Controller Design and Experimental Validation for Networked Control Systems with Time-Varying Random Delay

Chih-Chung Chen, Sandra Hirche and Martin Buss

Abstract

It is well known that the stability and performance of a networked control system (NCS) are strongly affected by the transmission delay, which is usually random in communication networks such as Ethernet. In order to cope with the random transmission delay and increase the control performance, a novel switching control approach for NCS is proposed. The random transmission delay is modelled by a Markov process. A controller able to monitor the transmission delay and synchronously switches with the delay is considered. The resulting closed-loop system is a Markovian jump linear system (MJLS) with time-varying random delay. In this paper a delay-dependent stability condition for stochastic exponential mean square stability is derived by using a Lyapunov-Krasovskii functional. The controller design algorithm for a switching controller is proposed. Experiments with the 3 degree-of-freedom (DoF) robotic manipulator ViSHaRD3 validate the proposed approach. A benchmark non-switching approach with buffering strategy at the controller side is implemented for comparison. The experimental results demonstrate significant performance improvements by the proposed switching control approach.

1 INTRODUCTION

A networked control system (NCS) is a feedback control system using a shared network for the communication between spatially distributed sensors, actuators and physical plants. An NCS has advantages such as low cost, high flexibility, easy installation and maintenance, which facilitate its applications in automation technology. Typical examples include unmanned aerial vehicles [1], Ethernet-based car control network [2] and teleoperation [3].

However, the use of a communication network comes at the price of non-ideal signal transmission: the sampled data sent through the network experience variable time delays and suffer transmission losses (or packet dropouts), as discussed in [4–6]. Particularly, the delay is well known as a source of instability and deteriorates the control performance [7, 8]. Various approaches have been proposed in the literature to cope with the delay, [4, 9–12]. In [9], the augmented state vector method for constant delay is proposed. In [4], the hybrid system analysis approach is applied to NCS for known delay and in [10] for uncertain delay. Time-varying delay and robust control are addressed in [11]. In [12], a delay compensation predictive control approach is proposed for the delay with known deviations. More approaches with deterministic delays can be found in [7, 13].

Studies with random time delay are also available in [14–19]. In [14], the delay is modelled as a Markov process and the effect of random delay is treated as a linear quadratic Gaussian (LQG) problem. However, the network-induced random delay must be less than one sampling interval. Therefore, this approach may be unsuitable for systems with a longer time delay. A stochastic hybrid system approach involving bounded random delay and switching feedback control laws is discussed in [15]. This approach results in a bilinear matrix inequality (BMI). An iteration algorithm is formulated for solving the BMI difficulties. The model-based NCS with random transmission delay is studied in [16]. The sufficient conditions for almost sure stability and stochastic exponential mean square stability are identified. In [17], a H_∞ control problem for Bernoulli binary random delay is considered, resulting a linear matrix inequality (LMI) for the analysis of stochastic exponential mean square stability. A mode-dependent controller is proposed in [18] by discrete-time Markovian jump linear system (MJLS) approach. However, only the stochastic stability is guaranteed. In [19], a set of memory controllers are designed such that the resulting closed-loop MJLS is exponential mean square stable. In this approach, the cone complementarity linearization is required to solve the BMI problems.

In this paper, stochastic exponential mean square stability for longer random transmission delay with upper bound is considered. The sensor-to-controller (SC) delay $\tau_{\mathrm{sc}}(r_t)$ is modelled by a Markovian process r_t, while the controller-to-actuator (CA) delay $\bar{\tau}_{\mathrm{ca}}$ is held constant by the buffering technique. The sampled-data system approach is applied and a delay-dependent switching state-feedback controller is proposed. The resulting delay contains a random component $\tau_{\mathrm{sc}}(r_t)$ related to network transmission and a linear time-varying component, which is uncertain but bounded by a sampling interval. The switching controller monitors the SC random delay and performs control laws switching synchronously. As a result, an MJLS with time-varying random delay is established. A delay-dependent stability condition for time-varying random delay is derived by Lyapunov-Krasovskii functionals. A simple switching controller design algorithm is proposed. All the results are presented in terms of LMI's. The proposed switching controller approach is experimentally validated using a 3 DoF robotic manipulator ViSHaRD3[1] [20]. A benchmark non-switching approach with buffering strategy, which renders the SC transmission delay constant, is implemented for comparison. The experimental results demonstrate significant performance improvements by the proposed switching control approach

This paper is organized into six sections. In section 2, the sampled-data MJLS is introduced. The

[1] Virtual Scenario Haptic Rendering Device

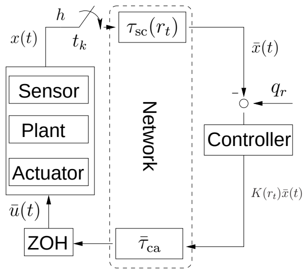

Figure 1: Illustration of NCS over communication network, the transmission delay from sensor-to-controller $\tau_{\text{sc}}(r_t)$ and from controller-to-actuator $\bar{\tau}_{\text{ca}}$.

system contains a time-varying random delay and a switching state-feedback controller. In section 3, a stochastic exponential mean square stability condition for MJLS is derived. In section 4, an LMI switching controller design algorithm is proposed. The experimental validation and performance comparison are discussed in section 5. Finally, section 6 provides a summary of this work.

Notation. In this paper $\lambda_{\max}(M)$ and $\lambda_{\min}(M)$ denote the maximal and the minimal eigenvalues of matrix M, whereas M^T and $||M||$ denote the transpose and induced Euclidean norm of matrix (or vector) M, respectively. The symbol $*$ denotes the transpose of the blocks outside the main diagonal block in sysmetric matrices. $||e(t_f)||_2$ denotes the L_2 norm of $e(t)$ on a given interval $[0\ t_f]$, where $||e(t_f)||_2 = \sqrt{\int_0^{t_f} e(t)e^T(t)dt}$. $\mathbb{E}$ stands for mathematical expectation and $\mathbf{P}$ for probability. $\{r_t, t \geq 0\}$ denotes a Markov process governing the mode switching in the finite set $\mathcal{S} := \{1, \ldots, N\}$ having the generator $\mathcal{A} = (\alpha_{i,j})$, $i, j \in \mathcal{S}$, $\alpha_{i,j} > 0$, $i \neq j$, $\alpha_{i,i} = -\sum_{i \neq j} \alpha_{i,j}$. The mode transition probability can be defined as

$$\mathbf{P}_{i,j}(r_{t+\Delta} = j | r_t = i) = \begin{cases} \alpha_{i,j}\Delta + o(\Delta^2), & i \neq j \\ 1 + \alpha_{i,i}\Delta + o(\Delta^2), & i = j, \end{cases}$$

where $\lim_{\Delta \to 0} o(\Delta^2)/\Delta = 0$.

