Joint Space-Time Interference Mitigation for Embedded Multi-Antenna GNSS Receivers

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BIOGRAPHY

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Emrah Taşdemir received his Dipl.-Ing. degree from RWTH Aachen University in 2009. Since then he has been working as a research associate at the Chair of Electrical Engineering and Computer Systems at RWTH Aachen University. His fields of research are multipath and interference mitigation techniques and energy- and area-efficient solutions for satellite navigation system receivers.

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Josef A. Nossek (IEEE S´ 72-M´ 74-SM´ 81-F´ 93) received the Dipl.-Ing. and Dr. techn. degrees in electrical engineering from Vienna University of Technology, Vienna, Austria, in 1974 and 1980, respectively. He joined Siemens AG, Munich, Germany, in 1974, where he was engaged in the design of both passive and active filters for communication systems. From 1987 to 1989, he was the Head of the Radio Systems Design Department, where he was instrumental in introducing high-speed VLSI signal processing into digital microwave radio. Since April

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1989, he has been a Professor of circuit theory and design with the Technische Universität München (TUM), Munich, Germany. Josef A. Nossek was the President Elect, President, and Past President of the IEEE Circuits and Systems Society in 2001, 2002, and 2003, respectively. He was the President of Verband der Elektrotechnik, Elektronik und Informationstechnik e.V. (VDE) in 2007 and 2008. He was the recipient of the ITG Best Paper Award in 1988, the Mannesmann Mobilfunk (currently Vodafone) Innovations Award in 1998, and the Award for Excellence in Teaching from the Bavarian Ministry for Science, Research and Art in 1998. From the IEEE Circuits and Systems Society, he received the Golden Jubilee Medal for "Outstanding Contributions to the Society" in 1999 and the Education Award in 2008. He was the recipient of the "Bundesverdienstkreuz am Bande" in 2008. In 2011 he received the IEEE Guillemin-Cauer Best Paper Award for his paper "Toward a Circuit Theory of Communication".

INTRODUCTION

Accurate synchronization and positioning in space and time is a key enabler for a wide variety of technical applications ranging from remote farming, automatic docking and landing to network synchronization and location-based services. Today, radio-based positioning systems like Global Satellite Navigation Systems (GNSS) offer the possibility to perform precise positioning by means of range measurements to a set of satellites. While long spreading codes are employed in order to attain a high spreading gain protecting against thermal noise, as well as intra- and intersystem interference, GNSS is inherently vulnerable to terrestrial *radio frequency interference* (RFI) due to the low receive power of the desired signal. Such interference might arise from wide-band communication systems operating in neighboring frequency bands or from jamming devices which are intentionally perturbing the GNSS signal acquisition and tracking. Due to the large distances between the satellites and GNSS receivers, the transmit signals of the satellites are strongly attenuated and hence, an interferer with relatively low transmit power suffices in order to drown the GNSS receivers in interference and hinder its operation. Therefore, one of the major nuisances for precise positioning with conventional GNSS receivers is RFI.

Typical approaches for RF interference mitigation exploit the signal structure of the interferer. For example *Frequency Domain Adaptive Filtering* (FDAF) methods are based on frequency-domain analysis for interference detection. FDAF can be applied effectively for narrow-band interference since this kind of interference signal can easily be identified by means of frequency spectrum analysis [1]. However, broad-band signals like strong, band-limited noise can hardly be identified since correlation in time-/frequency-domain is generally not evident. In order to counteract this kind of interference, filtering in spatial domain provides a powerful alternative. Using receivers with multiple contiguous antenna elements allows detection of spatially correlated signals. A single interferer is always concentrated in the spatial domain and can be detected based on estimates of the spatial correlation matrix. In [2], an effective method has been demonstrated for suppression of strong, broad-band interference signals. However, this approach does not consider temporal correlation of the interference signal. As shown in this paper, exclusively using the spatial correlation matrix in presence of temporally correlated interference leads to suboptimal filtering due to imperfect characterization of the receive signal model. The fact that with a reasonably small number of antenna elements, spatial filtering also mitigates signals of interest in a wide spatial area motivates a joint estimation of the spatio-temporal correlation matrix. Practically speaking, a narrowband interferer should be mitigated in the time-frequency domain whereas wideband interferers should preferably be cancelled in spatial domain.

In this work we develop an efficient method to jointly estimate the space-time correlation matrix of interference and noise with a multiple antenna receiver under the assumption that the space-time correlation matrix of the interference follows a Kronecker structure. Aided by the fact that *maximum likelihood* (ML) estimation of the signal parameters, as the time delay and angle of arrival, depend on the spatial and temporal correlation matrix, we show that optimum performance for signal parameter estimation is possible when space-time interference mitigation is based on correct estimates of the spatial and temporal correlation matrices of the interference and noise. In addition, we show the performance degradation for the case when mismatched filtering is performed based on the incorrect assumption of spatially and temporally white interference.

Notation

In this paper we define scalars, column vectors and matrices with lower case letters, lower case bold letters and upper case bold letters, respectively. The transposition, Hermitian (complex conjugation and transposition) and complex conjugation of a matrix $\mathbf{G}$ is denoted by $\mathbf{G}^{\mathrm{T}}$, $\mathbf{G}^{\mathrm{H}}$ and $\mathbf{G}^{*}$, respectively. The determinant of matrix $\mathbf{G}$ is given by $|\mathbf{G}|$. With $\mathbf{G} \in \mathbb{C}^{m\times n}$, the operator $\mathrm{vec}(\mathbf{G})$ returns the mn-dimensional vector resulting from stacking the columns of $\mathbf{G}$. The $m \times m$ identity matrix is given by $\mathbf{1}_m$. Given a vector $\mathbf{h} \in \mathbb{C}^m$, diag $(\mathbf{h})$ returns an $m \times m$ diagonal matrix with the elements of $\mathbf{h}$ on the diagonal. The Euclidean norm of the vector $\mathbf{h}$ is defined as $\|\mathbf{h}\|_2^2$. The operator $\otimes$ denotes the Kronecker product such that for two matrices $\mathbf{G}$ and $\mathbf{H}$ where $\mathbf{G} \in \mathbb{C}^{m\times n}$, we have

$$\mathbf{G}\otimes\mathbf{H} = \begin{bmatrix} G_{1,1}\mathbf{H} & \cdots & G_{1,n}\mathbf{H} \\ \vdots & \ldots & \vdots \\ G_{m,1}\mathbf{H} & \cdots & G_{m,n}\mathbf{H} \end{bmatrix},$$

with $G_{i,j}$ as the i,j-th element of the matrix $\mathbf{G}$.

Outline

This work is organized as follows. First we present the general system model. Afterwards we analyze the interference which is spatially and temporally correlated. We then present the maximum likelihood estimation of the signal parameters, which depend on the spatial and temporal correlation matrix of the interference. Thereafter, we discuss the estimation of the spatial and temporal correlation matrix. Afterwards, we provide simulation results, which show the gain in the accuracy of the time delay estimation which is achieved by considering the correct spatial and temporal correlation matrices of the interference. We conclude the paper with some closing remarks.

