COMPUTING FUNCTIONS VIA SIMO MULTIPLE-ACCESS CHANNELS: HOW MUCH CHANNEL KNOWLEDGE IS NEEDED?

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ABSTRACT

We view a wireless sensor network as a collection of sensor nodes that observe sources of information, process the picked up data and send it to a sink node, with the goal of computing a desired function of the measurements. To this end, we consider a previously proposed coding scheme that exploits the underlying fading multiple-access channel (MAC) to efficiently estimate the function values. The main problem addressed in this paper is how much channel state information (CSI) is needed at the sensor nodes to obtain sufficiently good estimates? First we show that there is no performance loss, independent of fading distributions, if, instead of perfect CSI, each sensor node has only access to the modulus of its channel coefficient. In the case of multiple antenna elements at the sink node and specific independent distributed fading environments, it is shown that CSI at sensor nodes is not necessary and a very simple correction of fading effects can be performed at the sink based on some statistical channel knowledge. In many cases, fading improves the estimation accuracy due to the multiple-access nature of the channel.

***Index Terms*—** Computation over MAC, communicating functions, in-network computation, wireless sensor networks

1. INTRODUCTION

In many sensor network applications, sensor nodes observe a physical phenomenon and transmit the sensed data over a wireless channel to a designated sink, which computes subsequently a function of measurement values such as "arithmetic mean", "maximum value", etc. In conventional sensor networks, this is typically achieved by transmitting the complete sensed data from all sensor nodes to the sink, using a widely-established access protocol like time division multiple access (TDMA). Such approaches are however highly inefficient with respect to energy consumption, complexity, latency, and cost, and hence also the network lifetime. An alternative approach is to combine the processes of data transmission and function computation into one step by exploiting channel collisions induced by the nature of the wireless multiple-access channel. This is known as computation over multiple-access channels [1, 2].

Motivated by the information-theoretical work in [1] as well as by the fact that the symbol-wise considerations in [1] seem not to be applicable in practice, References [2] and [3] proposed a simple analog scheme with high practical relevance to efficiently estimate some functions of the sensor measurements. This scheme outperforms TDMA-like protocols in a wide range of operating points [3].

A crucial assumption in [2] and [3] was the perfect knowledge of complex-valued CSI at sensor nodes prior to transmissions, called "Full CSI", so that every node was able to perfectly invert its own channel. In this paper, we analyze the impact of fading on the estimation of function values at the sink and address the question of how much CSI at the sensor nodes or the sink is needed. We show that, independent of fading distributions, the modulus of channel coefficients ("Modulus CSI") is sufficient to perform the estimation without performance loss in comparison to Full CSI. Furthermore, it is shown that for specific independent distributed fading environments, CSI at sensor nodes is not necessary and fading effects can be corrected by using second order statistical channel knowledge at the sink. Our results imply that the amount of channel knowledge at nodes can be significantly reduced without performance loss, which is equivalent to reduced complexity and much higher energy efficiency.

Notation: The transpose, Hermitian transpose, and conjugate are denoted by $(\cdot)^T$, $(\cdot)^H$, and $(\cdot)^*$. The distributions of normally distributed real, normally distributed proper complex random variables are described by $\mathcal{N}_{\mathbb{R}}(\cdot,\cdot)$, $\mathcal{N}_{\mathbb{C}}(\cdot,\cdot)$ respectively and $\oplus$ denotes the direct sum of matrices.

2. DEFINITIONS AND PROBLEM STATEMENT

A wireless sensor network consisting of $K \in \mathbb{N}$ identical spatially distributed single-antenna sensor nodes and one designated sink node, equipped with $n_R \in \mathbb{N}$ antenna elements, forms the basis of the considerations in this paper.

Let an appropriate probability space $(\Omega, \mathcal{A}, \mathbb{P})$ be given, with sample space Ω, σ-Algebra $\mathcal{A}$ and probability measure $\mathbb{P}$, over which all appearing random variables and stochastic processes are defined.