2 Problem Statement

2.1 NCS Model

Consider an LTI system as the plant

$$\dot{x}(t) = Ax(t) + B\bar{u}(t), \tag{1}$$

where $x \in \mathbb{R}^n$ is the state and $\bar{u} \in \mathbb{R}^m$ is the control input; A and B are constant matrices with appropriate dimensions and (A, B) is controllable. The plant is interconnected by a controller over a communication network, see Fig. 1. The sensor and controller are periodically sampled with the sampling interval h.

The SC transmission delay is considered as a Markovian time delay $\tau_{\text{sc}}(r_t)$. The mode switching is governed by a Markov process $r_t \in \mathcal{S}$ taking values from the finite set $\mathcal{S} := \{1, \ldots, N\}$. The switching rate from mode i to mode j is defined by $\alpha_{i,j}$. According to (1) and Fig. 1, the piecewise constant measurement from SC at the sampled time t_k is given by

$$\begin{aligned} \bar{x}(t) &= x(t_k - \tau_{\text{sc}}(r_t)) = x(t - \tau_1(t, r_t)), \\ \tau_1(t, r_t) &= t - t_k + \tau_{\text{sc}}(r_t), \quad t_k \leq t < t_{k+1}. \end{aligned} \tag{2}$$

Assume a remote state-feedback controller able to monitor the SC delay, e.g. using the time-stamping technique, and synchronously switches the feedback gains with the SC delay $\tau_{\text{sc}}(r_t)$. The control commands are fed back through the CA channel to the plant. Holding the CA delay constant $\bar{\tau}_{\text{ca}}$ by using buffering technique, the following control law is derived

$$\bar{u}(t) = K(r_t)\bar{x}(t) = K(r_t)x(t - \tau_1(t, r_t) - \bar{\tau}_{\text{ca}}). \tag{3}$$

Substitute (3) into system (1), the closed-loop system has the form

$$\dot{x}(t) = Ax(t) + BK(r_t)x(t - \tau(t, r_t)), \tag{4}$$

where $\tau(t, r_t) = \tau_1(t, r_t) + \bar{\tau}_{\text{ca}}$. System (4) is an MJLS with time-varying random delay $\tau(t, r_t)$.

2.2 Time Delay Model

The switching of transmission delays may result in sampled sequence disorder. In this paper the disordering in the sampled sequence is excluded, i.e. with the following assumption

A1: $\mathbf{P}(|\tau_{\text{sc}}(r_{t_{k+1}}) - \tau_{\text{sc}}(r_{t_k})| \geq h) = 0$.

The assumption A1 restricts that the switching difference of consecutive delays is less than one sampling interval. This assumption can be made as the current transmission delay in the real communication networks is usually correlated to the previous delay. In single-path networks the assumption is fulfilled.

The delay $\tau(t, r_t)$ contains a random piecewise constant component $\tau_{\text{sc}}(r_t)$ generated by the transmission delay and a periodically time-varying component $t - t_k$ generated by the inter-sampling effect as shown in Fig. 2. The periodically time-varying component is bounded by a sampling interval, i.e. $t - t_k \leq h$, and has the derivative $\dot{\tau} = 1$. For stability analysis, the upper bound h of the periodically time-varying component is considered, i.e. from now on we consider

$$\tau(r_t) = h + \tau_{\text{sc}}(r_t) + \bar{\tau}_{\text{ca}}. \tag{5}$$

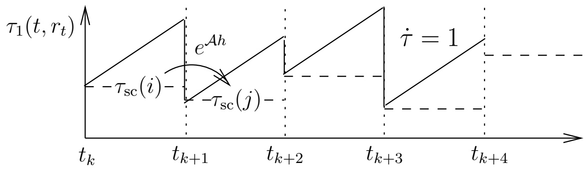

Figure 2: The evolution of time delay $\tau_1(t, r_t)$ for certain sample path of $\tau_{\rm sc}(r_t)$.

The associated upper and lower bounds of $\tau(r_t)$ are defined as

$$\bar{\tau} = h + \max_{i \in \mathcal{S}} \{\tau_{\rm sc}(i)\} + \bar{\tau}_{\rm ca}, \underline{\tau} = h + \min_{i \in \mathcal{S}} \{\tau_{\rm sc}(i)\} + \bar{\tau}_{\rm ca}.$$

Before the main result is introduced, the following definitions and lemmas have to be given.

Definition 1 *System (4) is stochastic exponential mean square stable if for any initial condition $x(t_0, r_{t_0})$, there exist positive constants b, and ρ such that for all $t \geq t_0$*

$$\mathbb{E}\big\{||x(t)||^2 | x(t_0, r_{t_0})\big\} \leq b||x(t_0, r_{t_0})||^2 e^{-\rho(t-t_0)}.$$

Definition 2 *[21] Let $\mathcal{L}$ be the weak infinitesimal operator and give a function $V(z(t), r_t)$, then the operator $\mathcal{L}$ acting on $V(z(t), r_t)$ is defined as*

$$\mathcal{L}V(z(t), r_t) = \lim_{\Delta \to 0^+} \frac{1}{\Delta} \Big\{ \mathbb{E}\{V(z(t+\Delta), r_{t+\Delta} | z(t), r_t = i)\} - V(z(t), r_t) \Big\}.$$

Lemma 1 *[22] Let X and Y be real constant matrices with appropriate dimensions. Then*

$$X^T Y + Y^T X \leq \varepsilon X^T X + \frac{1}{\varepsilon} Y^T Y$$

holds for any $\varepsilon > 0$.