SYSTEM MODEL

We assume a GNSS receiver equipped with a *uniform linear antenna* (ULA) array consisting of M isotropic radiators, separated by half the signal wavelength, such that receive/transmit array gain is independent of the beamforming direction [3]. The signal of a single satellite is impinging on the array with an angle θ_0 in the presence of $L \leq M$ interferers, which impact the sensor elements from directions θ_ℓ with $\ell = 1, \ldots, L$. For simplicity, all multipath effects are neglected. The complex baseband Doppler-compensated receive signal at the antenna array at time t and with bandwidth B can therefore be described by

$$\mathbf{y}(t) = \gamma_0\, \mathbf{a}(\theta_0)\, c(t - \tau_0) + \sum_{\ell=1}^{L} \mathbf{a}(\theta_\ell) b_\ell(t) + \mathbf{n}(t) \in \mathbb{C}^M, \tag{1}$$

where

$$\mathbf{a}(\theta) = \begin{bmatrix} 1 \\ \mathrm{e}^{-\mathrm{j}\pi \sin\theta} \\ \vdots \\ \mathrm{e}^{-\mathrm{j}(M-1)\pi \sin\theta} \end{bmatrix} \in \mathbb{C}^M, \tag{2}$$

is the array steering vector which depends on the angle of arrival θ. In addition, $\gamma_0 \in \mathbb{C}$ and $\tau_0 \in \mathbb{R}$ are the complex amplitude and delay of the receive satellite signal, respectively. Furthermore, $c(t)$ is the satellite signal (which is known at the GNSS receiver), $b_\ell(t) \in \mathbb{C}$ is the signal of the ℓ-th interferer (with unknown temporal structure and bandwidth $B_\ell < B$) and $\mathbf{n}(t) \in \mathbb{C}^M$ is the zero-mean *additive white Gaussian noise* (AWGN) at the M elements of the antenna array with

$$\mathrm{E}\left[\mathbf{n}(t)\mathbf{n}^{\mathrm{H}}(t)\right] = \sigma_{\mathrm{n}}^2 \mathbf{1}_M, \tag{3}$$

where σ_{n}^2 is the variance of the noise. Without loss of generality, we assume the variance of the signal $c(t)$ to be 1, such that the SNR of the satellite signal is given by

$$\mathrm{SNR} = \frac{|\gamma_0|^2}{\sigma_{\mathrm{n}}^2}. \tag{4}$$

Furthermore, we denote the variance of the zero-mean interferer as σ_ℓ^2, such that we define the *interference to noise ratio* (INR) of the ℓ-th interferer as

$$\mathrm{INR}_\ell = \frac{\sigma_\ell^2}{\sigma_{\mathrm{n}}^2}. \tag{5}$$

The receive signal is sampled with a sampling period $T_{\mathrm{s}} = \frac{1}{2B}$. The number of samples in each block is defined as N. The channel parameters are assumed to be constant for K blocks. We denote the n-th receive sample of the k-th block as $\mathbf{y}_{k,n}$, such that

$$\mathbf{y}_{k,n} = \mathbf{y}\left(((k-1)N + n)T_{\mathrm{s}}\right), \tag{6}$$

where $n = 1, \ldots, N$, $k = 1, \ldots, K$ and $\mathbf{y}(t)$ is given in (1).

We collect the N samples over the M antennas of the k-th block in the matrix $\mathbf{Y}_k \in \mathbb{C}^{M \times N}$ and thus, the *sampled* receive signal at the antenna array of the GNSS receiver for the k-th block period is denoted by

$$\begin{aligned} \mathbf{Y}_k &= \begin{bmatrix} \mathbf{y}_{k,1} & \cdots & \mathbf{y}_{k,N} \end{bmatrix} \\ &= \gamma_0\, \mathbf{a}(\theta_0)\, \mathbf{c}_k^{\mathrm{T}}(\tau_0) + \mathbf{A}\mathbf{B}_k + \mathbf{N}_k, \end{aligned} \tag{7}$$

where

$$\mathbf{A} = \begin{bmatrix} \mathbf{a}(\theta_1) & \cdots & \mathbf{a}(\theta_L) \end{bmatrix} \in \mathbb{C}^{M \times L} \tag{8}$$

$$\mathbf{B}_k = \begin{bmatrix} \mathbf{b}_{k,1}^{\mathrm{T}} \\ \vdots \\ \mathbf{b}_{k,L}^{\mathrm{T}} \end{bmatrix} \in \mathbb{C}^{L \times N} \tag{9}$$

$$\mathbf{N}_k = \begin{bmatrix} \mathbf{n}_{k,1} & \cdots & \mathbf{n}_{k,N} \end{bmatrix} \in \mathbb{C}^{M \times N} \tag{10}$$

with the N-dimensional vectors

$$\mathbf{c}_k(\tau_0) = \left[c(((k-1)N+1)T_{\mathrm{s}} - \tau_0) \;\cdots\; c(kNT_{\mathrm{s}} - \tau_0)\right]^{\mathrm{T}} \tag{11}$$

$$\mathbf{b}_{k,\ell} = \begin{bmatrix} b_\ell(((k-1)N+1)T_{\mathrm{s}}) & \cdots & b_\ell(kNT_{\mathrm{s}}) \end{bmatrix}^{\mathrm{T}} \tag{12}$$

$$\mathbf{n}_{k,i} = \mathbf{n}(((k-1)N+i)T_{\mathrm{s}}) \qquad \text{for} \quad i = 1, \ldots, N. \tag{13}$$

With the previous assumptions, i.e. with B as the bandwidth of the receive signal and $T_{\mathrm{s}} = \frac{1}{2B}$ as the sampling period, the AWGN samples are *spatially* and *temporally* white, i.e.

$$\mathrm{E}\left[\mathbf{n}_{k,i}\mathbf{n}_{k,i}^{\mathrm{H}}\right] = \sigma_{\mathrm{n}}^2 \mathbf{1}_M \tag{14}$$

$$\mathrm{E}\left[\mathbf{n}_{k,i}\mathbf{n}_{k,j}^{\mathrm{H}}\right] = \mathbf{0}, \tag{15}$$

for $i \neq j$.

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CORRELATED INTERFERENCE

Temporal Correlation

For block k, the interference component in (7) is given by $\mathbf{AB}_k \in \mathbb{C}^{M\times N}$. We model the L interference signals $b_{k,\ell}$ as independent Gaussian random processes

$$\mathrm{E}\left[\mathbf{b}_{k,i}\mathbf{b}_{k,j}^{\mathrm{H}}\right] = \mathbf{0}, \quad \text{for} \quad i \neq j, \tag{16}$$

with zero mean

$$\mathrm{E}\left[\mathbf{b}_{k,\ell}\right] = \mathbf{0}, \tag{17}$$

and correlation matrix

$$\mathrm{E}\left[\mathbf{b}_{k,\ell}\mathbf{b}_{k,\ell}^{\mathrm{H}}\right] = \sigma_\ell^2\,\mathbf{R}_{\mathrm{T}}, \tag{18}$$

for $\ell = 1,\ldots,L$ and where σ_ℓ^2 is the variance of the ℓ-th interfering signal. Although each ℓ-th interfering signal has a distinct variance σ_ℓ^2, i.e. the L interferers share a common temporal structure, i.e. the temporal correlation matrix $\mathbf{R}_{\mathrm{T}}$. This assumption is justified under a scenario where the different sources of interference are transmitters from the same interfering system. The different variances σ_ℓ^2 for $\ell = 1,\ldots,L$ capture the different receive signal power of the interferers due to differences in transmit power and path loss, i.e. distance from the interfering sources and the GNSS receiver.

