SIMO/MISO MSE-Duality for Multi-User FBMC with Highly Frequency Selective Channels

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***Abstract*—*Filter Bank based Multicarrier* (FBMC) with *Offset-QAM* (O-QAM) modulation is one of the strong candidates for the physical layer of 5-th generation mobile communication systems. The combination of FBMC/OQAM with multiple antennas in a multi-user scheme poses some challenges towards the design of equalizers and precoders due to the frequency overlapping of the sub-carriers, especially in environments with highly frequency selective channels. In this work we first calculate a fractionally spaced *Minimum Mean Square Error* (MMSE)-based equalizer design to accommodate *multi-user* (MU) *Single Input Multiple Output* (SIMO)-FBMC. Then, we apply the *Mean Square Error* (MSE)-duality to calculate the corresponding precoders for MU *Multiple Input Single Output* (MISO)-FBMC. Furthermore, we present four possibilities for the power allocation between users and sub-carriers. We finally show the performance superiority of the new method compared to a recently proposed method.**

I. Introduction

In recent years multi-carrier systems have been at the forefront of communications systems due to their attractive properties at high data rates. *Orthogonal frequency division multiplexing* with a *cyclic prefix* (CP-OFDM) is a widely implemented solution for multi-carrier systems in standards such as IEEE 802.11 (WiFi), LTE or VDSL. Its popularity is partly due to the simple equalization enabled by the CP and the efficient implementation using *Fast Fourier Transform* (FFT and IFFT). However, this comes at the price of a loss in spectral efficiency due to the CP, which is extremely long with the presence of highly frequency selective channels. CP-OFDM additionally suffers from high out-of-band emissions and the necessity of perfect synchronization.

An alternative solution to CP-OFDM are FBMC/OQAM systems which are a strong contender for 5-th generation mobile communication systems. FBMC/OQAM systems have improved spectral efficiency due to the *Synthesis* and *Analysis Filter Banks* (SFB and AFB) at the transmitter and receiver [1], which guarantee higher selectivity in the frequency domain and a much lower out-of-band radiation compared with CP-OFDM [2], [3]. This form of pulse shaping limits the *Inter-Carrier Interference* (ICI), whilst simultaneously attributing to more *Inter-Symbol Interference* (ISI) within each individual sub-carrier. Furthermore, FBMC/OQAM systems are extremely efficient in the presence of highly frequency selective channels. These advantages over CP-OFDM come at the cost of slightly higher computational complexity, however, this is not problematic [4].

When considering a MU, multi-antenna FBMC system using *Space Division Multiple Access* (SDMA), we must introduce multi-tap, fractionally spaced *Finite Impulse Response* (FIR) precoder filters before the transmitter antennas or multi-tap, fractionally spaced FIR equalizer filters after the receiver antennas. These filters are designed not only to compensate the ISI and ICI introduced by the highly frequency selective channel, but also mitigate the *Multi-User Interference* (MUI).

Due to the inter-dependencies and inter-coupling between the precoder filters at the transmitter of a MU-FBMC system, an MMSE-based precoder design is quite complex. Although we only consider ICI with the neighboring sub-carriers, in a MU-FBMC system the precoder design must additionally consider the precoder filters of all other users and also the neighboring sub-carriers. In an MMSE-based equalizer this inter-coupling is not present and therefore the design is simpler.

In [5] an MMSE-based, multi-tap, equalizer filter design per sub-carrier for a *Single Input Single Output* (SISO) setting was introduced to compensate the ISI and ICI in the presence of highly frequency selective channels. However, the implementation in a MU-MISO FBMC system requires further MUI compensation and will be explored in Section III. In [6] a method to calculate *quasi*-MMSE-based precoder filters was investigated for MU-MISO FBMC systems. In [7], [8] further methods to directly calculate precoder filters for FBMC systems were investigated. We use this *quasi*-MMSE-based precoder design as a benchmark for the MSE-duality based precoder designs introduced in this work. It should be noted that the numerical calculation of the precoder filters in [6] suffered from ill conditioned matrices which is not the case in our proposed method for most SNR values.

This paper is organized as follows, in Section II we will introduce the general FBMC system model. In Section III we will explore the MU-SIMO FBMC system and define the *Uplink* (UL) MSE and calculate an MMSE-based equalizer filter. In Section IV we will explore the MU-MISO FBMC system and define the *Downlink* (DL) MSE. In Section V we will explore four different methods to transform out UL MU-SIMO system into an equivalent DL MU-MISO system. In Sections VI and VII we will interpret the simulation results and summarize the results whilst expressing an outlook into further work on this topic.

II. FBMC System Model

An abstract model of a SISO FBMC system is depicted in Fig. 1. The SFB combines the M, complex valued QAM

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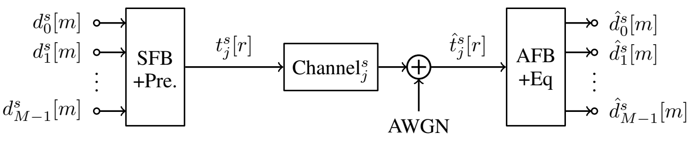

Figure 1. Abstract FBMC System Model

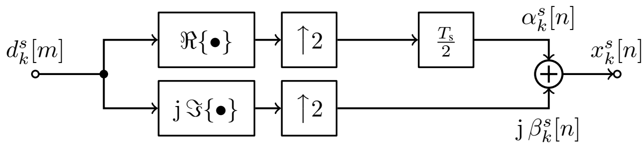

Figure 2. O-QAM staggering, $\mathcal{O}_k$, for odd indexed subcarrier

input signals $d_k^s[m]$ generated at a rate of $1/T_s$, into a single, complex valued signal $t_j^s[r]$ generated at a higher rate of M/T_s. The signal is transmitted across a highly frequency selective additive white Gaussian noise channel to the receiver. In our system, M corresponds to the number of sub-channels and M_u the number of sub-carriers we transmit across. The AFB separates the received signal back into its M components at a low rate per sub-carrier.