Lemma 2 *Consider a function*

$$V(z(t), r_t) = \int_{-\tau(r_t)}^{0} \int_{t+\theta}^{t} z^T(s) Q z(s) ds d\theta.$$

For $r_t = i$, $\mathcal{L}V(z(t), r_t)$ has the inequality

$$\begin{aligned} \mathcal{L}V(z(t), r_t) \leq{}& \tau(i) z^t(t) Q z(t) - \int_{t-\tau(r_t)}^{t} z^T(s) Q z(s) ds \\ &+ \bar{\alpha} \int_{-\bar{\tau}}^{-\underline{\tau}} \int_{t+\theta}^{t} z^T(s) Q z(s) ds d\theta.. \end{aligned} \tag{6}$$

Proof: Since

$$\mathbb{E}\{V(z(t+\Delta t), r_{t+\Delta t} | z(t), r_t = i)\} = \mathbb{E}\Big\{ \int_{-\tau(r_{t+\Delta t})}^{0} \int_{t+\Delta+\theta}^{t+\Delta} z^T(s) Q z(s) ds d\theta \Big| z(t), r_t = i \Big\}$$

$$= \mathbb{E}\Big\{ \int_{-\tau(r_{t+\Delta t})}^{0} \int_{t+\theta}^{t} z^T(s) Q z(s) ds d\theta + \int_{-\tau(r_{t+\Delta t})}^{0} \int_{t}^{t+\Delta} z^T(s) Q z(s) ds d\theta \Big| z(t), r_t = i \Big\}$$

$$- \mathbb{E}\Big\{ \int_{-\tau(r_{t+\Delta t})}^{0} \int_{t+\theta}^{t+\Delta+\theta} z^T(s) Q z(s) ds d\theta \Big| z(t), r_t = i \Big\},$$

According to Definition 2

$$
\begin{aligned}
&\mathbb{E}\{V(z(t+\Delta t), r_{t+\Delta}|z(t), r_t = i)\} - V(z(t), t) \\
&= \mathbb{E}\Big\{ \int_{-\tau(r_{t+\Delta})}^{0} \int_{t+\theta}^{t} z^T(s)Qz(s)dsd\theta \Big| z(t), r_t = i \Big\} + \mathbb{E}\Big\{ \int_{-\tau(r_{t+\Delta})}^{0} \int_{t}^{t+\Delta} z^T(s)Qz(s)dsd\theta \Big| z(t), r_t = i \Big\} \\
&\quad - \mathbb{E}\Big\{ \int_{-\tau(r_{t+\Delta})}^{0} \int_{t+\theta}^{t+\Delta+\theta} z^T(s)Qz(s)dsd\theta \Big| z(t), r_t = i \Big\} - \int_{-\tau(r_t)}^{0} \int_{t+\theta}^{t} z^T(s)Qz(s)dsd\theta \\
&= \sum_{j=1}^{N} \mathbf{P}(r_{t+\Delta} = j | r_t = i) \int_{-\tau(j)}^{0} \int_{t+\theta}^{t} z^T(s)Qz(s)dsd\theta + \Delta\tau(i)z^T(t)Qz(t) - \Delta \int_{-\tau(i)}^{0} z^T(t+\theta)Qz(t+\theta)d\theta \\
&\quad - \int_{-\tau(i)}^{0} \int_{t+\theta}^{t} z^T(s)Qz(s)dsd\theta \\
&= \Big(1 + \alpha_{i,i}\Delta + o(\Delta^2)\Big) \int_{-\tau(i)}^{0} \int_{t+\theta}^{t} z^T(s)Qz(s)dsd\theta + \Big(\sum_{j\neq i}^{N} \alpha_{i,j}\Delta + o(\Delta^2) \Big) \int_{-\tau(j)}^{0} \int_{t+\theta}^{t} z^T(s)Qz(s)dsd\theta \\
&\quad + \Delta\tau(i)z^T(t)Qz(t) - \Delta \int_{-\tau(i)}^{0} z^T(t+\theta)Qz(t+\theta)d\theta - \int_{-\tau(i)}^{0} \int_{t+\theta}^{t} z^T(s)Qz(s)dsd\theta \\
&= \sum_{j=1}^{N} \Big(\alpha_{i,j}\Delta + o(\Delta^2)\Big) \int_{-\tau(j)}^{0} \int_{t+\theta}^{t} z^T(s)Qz(s)dsd\theta + \Delta\tau(i)z^T(t)Qz(t) - \Delta \int_{-\tau(i)}^{0} z^T(t+\theta)Qz(t+\theta)d\theta.
\end{aligned} \tag{7}
$$

Dividing (7) by Δ and $\lim_{\Delta\to 0} o(\Delta^2)/\Delta = 0$, it yields

$$
\mathcal{L}V(z(t), r_t) = \tau(i)z^t(t)Qz(t) - \int_{t-\tau(r_t)}^{t} z^T(s)Qz(s)ds + \sum_{j=1}^{N} \alpha_{i,j} \int_{-\tau(j)}^{0} \int_{t+\theta}^{t} z^T(s)Qz(s)dsd\theta. \tag{8}
$$

Since $\alpha_{i,j} > 0$, $i \neq j$ and $-\alpha_{i,i} = \sum_{i\neq j}^{N} \alpha_{i,j}$ for $i, j \in \mathcal{S}$, equation (8) can be written as

$$
\begin{aligned}
\mathcal{L}V(z(t), r_t) &= \tau(i)z^t(t)Qz(t) - \int_{t-\tau(r_t)}^{t} z^T(s)Qz(s)ds + \alpha_{i,i} \int_{-\tau(i)}^{0} \int_{t+\theta}^{t} z^T(s)Qz(s)dsd\theta \\
&\quad + \sum_{i\neq j}^{N} \alpha_{i,j} \int_{-\tau(j)}^{0} \int_{t+\theta}^{t} z^T(s)Qz(s)dsd\theta \\
&\leq \tau(i)z^t(t)Qz(t) - \int_{t-\tau(r_t)}^{t} z^T(s)Qz(s)ds + \alpha_{i,i} \int_{-\underline{\tau}}^{0} \int_{t+\theta}^{t} z^T(s)Qz(s)dsd\theta \\
&\quad + \sum_{i\neq j}^{N} \alpha_{i,j} \int_{-\bar{\tau}}^{0} \int_{t+\theta}^{t} z^T(s)Qz(s)dsd\theta \\
&= \tau(i)z^t(t)Qz(t) - \int_{t-\tau(r_t)}^{t} z^T(s)Qz(s)ds + \alpha_{i,i} \int_{-\underline{\tau}}^{0} \int_{t+\theta}^{t} z^T(s)Qz(s)dsd\theta \\
&\quad + |\alpha_{i,i}| \int_{-\bar{\tau}}^{0} \int_{t+\theta}^{t} z^T(s)Qz(s)dsd\theta.
\end{aligned}
$$