Furthermore, a similar interference component as $\mathbf{AB}_k$ which fulfills the Kronecker model (29) can also result from a single interferer with multipath. Although, in this case the L multipath signals are correlated, spatial smoothing can be applied to decorrelate the signals [4] and obtain a spatial correlation matrix of full rank, i.e. L.

Let us assume the rank of the temporal correlation matrix $\mathbf{R}_{\mathrm{T}} \in \mathbb{C}^{N\times N}$ to be p. With the eigenvalue decomposition of the temporal correlation matrix $\mathbf{R}_{\mathrm{T}}$ given by

$$\mathbf{R}_{\mathrm{T}} = \mathbf{V}\mathbf{\Lambda}_{\mathrm{T}}\mathbf{V}^{\mathrm{H}}, \tag{19}$$

where $\mathbf{V} \in \mathbb{C}^{N\times p}$ contains the p eigenvectors which are orthonormal, i.e.

$$\mathbf{V}^{\mathrm{H}}\mathbf{V} = \mathbf{1}_p. \tag{20}$$

In addition,

$$\mathbf{\Lambda}_{\mathrm{T}} = \mathrm{diag}\left(\left[\lambda_{\mathrm{T},1},\cdots,\lambda_{\mathrm{T},p}\right]^{\mathrm{T}}\right), \tag{21}$$

where $\lambda_{\mathrm{T},i}$ for $i = 1,\ldots,p$ are the $p \leq N$ eigenvalues of the temporal correlation matrix $\mathbf{R}_{\mathrm{T}}$. Note that we allow the temporal correlation matrix to be rank *deficient* since p can be less than N, in which case $\mathbf{R}_{\mathrm{T}}$ is not invertible.

With the definition (18), the N elements on the diagonal of $\mathbf{R}_{\mathrm{T}}$ are all equal to one and hence,

$$\mathrm{tr}\left(\mathbf{R}_{\mathrm{T}}\right) = \mathrm{tr}\left(\mathbf{\Lambda}_{\mathrm{T}}\right) = N. \tag{22}$$

Furthermore the matrix $\mathbf{R}_{\mathrm{T}}$ has a Toeplitz structure, since the involved interference processes are taken to be stationary.

Spatial Correlation

The spatial correlation matrix of the interference component in (7) is given by

$$\mathbf{R}_{\mathrm{S}} = \mathbf{A}\mathbf{\Sigma}\mathbf{A}^{\mathrm{H}} \tag{23}$$

$$= \mathbf{U}\mathbf{\Lambda}_{\mathrm{S}}\mathbf{U}^{\mathrm{H}}, \tag{24}$$

where

$$\mathbf{\Sigma} = \mathrm{diag}\left(\left[\sigma_1^2,\cdots,\sigma_L^2\right]^{\mathrm{T}}\right). \tag{25}$$

In addition, (24) represents the eigenvalue decomposition of $\mathbf{R}_{\mathrm{S}}$, with $\mathbf{U} \in \mathbb{C}^{M\times L}$ containing the L eigenvectors which are orthonormal, i.e.

$$\mathbf{U}^{\mathrm{H}}\mathbf{U} = \mathbf{1}_L. \tag{26}$$

Moreover,

$$\mathbf{\Lambda}_{\mathrm{S}} = \mathrm{diag}\left(\left[\lambda_{\mathrm{S},1},\cdots,\lambda_{\mathrm{S},L}\right]^{\mathrm{T}}\right), \tag{27}$$

where $\lambda_{\mathrm{S},i}$ for $i = 1,\ldots,L$ are the $L \leq M$ eigenvalues of the spatial correlation matrix $\mathbf{R}_{\mathrm{S}}$.

Furthermore, we have

$$\mathrm{tr}\left(\mathbf{R}_{\mathrm{S}}\right) = \mathrm{tr}\left(\mathbf{\Lambda}_{\mathrm{S}}\right) = M\sum_{\ell=1}^{L}\sigma_\ell^2. \tag{28}$$

Similar to the temporal case, the matrix $\mathbf{R}_{\mathrm{S}}$ has a Toeplitz structure due to the Vandermonde structure of the steering vector (c.f. (2)).

Spatio-Temporally Correlated Interference

Based on the previous definitions and after stacking the interfering signals into a vector $\mathrm{vec}\left(\mathbf{AB}_k\right) \in \mathbb{C}^{MN}$, we observe that the correlation matrix of this resulting vector has the following structure

$$\mathrm{E}\left[\mathrm{vec}\left(\mathbf{AB}_k\right)\mathrm{vec}\left(\mathbf{AB}_k\right)^{\mathrm{H}}\right] = \mathbf{R}_{\mathrm{T}} \otimes \mathbf{R}_{\mathrm{S}}, \tag{29}$$

i.e. it results from Kronecker product of two correlation matrices, namely the temporal and spatial correlation matrices. In such case, the interference component is defined by processes which are separable [5].

Based on $\mathbf{AB}_k$, the temporal correlation matrix depends on the spatial correlation matrix (see eigenvalue decomposition of $\mathbf{R}_{\mathrm{T}}$ in (19)) as follows

$$\mathbf{R}_{\mathrm{S}} = \mathrm{E}\left[\left(\mathbf{AB}_k\right)\mathbf{V}\mathbf{\Lambda}_{\mathrm{T}}^{-1}\mathbf{V}^{\mathrm{H}}\left(\mathbf{AB}_k\right)^{\mathrm{H}}\right], \tag{30}$$

and similarly the spatial correlation matrix depends on the temporal correlation matrix (see eigenvalue decomposition of $\mathbf{R}_{\mathrm{S}}$ in (24)) as follows

$$\mathbf{R}_{\mathrm{T}} = \mathrm{E}\left[\left(\mathbf{AB}_k\right)^{\mathrm{H}}\mathbf{U}\mathbf{\Lambda}_{\mathrm{S}}^{-1}\mathbf{U}^{\mathrm{H}}\left(\mathbf{AB}_k\right)\right]. \tag{31}$$

Since we allow the matrices $\mathbf{R}_\mathrm{S}$ and $\mathbf{R}_\mathrm{T}$ to be rank deficient, i.e. $p \leq N$ and $L \leq M$, note that $\mathbf{V}\mathbf{\Lambda}_\mathrm{T}^{-1}\mathbf{V}^\mathrm{H}$ and $\mathbf{U}\mathbf{\Lambda}_\mathrm{S}^{-1}\mathbf{U}^\mathrm{H}$ are *not* necessarily the inverses of $\mathbf{R}_\mathrm{T}$ and $\mathbf{R}_\mathrm{S}$, respectively, since the inverse does not actually exist. The computation of (30) and (31) based on the temporal and spatial correlation matrix can be interpreted as performing a spatial and temporal *prewhitening*, respectively.