The first operation in the SFB is the O-QAM staggering ($\mathcal{O}_k$) of the input symbols $d_k^s[m]$. The structure of the O-QAM operation is depicted in Fig. 2. The input symbol $d_k^s[m]$ is split into its real and imaginary parts, up-sampled by a factor of 2, then depending on which sub-carrier we observe, either the $\Re\{d_k^s[m]\}$ or $\mathrm{j}\,\Im\{d_k^s[m]\}$ symbol is delayed by exactly $T_s/2$ and finally these components are added together. When the sub-carrier index k is even, the $\Re\{d_k^s[m]\}$ symbol is delayed and when the sub-carrier index is odd, the $\mathrm{j}\,\Im\{d_k^s[m]\}$ symbol is delayed. At the output of our $\mathcal{O}_k$ operation we receive a symbol $x_k^s[n]$, which has an O-QAM structure, i.e. each symbol is either purely real or purely imaginary at a double symbol rate relative to the input signals $d_k^s[m]$. Due to this characteristic of the O-QAM symbols, there is a phase change of $\pi/2$ between immediately adjacent sub-carriers, ensuring orthogonality between each sub-carrier. At the receiver, the AFB applies O-QAM de-staggering to reconstruct the complex QAM $\hat{d}_k^s[m]$ symbols from the equalized $\hat{x}_k^s[n]$ symbols.

Since an implementation as described above is not very efficient due to the extremely high data rate, an implementation as a *Modified DFT* (MDFT) filter bank is much more efficient. A MDFT filter bank takes advantage of exponentially modulated, pulse shaping filters given by

$$h_k[r] = h_\text{p}[r]\,\exp\left(\mathrm{j}\,\frac{2\pi}{M}\,k\,\left(r - \frac{L_\text{p}-1}{2}\right)\right),\ r = 0,\ldots,L_\text{p}-1,$$

where $h_\text{p}[r]$ is a lowpass prototype filter with length $L_\text{p} = KM+1$, with K representing the overlapping factor to indicate the number of symbols which overlap in the time domain. K should be kept as small as possible not only to limit the complexity but also to reduce the time-domain spreading of the symbols and the transmission latency. Furthermore, MDFT takes advantage of the polyphase decomposition of $h_\text{p}[r]$ so that the filtering can be performed at a rate of only $2/T_s$.

To minimize the complexity in the calculations of the MU-SIMO equalizer vectors and MU-MISO precoder vectors, we set $K = 4$ and the roll-off factor of our root raised cosine filter equal to one. Thus the frequency response of the filter $\mathbf{h}_k$ only significantly overlaps with the two adjacent filters.

To simplify the ICI notation we define the following filtering operation, $h_{l,j}^s[n] = \left(h_l * h_{\text{ch},j}^s * h_k\right)[r]\,|_{r=n\frac{M}{2}}$. This represents the interference from the transmitter antenna j in sub-carrier l into the receiver antenna of user s in sub-carrier k, with $l \in \{k-1, k, k+1\}$. To simplify notation we do not include the sub-script index of the receiver sub-carrier since the interference is always relative to k. The length of the resulting filter is

$$Q = \left\lceil \frac{2(L_\text{p}-1) + L_\text{ch}}{M/2} \right\rceil .$$

After the O-QAM staggering operation, the input sequences $\mathbf{x}_k^s[n]$ have the structure

$$\mathbf{x}_k^s[n] = \begin{cases} \begin{bmatrix} \alpha_k^s[m] & \mathrm{j}\beta_k^s[m] & \alpha_k^s[m-1] & \cdots \end{bmatrix}^T, & k \text{ is odd,} \\ \begin{bmatrix} \mathrm{j}\beta_k^s[m] & \alpha_k^s[m] & \mathrm{j}\beta_k^s[m-1] & \cdots \end{bmatrix}^T, & k \text{ is even,} \end{cases}$$

where $\alpha_k^s[m]$ and $\beta_k^s[m]$ represent the real and imaginary part of complex modulated QAM input symbol. In the following Sections we work with a purely real notation and therefore define a purely real input sequence as $\mathbf{x}_k^s[n] = \mathbf{J}_k\tilde{\mathbf{x}}_k^s[n]$ with

$$\mathbf{J}_k = \begin{cases} \text{diag}\begin{bmatrix} 1 & \mathrm{j} & 1 & \mathrm{j} & \cdots \end{bmatrix}, & k \text{ is odd,} \\ \text{diag}\begin{bmatrix} \mathrm{j} & 1 & \mathrm{j} & 1 & \cdots \end{bmatrix}, & k \text{ is even.} \end{cases}$$

This extracts the imaginary j's from the input signal. We then multiply the transposed convolution matrix of $h_{l,j}^s[n]$ by $\mathbf{J}_k$ and are left with $\tilde{\mathbf{H}}_{l,j}^s = \mathbf{H}_{l,j}^s\mathbf{J}_k$. It can be shown, [6] and [5], that calculating the precoder or equalizer filters with either the real or imaginary part of the input symbol both result in the same filters.

