of data streams of user k is given by $S_k \leq \min\{M, N_k\}$. As described in [27], the multiplexing gain might be degraded if too many streams are transmitted. Therefore, it can make sense to choose S_k smaller than the highest possible value. To account for this fact, we consider a successive stream allocation in Section IV.

The data rate of a user can be computed as a function of the beamforming matrices in the dual uplink:

$$r_k(\boldsymbol{T}_1, \ldots, \boldsymbol{T}_K) = \log_2 \det\left(\mathbf{I}_{S_k} + \boldsymbol{T}_k^{\mathrm{H}} \boldsymbol{H}_k^{\mathrm{H}} \boldsymbol{X}_k^{-1} \boldsymbol{H}_k \boldsymbol{T}_k\right) \tag{2}$$

with

$$\boldsymbol{X}_k = \mathbf{I}_M + \sum_{\ell \neq k} \boldsymbol{H}_\ell \boldsymbol{T}_\ell \boldsymbol{T}_\ell^{\mathrm{H}} \boldsymbol{H}_\ell^{\mathrm{H}}. \tag{3}$$

The sum transmit power

$$P = \sum_{k=1}^{K} \mathrm{trace}[\boldsymbol{T}_k \boldsymbol{T}_k^{\mathrm{H}}] \tag{4}$$

is a function of the beamforming matrices as well.

Our aim is to maximize the energy efficiency, which is the reciprocal value of the energy per bit. Therefore, the problem is equivalent to minimizing the energy per bit E_b, which is given by

$$E_{\mathrm{b}} = \frac{P_{\mathrm{total}} T}{RT} = \frac{P_{\mathrm{total}}}{R} = \alpha \frac{P + c}{R}. \tag{5}$$

In (5), T is the total transmission time, and $R = \sum_{k=1}^{K} r_k(\boldsymbol{T}_1, \ldots, \boldsymbol{T}_K)$ is the sum rate. The total power P_{total} is expressed as the sum of a scaled version of the transmit power αP and a constant term αc modeling the power consumed by the circuit electronics apart from the power amplifier (cf., e.g., [4]). Without loss of generality, we will assume that $\alpha = 1$. The optimization problem can be written as

$$\min_{\boldsymbol{T}_1, \ldots, \boldsymbol{T}_K} \frac{P(\boldsymbol{T}_1, \ldots, \boldsymbol{T}_K) + c}{R(\boldsymbol{T}_1, \ldots, \boldsymbol{T}_K)} \tag{6}$$

which is a nonconvex problem. Unlike the energy efficiency optimization problems in many other publications (e.g. [4], [7], [13]–[15]), this problem is not a convex-concave fractional program since the rate in (2) is not a concave function due to the use of linear transceivers.

III. Gradient Algorithm

The derivative of the sum rate R with respect to the complex conjugate of the beamforming matrix $\boldsymbol{T}_k^*$ is given by [26], [27]

$$\frac{\partial R}{\partial \boldsymbol{T}_k^*} = \frac{1}{\ln 2} \boldsymbol{H}_k^{\mathrm{H}} \left(K \boldsymbol{X}^{-1} - \sum_{\ell \neq k} \boldsymbol{X}_\ell^{-1}\right) \boldsymbol{H}_k \boldsymbol{T}_k \tag{7}$$

where $\boldsymbol{X}_\ell$ is defined as in (3), and

$$\boldsymbol{X} = \mathbf{I}_M + \sum_{k=1}^{K} \boldsymbol{H}_k \boldsymbol{T}_k \boldsymbol{T}_k^{\mathrm{H}} \boldsymbol{H}_k^{\mathrm{H}}. \tag{8}$$

Using the quotient rule, we get the conjugate of the gradient matrices of the energy per bit, which read

$$\frac{\partial E_{\mathrm{b}}}{\partial \boldsymbol{T}_k^*} = \frac{1}{R^2} \left(R \boldsymbol{T}_k - (P + c) \frac{\partial R}{\partial \boldsymbol{T}_k^*}\right). \tag{9}$$

We study the application of a gradient descent algorithm with the update rule

$$\boldsymbol{T}_k \leftarrow \boldsymbol{T}_k + d \frac{\partial E_{\mathrm{b}}}{\partial \boldsymbol{T}_k^*} \tag{10}$$

which is performed for all users in a synchronous manner. For the step size d, a simple step size adaptation is implemented: in each iteration, we start with the initial step size d_{init}, and whenever the gradient step would lead to an increased energy per bit, we repeatedly reduce the step size by a factor of two and retry the gradient step until a decrease in energy can eventually be achieved.

Unlike in [26], [27], a projection is not necessary after the gradient step since (6) is an unconstrained optimization.