Each sensor node has the challenge to observe a certain physical phenomenon (temperature, pressure, ...), and we model these observations as time-discrete stochastic processes $X_k(t) \in \mathcal{X}$, $k = 1, \dots, K$, $t \in \mathbb{Z}_+$, where $\mathcal{X} = [x_{\min}, x_{\max}] \subset \mathbb{R}$ denotes the *physical measurement range*, i. e. the range in which measurement outcomes from physical phenomena observations are. Finally, without loss of generality, we assume that the sensor readings $\boldsymbol{x}(t) := (X_1(t), \dots, X_K(t))^T \in \mathcal{X}^K$ are independent and identically distributed (i. i. d.), like in a scenario where the sensors observe identical values, subject to i. i. d. observation noise.

With these ingredients we are ready to define the most important building blocks of our considerations in a precise form.

Definition 1 (SIMO-WS-MAC)**.** Let $\boldsymbol{x}(t) \in \mathcal{X}^K$, be the sensed data, $n_R \in \mathbb{N}$ the number of receive antennas at the sink node, and let sensor nodes be restricted to peak power constraint $P_{\max} \in \mathbb{R}_{++}$. Let $H_{nk}(t)$, $n = 1, \dots, n_R$; $k = 1, \dots, K$, be a complex-valued flat-fading process between the kth sensor and the nth receive antenna element and $N_n(t) \in \mathbb{C}$ the time-discrete receiver noise process at antenna element n. Assume that the data, the fading and the noise are mutually independent. Then, we refer to the vector-valued map $(X_1(t), \dots, X_K(t)) \longmapsto (Y_1(t), \dots, Y_{n_R}(t)) \in \mathbb{C}^{n_R}$,

$$Y_n(t) = \sum_{k=1}^{K} H_{nk}(t) X_k(t) + N_n(t),\ n = 1, \dots, n_R\ , \quad (1)$$

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as the *SIMO-Wireless Sensor Multiple-Access Channel (SIMO-WS-MAC)*. For $n_R = 1$ we simply say WS-MAC.

The SIMO-WS-MAC is a collection of K SIMO-links, which share the common radio interface per multiple-access. Eq. (1) offers the mathematical characteristic of the WS-MAC, namely *summation*, which can be explicitly used for *desired function computation* if there is a match between the desired function and the underlying multiple-access channel [1].

Definition 2 (Desired Function)**.** $\mathcal{F}_{\mathrm{D}}$ is the set of *desired functions* $f : \mathcal{X}^K \to \mathbb{R}$ of measured sensor data.

Definition 3 (Pre-processing Functions)**.** We define the functions $\varphi_k : \mathcal{X} \to \mathbb{R}$, $k = 1, \ldots, K$, which operate on the sensed data $X_k(t) \in \mathcal{X}$, as the *Pre-Processing Functions*.

Definition 4 (Post-Processing Function)**.** Let $Y(t) \in \mathbb{C}$ be the output of the WS-MAC. Then we define the injective function $\psi : \mathbb{R} \to \mathbb{R}$, which operate on $Y(t)$, as the *Post-Processing Function*.

Remark 1. The pre- and post-processing functions, which obviously depend on the desired function, transform the WS-MAC in such a way that the resulting mathematical characteristic of the overall channel matches the characteristic of the desired function. For example in the case of geometric mean as the desired function (cf. Example 1), the overall channel is a *multiplicative* MAC.

Example 1. (i) Arithmetic mean: $f(\boldsymbol{x}(t)) = \frac{1}{K}\sum_{k=1}^{K} X_k(t)$ with pre-processing functions $\varphi_k(X_k(t)) = \varphi(X_k(t)) = X_k(t)$, $k = 1, \ldots, K$, and post-processing function $\psi(Y(t)) = \frac{1}{K}Y(t)$. (ii) Geometric mean: $f(\boldsymbol{x}(t)) = (\prod_{k=1}^{K} X_k(t))^{\frac{1}{K}}$, $\forall k, t\ X_k(t) > 0$, with pre-processing functions $\varphi_k(X_k(t)) = \varphi(X_k(t)) = \log_a(X_k(t))$, $k = 1, \ldots, K$, a an arbitrary base, and post-processing function $\psi(Y(t)) = a^{\frac{1}{K}Y(t)}$.