With (30) and (31) and employing the eigenvalue decomposition of $\mathbf{R}_\mathrm{S}$ (24) and $\mathbf{R}_\mathrm{T}$ (19), we can model the interference component in (7) as

$$\mathbf{A}\mathbf{B}_k = \mathbf{U}\mathbf{\Lambda}_\mathrm{S}^{\frac{1}{2}}\mathbf{Z}_k\mathbf{\Lambda}_\mathrm{T}^{\frac{1}{2}}\mathbf{V}^\mathrm{H}, \tag{32}$$

where the matrix $\mathbf{Z}_k \in \mathbb{C}^{L\times p}$ contains i.i.d. complex Gaussian random variables with zero-mean and unit-variance such that defining $\mathbf{Z}_k$ based on its columns and rows

$$\mathbf{Z}_k = \begin{bmatrix} \mathbf{z}'_{k,1} & \cdots & \mathbf{z}'_{k,p} \end{bmatrix}, \tag{33}$$

$$= \begin{bmatrix} \mathbf{z}_{k,1}^\mathrm{T} \\ \vdots \\ \mathbf{z}_{k,L}^\mathrm{T} \end{bmatrix} \tag{34}$$

the following holds over the columns of $\mathbf{Z}_k$

$$\mathrm{E}\left[\mathbf{z}'_{k,m}\mathbf{z}'^{,\mathrm{H}}_{k,m}\right] = \mathbf{1}_L \tag{35}$$

$$\mathrm{E}\left[\mathbf{z}'_{k,m}\mathbf{z}'^{,\mathrm{H}}_{k,n}\right] = \mathbf{0}, \tag{36}$$

for $m, n = 1, \ldots, p$ with $m \neq n$, and over the rows of $\mathbf{Z}_k$

$$\mathrm{E}\left[\mathbf{z}_{k,i}\mathbf{z}_{k,i}^\mathrm{H}\right] = \mathbf{1}_p \tag{37}$$

$$\mathrm{E}\left[\mathbf{z}_{k,i}\mathbf{z}_{k,j}^\mathrm{H}\right] = \mathbf{0}, \tag{38}$$

for $i, j = 1, \ldots, L$ with $j \neq i$.

MAXIMUM LIKELIHOOD ESTIMATION

With the fact that the noise and the interference $\mathbf{A}\mathbf{B}_k$ are independent and Gaussian distributed, the correlation matrix of the effective noise vector $\mathrm{vec}\left(\mathbf{A}\mathbf{B}_k + \mathbf{N}_k\right) \in \mathbb{C}^{MN}$, i.e. of the interference plus noise, reads as

$$\mathbf{R} = \mathbf{R}_\mathrm{T} \otimes \mathbf{R}_\mathrm{S} + \sigma_\mathrm{n}^2\mathbf{1}_{MN}, \tag{39}$$

where we used (29).

Given the observation $\mathbf{Y}_k$ and the correlation matrix $\mathbf{R}$, the likelihood function with respect to signal parameters, i.e. the channel coefficient γ_0, the time delay τ_0 and the angle of arrival θ_0, is given by

$$\frac{\mathrm{e}^{\left(-\mathrm{vec}\left(\mathbf{Y}_k - \gamma_0\,\mathbf{a}(\theta_0)\mathbf{c}_k^\mathrm{T}(\tau_0)\right)^\mathrm{H}\mathbf{R}^{-1}\mathrm{vec}\left(\mathbf{Y}_k - \gamma_0\,\mathbf{a}(\theta_0)\mathbf{c}_k^\mathrm{T}(\tau_0)\right)\right)}}{\pi^{MN}|\mathbf{R}|}.$$

Furthermore, taking the natural logarithm of the previous likelihood function and employing

$$\mathrm{vec}\left(\gamma_0\,\mathbf{a}(\theta_0)\,\mathbf{c}_k^\mathrm{T}(\tau_0)\right) = \gamma_0\mathbf{c}_k(\tau_0) \otimes \mathbf{a}(\theta_0),$$

the log-likelihood function without the constant terms reads as

$$L(\mathbf{Y}_k, \gamma_0, \tau_0, \theta_0) = -\Big(\mathrm{vec}(\mathbf{Y}_k)^\mathrm{H} - \gamma_0^*\left(\mathbf{c}_k(\tau_0) \otimes \mathbf{a}(\theta_0)\right)^\mathrm{H}\Big) \times \mathbf{R}^{-1}\left(\mathrm{vec}(\mathbf{Y}_k) - \gamma_0\left(\mathbf{c}_k(\tau_0) \otimes \mathbf{a}(\theta_0)\right)\right). \tag{40}$$

Hence, the *maximum likelihood* (ML) estimates of the signal parameters can be obtained from

$$\left\{\hat{\gamma}_0, \hat{\theta}_0, \hat{\tau}_0\right\} = \arg\max_{\gamma_0,\theta_0,\tau_0} \quad L(\mathbf{Y}_k, \gamma_0, \tau_0, \theta_0). \tag{41}$$

Taking the derivative of (41) with respect to γ_0 and setting it to zero, we obtain the maximum likelihood estimate of the channel coefficient

$$\hat{\gamma_0} = \frac{\left(\mathbf{c}_k(\tau_0) \otimes \mathbf{a}(\theta_0)\right)^\mathrm{H}\mathbf{R}^{-1}\mathrm{vec}\left(\mathbf{Y}_k\right)}{\left(\mathbf{c}_k(\tau_0) \otimes \mathbf{a}(\theta_0)\right)^\mathrm{H}\hat{\mathbf{R}}^{-1}\left(\mathbf{c}_k(\tau_0) \otimes \mathbf{a}(\theta_0)\right)}. \tag{42}$$

Substituting this estimate for γ_0 in the log-likelihood function (41) gives us the maximum likelihood estimates of θ_0 and τ_0

$$\left\{\hat{\theta}_0, \hat{\tau}_0\right\} = \arg\max_{\theta_0,\tau_0} \frac{\left|\left(\mathbf{c}_k(\tau_0) \otimes \mathbf{a}(\theta_0)\right)^\mathrm{H}\mathbf{R}^{-1}\mathrm{vec}\left(\mathbf{Y}_k\right)\right|^2}{\left(\mathbf{c}_k(\tau_0)\otimes\mathbf{a}(\theta_0)\right)^\mathrm{H}\mathbf{R}^{-1}\left(\mathbf{c}_k(\tau_0)\otimes\mathbf{a}(\theta_0)\right)}. \tag{43}$$

However, in order to compute the ML estimates (42) and (43), we need to know $\mathbf{R}$ defined in (39). This implies that we need to estimate the spatial correlation matrix $\mathbf{R}_\mathrm{S}$ and temporal correlation matrix $\mathbf{R}_\mathrm{T}$ of the interference. Given an estimate of the spatial and temporal correlation matrix $\hat{\mathbf{R}}_\mathrm{T}$ and $\hat{\mathbf{R}}_\mathrm{S}$, we can compute an estimate of $\mathbf{R}$ as follows

$$\hat{\mathbf{R}} = \hat{\mathbf{R}}_\mathrm{T} \otimes \hat{\mathbf{R}}_\mathrm{S} + \sigma_\mathrm{n}^2\mathbf{1}_{MN}. \tag{44}$$

The estimate $\hat{\mathbf{R}}$ of the correlation matrix $\mathbf{R}$ can thus be employed in (42) and (43) to obtain estimates of the signal parameters γ_0, θ_0 and τ_0. In the following section, we address the issue of the estimation of the spatial and temporal correlation matrix.

SPATIO-TEMPORAL COVARIANCE MATRIX ESTIMATION

The estimation of correlation matrices is of vital importance in many signal processing applications. In several situations, the correlation matrix has a certain structure like the Kronecker product of two smaller correlation matrices as in the considered scenario (29). Further examples of such processes can be found in communications, where *multiple-input multiple-output* (MIMO) channels resulting from multiple antennas at the transmitter and receiver can be described by the Kronecker model [6, 7] due to the spatial correlations at the transmitter and receiver. In other applications, we have spatio-temporal correlations [8] such as the noise processes in the signal modelling of *magnetoencephalography* (MEG) and *electroencephalography* (EEG) data [9, 10]. Similarly, the spatio-temporal correlations follow a Kronecker product in radar applications [11].