Define $\bar{\alpha} = \max_{i\in\mathcal{S}}\{|\alpha_{i,i}|\}$, it becomes

$$
\mathcal{L}V(z(t), r_t) \leq \tau(i)z^t(t)Qz(t) - \int_{t-\tau(r_t)}^{t} z^T(s)Qz(s)ds + \bar{\alpha} \int_{-\bar{\tau}}^{-\underline{\tau}} \int_{t+\theta}^{t} z^T(s)Qz(s)dsd\theta.
$$

and completes the proof. ■

3 Main Result

In this section, a delay-dependent stability condition for an NCS with time-varying random delay is presented. The approach is derived by using a Lyapunov-Krasovskii functional and descriptor transformation. Choose any $\gamma > 0$ and set a new variable $z(t) = e^{\gamma t}x(t)$ substituted into (4), it becomes

$$\dot{z}(t) = (A + \gamma I)z(t) + e^{\gamma\tau(r_t)}BK(r_t)z(t - \tau(r_t)). \tag{9}$$

Note that

$$z(t) - z(t - \tau(r_t)) = \int_{t-\tau(r_t)}^{t} \dot{z}(s)ds.$$

By substituting this into (9), $\dot{z}(t)$ becomes

$$\dot{z}(t) = \big(\hat{A} + \hat{A}_1(r_t)\big)z(t) - \hat{A}_1(r_t)\int_{t-\tau(r_t)}^{t} \dot{z}(s)ds, \tag{10}$$

where $\hat{A} = A + \gamma I$ and $\hat{A}_1(r_t) = e^{\gamma\tau(r_t)}BK(r_t)$.

Let $\xi^T(t) = [z^T(t) \quad \dot{z}^T(t)]^T$, system (10) has the descriptor form

$$E\dot{\xi}(t) = \big(\bar{A} + \bar{A}_1(r_t)\big)\xi(t) - \bar{A}_2(r_t)\int_{t-\tau(r_t)}^{t} \xi(s)ds, \tag{11}$$

where

$$E = \begin{bmatrix} I & 0 \\ 0 & 0 \end{bmatrix}, \qquad \bar{A} = \begin{bmatrix} 0 & I \\ \hat{A} & -I \end{bmatrix},$$
$$\bar{A}_1(r_t) = \begin{bmatrix} 0 & 0 \\ \hat{A}_1(r_t) & 0 \end{bmatrix}, \ \bar{A}_2(r_t) = \begin{bmatrix} 0 & 0 \\ 0 & \hat{A}_1(r_t) \end{bmatrix}.$$

Theorem 1 *For the closed-loop system (4) with a given $\gamma > 0$, if there exist matrices $Q > 0$, $W > 0$ and $X(i) = X^T(i) > 0$, $i \in \mathcal{S}$ such that the following LMI's hold*

$$\begin{bmatrix} Q & \bar{A}_2^T(i) \\ * & W \end{bmatrix} \geq 0, \tag{12}$$

$$\begin{bmatrix} \Psi_1(i) & \Psi_2(i) & \Psi_3(i) \\ * & -\hat{\tau}(i)Q^{-1} & 0 \\ * & * & -\chi(i) \end{bmatrix} < 0, \tag{13}$$

where

$$\hat{\tau}(i) = \tau(i) + \frac{1}{2}\bar{\alpha}(\bar{\tau}^2 - \underline{\tau}^2), \quad \bar{\alpha} = \max_{i\in\mathcal{S}}\{|\alpha_{i,i}|\},$$
$$\Psi_1(i) = \big(\bar{A} + \bar{A}_1(i)\big)X(i) + X^T(i)\big(\bar{A} + \bar{A}_1(i)\big)^T$$
$$+ \tau(i)W + \alpha_{i,i}EX^T(i),$$
$$\Psi_2(i) = \hat{\tau}(i)X^T(i),$$
$$\Psi_3(i) = \big[\sqrt{\alpha_{i,1}}EX^T(i)\cdots\sqrt{\alpha_{i,N}}EX^T(i)\big],$$
$$\chi(i) = diag\big\{X(1),\ldots,X(N)\big\},$$

then the system is stochastic exponential mean square stable.

Proof: The state $\{\xi(t), r_t, t \geq 0\}$ depends on the history $\xi(t+\theta)$, $\theta \in [-2\tau(r_t), 0]$, which implies $\{\xi(t), r_t, t \geq 0\}$ is not a Markov process. The problem is modified into a new Markov process $\{\Xi(t), r_t, t \geq 0\}$ in accordance to [22] with the following values

$$\Xi(t) = \xi(s+t),\ s \in \big[t - 2\tau(r_t), t\big].$$

Define a set of positive definite matrices $P(r_t) = X^{-1}(r_t)$ and consider a Lyapunov candidate as follows

$$V(\Xi(t), r_t) = V_1(\Xi(t), r_t) + V_2(\Xi(t), r_t) + V_3(\Xi(t), r_t), \tag{14}$$

where

$$V_1(\Xi(t), r_t) = \xi^T(t)EP(r_t)\xi(t),$$
$$V_2(\Xi(t), r_t) = \int_{-\tau(r_t)}^{0}\int_{t+\theta}^{t}\xi^T(s)Q\xi(s)dsd\theta,$$
$$V_3(\Xi(t), r_t) = \bar{\alpha}\int_{-\bar{\tau}}^{-\underline{\tau}}\int_{t+\theta}^{t}\xi^T(s)Q\xi(s)(s-t-\theta)dsd\theta.$$

Suppose $r_t = i \in \mathcal{S}$, then

$$\mathcal{L}V_1(\Xi(t), r_t) = \dot{\xi}^T(t)EP(r_t)\xi(t) + \xi^T(t)P^T(r_t)E\dot{\xi}(t)$$
$$= \xi^T(t)\Big[\big(\bar{A} + \bar{A}_1(r_t)\big)^T P(r_t) + P^T(r_t)\big(\bar{A} + \bar{A}_1(r_t)\big)$$
$$+ \sum_{j=1}^{N}\alpha_{i,j}EP(j)\Big]\xi(t) - 2\xi^T(t)P^T(r_t)\bar{A}_2(r_t)\int_{t-\tau(r_t)}^{t}\xi(s)ds.$$