Now the problem which arises is: How can we compute desired functions by means of the SIMO-WS-MAC in an efficient way with a minimum amount of required channel knowledge?

3. ROBUST COMPUTATION OF DESIRED FUNCTIONS OVER A SIMO-WS-MAC

Since a precise symbol- and phase synchronization, as desired in [1], is illusive in large-scale sensor networks, in [2] and [3] we proposed an approach, in which for function value transmission at time t, any sensor node generates a complex transmit sequence of length $M \in \mathbb{N}$ with unit norm, i. e. for the kth sensor $\boldsymbol{s}_k(t) = (S_{k1}(t), \ldots, S_{km}(t))^T \in \mathbb{C}^M$, $k = 1 \ldots, K$, and $\|\boldsymbol{s}_k\|_2^2 = 1\,\forall\, k$. The pre-processed sensor information $\varphi_k(X_k(t))$ is then used as a transmit energy for the generated sequence $\boldsymbol{s}_k(t)\ \forall k$. Since transmit powers are positive real numbers, we have to ensure that $\forall k, t\ \varphi_k(X_k(t)) \geq 0$. Therefore we change the domains of φ_k by a bijective function $g : \mathcal{X} \to \mathcal{R}$, i. e. $R_k(t) := g(X_k(t))$, which depends on φ, such that $\mathcal{R}$ is the new domain that fulfills the requirement for all k, t. For more details we refer to [3].

For simplicity we assume perfect block-synchronism in the following, so that the mth output symbol of the SIMO-WS-MAC at antenna n can be written as

$$Y_{nm}(t) = \sum_{k=1}^{K} H_{nk}^{(m)}(t)\sqrt{\alpha\varphi_k(R_k(t))}S_{km}(t) + N_{nm}(t), \quad (2)$$

$n = 1, \ldots, n_R$; $m = 1, \ldots, M$. Note that the synchronism assumption is not necessary, because the described approach is relatively robust against imperfections in block-synchronization [2]. The constant $\alpha > 0$ ensures transmit power constraints, i. e. $0 \leq \frac{\alpha\varphi(R_k(t))}{M} \leq P_{\max}$, while $\forall n, m, t\ N_{nm}(t) \sim \mathcal{N}_{\mathbb{C}}(0, \frac{1}{M}\sigma_N^2)$, with independent real and imaginary parts each with variance $\frac{1}{2M}\sigma_N^2$, describes the stationary receiver noise process on the nth antenna and mth symbol. We arrange the channel outputs into the matrix $\boldsymbol{Y}(t) = (Y_{nm}(t)) \in \mathbb{C}^{n_R \times M}$, the channel coefficients into a sequence of M matrices $\boldsymbol{H}_m(t) = (H_{nk}^{(m)}(t)) \in \mathbb{C}^{n_R \times K}$, the K transmit sequences into a matrix $\boldsymbol{S}(t) := (\boldsymbol{s}_1(t), \ldots, \boldsymbol{s}_K(t))^T \in \mathbb{C}^{K \times M}$, the additive noise terms into $\boldsymbol{N}(t) = (N_{nm}(t)) \in \mathbb{C}^{n_R \times M}$ and the K transmit energies into the diagonal matrix $\boldsymbol{\Phi}(t) := \mathrm{diag}(\Phi_1(t), \ldots, \Phi_K(t)) \in \mathbb{R}_+^{K \times K}$, with $\Phi_k(t) := \sqrt{\alpha\varphi_k(R_k(t))}$, $k = 1, \ldots, K$, so that we can capture all receive signals (2) into one vector equation

$$\mathrm{vec}\big(\boldsymbol{Y}(t)\big) = \left(\bigoplus_{m=1}^{M} \boldsymbol{H}_m(t)\right) \mathrm{vec}\big(\boldsymbol{\Phi}(t)\boldsymbol{S}(t)\big) + \mathrm{vec}\big(\boldsymbol{N}(t)\big)\,. \quad (3)$$