In the literature, several methods are proposed for the estimation of the correlation matrix following a Kronecker product structure [12, 13]. A very simple approach consists in simply computing the unstructured sample correlation matrix. Nevertheless, the shortcomings of computing the unstructured sample correlation matrix is that it ignores the inherent structure of the correlation matrix and therefore, requires a large number of samples to have a meaningful estimate. Certain structure and characteristics of the matrix, as for instance a Toeplitz structure, can be taken into consideration by finding, for instance, the closest correlation matrix in the Frobenius norm sense with a such structure [14].

Another possible solution is to consider an iterative approach, like the so called *flip-flop* algorithm, where one of the involved correlation matrices is computed at a time based on a previous estimate of the other correlation matrix [12, 13]. The algorithm is repeated until convergence based on several possible criteria, like in the Frobenius norm sense or according to the likelihood function. The iterative nature of the algorithm is due to the fact that the estimation of the correlation matrices are coupled (see (30) and (31)).

In contrast to the previous iterative solutions, in this work we present an approach where the spatial and temporal correlation matrix can be computed directly. This algorithm is detailed in the following.

Proposed Estimation Algorithm

It can be shown that $\mathrm{E}\left[\mathbf{Y}_k\mathbf{Y}_k^{\mathrm{H}}\right]$, results in

$$\begin{aligned}
\mathrm{E}\left[\mathbf{Y}_k\mathbf{Y}_k^{\mathrm{H}}\right] &= \mathrm{E}\left[\left(\gamma_0\,\mathbf{a}(\theta_0)\,\mathbf{c}_k^{\mathrm{T}}(\tau_0) + \mathbf{A}\mathbf{B}_k + \mathbf{N}_k\right) \times \right.\\
&\qquad \left.\left(\gamma_0^*\,\mathbf{c}_k^*(\tau_0)\,\mathbf{a}^{\mathrm{H}}(\theta_0) + \mathbf{B}_k^{\mathrm{H}}\mathbf{A}^{\mathrm{H}} + \mathbf{N}_k^{\mathrm{H}}\right)\right]\\
&= \mathrm{E}\left[\left(\gamma_0\,\mathbf{a}(\theta_0)\mathbf{c}_k^{\mathrm{T}}(\tau_0) + \mathbf{U}\boldsymbol{\Lambda}_{\mathrm{S}}^{\frac{1}{2}}\mathbf{Z}_k\boldsymbol{\Lambda}_{\mathrm{T}}^{\frac{1}{2}}\mathbf{V}^{\mathrm{H}} + \mathbf{N}_k\right)\times\right.\\
&\qquad \left.\left(\gamma_0^*\,\mathbf{c}_k^*(\tau_0)\,\mathbf{a}^{\mathrm{H}}(\theta_0) + \mathbf{V}\boldsymbol{\Lambda}_{\mathrm{T}}^{\frac{1}{2}}\mathbf{Z}_k^{\mathrm{H}}\boldsymbol{\Lambda}_{\mathrm{S}}^{\frac{1}{2}}\mathbf{U}^{\mathrm{H}} + \mathbf{N}_k^{\mathrm{H}}\right)\right]\\
&= |\gamma_0|^2\;\mathbf{a}(\theta_0)\,\mathrm{E}\left[\mathbf{c}_k^{\mathrm{T}}(\tau_0)\mathbf{c}_k^*(\tau_0)\right]\mathbf{a}^{\mathrm{H}}(\theta_0)\\
&\qquad + \mathbf{U}\boldsymbol{\Lambda}_{\mathrm{S}}^{\frac{1}{2}}\,\mathrm{E}\left[\mathbf{Z}_k\boldsymbol{\Lambda}_{\mathrm{T}}\mathbf{Z}_k^{\mathrm{H}}\right]\boldsymbol{\Lambda}_{\mathrm{S}}^{\frac{1}{2}}\mathbf{U}^{\mathrm{H}} + \mathrm{E}\left[\mathbf{N}_k\mathbf{N}_k^{\mathrm{H}}\right]\\
&\approx \mathbf{U}\boldsymbol{\Lambda}_{\mathrm{S}}^{\frac{1}{2}}\,\mathrm{E}\left[\mathbf{Z}_k\boldsymbol{\Lambda}_{\mathrm{T}}\mathbf{Z}_k^{\mathrm{H}}\right]\boldsymbol{\Lambda}_{\mathrm{S}}^{\frac{1}{2}}\mathbf{U}^{\mathrm{H}} + \mathrm{E}\left[\mathbf{N}_k\mathbf{N}_k^{\mathrm{H}}\right]\\
&= \mathbf{U}\boldsymbol{\Lambda}_{\mathrm{S}}^{\frac{1}{2}}\left(\sum_{i=1}^{p}\lambda_{\mathrm{T},i}\;\mathrm{E}\left[\mathbf{z}'_{k,i}\mathbf{z}'^{,\mathrm{H}}_{k,i}\right]\right)\boldsymbol{\Lambda}_{\mathrm{S}}^{\frac{1}{2}}\mathbf{U}^{\mathrm{H}}\\
&\qquad + \sum_{j=1}^{N}\;\mathrm{E}\left[\mathbf{n}_{k,j}\mathbf{n}_{k,j}^{\mathrm{H}}\right]\\
&= \mathrm{tr}\left(\boldsymbol{\Lambda}_{\mathrm{T}}\right)\mathbf{U}\boldsymbol{\Lambda}_{\mathrm{S}}\mathbf{U}^{\mathrm{H}} + N\sigma_{\mathrm{n}}^2\mathbf{1}_M\\
&= \mathrm{tr}\left(\boldsymbol{\Lambda}_{\mathrm{T}}\right)\mathbf{R}_{\mathrm{S}} + N\sigma_{\mathrm{n}}^2\mathbf{1}_M\\
&= N\left(\mathbf{R}_{\mathrm{S}} + \sigma_{\mathrm{n}}^2\mathbf{1}_M\right),
\end{aligned} \tag{45}$$

where in the first and second expression we employ (7) and (32). In the third expression we use the fact that the signal, interference and noise are independent from one another. The approximation results from the fact that $|\gamma_0|^2 \ll \sum_{\ell=1}^{L}\sigma_\ell^2$ and $|\gamma_0|^2 \ll \sigma_{\mathrm{n}}^2$, i.e. the satellite signal is much weaker than the interfering signals and noise at the antenna array, which happens to be the case for GNSS signals. In the fifth expression we have used (33), (21) and (10). For the following step we employ (35) and (14). The last two steps result from (22) and (24).

From (45), we can obtain $\mathbf{R}_{\mathrm{S}}$ based on $\mathrm{E}\left[\mathbf{Y}_k\mathbf{Y}_k^{\mathrm{H}}\right]$. If we approximate the expectation in $\mathrm{E}\left[\mathbf{Y}_k\mathbf{Y}_k^{\mathrm{H}}\right]$ with the sample correlation matrix given K blocks:

$$\mathrm{E}\left[\mathbf{Y}_k\mathbf{Y}_k^{\mathrm{H}}\right] \approx \frac{1}{K}\sum_{k=1}^{K}\mathbf{Y}_k\mathbf{Y}_k^{\mathrm{H}}, \tag{46}$$

this implies that we can find an estimate of the spatial correlation matrix as follows

$$\hat{\mathbf{R}}_{\mathrm{S}} \approx \frac{1}{KN}\sum_{k=1}^{K}\mathbf{Y}_k\mathbf{Y}_k^{\mathrm{H}} - \sigma_{\mathrm{n}}^2\mathbf{1}_M, \tag{47}$$

which is obtained without iterating.