According to Lemma 1, $\mathcal{L}V_1(\Xi(t), r_t)$ becomes

$$\mathcal{L}V_1(\Xi(t), r_t)$$
$$\leq \xi^T(t)\Big[\big(\bar{A} + \bar{A}_1(r_t)\big)^T P(r_t) + P^T(r_t)\big(\bar{A} + \bar{A}_1(r_t)\big)$$
$$+ \sum_{j=1}^{N}\alpha_{i,j}EP(j)\Big]\xi(t) + \tau(r_t)\xi^T(t)P^T(r_t)WP(r_t)\xi(t)$$
$$+ \int_{t-\tau(r_t)}^{t}\xi^T(s)\bar{A}_2(r_t)W^{-1}\bar{A}_2^T(r_t)\xi(s)ds.$$

Set

$$Q \geq \bar{A}_2^T(r_t)W^{-1}\bar{A}_2(r_t), \quad \forall r_t \in \mathcal{S}, \tag{15}$$

Then $\mathcal{L}V_1(\Xi(t), r_t)$ yields

$$\begin{aligned}
&\mathcal{L}V_1(\Xi(t), r_t) \\
&\leq \xi^T(t)\Big[\big(\bar{A} + \bar{A}_1(r_t)\big)^T P(r_t) + P^T(r_t)\big(\bar{A} + \bar{A}_1(r_t)\big) \\
&+ \sum_{j=1}^{N} \alpha_{i,j} EP(j)\Big]\xi(t) + \tau(r_t)\xi^T(t)P^T(r_t)WP(r_t)\xi(t) \\
&+ \int_{t-\tau(r_t)}^{t} \xi^T(s)Q\xi(s)ds.
\end{aligned} \tag{16}$$

According to Lemma 2,

$$\begin{aligned}
&\mathcal{L}V_2(\Xi(t), r_t) \\
&\leq \tau(r_t)\xi^T(t)Q\xi(t) - \int_{t-\tau(r_t)}^{t} \xi^T(s)Q\xi(s)ds \\
&+ \bar{\alpha}\int_{-\bar{\tau}}^{-\underline{\tau}}\int_{t+\theta}^{t} \xi^T(s)Q\xi(s)dsd\theta.
\end{aligned} \tag{17}$$

$$\begin{aligned}
\mathcal{L}V_3(\Xi(t), r_t) = &\frac{1}{2}\bar{\alpha}(\bar{\tau}^2 - \underline{\tau}^2)\xi^T(t)Q\xi(t) \\
&- \bar{\alpha}\int_{-\bar{\tau}}^{-\underline{\tau}}\int_{t+\theta}^{t} \xi^T(s)Q\xi(s)dsd\theta.
\end{aligned} \tag{18}$$

Combining (16)-(18) results in

$$\begin{aligned}
&\mathcal{L}V(\Xi(t), r_t) \\
&\leq \xi^T(t)\Big[\big(\bar{A} + \bar{A}_1(r_t)\big)^T P(r_t) + P^T(r_t)\big(\bar{A} + \bar{A}_1(r_t)\big) \\
&+ \sum_{j=1}^{N} \alpha_{i,j} EP(j)\Big]\xi(t) + \tau(r_t)\xi^T(t)P^T(r_t)WP(r_t)\xi(t) \\
&+ \Big(\tau(r_t) + \frac{1}{2}\bar{\alpha}(\bar{\tau}^2 - \underline{\tau}^2)\Big)\xi^T(t)Q\xi(t) \\
&= \xi^T(t)\Theta(r_t)\xi(t).
\end{aligned} \tag{19}$$

Pre- and post-multiply $\Theta(r_t)$ by $X^T(r_t)$ and $X(r_t)$ give

$$\begin{aligned}
&\big(\bar{A} + \bar{A}_1(r_t)\big)X(r_t) + X^T(r_t)\big(\bar{A} + \bar{A}_1(r_t)\big)^T + \tau(r_t)W \\
&+ \alpha_{i,i}X^T(i)E + \sum_{j \neq i}^{N} \alpha_{i,j}X^T(i)EX^{-1}(j)X(i) \\
&+ \hat{\tau}(r_t)X^T(r_t)QX(r_t) < 0.
\end{aligned} \tag{20}$$

Applying Schur complement to (15) and (20) results in (12) and (13).

Since $\max_{\theta\in[-2\tau,0]}\{||\xi(t+\theta)||\} \leq \varphi||\xi(t)||$ for some $\varphi > 0$ [23], the following can be established

$$\begin{aligned} V(\Xi(t), r_t) &\leq \Big[\lambda_{\max}(EP(r_t)) + \varphi\lambda_{\max}(Q)\Big]||\xi(t)||^2 \\ &\leq \Lambda_{\max}(r_t)||\xi(t)||^2, \end{aligned} \tag{21}$$

where

$$\varphi = \frac{1}{2}\bar{\tau}^2 + \frac{1}{6}(\bar{\tau}^3 - \underline{\tau}^3)\bar{\alpha}$$

$$\Lambda_{\max}(r_t) = \lambda_{\max}(EP(r_t)) + \varphi\lambda_{\max}(R).$$

Combining (19) and (21) yields

$$\frac{\mathcal{L}V(\Xi(t), r_t)}{V(\Xi(t), r_t)} \leq -\min_{r_t\in\mathcal{S}}\left\{\frac{\lambda_{\min}(-\Theta(r_t))}{\Lambda_{\max}(r_t)}\right\} \triangleq -\rho_0$$

and

$$\mathbb{E}\mathcal{L}V(\Xi(t), r_t) \leq -\rho_0\mathbb{E}V(\Xi(t), r_t). \tag{22}$$

By applying Dynkin's formula into (22) it becomes

$$\begin{aligned} &\mathbb{E}V(\Xi(t), r_t) - \mathbb{E}V(\Xi(0), r_0) \\ &= \mathbb{E}\Big[\int_0^t \mathcal{L}V(\Xi(s), r_s)ds\Big] \leq \\ &-\rho_0\int_0^t \mathbb{E}\mathcal{L}V(\Xi(s), r_s)ds. \end{aligned} \tag{23}$$

Using the Gronwall-Bellman lemma, (23) results in

$$\mathbb{E}V(\Xi, r_t) \leq e^{-\rho_0 t}\mathbb{E}V(\Xi(0), r_0).$$

Since

$$\begin{aligned} V(\Xi(t), r_t) &\geq \Big[\lambda_{\min}(EP(r_t)) + \varphi\lambda_{\min}(R)\Big]||\xi(t)||^2 \\ &= \Lambda_{\min}(r_t)||\xi(t)||^2, \end{aligned}$$

it is established that

$$\mathbb{E}||\xi(t)||^2 \leq e^{-\rho_0 t}\frac{\mathbb{E}V(\Xi(0), r_0)}{\min_{r_t\in\mathcal{S}}\{\Lambda_{\min}(r_t)\}}. \tag{24}$$