If we define further $\tilde{\boldsymbol{y}}(t) := \mathrm{vec}(\boldsymbol{Y}(t)) \in \mathbb{C}^{n_R M}$, $\tilde{\boldsymbol{s}}(t) := \mathrm{vec}(\boldsymbol{\Phi}(t)\boldsymbol{S}(t)) \in \mathbb{C}^{MK}$, $\tilde{\boldsymbol{n}} := \mathrm{vec}(\boldsymbol{N}(t)) \in \mathbb{C}^{n_R M}$, and

$$\tilde{\boldsymbol{H}}(t) := \bigoplus_{m=1}^{M} \boldsymbol{H}_m(t) = \begin{pmatrix} \boldsymbol{H}_1(t) & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{H}_2(t) & \boldsymbol{0} & \boldsymbol{0} \\ \vdots & \vdots & \ddots & \vdots \\ \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{H}_M(t) \end{pmatrix}, \quad (4)$$

which is an element of $\mathbb{C}^{Mn_R \times MK}$, (3) reduces to

$$\tilde{\boldsymbol{y}}(t) = \tilde{\boldsymbol{H}}(t)\tilde{\boldsymbol{s}}(t) + \tilde{\boldsymbol{n}}(t)\,. \quad (5)$$

In the following we suppress an explicit designation of measurement time instance t.

To recover the desired function value from (5), first one has to calculate the received sum energy (sum over all antennas and symbols), i. e.

$$\begin{aligned} \tilde{\boldsymbol{y}}^H\tilde{\boldsymbol{y}} =& \frac{\alpha}{M}\sum_{n=1}^{n_R}\sum_{m=1}^{M}\sum_{k=1}^{K} |H_{nk}^{(m)}|^2\varphi_k(R_k) + \sum_{n=1}^{n_R}\Delta_{1,n}(\tilde{\boldsymbol{H}}, \boldsymbol{\Phi}, \boldsymbol{S}) \\ &+ \sum_{n=1}^{n_R}\Delta_{2,n}(\tilde{\boldsymbol{H}}, \boldsymbol{\Phi}, \boldsymbol{S}, \boldsymbol{N}) + \sum_{n=1}^{n_R}\Delta_{3,n}(\boldsymbol{N})\,, \end{aligned} \quad (6)$$

followed by simple calculations (appliance of ψ, g^{-1}). The error terms in (6) are

$$\Delta_{1,n}(\tilde{\boldsymbol{H}}, \boldsymbol{\Phi}, \boldsymbol{S}) := \sum_{m=1}^{M}\sum_{k=1}^{K}\sum_{\substack{\ell=1 \\ \ell\neq k}}^{K} (H_{nk}^{(m)})^* H_{n\ell}^{(m)}\Phi_k\Phi_\ell S_{km}^* S_{\ell m} \quad (7)$$

$$\Delta_{2,n}(\tilde{\boldsymbol{H}}, \boldsymbol{\Phi}, \boldsymbol{S}, \boldsymbol{N}) := 2\sum_{m=1}^{M}\sum_{k=1}^{K}\Phi_k\,\mathrm{Re}\big\{H_{nk}^{(m)}S_{km}N_{nm}^*\big\} \quad (8)$$

$$\Delta_{3,n}(\boldsymbol{N}) := \sum_{m=1}^{M}|N_{nm}|^2\,, \quad (9)$$

which we combine to the overall error summand

$$\tilde{\Delta}(\tilde{\boldsymbol{H}}, \boldsymbol{\Phi}, \boldsymbol{S}, \boldsymbol{N}) := \textstyle\sum_{n=1}^{n_R}(\Delta_{1,n} + \Delta_{2,n} + \Delta_{3,n})\,. \quad (10)$$

An adequate generation of sequences $\boldsymbol{s}_k$, $k = 1, \ldots, K$, which reduces the error terms $\Delta_{1,n}$, $\Delta_{2,n}$ simultaneously, is a matter of sequence design, which will be not considered in this paper. Another possibility is to choose the elements of $\boldsymbol{s}_k(t)$ in a way that they act like uncorrelated noise such that $\Delta_{1,n}$, $\Delta_{2,n}$ disappear on average. Therefore the nodes generate for any $m = 1, \dots, M$ the sequence elements $S_{km}(t) = M^{-1/2} \mathrm{e}^{i\Theta_{km}(t)}$, with $i^2 = -1$ and $\forall\, k, m, t$ $\Theta_{km}(t)$ i. i. d. uniformly distributed in $[0, 2\pi)$.