In a similar fashion, we can show that

$$\begin{aligned}
\mathrm{E}\left[\mathbf{Y}_k^{\mathrm{H}}\mathbf{Y}_k\right] &= \mathrm{E}\left[\left(\gamma_0^*\mathbf{c}_k^*(\tau_0)\mathbf{a}^{\mathrm{H}}(\theta_0) + \mathbf{B}_k^{\mathrm{H}}\mathbf{A}^{\mathrm{H}} + \mathbf{N}_k^{\mathrm{H}}\right) \times \right.\\
&\qquad \left.\left(\gamma_0\,\mathbf{a}(\theta_0)\mathbf{c}_k^{\mathrm{T}}(\tau_0) + \mathbf{A}\mathbf{B}_k + \mathbf{N}_k\right)\right]\\
&= \mathrm{E}\left[\left(\gamma_0^*\mathbf{c}_k^*(\tau_0)\mathbf{a}^{\mathrm{H}}(\theta_0) + \mathbf{V}\boldsymbol{\Lambda}_{\mathrm{T}}^{\frac{1}{2}}\mathbf{Z}_k^{\mathrm{H}}\boldsymbol{\Lambda}_{\mathrm{S}}^{\frac{1}{2}}\mathbf{U}^{\mathrm{H}} + \mathbf{N}_k^{\mathrm{H}}\right)\times\right.\\
&\qquad \left.\left(\gamma_0\,\mathbf{a}(\theta_0)\mathbf{c}_k^{\mathrm{T}}(\tau_0) + \mathbf{U}\boldsymbol{\Lambda}_{\mathrm{S}}^{\frac{1}{2}}\mathbf{Z}_k\boldsymbol{\Lambda}_{\mathrm{T}}^{\frac{1}{2}}\mathbf{V}^{\mathrm{H}} + \mathbf{N}_k\right)\right]\\
&= \|\mathbf{a}(\theta_0)\|_2^2\;|\gamma_0|^2\;\mathrm{E}\left[\mathbf{c}_k^*(\tau_0)\mathbf{c}_k^{\mathrm{T}}(\tau_0)\right]\\
&\qquad + \mathbf{V}\boldsymbol{\Lambda}_{\mathrm{T}}^{\frac{1}{2}}\;\mathrm{E}\left[\mathbf{Z}_k^{\mathrm{H}}\boldsymbol{\Lambda}_{\mathrm{S}}\mathbf{Z}_k\right]\boldsymbol{\Lambda}_{\mathrm{T}}^{\frac{1}{2}}\mathbf{V}^{\mathrm{H}} + \mathrm{E}\left[\mathbf{N}_k^{\mathrm{H}}\mathbf{N}_k\right]\\
&\approx \mathbf{V}\boldsymbol{\Lambda}_{\mathrm{T}}^{\frac{1}{2}}\;\mathrm{E}\left[\mathbf{Z}_k^{\mathrm{H}}\boldsymbol{\Lambda}_{\mathrm{S}}\mathbf{Z}_k\right]\boldsymbol{\Lambda}_{\mathrm{T}}^{\frac{1}{2}}\mathbf{V}^{\mathrm{H}} + \mathrm{E}\left[\mathbf{N}_k^{\mathrm{H}}\mathbf{N}_k\right]\\
&= \mathbf{V}\boldsymbol{\Lambda}_{\mathrm{T}}^{\frac{1}{2}}\left(\sum_{\ell=1}^{L}\lambda_{\mathrm{S},\ell}\;\mathrm{E}\left[\mathbf{z}_{k,j}^*\mathbf{z}_{k,j}^{\mathrm{T}}\right]\right)\boldsymbol{\Lambda}_{\mathrm{T}}^{\frac{1}{2}}\mathbf{V}^{\mathrm{H}}\\
&\qquad + M\sigma_{\mathrm{n}}^2\mathbf{1}_N\\
&= \mathrm{tr}\left(\boldsymbol{\Lambda}_{\mathrm{S}}\right)\mathbf{V}\boldsymbol{\Lambda}_{\mathrm{T}}\mathbf{V}^{\mathrm{H}} + M\sigma_{\mathrm{n}}^2\mathbf{1}_N\\
&= \mathrm{tr}\left(\boldsymbol{\Lambda}_{\mathrm{S}}\right)\mathbf{R}_{\mathrm{T}} + M\sigma_{\mathrm{n}}^2\mathbf{1}_N
\end{aligned} \tag{48}$$

which basically follows the same steps as the derivation (45) by using (7), (32), (34), (37) and (28).

From (48), we obtain $\mathbf{R}_{\mathrm{T}}$ based on $\mathrm{E}\left[\mathbf{Y}_k^{\mathrm{H}}\mathbf{Y}_k\right]$, which given K blocks, can be approximated by the sample correlation matrix :

$$\mathrm{E}\left[\mathbf{Y}_k^{\mathrm{H}}\mathbf{Y}_k\right] \approx \frac{1}{K}\sum_{k=1}^{K}\mathbf{Y}_k^{\mathrm{H}}\mathbf{Y}_k. \tag{49}$$

Hence, an estimate of the temporal correlation matrix is given by

$$\hat{\mathbf{R}}_{\mathrm{T}} \approx \frac{1}{\mathrm{tr}\left(\boldsymbol{\Lambda}_{\mathrm{S}}\right)}\left(\frac{1}{K}\sum_{k=1}^{K}\mathbf{Y}_k^{\mathrm{H}}\mathbf{Y}_k - M\sigma_{\mathrm{n}}^2\mathbf{1}_M\right), \tag{50}$$

which is obtained without iterating and where we can approximate $\mathrm{tr}\left(\mathbf{\Lambda}_{\mathrm{S}}\right)$ given the estimate of the spatial correlation matrix (47)

$$\mathrm{tr}\left(\mathbf{\Lambda}_{\mathrm{S}}\right) \approx \mathrm{tr}\left(\hat{\mathbf{R}}_{\mathrm{S}}\right).$$