Equation (24) provides the proof for stochastic exponential mean square stability. ∎

Remark 1 *The delay $\tau(r_t)$ contains the transmission delay and uncertain time-varying component bounded by a sampling interval, see (5). Accordingly, the transmission delay as well as the sampling interval are conjointly treated by a single stability condition in Theorem 1. The solution of Theorem 1 indicates the trade-off between transmission delays $\tau_{sc}(r_t) + \bar{\tau}_{ca}$ and the sampling interval h for which the stochastic exponential mean square stability can be guaranteed by the proposed approach.*

Remark 2 *In case of constant transmission delay, i.e.* $\tau_{sc}(r_t) = \tau_{sc}$ *and* $\alpha_{i,j} = 0$*, Theorem 1 is applicable to systems with constant delay. For random SC and CA transmission delays, the extended results can be found in [24].*

Remark 3 *It is noted that* $\mathbb{E}||\xi(t)||^2 \geq \mathbb{E}||z(t)||^2$*, and* $z(t) = e^{\gamma t}x(t)$*. Therefore, the inequality (24) can be rewritten as*

$$\mathbb{E}||x(t)||^2 \leq e^{-(\rho_0+2\gamma)t}\frac{\mathbb{E}V(\Xi(0), r_0)}{\min_{r_t\in\mathcal{S}}\left\{\Lambda_{\min}(r_t)\right\}}. \tag{25}$$

As shown in (25) , the given γ *in Theorem 1 ensures the decay rate of trajectory* $\mathbb{E}||x(t)||^2$ *and determines the control performance of the closed-loop system (4).*

4 Controller Design

The difficulty in solving switching feedback gain $K(i)$ in the matrix inequality (13) involves nonlinear terms, i.e. $\bar{A}_1(i)X(i)$ in $\Psi_1(i)$ and cannot be considered as an LMI problem. However, by introducing special settings of $X(i)$ the nonlinear terms can be eliminated and the LMI problem is recovered.

Theorem 2 *For given positive scalars* $n_1(i)$*,* $n_2(i)$*,* ε *and* γ*, if there exist matrices* $W > 0$ *and* $X_{11}(i) = X_{11}^T(i) > 0$*,* $i \in \mathcal{S}$ *satisfying*

$$X(i) = \begin{bmatrix} X_{11}(i) & 0 \\ -n_1(i)X_{11}(i) & n_2(i)X_{11}(i) \end{bmatrix} \tag{26}$$

such that

$$\begin{bmatrix} \hat{\Psi}_1(i) & \hat{\Psi}_2(i) & \hat{\Psi}_3(i) \\ * & -\varepsilon\hat{\tau}(i)W & 0 \\ * & * & -\chi(i) \end{bmatrix} < 0, \tag{27}$$

where

$$\hat{\tau}(i) = \tau(i) + \frac{1}{2}\bar{\alpha}(\bar{\tau}^2 - \underline{\tau}^2), \quad \bar{\alpha} = \max\{|\alpha_{i,i}|\},$$

$$\hat{\Psi}_1(i) = \bar{A}X(i) + X^T(i)\bar{A}^T + \begin{bmatrix} 0 & e^{\gamma\tau(i)}Y^T(i)B^T \\ e^{\gamma\tau(i)}BY(i) & 0 \end{bmatrix}$$
$$+ \tau(i)W + \alpha_{i,i}EX^T(i),$$

$$\hat{\Psi}_2(i) = \varepsilon\hat{\tau}(i)\begin{bmatrix} 0 & -n_1(i)e^{\gamma\tau(i)}Y^T(i)B^T \\ 0 & n_2(i)e^{\gamma\tau(i)}Y^T(i)B^T \end{bmatrix},$$

$$\hat{\Psi}_3(i) = \left[\sqrt{\alpha_{i,1}}EX^T(i)\cdots\sqrt{\alpha_{i,N}}EX^T(i)\right],$$

$$\chi(i) = diag\left\{X(1), \ldots, X(N)\right\}$$

holds, the closed-loop system (4) is stochastic exponential mean square stable with the feedback gain

$$K(i) = Y(i)X_{11}^{-1}(i). \tag{28}$$

Proof: According to Theorem 1, the switching controller (3) stabilizes the closed-loop system (4) if the inequalities (15) and (20) are satisfied. Choose a $\varepsilon > 0$ and let $Q = \varepsilon \bar{A}_2^T(r_t)W^{-1}A_2(r_t)$, (15) becomes

$$\varepsilon \bar{A}_2^T(r_t)W^{-1}A_2(r_t) \geq \bar{A}_2^T(r_t)W^{-1}A_2(r_t). \tag{29}$$

Subsitute (29) and (26) into (20) and let $Y(i) = K(i)X_{11}(i)$. The nonlinear terms in (13) are eliminated and the LMI (27) is derived. ■

Remark 4 *The restrictions on matrix $X(i)$ in (26) introduce certain conservatism in the controller design. However, the conservatism seems to be acceptable in the experimental validation as convincingly demonstrated in the subsequent section on experimental validation..*

5 Experimental Validation

In order to validate the proposed switching control approach, experiments of the position control for a 3 DoF robotic manipulator ViSHaRD3 [20] are conducted. The device is equipped with a fixed end-effector and three revolute joints as shown in Fig. 3. Each joint is actuated by a Maxon RE40 DC motor coupled with a harmonic drive gear (gear ratio 1:100). The DC-motor torque is modulated by the PWM amplifier operated under current control. The reference signal is given by voltage from a D/A converter and is an output of the I/O board. The ViSHaRD3 device is connected to a PC with real-time Linux. The distributions of transmission delay are generated randomly by a known probability transition rate and saved in a look-up table. According to the look-up table, the switching of controller is performed. The control loop and the communication network, i.e. the transmission delay, are implemented in MATLAB/SIMULINK blocksets. Standalone real-time code is generated directly from the SIMULINK models.

5.1 Experimental System Model

Due to the requirement of the proposed approach, the ViSHaRD3 device is linearized by computed torque feedforward approach [25]. Combined with friction compensation, the linearized ViSHaRD3 system is decoupled into three systems

$$\frac{d}{dt}\begin{bmatrix} q_i \\ \dot{q}_i \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 1 & -50 \end{bmatrix} \begin{bmatrix} q_i \\ \dot{q}_i \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u_i \tag{30}$$

$i = 1, 2$ for joint 1, 2

$$\frac{d}{dt}\begin{bmatrix} q_3 \\ \dot{q}_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 1 & -40 \end{bmatrix} \begin{bmatrix} q_3 \\ \dot{q}_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u_3 \tag{31}$$

for joint 3, where $q = [q_1,\ q_2,\ q_3]^T$.