4. FULL CSI VS. MODULUS AND NO CSI AT NODES

In this section we analyze the impact of fading effects on the computation of functions over a SIMO-WS-MAC, using the scheme described in Section 3. In this context, we make different assumptions with regard to the channel knowledge at sensor nodes and the sink. Moreover, we legitimate the consideration of multiple antennas at the receiver, because as it is shown that in specific fading environments, multiple antennas improve the function reconstruction quality and besides reduce the required amount of channel knowledge.

4.1. Full CSI vs. Modulus CSI at sensor nodes

First, we consider the special case $n_R = 1$ in this subsection. The behavior of the WS-MAC (cf. Definition 1) reveals that any kind of *instantaneous* channel knowledge at the receiver side cannot be used to correct fading effects, since the sink node has only access to a noisy linear combination (2), but no access to any individual term in the sum. Therefore, in the previous work we suggested estimating the "complex" channel coefficients at sensor nodes[1] to invert the channel prior to transmission, which we refer to as "Full CSI". If we use the scheme described in Section 3, this means that the kth sensor, $k = 1, \dots, K$, transmits $\forall m$ $\sqrt{\alpha\varphi_k(R_k)} S_{km}/H_{1k}^{(m)}$, $|H_{1k}^{(m)}| \neq 0$, which will serve as a benchmark in the following. Note that we have to ensure through a subtle choice of $\alpha > 0$, that $\forall k, m, t$ $\frac{\alpha\varphi_k(R_k(t))}{M|H_{1k}^{(m)}|^2} \leq P_{\max}$, in order to satisfy transmit power constraints.

The approach of setting the transmit energy of the random sequences equal to the pre-processed sensor data has the advantage, that the first summand in (6), i. e. $\frac{\alpha}{M}\sum_{n,m,k} |H_{nk}^{(m)}|^2 \varphi_k(R_k)$, which is the term of interest in the entire receive energy, is only affected by the "squared modulus" of instantaneous channel coefficients. Hence, the question which arises is: Is an estimation of complex channel coefficients (this requires a sensitive phase estimation) on sensor nodes necessary, or is it sufficient to estimate the absolute values of channel coefficients only, which is called "Modulus CSI"? This would be obviously an improvement with respect to channel estimation effort and accuracy.

Essential for the case of perfect channel inversion through Full CSI is the fact that the entire error term (10) has expectation value σ_N^2, because it can be easily shown that $\Delta_{1,1}$, $\Delta_{2,1}$ are zero mean, and $\mathbb{E}\{\Delta_{3,1}\} = \sigma_N^2$ [2]. This is necessary to formulate an unbiased estimator $\hat{f}$ for desired function f at the sink on the basis of (6), since σ_N^2 is known to the sink and can be simply subtracted. But is such an estimator also unbiased for Modulus CSI, i. e. the kth sensor transmits $\sqrt{\alpha\varphi_k(R_k)} S_{km}/|H_{1k}^{(m)}|$ instead of $\sqrt{\alpha\varphi_k(R_k)} S_{km}/H_{1k}^{(m)}$? To answer this question, we have to analyze the error terms $\Delta_{1,1}$ (7) and $\Delta_{2,1}$ (8), which depend on channel coefficients. It is obvious that $\mathbb{E}\{\Delta_{2,1}\} \equiv 0$, since the zero mean noise terms N_{1m}, $m = 1, \dots, M$, are independent of sensor readings and fading. For $\mathbb{E}\{\Delta_{1,1}\}$, this is not immediately clear, so we have to prove it.

Proposition 1. *Let $H_{1k}^{(m)}$, $|H_{1k}^{(m)}| > 0$, be the random complex channel coefficient between the kth sensor node and the sink at receive symbol m. Then, without any performance loss, channels can be inverted by the Modulus $|H_{1k}^{(m)}|$ prior to transmissions for all k and m, independent of the fading distributions.*

To prove the proposition, the following lemma is useful.