III. MU-SIMO System

For the MU-SIMO system we define a multi-tap equalizer vector $\mathbf{w}_{k,j}^s \in \mathbb{C}^B$ per user, per sub-carrier and per receiver antenna and a single-tap precoder scalar $b_k^{v,\text{UL}} \in \mathbb{R}_+$ per user and sub-carrier. In the MU-SIMO system we have U decentralized users who transmit the $\mathbf{x}^1, \cdots, \mathbf{x}^U$ input signals over M_u sub-carriers to N_r centralized receiver antennas. We assume there is no channel state information in the transmitter antennas and therefore set the MU-SIMO precoder scalar equal to one $b_k^{v,\text{UL}} = 1$. The real part of our receive signal for user v in sub-carrier k is defined as

$$\hat{\alpha}_k^{v,\text{UL}}[n] = \sum_{j=1}^{N_\text{r}} \bar{\mathbf{w}}_{k,j}^{v,T}\left(\sum_{s=1}^{U}\sum_{l=k-1}^{k+1}\bar{\mathbf{H}}_{l,j}^s\tilde{\mathbf{x}}_l^s[n] + \bar{\boldsymbol{\Xi}}_{k,j}\right), \quad (1)$$

where we define the stacking vectors as

$$\bar{\mathbf{w}}_{k,j}^{v,T} = \begin{bmatrix} \left(\mathbf{w}_{k,j}^{v,T}\right)^{(R)}, & \left(\mathbf{w}_{k,j}^{v,T}\right)^{(I)} \end{bmatrix} \in \mathbb{R}^{1\times 2B},$$

$$\bar{\mathbf{H}}_{l,j}^s = \begin{bmatrix} \left(\tilde{\mathbf{H}}_{l,j}^s\right)^{(R)}, & \left(\tilde{\mathbf{H}}_{l,j}^s\right)^{(I)} \end{bmatrix}^T \in \mathbb{R}^{2B\times(B+Q-1)},$$

$$\bar{\boldsymbol{\Xi}}_{k,j} = \begin{bmatrix} \left(\boldsymbol{\Gamma}_k\boldsymbol{\eta}_j\right)^{(R)}, & \left(\boldsymbol{\Gamma}_k\boldsymbol{\eta}_j\right)^{(I)} \end{bmatrix}^T \in \mathbb{R}^{2B\times 1},$$

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where we define $(\bullet)^{(R)} = \Re\{\bullet\}$, $(\bullet)^{(I)} = \Im\{\bullet\}$ and $\mathbf{\Gamma}_k$ as an $M/2$ downsampled, transposed convolution matrix of $\mathbf{h}_k$ which filters the noise $\boldsymbol{\eta}_j$. For the following derivations we assume the input signals to be *independent and identically distributed* (i.i.d.) and Gaussian distributed. The covariance matrix before O-QAM staggering is defined as

$$\mathrm{E}\left[\mathbf{d}_k^s[m]\mathbf{d}_k^{s,H}[m]\right] = (\sigma_{\mathrm{d}}^2/U)\mathbf{I}$$

and after the O-QAM staggering as

$$\mathrm{E}\left[\tilde{\mathbf{x}}_k^s[n]\tilde{\mathbf{x}}_k^{s,T}[n]\right] = (\sigma_{\mathrm{d}}^2/(2U))\mathbf{I} = \sigma_{\mathrm{M}}^2\mathbf{I}.$$

Furthermore, we assume the additive noise is Gaussian distributed with $\boldsymbol{\eta}_j[n] \sim \mathcal{N}_{\mathbb{C}}\left(\mathbf{0}, \sigma_\eta^2\mathbf{I}\right)$ for each receiver antenna. Additionally, the input signals and the noise are assumed to be uncorrelated as well as the noise between receiver antennas, i.e. $\mathrm{E}\left[\mathbf{x}_k^s\boldsymbol{\eta}_j^H\right] = \mathbf{0}\ \forall\ s,k,j$ and $\mathrm{E}\left[\boldsymbol{\eta}_i\boldsymbol{\eta}_j^H\right] = \mathbf{0}$ for $i \neq j$, respectively. The optimization problem we wish to minimize is expressed with respect to the UL MSE $\epsilon_k^{v,\mathrm{UL}}$ as

$$\begin{aligned}\hat{\mathbf{w}}_k^v &= \underset{\hat{\mathbf{w}}_k^v}{\arg\min}\,\mathrm{E}\left[\left|\hat{\alpha}_k^{v,\mathrm{UL}}[n] - \alpha_k^{v,\mathrm{UL}}[n-\nu]\right|^2\right],\\ &= \underset{\hat{\mathbf{w}}_k^v}{\arg\min}\,\epsilon_k^{v,\mathrm{UL}},\end{aligned} \tag{2}$$

where we define ν as the transmission latency in our system. We solve the optimization problem in (2) similar to [5], arriving at an MMSE-based equalizer filter for all receiver antennas

$$\hat{\mathbf{w}}_k^v = \sigma_{\mathrm{M}}^2\left(\sum_{s=1}^{U}\sum_{l=k-1}^{k+1}\sigma_{\mathrm{M}}^2\hat{\mathbf{H}}_l^s\hat{\mathbf{H}}_l^{s,T} + \hat{\mathbf{R}}_\eta\right)^{-1}\hat{\mathbf{H}}_k^v\mathbf{e}_\nu. \tag{3}$$