The joint vector q of ViSHaRD3 is fed to the remote controller through a communication network having the SC delay $\tau_{\rm sc}(r_t) \in \{6, 8, 12\}$ ms and the transition rate

$$\mathcal{A} = \begin{bmatrix} -3 & 2 & 1 \\ 1 & -3 & 2 \\ 3 & 1 & -4 \end{bmatrix}.$$

The ViSHaRD3 system is stabilized by a set of PD controllers, which are synchronously switched with the SC delay. Combine the switching PD controller into (30) and (31), it yields

$$\dot{q}(t) = A_i q(t) + \bar{K}(r_t) q(t - \tau(r_t)), \tag{32}$$

where $i = 1, 2, 3$ and

$$A_1 = A_2 = \begin{bmatrix} 0 & 1 \\ 1 & -50 \end{bmatrix},\ A_3 = \begin{bmatrix} 0 & 1 \\ 1 & -40 \end{bmatrix},$$
$$\bar{K} = \begin{bmatrix} 0 & 0 \\ -K_P(r_t) & -K_D(r_t) \end{bmatrix}.$$

The PD gains in (32) are computed by (27) using the `Yalmip toolbox` [26] in MATLAB. With the sampling interval $h = 5$ ms, CA transmission delay $\bar{\tau}_{\rm ca} = 1$ ms, the resulting delay in (5) has the values $\tau(1) = 12$ ms, $\tau(2) = 14$ ms, $\tau(3) = 18$ ms. The LMI (27) in Theorem 2 is solved for the decay rate of $\gamma = 2.5$, by using brute-force search of $n_1(i),\ n_2(i)$, where $i \in \mathcal{S} := \{1, 2, 3\}$. The feasible PD gains are computed with $n_1(1) = n_2(1) = 1.6 \times 10^3$, $n_1(2) = n_2(2) = 3.6 \times 10^3$ and $n_1(3) = n_2(3) = 1.7 \times 10^6$ and summarized in Table I.

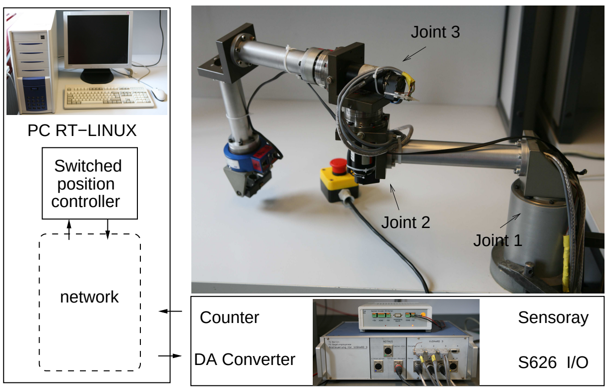

Figure 3: Experimental 3 DoF ViSHaRD3 system.

Table 1:
The feasible switching PD controller for ViSHaRD3 device

	Joint 1/2	Joint 3
$\tau(1) = 12\,\text{ms}$	$K_P(1) = 259.65$	$K_P(1) = 208.95$
$\tau_{\text{sc}}(1) = 1\,\text{ms}$	$K_D(1) = 5.19$	$K_D(1) = 5.21$
$\tau(2) = 14\,\text{ms}$	$K_P(2) = 96.54$	$K_P(2) = 77.39$
$\tau_{\text{sc}}(2) = 3\,\text{ms}$	$K_D(2) = 1.93$	$K_D(2) = 1.93$
$\tau(3) = 18\,\text{ms}$	$K_P(3) = 77.21$	$K_P(3) = 23.26$
$\tau_{\text{sc}}(3) = 7\,\text{ms}$	$K_D(3) = 1.55$	$K_D(3) = 1.21$

5.2 Experimental Results

The initial joint vector of ViSHaRD3 is set to $q^T(t_0) = [0\ 0\ \ -0.5\pi]$ rad and $\dot{q}^T(t_0) = [0\ 0\ 0]$ rad/s. A sinusoidal function, which has the amplitude 0.2 and frequency 0.5 rad/s, serves as position reference q_r to the system. The experiments are run 10 times with random initial distribution probabilities of SC delay. A sample path of the SC delay is shown in Fig 4 (a). Two approaches are investigated. In the proposed switching control approach, the delay is monitored using the time-stamping technique and the remote controller is synchronously switched with the SC delay. The second approach holds the SC delay constant by using the buffering technique at the controller side, i.e. the controller is designed with the higher delay $\tau_{\text{sc}}(3)$. The evolutions of normalized mean control error[2] $\bar{e}(t)$ are shown in Fig. 4 (b) for comparison. It is observed that the normalized mean control errors of the proposed approach (solid line) are smaller than the non-switching approach (dashed line). The L_2 norm of normalized mean control error over the experimental time horizon $[0\ t_f]$, $t_f = 10\,\text{s}$, is measured to be $||\bar{e}(t_f)||_2 = 1.42$ for the proposed switching control approach and $||\bar{e}(t_f)||_2 = 3.92$ for non-switching approach. The switching control approach has superior performance benefits over the non-switching counterpart even when the total switching difference of delay is small, as in this case of only 6 ms. In case of larger switching delay differences, the performance benefit is likely to be more obvious. The experimental results show that the controller design algorithm in Theorem 2 enables a good control performance even with sufficient stability condition and restrictions on $X(i)$.

Open issues remain to be addressed in future research concerning control performance optimization and packet dropout.

[2] The normalized control error is defined as $\bar{e}(t) = \frac{q(t)-q_r(t-\tau(r_t))}{\max\{||q_r(t)||\}}$.