Convergence of the cost function value is guaranteed since the energy per bit is bounded by the optimal value and since above step size adaptation guarantees a decrease of the energy per bit in each iteration. To detect convergence, the decrease of the energy per bit from one step to the next is tracked.

As initialization, we use truncated identity matrices of size $N_k \times S_k$ as in [27]. For the basic algorithm, we choose $S_k = \min\{M, N_k\}$. A modified algorithm, which starts with $S_k = 0 \; \forall k$ and successively increases S_k, is introduced in the following section.

IV. Successive Stream Allocation

In [27], a successive stream allocation was proposed to improve the solutions obtained using the gradient projection method for the sum rate maximization. We adopt the principle idea of this algorithm and adapt it to the energy efficiency problem.

Let $\boldsymbol{s} = [S_1, \ldots, S_K]^{\mathrm{T}}$ be the stream allocation in step i of the allocation procedure. Then, for each user k with $S_k < \min\{M, N_k\}$, a stream allocation $\boldsymbol{s}(k) = \boldsymbol{s} + \boldsymbol{e}_k$ is created, where $\boldsymbol{e}_k$ is the kth canonical unit vector, which has a one in the kth component and zeros elsewhere. For each such allocation $\boldsymbol{s}(k)$, the resulting energy per bit $E_{\mathrm{b}}(k)$ is computed by executing the gradient algorithm described in the previous section using a truncated identity matrix of size $N_\ell \times S_\ell(k)$ for user $\ell \in \{1, \ldots, K\}$ where $S_\ell(k)$ are the components of $\boldsymbol{s}(k)$. Finally, the allocation $\boldsymbol{s}_k$ that leads to the lowest energy per bit in step i is used as initial allocation $\boldsymbol{s}$ in the next allocation step $i+1$.

The allocation procedure terminates as soon as the energy per bit no longer decreases or even starts to increase. As initialization, $\boldsymbol{s} = \mathbf{0}$ is used.

V. Reference Algorithms

Since a reference implementation is necessary to judge the quality of the gradient-based solutions, we implement a globally optimal solver based on monotonic optimization for the special case of single-antenna receivers, i.e., for the multiple-input single-output (MISO) case. Monotonic optimization has already been applied to various communication systems (e.g., [29]–[31]). The drawback of approaches based on monotonic optimization is a computational complexity that is exponential in the number of optimization varibables, in our case the number of users. Therefore, such an approach is not meant to be implemented in a real system, but is only meant to serve as a reference algorithm in numerical simulations.

For the special case of single-antenna receivers, we have $S_k \leq 1 \; \forall k$, and the transmit filters become scalars t_k. The transmit strategy can be completely described by the uplink powers $p_k = |t_k|^2$, which are the squared absolute values of these scalars. The data rate of user k is then given by

$$r_k(\boldsymbol{p}) = \log_2 \left(1 + p_k \boldsymbol{h}_k^{\mathrm{H}} \left(\mathbf{I}_M + \sum_{\ell \neq k} p_\ell \boldsymbol{h}_\ell \boldsymbol{h}_\ell^{\mathrm{H}} \right)^{-1} \boldsymbol{h}_k \right) \quad (11)$$

where $\boldsymbol{p} = [p_1, \ldots, p_K]^{\mathrm{T}}$. Let $\boldsymbol{r}(\boldsymbol{p}) = [r_1(\boldsymbol{p}), \ldots, r_K(\boldsymbol{p})]^{\mathrm{T}}$, and let

$$\boldsymbol{q}(\boldsymbol{\rho}) = \begin{cases} \boldsymbol{r}^{-1}(\boldsymbol{\rho}) & \text{if } \boldsymbol{\rho} \in \mathcal{R}, \\ [\infty, \ldots, \infty]^{\mathrm{T}} & \text{otherwise} \end{cases} \quad (12)$$

where $\mathcal{R} = \{\boldsymbol{r}(\boldsymbol{p}) \mid \boldsymbol{p} \geq \mathbf{0}\}$ denotes the set of rate vectors achievable with finite sum power. According to [32], the inverse function $\boldsymbol{r}^{-1}$ exists and can be evaluated, e.g., using one of the algorithms in [33], [34]. The feasibility condition $\boldsymbol{\rho} \in \mathcal{R}$ can be checked with the feasibility test described in [35].