Given the MMSE-based equalizer we are left with a simplified, closed form expression for the UL MSE per user and per sub-carrier defined as

$$\epsilon_k^{v,\mathrm{UL}} = \sigma_{\mathrm{M}}^2\left(1 - \mathbf{e}_\nu^T\hat{\mathbf{H}}_k^{v,T}\hat{\mathbf{w}}_k^v\right), \tag{4}$$

with the stacking matrices

$$\begin{aligned}\hat{\mathbf{w}}_k^{v,T} &= \left[\begin{array}{ccc}\bar{\mathbf{w}}_{k,1}^{v,T} & \cdots & \bar{\mathbf{w}}_{k,N_{\mathrm{r}}}^{v,T}\end{array}\right] \in \mathbb{R}^{1\times 2BN_{\mathrm{r}}},\\ \hat{\mathbf{H}}_l^s &= \left[\begin{array}{ccc}\bar{\mathbf{H}}_{l,1}^s & \cdots & \bar{\mathbf{H}}_{l,N_{\mathrm{r}}}^s\end{array}\right]^T \in \mathbb{R}^{2BN_{\mathrm{r}}\times(B+Q-1)},\\ \mathbf{e}_\nu &= \left[\begin{array}{ccccc}0 & \cdots & 0\ 1\ 0 & \cdots & 0\end{array}\right]^T \in \mathbb{R}^{(B+Q-1)\times 1},\\ \hat{\mathbf{R}}_\eta &= \text{blockdiag}\left[\begin{array}{ccc}\bar{\mathbf{R}}_{\eta_1} & \cdots & \bar{\mathbf{R}}_{\eta_{N_{\mathrm{r}}}}\end{array}\right] \in \mathbb{R}^{2BN_{\mathrm{r}}\times 2BN_{\mathrm{r}}},\\ \bar{\mathbf{R}}_{\eta,k} &= \begin{bmatrix}\mathbf{R}_{\eta,k,1} & \mathbf{R}_{\eta,k,2}\\ -\mathbf{R}_{\eta,k,2} & \mathbf{R}_{\eta,k,1}\end{bmatrix} \in \mathbb{R}^{2B\times 2B},\\ \text{with } \mathbf{R}_{\eta,k,1} &= \frac{\sigma_\eta^2}{2}\left(\mathbf{\Gamma}_k^{(R)}\mathbf{\Gamma}_k^{(R),T} + \mathbf{\Gamma}_k^{(I)}\mathbf{\Gamma}_k^{(I),T}\right) \in \mathbb{R}^{B\times B},\\ \mathbf{R}_{\eta,k,2} &= \frac{\sigma_\eta^2}{2}\left(\mathbf{\Gamma}_k^{(R)}\mathbf{\Gamma}_k^{(I),T} - \mathbf{\Gamma}_k^{(I)}\mathbf{\Gamma}_k^{(R),T}\right) \in \mathbb{R}^{B\times B}.\end{aligned}$$

IV. MU-MISO SYSTEM

For the MU-MISO system we define a multi-tap precoder vector $\mathbf{b}_{k,j}^s \in \mathbb{C}^B$ per user, per sub-carrier and per transmitter antenna and a single-tap equalizer scalar $w_k^{v,\mathrm{DL}} \in \mathbb{R}_+$ per user and sub-carrier. In the MU-MISO system we have N_{t} centralized transmitter antennas which transmit the $\mathbf{x}^1, \cdots, \mathbf{x}^U$ input signals over M_{u} sub-carriers to the U decentralized users. The real part of our receive signal for user v in sub-carrier k is defined as

$$\hat{\alpha}_k^{v,\mathrm{DL}}[n] = w_k^{v,\mathrm{DL}}\left(\sum_{s=1}^{U}\sum_{l=k-1}^{k+1}\hat{\mathbf{b}}_l^{s,T}\hat{\mathbf{H}}_l^v\tilde{\mathbf{x}}_l^s[n] + \Re\{\mathbf{h}_k^H\boldsymbol{\eta}^v\}\right), \tag{5}$$

where we define $\hat{\mathbf{b}}_l^s$ and $\hat{\mathbf{H}}_l^v$ as the equivalent stacking vectors of $\hat{\mathbf{w}}_l^s$ and $\hat{\mathbf{H}}_l^v$ from Section III respectively, except with $N_{\mathrm{r}} = N_{\mathrm{t}}$. Again we assume that the input signals $\tilde{\mathbf{x}}_k^s$ are i.i.d. and Gaussian distributed with an equivalent distribution to that defined in Section III. Furthermore, we assume the additive noise is Gaussian distributed with $\boldsymbol{\eta}^s[n] \sim \mathcal{N}_{\mathbb{C}}\left(\mathbf{0}, \sigma_\eta^2\mathbf{I}\right)$ for each user. Additionally, the input signals and the noise are uncorrelated as well as the noise between users, i.e. $\mathrm{E}\left[\mathbf{x}_k^s\boldsymbol{\eta}^{s,H}\right] = \mathbf{0}\ \forall\ s,k$ and $\mathrm{E}\left[\boldsymbol{\eta}^s\boldsymbol{\eta}^{v,H}\right] = \mathbf{0}$ for $s \neq v$, respectively.

The DL MSE expression we wish to minimize is defined as

$$\begin{aligned}\hat{\mathbf{b}}_k^v &= \underset{\hat{\mathbf{b}}_k^v}{\arg\min}\,\mathrm{E}\left[\left|\hat{\alpha}_k^{v,\mathrm{DL}}[n] - \alpha_k^{v,\mathrm{DL}}[n-\nu]\right|^2\right],\\ &= \underset{\hat{\mathbf{b}}_k^v}{\arg\min}\,\epsilon_k^{v,\mathrm{DL}} \quad \text{s. t.} \sum_{v=1}^{U}\sum_{k=1}^{M_{\mathrm{u}}}\left\|\hat{\mathbf{b}}_k^v\right\|_2^2 \leq M_{\mathrm{u}}U.\end{aligned}$$