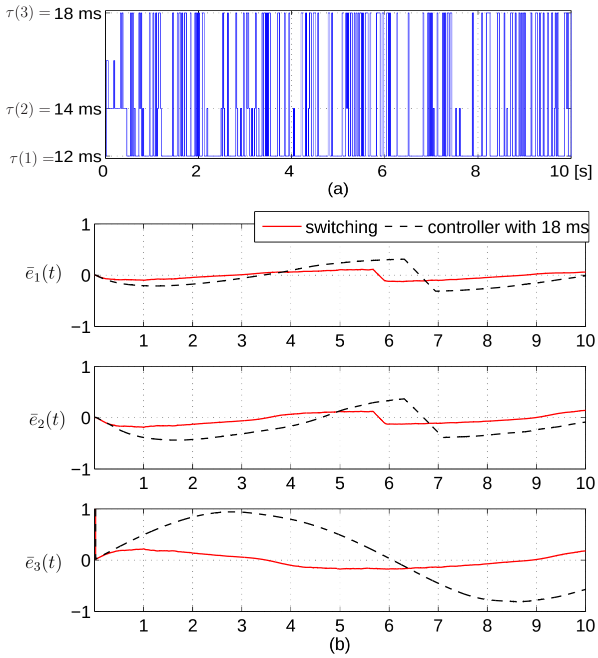

Figure 4: The sample path of Markovian delay for the experiment (a) and mean control error evolutions with switching controller (solid line) and non-switching controller (dashed line) (b)

6 Conclusions

This paper introduces a novel control approach for networked control systems (NCS). The control approach is based on Markovian jump linear systems with time-varying random delay. It gives a sufficient stability condition and controller design algorithm in terms of linear matrix inequalities (LMI's). Stochastic exponential mean square stability is guaranteed for longer random transmission delay by using the Lyapunov-Krasovskii approach. A switching state-feedback controller is proposed and validated by a 3 DoF robotic manipulator ViSHaRD3. The experimental results demonstrate superior performance benefits of the proposed switching controller over the non-switching counterpart.

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References

[1] P. Seiler, *Coordinated control of unmanned aerial vehicles*. PhD thesis, Univ. California, Berkeley, 2001.

[2] R. Daoud, H. Amer, H. Elsayed, and Y. Sallez, "Ethernet-based car control network," in *IEEE CCECE/CCGEI*, (Ottawa), 2006.

[3] S. Hirche, *Haptic Telepresence in Packet Switched Communication Networks*. PhD thesis, Technische Universität München, Institute of automatic control engineering, 2005.

[4] W. Zhang, M. S. Branicky, and S. M. Philips, "Stability of network control systems," *IEEE Control Systems Magazine*, vol. 21, pp. 84–99, February 2001.

[5] J. Baillieul and P. Antsaklis, "Control and communication challenges in networked real-time systems," *Proceedings of the IEEE*, vol. 95, pp. 9–28, Jan. 2007.

[6] J. Hespanha, P. Naghshtabrizi, and Y. Xu, "A survey of recent results in networked control systems," *Proc. of IEEE Special Issue on Technology of Networked Control Systems*, vol. 95, pp. 137–162, Jan. 2007.

[7] J.-P. Richard, "Time-delay systems: an overview of some recent advances and open problems," *Automatica*, vol. 39, pp. 1667–1694, 2003.

[8] K. Gu, V. L. Kharitonov, and J. Chen, *Stability of Time-Delay Systems*. Boston: Birkhäuser, 2003.

[9] A. Ray and Y. Galevi, "Integrated communication and control systems: Part II-design considerations," *Journal of Dynamic Systems, Measurements, and Control*, vol. 110, pp. 374–381, 1988.

[10] H. Lin, G. Zhai, and P. J. Antsaklis, "Robust stability and disturbance attenuation analysis of a class of networked control systems," in *42nd IEEE Conference on Decision and Control*, (Maui), pp. 1182–1187, 2003.

[11] E. Fridman, A. Seuret, and J.-P. Richard, "Robust sampled-data stabilization of linear systems: an input delay approach," *Automatica*, vol. 40, pp. 1441–1446, 2004.

[12] E. Witrant, C. C. de Wit, D. Georges, and M. Alamir, "Remote stabilization via communication networks with a distributed control law," *IEEE Transactions on Automatic Control*, vol. 52, no. 8, pp. 1480–1485, 2007.

[13] V. L. Kharitonov, "Robust stability analysis of time delay systems: A survey," *Annual Reviews in Control*, vol. 23, pp. 185–196, 1999.

[14] J. Nilsson, *Real-time control systems with delay.* PhD thesis, Lund Institute of Technology, 1998.

[15] L. Xiao, A. Hassibi, and J. P. How, "Control with random communication delays via a discrete-time jump system approach," in *Proceedings of the American Control Conference*, (Chicago, Illinois), June 2000.

[16] L. Montestruque and P. Antsaklis, "Stability of model-based networked control systems with time-varying transmission times," *IEEE Transactions on Automatic Control*, vol. 49, no. 9, pp. 1562–1572, 2004.

[17] F. Yang, Z. Wang, Y. S. Hung, and M. Gani, "H_∞ control for networked control systems with random communication delays," *IEEE Transactions on Automatics Control*, vol. 51, pp. 511–518, Mar. 2006.

[18] L. Zhang, Y. Shi, T. Chen, and B. Huang, "A new method for stabilization of networked control systems with random delays," *IEEE Transactions on Automatic Control*, vol. 20, no. 8, pp. 1177–1181, 2005.

[19] M. Sun, J. Lam, S. Xu, and Y. Zou, "Robust exponential stabilization for Markovian jump systems with mode-dependent input delay," *Automatica*, vol. 43, pp. 1799–1807, 2007.

[20] M. Ueberle, *Design, Control, and Evaluation of a Family of Kinematic Haptic Interfaces.* PhD thesis, Technische Universität München, Institute of automatic control engineering, 2006.

[21] X. Mao, "Exponential stability of stochastic delay interval systems with Markovian switching," *IEEE Transactions on Automatic Control*, vol. 47, no. 10, pp. 1604–1612, 2002.

[22] E.-K. Boukas and Z.-K. Liu, *Deterministic and Stochastic Time Delay Systems.* Boston: Birkhäuser, 2002.

[23] M. S. Mahmoud and N. F. Al-Muthairi, "Design of robust controller for time-delay systems," *IEEE Transactions on Automatic Control*, vol. 39, pp. 995–999, 1984.

[24] C.-C. Chen, S. Hirche, and M. Buss, "Sampled-data networked control systems with random time delay," in *IFAC 2008*, (Seoul), July 2008.

[25] L. Sciavicco and B. Siciliano, *Modelling and Control of Robot Manipulators.* London: Springer, 2 ed., 2001.

[26] J. Löfberg, "Yalmip : A toolbox for modeling and optimization in MATLAB," in *Proceedings of the CACSD Conference*, (Taipei, Taiwan), 2004.