SIMULATION RESULTS

In order to visualize the potential of mitigating the interference by taking into account the spatio-temporal correlation matrix, we simulate the *maximum likelihood estimation* (MLE) of the time delay (43). To this end, we consider a GPS signal with C/A code described as follows

$$\sum_{n=0}^{1022} c_n\, \delta(t - nT_{\mathrm{c}}), \tag{51}$$

where c_n for $n = 0, \cdots, 1022$ is the chip sequence of a given satellite consisting of ± 1. Furthermore, T_{c} is the duration of a chip, which for a GPS signal is $T_{\mathrm{c}} = \frac{1\times 10^{-3}\,\mathrm{s}}{1023} = 978.5$ ns, where 1023 is the number of chips in the chip sequence. With $p\,(t)$ as the pulse function, we have that the satellite signal $c(t)$ is

$$\begin{aligned} c(t) &= p\,(t) \star \sum_{n=0}^{1022} c_n\, \delta\left(t - nT_{\mathrm{c}}\right) \\ &= \sum_{n=0}^{N-1} c_n\, p\left(t - nT_{\mathrm{c}}\right), \end{aligned} \tag{52}$$

where $\star$ denotes the convolution operation. As mentioned before, the receive signal has been passed through an ideal lowpass filter with bandwidth B. We assume the bandwidth of the receive signal to be $B = 1.023$ MHz. In this case, the pulse function $p\,(t)$ given in (52) for a finite bandwidth of 1.023 MHz is given as [15]

$$p(t) = \frac{1}{\pi T_{\mathrm{c}}}\left(\mathrm{Si}\left(2\pi\left(\frac{t}{T_{\mathrm{c}}} + \frac{1}{2}\right)\right) - \mathrm{Si}\left(2\pi\left(\frac{t}{T_{\mathrm{c}}} - \frac{1}{2}\right)\right)\right), \tag{53}$$

where $\mathrm{Si}(\bullet)$ is the sine integral. With the previous expressions we are able to characterize the satellite signal and hence, also $\mathbf{c}_k(\tau_0)$ defined in (11). The 1023 chips of a GPS signal span 1 ms, which we assume to be the duration of one block. Thus, with the sampling period $T_{\mathrm{s}} = \frac{1}{2B}$ with $B = 1.023$ MHz we have that the receive signal is sampled at twice the chip rate such that the number of samples per block spanning 1 ms is $N = 2046$.

As for the GNSS receiver, we consider an antenna array with $M = 2$ antennas, upon which the GPS satellite signal is impinging from the front-fire, i.e. with angle of arrival $\theta_0 = 0$, as shown in Figure 1. The SNR of the signal is $\mathrm{SNR} = -10$ dB such that the amplitude of the channel gain γ_0 in (7) is $|\gamma_0| = 0.1$. We assume the phase of the channel gain γ_0, i.e. $\arg(\gamma_0)$, to be uniformly distributed in the interval $[0, 2\pi[$.

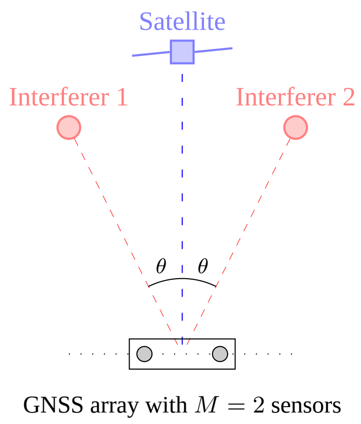

Fig. 1 Simulation Setup

For the interference, we consider $L = 2$ interfering Gaussian signals are impinging on the array from angles θ_1 and θ_2, such that

$$\theta_1 = \theta \tag{54}$$

$$\theta_2 = -\theta, \tag{55}$$

as depicted in Figure 1. In addition, we assume the interference signals to have the same variance, i.e. $\sigma_1^2 = \sigma_2^2$ and hence,

$$\mathrm{INR}_1 = \mathrm{INR}_2.$$

Furthermore, as mentioned in the description of the interference model, both interfering signals have the same temporal correlations described by the correlation matrix $\mathbf{R}_{\mathrm{T}}$.

We assume both interfering signals to occupy 80% of the signal bandwidth, i.e. the bandwidth B_1 and B_2 of interfering signals is $B_1 = B_2 = 0.8 \cdot 1.023$ MHz. In addition, the spectrum of the interfering signals is assumed to be flat over this bandwidth. Based on this, we can determine the temporal correlation matrix $\mathbf{R}_{\mathrm{T}}$. Given the previous assumptions, the autocorrelation function $r(t)$ of the frequency-flat wideband interfering signal is [16]

$$r(t) = \sigma_1^2\, \mathrm{sinc}(2B_1 t), \tag{56}$$

where

$$\mathrm{sinc}(x) = \frac{\sin \pi x}{\pi x}.$$

Given (56), the i,j-th element of the temporal correlation matrix $\mathbf{R}_{\mathrm{T}}$ (see (18)) for $i,j = 1,\ldots,N$ is determined by

$$R_{\mathrm{T},i,j} = \mathrm{sinc}(2B_1|i-j|),$$

where B_1 was defined above.

As a figure of merit we consider the *root mean square error* (RMSE) of the time delay averaged over $K = 20000$ blocks and multiplied by the speed of light c, in order to characterize the positioning error in meters:

$$\mathrm{RMSE} = c \cdot \sqrt{\frac{1}{K}\sum_{k=1}^{K}(\hat{\tau}_{0,k} - \tau_0)}, \tag{57}$$

where $\hat{\tau}_{0,k}$ represents the time delay ML estimate of the k-th block. Without loss of generality, we assume $\tau_0 = 0$. We evaluate the performance of the delay estimation as a function of the total interfering power. Recalling that we have assumed the same variance for the $L = 2$ interferers, the total interfering power is given by

$$\mathrm{INR} = \mathrm{INR}_1 + \mathrm{INR}_2 = 2\frac{\sigma_1}{\sigma_\mathrm{n}^2}.$$

where we used (5).

For the first simulation scenario, we compare the time delay ML estimation (43) when we employ the correct spatial and temporal correlation matrices $\mathbf{R}_\mathrm{S}$ and $\mathbf{R}_\mathrm{T}$ and the ML estimation with known spatial correlation matrix $\mathbf{R}_\mathrm{S}$ but under the assumption of temporally uncorrelated interference, i.e. the interference is assumed to be temporally white

$$\mathbf{R}_\mathrm{T} = \mathbf{1}_N.$$

To this end, we plot in Figure 2, the RMSE as a function of the INR for different values of θ. We can observe that considering solely the spatial correlation matrix in presence of temporally correlated interference leads to suboptimal parameter estimation due to imperfect characterization of the interference. For angles $\theta \to \frac{\pi}{2}$, the gain resulting from employing the correct temporal correlation matrix for the interference becomes less significant since for the considered scenario depicted in Figure 1, the interfering signals are spatially orthogonal to the satellite signal for $\theta = \frac{\pi}{2}$, i.e., for instance, for the first interferer:

$$\mathbf{a}^\mathrm{H}(\theta_0 = 0) \cdot \mathbf{a}\left(\theta_1 = \frac{\pi}{2}\right) = 0, \qquad (58)$$

which follows from the steering vector (2) with $M = 2$ and $\mathrm{e}^{-\mathrm{j}\pi \sin 0} = 1$ and $\mathrm{e}^{-\mathrm{j}\pi \sin \frac{\pi}{2}} = -1$. Hence, for the given scenario interfering signals impinging from angles $\theta_{1,2} \to \frac{\pi}{2}$ do not influence the estimation performance as can be observed in Figure 2, i.e. the RMSE is independent of the power of the interfering signals. For $\theta < \frac{\pi}{2}$, the estimation becomes worse in general as the INR increases due the increased interfering power. Nonetheless, even for these cases employing the correct temporal correlation matrix leads to a performance gain compared to the case when the temporal correlations are ignored.

Now let us consider the case when we perform the ML estimation with the correct temporal correlations for the interference but assuming the interfering signals to be spatially white, i.e. $\mathbf{R}_\mathrm{S}$ is assumed to be a weighted identity matrix. The RMSE as a function of the INR resulting from this setup is depicted in Figure 2 along with the ML estimation with the correct spatio-temporal correlations. The mismatched spatial correlations lead to a performance degradation specially for $\theta \to \frac{\pi}{2}$. This results from the fact that for $\theta = \frac{\pi}{2}$, the interfering signals are impinging on the array from both directions of the end-fire and hence, the signals are spatially coherent and not spatially orthogonal as assumed with a diagonal spatial correlation matrix $\mathbf{R}_\mathrm{S}$.