Lemma 1. *Let A, B be real independent random variables. If one of both is uniformly distributed in $[0, 2\pi)$, then the reduced sum $C = (A+B) \bmod 2\pi$ is also uniformly distributed in $[0, 2\pi)$, independent of the distribution of the other random variable.*

Proof. The proof, based on [4], is omitted for lack of space. ■

Proof of Proposition 1. If we write the complex fading coefficient between the kth sensor, $k = 1, \dots, K$, and the sink node at symbol m, $m = 1, \dots, M$, in polar form, i. e. $H_{1k}^{(m)} = |H_{1k}^{(m)}|\, \mathrm{e}^{i\Lambda_{1k}^{(m)}}$, with $\Lambda_{1k}^{(m)}$ the corresponding random phase, (7) under Modulus CSI can be written as

$$\Delta_{1,1} = \frac{2}{M}\sum_{\ell=2}^{K}\sum_{k=1}^{\ell-1}\sum_{m=1}^{M} \Phi_\ell \Phi_k \cos\big(\underbrace{\Lambda_{1\ell}^{(m)} - \Lambda_{1k}^{(m)}}_{=:\Delta\Lambda_{\ell k}^{(m)}} + \underbrace{\Theta_{\ell m} - \Theta_{km}}_{=:\Delta\Theta_{\ell k}^{(m)}}\big). \tag{11}$$

It is not surprising that the absolute values of channel coefficients are eliminated, but the random phases of fading coefficients still influence the function value quality. Let $Z_{\ell k}^{(m)} := \Delta\Lambda_{\ell k}^{(m)} + \Delta\Theta_{\ell k}^{(m)}$, $C_{\ell k}^{(m)} := \cos(Z_{\ell k}^{(m)})$, and note that the $Z_{\ell k}^{(m)}$ are random variables reduced mod 2π.

A sufficient condition for $\mathbb{E}\{\Delta_{1,1}\} = 0$ is $\forall \ell, k, m\, \mathbb{E}\{C_{\ell k}^{(m)}\} = 0$, which should be valid for arbitrary distributions of phase differences $\Delta\Lambda_{\ell k}^{(m)}$.

Since $\forall m, k, \ell \neq k$ $\Theta_{\ell m}, \Theta_{km}$ are uniformly i. i. d. in $[0, 2\pi)$, according to Lemma 1, the differences $\Delta\Theta_{\ell k}^{(m)}$ are also uniformly distributed in $[0, 2\pi)$. Moreover, $\forall m, k, \ell \neq k$ $\Delta\Theta_{\ell k}^{(m)}$ and $\Delta\Lambda_{\ell k}^{(m)}$ in (11) are stochastically independent, a repeated application of Lemma 1 shows that all $Z_{\ell k}^{(m)}$ are uniformly distributed in $[0, 2\pi)$. Therefore, $\forall m, k, \ell \neq k$ $\mathbb{E}\{C_{\ell k}^{(m)}\} = 0$, since the densities of cosines with in $[0, 2\pi)$ uniformly distributed random arguments are symmetric around zero, which can be proven by common random variable transformation. Finally, from the linearity of expectation operator and the independence between the $C_{\ell k}^{(m)}$ and the sensor readings, it follows that $\mathbb{E}\{\Delta_{1,1}\} \equiv 0$. ■

4.2. No CSI at sensor nodes

In this section we show that, in the case of *independent* fading distributions (correlated fading will not be considered in this paper), no channel knowledge on sensor nodes is necessary if the sink node possesses some statistical knowledge about fading coefficients. More precisely, if we consider only elements of $\mathcal{F}_{\mathrm{D}}$ with the property $\varphi_1 = \cdots = \varphi_K = \varphi$, so that besides the mapped i. i. d. sensor readings $R_k = g(X_k)$ also the pre-processed sensor data $\varphi(R_k)$ are i. i. d, the averaging behavior of the SIMO-WS-MAC itself helps to dramatically reduce channel estimation effort[2].

4.2.1. Block-Fading

Let us suppose that the fading coefficients are constant during the transmission of any sequence of length M. Note that for the special case of block-fading, the direct sum (4) reduces to the Kronecker product $\tilde{\boldsymbol{H}} = \boldsymbol{I}_M \otimes \boldsymbol{H}$, with $\boldsymbol{H} \in \mathbb{C}^{n_R \times K}$ and $\boldsymbol{I}_M$ the

[1] For example this can be done if the sink node initiates function value transmissions through pilot sequences.