By plugging (5) into the argument of our optimization problem, we arrive at a close formed expression for the DL MSE per user and per sub-carrier as

$$\begin{aligned}\epsilon_k^{v,\mathrm{DL}} = \sigma_{\mathrm{M}}^2\Bigg(&\left(w_k^{v,\mathrm{DL}}\right)^2\sum_{s=1}^{U}\sum_{l=k-1}^{k+1}\hat{\mathbf{b}}_l^{s,T}\hat{\mathbf{H}}_l^v\hat{\mathbf{H}}_l^{v,T}\hat{\mathbf{b}}_l^s + 1\\ &- 2w_k^{v,\mathrm{DL}}\hat{\mathbf{b}}_k^{v,T}\hat{\mathbf{H}}_k^v\mathbf{e}_\nu\Bigg) + \left(w_k^{v,\mathrm{DL}}\right)^2\sigma_\eta^2\left\|\mathbf{h}_{\mathrm{p}}\right\|_2^2,\end{aligned} \tag{6}$$

where $\mathbf{h}_{\mathrm{p}} \in \mathbb{R}^{L_{\mathrm{p}}}$ is a vector containing prototype filter coefficients $h_{\mathrm{p}}[r]$.

We observe in (6) that the DL MSE expression for user v in sub-carrier k not only depends on the precoder vector $\hat{\mathbf{b}}_k^v$, but it additionally depends on the precoder vectors of the neighboring sub-carriers as well as the precoder vectors from all other users in the system. This interdependency between precoder vectors makes the minimization of the MSE more difficult than in the MU-SIMO system.

V. MSE-DUALITY TRANSFORMATION

In this Section we investigate four different methods of transforming our UL SIMO system into an equivalent DL MISO system using the duality principle as introduced in [9] and [10]. The basic principle behind an MSE-duality transformation is to switch the roles of the UL and DL filters, i.e. we interchange each receiver filter in the UL system with the respective transmitter filter in the DL system. As the dual DL system has purely transmitter processing, we must ensure that the transmit power is subsequently limited, thus we weigh every transmitter filter with a strictly real value and multiply the receiver with the inverse weighting factor.

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We investigate the different levels of duality transformations similar to those defined in [10]. These different duality transformations are summarized as follows:

- In Subsection V-A we attempt to preserve the *System-Wide Sum-MSE*, equivalent to a *Level 1* transformation from [10]. This is the simplest form of duality where we keep the total sum-MSE for all users and sub-carriers equal when transforming the UL to a DL system. We only require a single scaling factor which leads to relatively low computational complexity.
- In Subsection V-B we attempt to preserve the *User-Wise Sum-MSE*, equivalent to a *Level 2* transformation from [10]. This method preserves the sum-MSE per user resulting in an individual scaling factor per user for all sub-carriers and transmitter antennas. We have to solve a linear system of equations for U scaling factors, thus leading to a higher computational complexity than a *Level 1* transformation.
- In Subsection V-C we attempt to preserve the *Sub-Carrier-Wise Sum-MSE*, equivalent to a *Level 2* transformation from [10]. This method preserves the sum-MSE per sub-carrier resulting in a distinct scaling factor per sub-carrier for all users and transmitter antennas. Again, we have to solve a linear system of equations for the M_u scaling factors, which leads to a higher computational complexity than a *User-Wise Sum-MSE* and *Level 1* transformation.
- In Subsection V-D we attempt to preserve the *User and Sub-Carrier-Wise MSE*, equivalent to a *Level 3* transformation from [10]. This method preserves the individual MSE for every single user and sub-carrier. This results in a scaling factor per user and per sub-carrier for all transmitter antennas. Evidently, a *Level 3* duality transformation requires the highest computational complexity because we must solve for $M_\mathrm{u} \times U$ different scaling factors. However, this method guarantees that the individual MSE per user and per sub-carrier stays equal in the UL SIMO and the DL MISO systems.

In all these MSE-duality transformations the total power is preserved [9], [10], i.e. $\sum_{v=1}^{U}\sum_{k=1}^{M_\mathrm{u}} \left\|\hat{\mathbf{b}}_k^v\right\|_2^2 \leq M_\mathrm{u} U$.

A. System-Wide Sum-MSE

First, we define a relation between the UL and DL filters with a single, real-valued scaling factor such that

$$\begin{aligned} \hat{\mathbf{b}}_k^s &= \gamma \hat{\mathbf{w}}_k^s, \\ w_k^{s,\mathrm{DL}} &= \gamma^{-1}, \end{aligned} \tag{7}$$

with $\gamma \in \mathbb{R}_+$ and recalling that the UL precoder scalar is set such that $b_k^{v,\mathrm{UL}} = 1,\ \forall v, k$. In the next step we set the system-wide sum-MSE equal between the UL and the DL system, i.e. we sum over all users and all sub-carriers and set these MSE values to be equal

$$\sum_{v=1}^{U}\sum_{k=1}^{M_\mathrm{u}} \epsilon_k^{v,\mathrm{DL}} \overset{!}{=} \sum_{v=1}^{U}\sum_{k=1}^{M_\mathrm{u}} \epsilon_k^{v,\mathrm{UL}}, \tag{8}$$

where the relation $\overset{!}{=}$ implies both sides of the equation must be equal. By solving (8) we can calculate a single scaling factor γ for all users, sub-carriers and transmitter antennas

$$\gamma^2 = \frac{M_\mathrm{u} U \sigma_\eta^2 \|\mathbf{h}_\mathrm{p}\|_2^2}{\sum_{v=1}^{U}\sum_{k=1}^{M_\mathrm{u}} \sigma_\mathrm{M}^2 \left(\hat{\mathbf{w}}_k^{v,T}\hat{\mathbf{H}}_k^v \mathbf{e}_\nu - \sum_{s=1}^{U}\sum_{l=k-1}^{k+1} \hat{\mathbf{w}}_l^{s,T}\hat{\mathbf{H}}_l^v \hat{\mathbf{H}}_l^{v,T}\hat{\mathbf{w}}_l^s\right)}. \tag{9}$$