Using above definitions, the energy efficiency problem (6) can be rewritten as

$$\min_{\boldsymbol{\rho} \geq \mathbf{0}} \quad \log\left(\mathbf{1}^{\mathrm{T}} \boldsymbol{q}(\boldsymbol{\rho}) + c\right) - \log\left(\mathbf{1}^{\mathrm{T}} \boldsymbol{\rho}\right) \quad (13)$$

where $\mathbf{1}$ denotes the all-ones vector. It was shown in [32] that $\mathbf{1}^{\mathrm{T}} \boldsymbol{q}(\boldsymbol{\rho})$ is monotonic in each component of $\boldsymbol{\rho}$. Therefore, the objective function of (13) is a difference of monotonic functions, and the globally optimal rate vector $\boldsymbol{\rho}$ can be found using the branch-reduce-and-bound method from [36].

A similar approach was used for the problem of energy-efficient rate balancing (with fairness considerations) in our previous work [10]. An important difference is that the solution in [10] can be found in polynomial time due to the special structure of energy-efficient rate balancing in MISO broadcast channels with linear transceivers. This is not true for the corresponding energy efficiency problem without fairness constraints so that the method introduced in this section is prohibitively complex for practical implementation.

For the case of multiple antennas at the receivers, i.e., for the MIMO case, above procedure cannot be applied. However, we can use the following loose lower bound to the achievable energy per bit. The optimal energy-efficient transmit strategy in MIMO broadcast channels relies on dirty paper coding and can be computed as described in [5]–[7]. With linear transceivers, the energy per bit must always lie above this globally optimal energy per bit. However, it is not clear which part of the energy gap between the suboptimal solution and the lower bound is inherent to the restriction to linear transceivers and which part is due to the incapability of the gradient algorithm to find the best linear strategy.

VI. Numerical Results

To evaluate the quality of the solutions obtained from the gradient-based optimization methods, we present numerical simulation results that are averaged over 1000 channel realizations with i.i.d. circularly symmetric Gaussian coefficients. The noise is assumed to be white with constant power across all receive antennas, and we normalize the transmit power and the circuit power to the noise power. Thus, the noise covariances are $\boldsymbol{C}_k = \mathbf{I}_{N_k}$.

The first considered system is a MISO system with $K = 3$ users and $M = 3$ transmit antennas. For such a system, the globally optimal linear strategy can be computed as described in Section V. In Fig. 1, we compare this globally optimal solution to the basic gradient method, which is initialized with $S_k = \min\{M, N_k\}$ data streams per user, as well as to the combination of successive stream allocation and gradient descent. To this end, we have plotted the relative increase of the average energy per bit when using the gradient algorithms instead of the globally optimal linear strategy for various

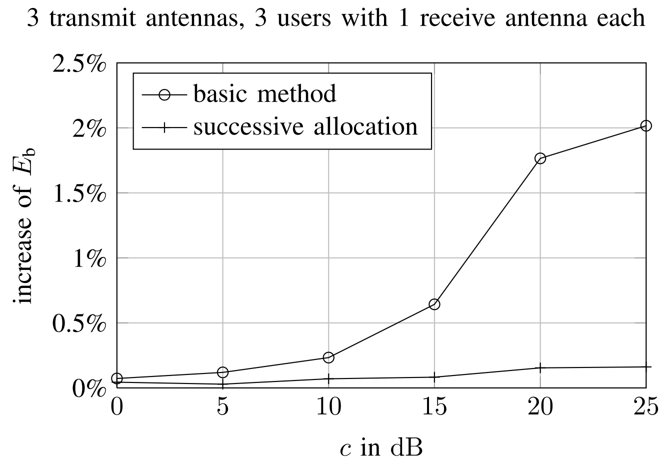

Fig. 1. Increase of the energy per bit when using the gradient-based solutions instead of the optimal linear strategy.

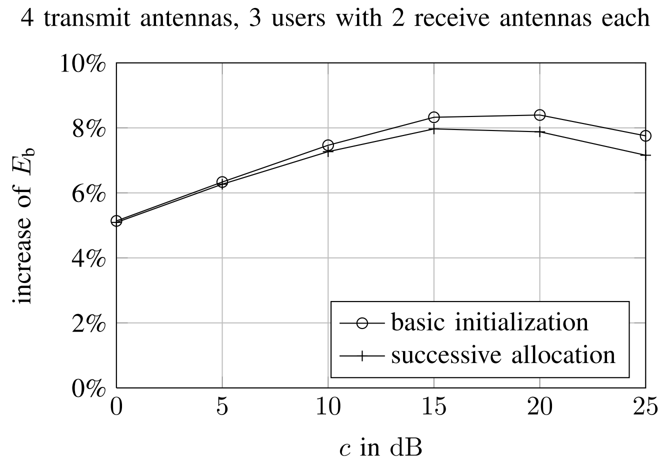

Fig. 2. Increase of the energy per bit when using the gradient-based linear solutions instead of the optimal nonlinear DPC strategy.

values of the circuit power constant c, which is normalized to the noise power and given in dB.