[2] Proofs of the results in this section, which are based on variations of the strong law of large numbers, are omitted for space constraints.

$M \times M$ identity matrix. Furthermore, let us suppose that the fading elements H_{nk} of $\boldsymbol{H}$, which are now independent of m, are i. i. d. random variables with $\mathbb{E}\{H_{11}\} = \cdots = \mathbb{E}\{H_{nk}\} = \mu_H \in \mathbb{C}$, $\mathrm{Re}\{\mu_H\}, \mathrm{Im}\{\mu_H\} < \infty$, and finite variances $\mathbb{V}\mathrm{ar}\{H_{11}\} = \cdots = \mathbb{V}\mathrm{ar}\{H_{nk}\} = \sigma_H^2 > 0$. Then, the mathematical characteristic of the SIMO-WS-MAC can be explicitly used by the sink to correct fading effects in the first term of (6), which reduces the channel estimation effort significantly in comparison to Full and Modulus CSI.

Proposition 2. *Suppose that both the first absolute moment of $|H_{11}|^2\varphi(R_1)$ and the expected value $\mathbb{E}\{\varphi(R_1)\}$ exist. Further suppose that the sink knows $\mathbb{E}\{|H_{11}|^2\} = \sigma_H^2 + |\mu_H|^2$ and divides (6) by this expected value. Then, the performance loss due to the lack of CSI at nodes is arbitrarily small provided that $n_R + K$ is sufficiently large.*

Corollary 1. *For the special case $\sigma_H^2 + |\mu_H|^2 = 1$, $n_R + K$ sufficiently large, no channel correction is necessary.*

Remark 2. Proposition 2 and Corollary 1 state only results for the behavior of the first term in (6), which is the term of interest, and say nothing about the behavior of error terms $\Delta_{1,n}$, $\Delta_{2,n}$, $n = 1, \ldots, n_R$. But if there are for example deterministic components in the channel statistics, i. e. $\mathbb{E}\{\mathrm{Re}\{H_{nk}^{(m)}\}\} \neq 0$ and/or $\mathbb{E}\{\mathrm{Im}\{H_{nk}^{(m)}\}\} \neq 0$, it can be shown by simple calculations that $\forall n\ \mathbb{E}\{\Delta_{1,n}\}$, $\mathbb{E}\{\Delta_{2,n}\} \equiv 0$ still holds, independent of fading distributions, so that no systematic error occurs.

The results above indicate that for i. i. d. fading coefficients which are constant over time for some given channel realization, CSI at sensor nodes is not necessary and fading effects can be corrected at the sink by some second order statistical knowledge. Furthermore, the number n_R of receive antenna elements affect the rate of convergence in the law of large numbers, because the averaging encompasses $J = n_R K$ summands. Thus, already $n_R = 2$ generates a noticeable performance gain (cf. Example 2).

Remark 3. By the way, Proposition 2 gives hints for an adequate estimation of $\sigma_H^2 + |\mu_H|^2$, which is required by the sink, i. e. during network initializations, all nodes transmit $\varphi(R_k) = 1$ for large enough M so that the sink immediately receives a satisfactory estimation of the second moment.

4.2.2. I. i. d. Fading

In Section 4.2.1 the fading coefficients were constant for a frame of M symbols so that the first term in (6) reduces to a double sum over antennas n and sensors k. Now we consider the other extreme, where the fading is not only i. i. d. over sensors and antennas, but also i. i. d. over time, as in a fast fading situation. In comparison to the block-fading scenario, the first triple sum in (6) has then $J = n_R K M$ i. i. d. summands. Hence, Proposition 2 can be used again, but the rate of convergence is increased by a factor M, so that in the context of function computation, a fast fading situation is beneficial.