By calculating only a single weighting factor this method of duality transformation guarantees that the system-wide sum-MSE remains equal after the transformation, however, each individual user MSE and sub-carrier MSE can change. Furthermore, this method considers all users, sub-carriers and transmitter antennas when spreading the total available transmit power amongst them. This means that the users and sub-carriers with the overall worst channels in the system get more power to compensate for their channels. However, it should be noted that if a certain user in a certain sub-carrier suffers from extremely large MSE, this user will obtain a disproportionate amount of transmit power with respect to the other users and sub-carriers.

B. User-Wise Sum-MSE

Next, we define a relation between the UL and DL filters with a real-valued scaling factor per user such that

$$\begin{aligned} \hat{\mathbf{b}}_k^s &= \gamma^s \hat{\mathbf{w}}_k^s, \\ w_k^{s,\mathrm{DL}} &= \left(\gamma^s\right)^{-1}, \end{aligned} \tag{10}$$

with $\gamma^s \in \mathbb{R}_+$ and recalling that the UL precoder scalar is set such that $b_k^{v,\mathrm{UL}} = 1,\ \forall v, k$. Following this, we set the user-wise sum-MSE equal between the UL and the DL system, i.e. we sum over all sub-carriers and set them equal per user

$$\sum_{k=1}^{M_\mathrm{u}} \epsilon_k^{v,\mathrm{DL}} \overset{!}{=} \sum_{k=1}^{M_\mathrm{u}} \epsilon_k^{v,\mathrm{UL}}, \quad \forall v. \tag{11}$$

We end up with a system of linear equations to solve for U scaling factors γ^s,

$$\mathbf{A}^s \begin{bmatrix} \left(\gamma^1\right)^2 \\ \vdots \\ \left(\gamma^U\right)^2 \end{bmatrix} = M_\mathrm{u} \sigma_\eta^2 \|\mathbf{h}_\mathrm{p}\|_2^2 \mathbf{1}_U, \tag{12}$$

where the matrix $\mathbf{A}^s \in \mathbb{R}^{U \times U}$ has strictly positive main diagonal elements and strictly negative off-diagonal elements and we define $\mathbf{1}_U$ as the all-one vector of length U. The matrix $\mathbf{A}^s$ is defined as

$$\left[\mathbf{A}^s\right]_{v,y} = \begin{cases} \sum_{k=1}^{M_\mathrm{u}}\sum_{l=k-1}^{k+1} \sigma_\mathrm{M}^2 \left(\hat{\mathbf{w}}_k^{v,T}\hat{\mathbf{H}}_k^v \mathbf{e}_\nu \right. \\ \qquad \left. - \hat{\mathbf{w}}_l^{v,T}\hat{\mathbf{H}}_l^v \hat{\mathbf{H}}_l^{v,T}\hat{\mathbf{w}}_l^v\right), & \text{if } v = y, \\ -\sum_{k=1}^{M_\mathrm{u}}\sum_{l=k-1}^{k+1} \sigma_\mathrm{M}^2 \hat{\mathbf{w}}_k^{y,T}\hat{\mathbf{H}}_l^v \hat{\mathbf{H}}_l^{v,T}\hat{\mathbf{w}}_k^y, & \text{if } v \neq y. \end{cases} \tag{13}$$

This form of duality transformation guarantees that each individual user's sum-MSE stays equal in the UL and the DL system. Therefore, this method can be interpreted as allocating an equal amount of transmit power to each user whilst allowing each user to spread this transmit power across the sub-carriers

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as required. Again, the disadvantage of this method arises if the MSE of certain sub-carriers is disproportionately large. This leads to these sub-carriers obtaining a greater amount of transmit power to compensate their channels.

C. Sub-Carrier-Wise Sum-MSE

Next, we define a relation between the UL and DL filters with a real-valued scaling factor per sub-carrier such that

$$\begin{aligned} \hat{\mathbf{b}}_k^s &= \gamma_k \hat{\mathbf{w}}_k^s, \\ w_k^{s,\mathrm{DL}} &= \gamma_k^{-1}, \end{aligned} \tag{14}$$

with $\gamma_k \in \mathbb{R}_+$ and recalling that the UL precoder scalar is set such that $b_k^{v,\mathrm{UL}} = 1,\ \forall v, k$. Next we set the sub-carrier-wise sum MSE equal between the UL and the DL system, i.e. we sum over all users and set the sum-MSE values equal per sub-carrier

$$\sum_{v=1}^{U} \epsilon_k^{v,\mathrm{DL}} \overset{!}{=} \sum_{v=1}^{U} \epsilon_k^{v,\mathrm{UL}}, \quad \forall k. \tag{15}$$