It can be seen that the energy increase is most notable when using the gradient method with the basic allocation in the case of a high circuit power constant. Since a high circuit power penalizes slow transmission (cf. (5) and, e.g., [9]), the optimal solution requires transmission with a high data rate in this case. This corresponds to transmission in the high SNR regime. For this regime, a significant performance loss was also observed for the case of sum rate maximization with a pure gradient method in [27]. We observe that most of the gap between the gradient scheme and the globally optimal linear solution can be closed by performing a successive stream allocation as explained in Section IV.

This improvement in energy efficiency comes at the cost of a significantly increased computational complexity since the gradient algorithm has to be executed not only once, but once for each considered stream allocation $\boldsymbol{s}(k)$ in each allocation step i. According to the discussion of the multiplexing gain degradation in [27], we expect that a decrease of the energy per bit is observed only during the first M allocation steps. Therefore, we expect no more than $(M+1)K$ executions of the gradient algorithm, i.e., the extended version has up to $(M+1)K$ times the complexity of the basic gradient method.

However, the most important observation in Fig. 1 is that both gradient methods perform very close to the optimal solution. Even for high values of the circuit power constant, the computationally efficient basic gradient method requires only 2% more energy than the globally optimal solution. Therefore, for a practical application, it might not be sensible to invest the additional computational complexity required by the successive stream allocation.

We get a similar situation in the case of multiple receive antennas, for which we have simulated two different scenarios. In Fig. 2, we have plotted simulation results for a setting with $M = 4$ transmit antennas and $K = 3$ users with $N_k = 2$ antennas each, which was already considered for the sum rate maximization in [27]. We again observe a difference between the basic gradient descent and the successive stream allocation with gradient descent especially for high values of the circuit power constant c, i.e., in the high SNR regime. However, this difference is again relatively small.

In the MIMO case, the reference value is not the globally optimal linear strategy since the algorithm from Section V is not applicable in case of multiple receive antennas. Instead, the increase in energy is given in relation to the optimal nonlinear strategy which applies dirty paper coding [5]–[7]. In the MISO case, we have observed a very small gap between the gradient method with successive allocation and the optimal linear strategy. Therefore, we conjecture that also in the MIMO case, the gradient algorithms perform close to the optimal linear solution, i.e., a large portion of the energy gap comes from the difference between linear transceivers and nonlinear DPC (and not from the suboptimality of the gradient method).

We have to keep in mind that linear transceivers as well as gradient methods can be easily implemented in practice while DPC is not appropriate for implementation in a practical system due to the computationally complex coding operation [18]. Therefore, we find it remarkable that the linear strategy obtained using the basic gradient method without successive allocation increases the energy per bit only by about 5% to 8% compared to the optimal DPC solution.

For the larger MIMO system with $M = 10$ transmit antennas, $K = 4$ users, and $N_k = 5$ receive antennas at each user terminal, which was considered for the simulation results in Fig. 3, the qualitative results are the same.

VII. Conclusion

For computationally efficient optimization of the energy efficiency of MIMO broadcast channels under the assumption of linear transceivers, only suboptimal algorithms are available. However, the numerical studies in this paper show that suboptimal solutions obtained using the gradient-descent method perform very close to the optimal linear strategy. Moreover,

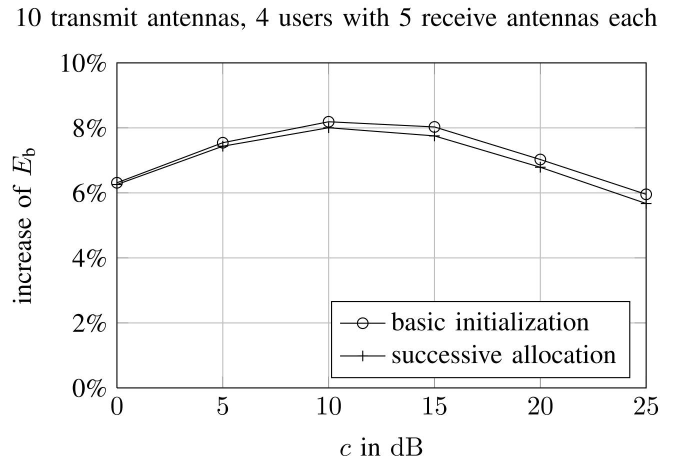

Fig. 3. Increase of the energy per bit when using the gradient-based linear solutions instead of the optimal nonlinear DPC strategy.

the size of the energy gap between the obtained solutions and the optimal nonlinear dirty paper coding is satisfactory, too, when keeping in mind the prohibitive complexity of practical implementations of dirty paper coding.

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