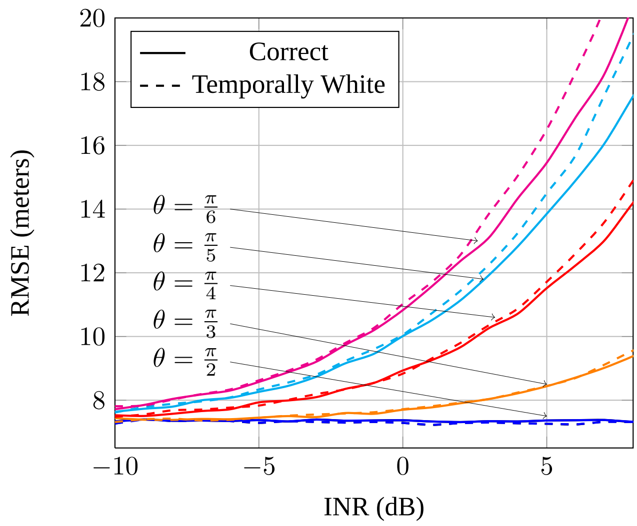

Fig. 2 RMSE vs. INR, Setup 1

Note that for $\theta = \frac{\pi}{6}$, the resulting RMSE for both setups are equivalent. This is due to the fact that in this case the assumption about $\mathbf{R}_\mathrm{S}$ being diagonal is actually correct since the interfering signals are spatially orthogonal. For $\theta = \frac{\pi}{6}$, we have that $\mathbf{A}$ from (8) is

$$\begin{aligned}\mathbf{A} &= \begin{bmatrix} 1 & 1 \\ \mathrm{e}^{-\mathrm{j}\pi \sin \frac{\pi}{6}} & \mathrm{e}^{-\mathrm{j}\pi \sin -\frac{\pi}{6}} \end{bmatrix} \\ &= \begin{bmatrix} 1 & 1 \\ -\mathrm{j} & \mathrm{j} \end{bmatrix}, \end{aligned} \qquad (59)$$

since $\sin \pm\frac{\pi}{6} = \pm\frac{1}{2}$, such that the columns of $\mathbf{A}$ are orthogonal and hence $\mathbf{R}_\mathrm{S}$ is a weighted identity matrix (see (23)).

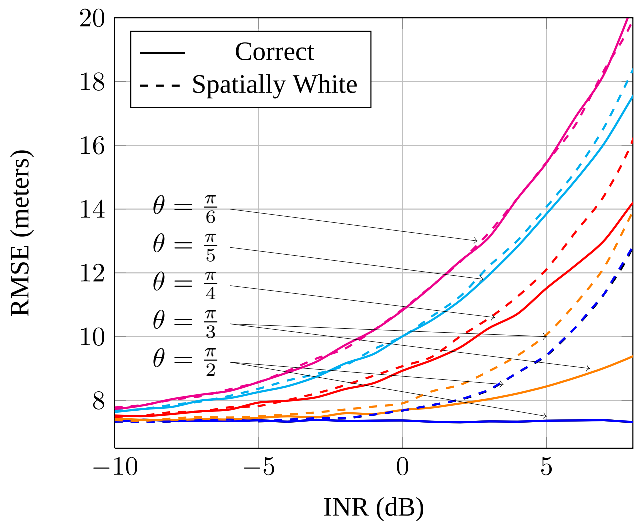

Fig. 3 RMSE vs. INR, Setup 2

For the sake of completeness, we compare the ML estimation with the correct spatial and temporal correlation matrices of the interference with the ML estimation under the assumption that the interfering signals are spatially and temporally white, i.e. both $\mathbf{R}_\mathrm{S}$ and $\mathbf{R}_\mathrm{T}$ are weighted identity matrices. The comparison is shown in Figure 4. As expected, the degradation due to the mismatched spatial and temporal correlations can be observed for all curves.

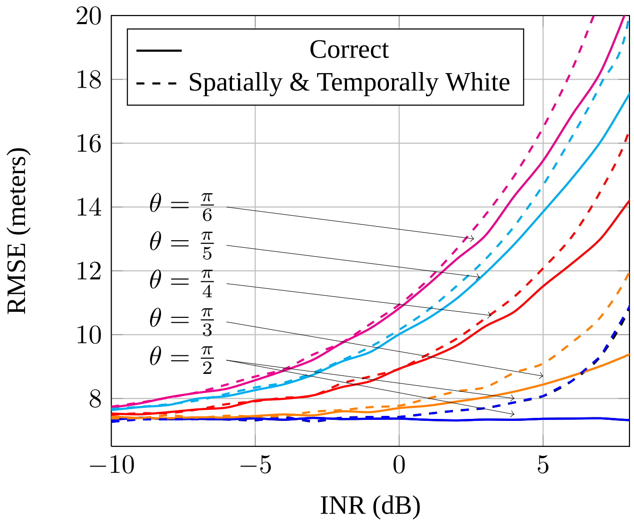

Fig. 4 RMSE vs. INR, Setup 3

Estimated Correlation Matrices

Let us now observe the performance of the ML estimation with the correct spatial and temporal correlation matrices with that of the ML estimation based on the estimated spatial and temporal correlation. To this end, we consider the estimation of the spatial correlation matrix and of the temporal correlation matrix proposed and derived before, i.e. using (47) and (50), respectively. In Figure 5 we depict the RMSE as a function of the INR for the ML estimation with correct and estimated correlation matrices. We can observe that the spatial and temporal correlation are estimated well enough, such that there is a minimal performance loss as compared to the case when the correct correlations matrices are employed.

CONCLUSION

In this work we have analyzed interference on GNSS receive antenna arrays, which have a spatio-temporal correlation matrix following a Kronecker structure. We have proposed a non-iterative method for the estimation of the spatial and temporal correlation matrices. Considering the maximum likelihood parameter estimation of the time delay, we have shown that taking into account solely one of

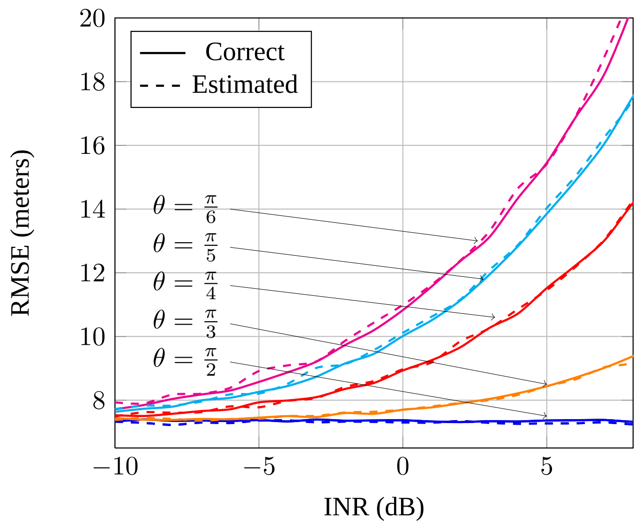

Fig. 5 RMSE vs. INR, Estimated Correlation Matrices

the correlations (spatial or temporal), leads to suboptimal parameter estimation.

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