We end up with a system of linear equations to solve for M_u scaling factors γ_k

$$\mathbf{A}^k \begin{bmatrix} \gamma_1^2 \\ \vdots \\ \gamma_{M_\mathrm{u}}^2 \end{bmatrix} = U\sigma_\eta^2 \|\mathbf{h}_\mathrm{p}\|_2^2 \mathbf{1}_{M_\mathrm{u}}, \tag{16}$$

where the tri-diagonal matrix $\mathbf{A}^k \in \mathbb{R}^{M_\mathrm{u} \times M_\mathrm{u}}$ has strictly positive elements on the main diagonal and strictly negative off-diagonal elements. This matrix is defined as

$$\left[\mathbf{A}^k\right]_{k,m} = \begin{cases} \sum\limits_{v,s=1}^{U} \sigma_\mathrm{M}^2 \left(\hat{\mathbf{w}}_k^{v,T} \hat{\mathbf{H}}_k^v \mathbf{e}_\nu \right. \\ \qquad \left. - \hat{\mathbf{w}}_k^{s,T} \hat{\mathbf{H}}_k^v \hat{\mathbf{H}}_k^{v,T} \hat{\mathbf{w}}_k^s \right), & \text{if } k = m, \\ -\sum\limits_{v,s=1}^{U} \sigma_\mathrm{M}^2 \hat{\mathbf{w}}_{k+1}^{s,T} \hat{\mathbf{H}}_{k+1}^v \hat{\mathbf{H}}_{k+1}^{v,T} \hat{\mathbf{w}}_{k+1}^s, & \text{if } m = k+1, \\ -\sum\limits_{v,s=1}^{U} \sigma_\mathrm{M}^2 \hat{\mathbf{w}}_{k-1}^{s,T} \hat{\mathbf{H}}_{k-1}^v \hat{\mathbf{H}}_{k-1}^{v,T} \hat{\mathbf{w}}_{k-1}^s, & \text{if } m = k-1, \\ 0 & \text{else.} \end{cases} \tag{17}$$

This method of duality transformation guarantees that the sum-MSE per sub-carrier stays equal in the UL and the DL system. Therefore, by applying this duality transformation we allocate an equal amount of transmit power to each sub-carrier but allow the transmit power to be spread across the users transmitting in that sub-carrier. Again, this transformation has the disadvantage that if a certain user suffers from high MSE in a certain sub-carrier, that user will obtain a disproportionate amount of transmit power with respect to the other users.

D. User and Sub-Carrier-Wise MSE

Finally, we define a relation between the UL and DL filters with a real-valued scaling factor per user and per sub-carrier such that

$$\begin{aligned} \hat{\mathbf{b}}_k^s &= \gamma_k^s \hat{\mathbf{w}}_k^s, \\ w_k^{s,\mathrm{DL}} &= \left(\gamma_k^s\right)^{-1}, \end{aligned} \tag{18}$$

with $\gamma_k^s \in \mathbb{R}_+$ and recalling that the UL precoder is set such that $b_k^{v,\mathrm{UL}} = 1,\ \forall v, k$. We then set the user and sub-carrier-wise sum-MSE equal between the UL and the DL system, i.e. we set the individual MSE expressions per user and per sub-carrier equal such that

$$\epsilon_k^{v,\mathrm{DL}} \overset{!}{=} \epsilon_k^{v,\mathrm{UL}}, \quad \forall v, k. \tag{19}$$

We end up with a system of linear equations to solve for $U \times M_\mathrm{u}$ scaling factors γ_k^s

$$\begin{bmatrix} \mathbf{A}^{1,1} & \mathbf{A}^{1,2} & \cdots & \mathbf{A}^{1,U} \\ \mathbf{A}^{2,1} & \mathbf{A}^{2,2} & \cdots & \mathbf{A}^{2,U} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{A}^{U,1} & \mathbf{A}^{U,2} & \cdots & \mathbf{A}^{U,U} \end{bmatrix} \begin{bmatrix} \boldsymbol{\gamma}^1 \\ \boldsymbol{\gamma}^2 \\ \vdots \\ \boldsymbol{\gamma}^U \end{bmatrix} = \sigma_\eta^2 \|\mathbf{h}_\mathrm{p}\|_2^2 \mathbf{1}_{M_\mathrm{u} \times U}, \tag{20}$$

where $\mathbf{A}^{v,s} \in \mathbb{R}^{M_\mathrm{u} \times M_\mathrm{u}}$ and $\boldsymbol{\gamma}^v \in \mathbb{R}_+^{M_\mathrm{u}}$. We stack the scaling factors per user and define the following vector

$$\boldsymbol{\gamma}^v = \left[\ (\gamma_1^v)^2, \quad (\gamma_2^v)^2, \quad \cdots, \quad (\gamma_{M_\mathrm{u}}^v)^2 \ \right]^T. \tag{21}$$

We define the tri-diagonal matrices $\mathbf{A}^{v,v}$ and $\mathbf{A}^{v,s}$ for $s \neq v$ as follows

$$\left[\mathbf{A}^{v,v}\right]_{k,j} = \begin{cases} \sigma_\mathrm{M}^2 \left(\hat{\mathbf{w}}_k^{v,T} \hat{\mathbf{H}}_k^v \mathbf{e}_\nu - \hat{\mathbf{w}}_k^{v,T} \hat{\mathbf{H}}_k^v \hat{\mathbf{H}}_k^{v,T} \hat{\mathbf{w}}_k^v \right), & \text{if } k = j, \\ -\sigma_\mathrm{M}^2 \hat{\mathbf{w}}_{k-1}^{v,T} \hat{\mathbf{H}}_{k-1}^v \hat{\mathbf{H}}_{k-1}^{v,T} \hat{\mathbf{w}}_{k-1}^v, & \text{if } j = k-1, \\ -\sigma_\mathrm{M}^2 \hat{\mathbf{w}}_{k+1}^{v,T} \hat{\mathbf{H}}_{k+1}^v \hat{\mathbf{H}}_{k+1}^{v,T} \hat{\mathbf{w}}_{k+1}^v, & \text{if } j = k+1, \\ 0, & \text{else.} \end{cases} \tag{22}$$