Example 2 (Numerical Example)**.** Consider the widely-used special case of uncorrelated Rician fading: $\forall n, k$ and $\forall m\ H_{nk}^{(m)} \sim \mathcal{N}_{\mathbb{C}}(\sqrt{.125}(1+i), .75)$, where $\sigma_H^2 + |\mu_H|^2 = 1$ is fulfilled. The network example consists of $K = 25$ nodes with sequence length $M = 15$. The sensor readings are i. i. d. uniformly distributed in $\mathcal{X} = [2, 14]$, the desired function is "arithmetic mean", $\sigma_N^2 = 1$, and the performance measure is $\mathbb{P}(|E| \geq \epsilon)$, $\epsilon > 0$, i. e. the probability that the relative estimation error $|E| := |(\hat{f} - f)/f| \geq \epsilon$. A comparison of the block-fading and the i. i. d. case for $n_R = 2, 4$ by Monte Carlo simulations are depicted in Fig. 1.

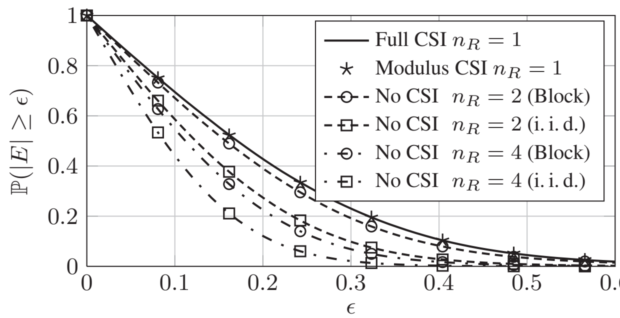

Fig. 1. Full CSI ($n_R = 1$) vs. Modulus CSI ($n_R = 1$) vs. No CSI ($n_R = 2, 4$) for Rician Block- and Rician i. i. d.-fading.

Fig. 1 confirms that Full CSI provides no performance gain in comparison with Modulus CSI. Besides, No CSI outperforms, already for $n_R = 2$ and *small* K, M, the Full CSI case ($n_R = 1$). This indicates that independent fading provides also for the error terms $\Delta_{1,n}$, $\Delta_{2,n}$ a faster convergence to their zero averages.

Remark 4. The plots of Fig. 1 suggest that for the specific Example 2, Corollary 1 could probably strengthened to $\mathbb{P}(|E| \geq \epsilon) \to 0$ for $n_R \to \infty$, since for $n_R > 1$ there are obviously even gains for No CSI in comparison with Full CSI. However, it is currently not clear if such a statement holds for arbitrary desired functions.

4.2.3. Independent but not Identically Distributed Fading

To extend the results from Sections 4.2.1 and 4.2.2, we now consider more general fading distributions, which are still independent but no longer necessarily identically distributed. Hence, it is possible that the elements $H_{nk}^{(m)}$ of (4) have different distributions with existent first moments $\mathbb{E}\{H_{nk}^{(m)}\} = \mu_{nk}^{(m)} \in \mathbb{C}$, $\forall n, k, m$ $\mathrm{Re}\{\mu_{nk}^{(m)}\}, \mathrm{Im}\{\mu_{nk}^{(m)}\} < \infty$, and $\forall n, k, m\ 0 < \mathbb{V}\mathrm{ar}\{H_{nk}^{(m)}\} = (\sigma_{nk}^{(m)})^2 < \infty$. So we can generalize the result of Proposition 2.

Proposition 3. *Let $\forall n, k, m\ W_{nk}^{(m)} := |H_{nk}^{(m)}|^2\varphi(R_k)$ and let their second moments exist and be finite. If n_R, K, M sufficiently large and the fading distributions fulfill $\sum_{n,k,m} \frac{\mathbb{V}\mathrm{ar}\{W_{nk}^{(m)}\}}{(nkm)^2} < \infty$ for $n, k, m \to \infty$, the performance loss due to the lack of CSI at nodes is arbitrary small, provided that fading effects are corrected at the sink by dividing (6) with $\sum_{n,k,m}((\sigma_{nk}^{(m)})^2 + |\mu_{nk}^{(m)}|^2)$.*

Remark 5. It is important to emphasize that the results in Section 4.2 cover a wide range of independent fading distributions, why they are of much interest for application in practical sensor networks.

5. REFERENCES

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