$$\left[\mathbf{A}^{v,s}\right]_{k,j} = \begin{cases} -\sigma_\mathrm{M}^2 \hat{\mathbf{w}}_k^{s,T} \hat{\mathbf{H}}_k^v \hat{\mathbf{H}}_k^{v,T} \hat{\mathbf{w}}_k^s, & \text{if } k = j, \\ -\sigma_\mathrm{M}^2 \hat{\mathbf{w}}_{k-1}^{s,T} \hat{\mathbf{H}}_{k-1}^v \hat{\mathbf{H}}_{k-1}^{v,T} \hat{\mathbf{w}}_{k-1}^s, & \text{if } j = k-1, \\ -\sigma_\mathrm{M}^2 \hat{\mathbf{w}}_{k+1}^{s,T} \hat{\mathbf{H}}_{k+1}^v \hat{\mathbf{H}}_{k+1}^{v,T} \hat{\mathbf{w}}_{k+1}^s, & \text{if } j = k+1, \\ 0 & \text{else.} \end{cases} \tag{23}$$

By calculating a weighting factor per user and per sub-carrier, this method of duality transformation guarantees that the individual user and sub-carrier MSE stays equal between the UL and the DL system. Therefore by applying this method of duality transformation we cannot spread the transmit power amongst the users or sub-carriers but instead we normalize the filter in each sub-carrier for each user individually.

VI. Simulation Results

In this Section we discuss the simulation results of all four MSE-duality transformations taking the MMSE-based precoder design from [6] as a reference. For the MU-MISO FBMC system we used channel realizations from the *Wireless World Initiative New Radio* (WINNER II) project which is an extension to the *Spacial Channel Model* (SCM) [11] developed by *3rd Generation Partnership Project* (3GPP) . Using WINNER II in MATLAB, we had an array of libraries to generate both different layouts and channels, dependent on the sampling rate, f_s, the number of transmitter antennas, N_t, and the total number of users, U.

Throughout our simulations we transmitted data across $M_\mathrm{u} = 210$ of the available $M = 256$ sub-carriers per user and per transmitter antenna. We used a sampling rate

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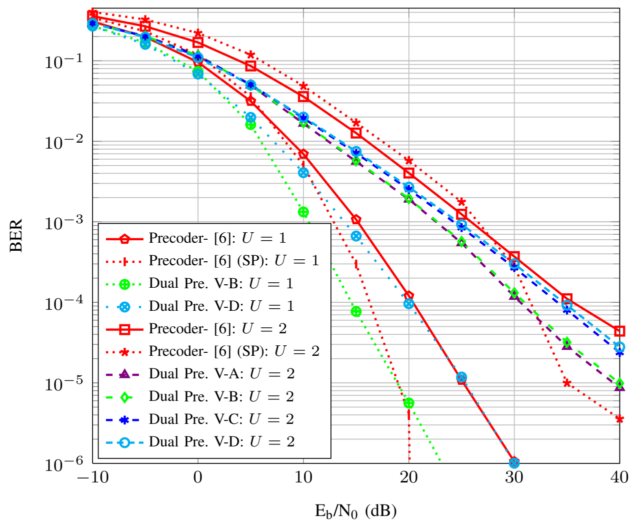

Figure 3. BER of the Precoder- [6] and the four Dual Precoder designs for $N_t = 4$ and varying $U = 1$ and $U = 2$

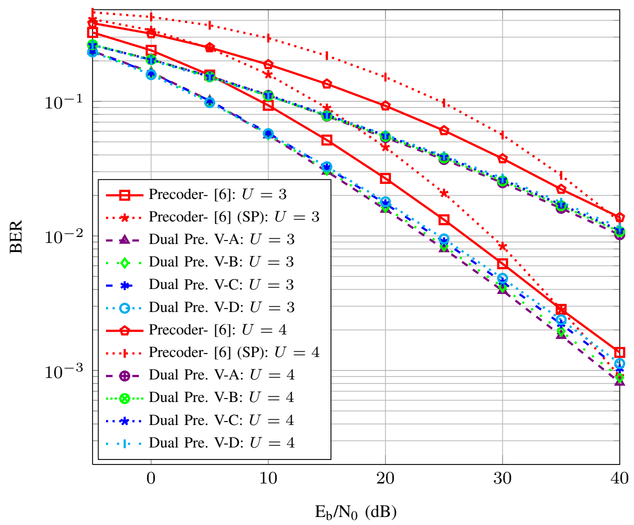

Figure 4. BER of the Precoder- [6] and the four Dual Precoder designs for $N_t = 4$ and varying $U = 3$ and $U = 4$

of $f_s = 11.2$MHz. We used randomly generated 16-QAM symbols and took a block length of 1000 symbols per sub-carrier. We had a channel impulse response of $L_{ch} = 124$ taps. With the chosen system configurations, especially due to $L_{ch} = 124$ and the highly frequency selective channel, a CP-OFDM system would have required a CP with a minimum length of 123 taps [2], [3]. This would have limited the data-throughput of the CP-OFDM to almost 50%, therefore we have not included a direct comparison in the simulation results. Throughout the simulations we took the quantity of E_b/N_0 to be a pseudo-signal-to-noise ratio per user for the MU-MISO simulations. We took the *Bit Error Rate* (BER) and MSE as an average over all users. We took an average over 500 randomly generated channel realizations.

We investigated a system with a precoder vector of length $B = 5$ taps. We set $N_t = 4$, whilst varying the number of users $U \in \{1, 2, 3, 4\}$ in the system. For the MSE plots (Figure 5, Figure 6) we used a solid line to represent the measured MSE

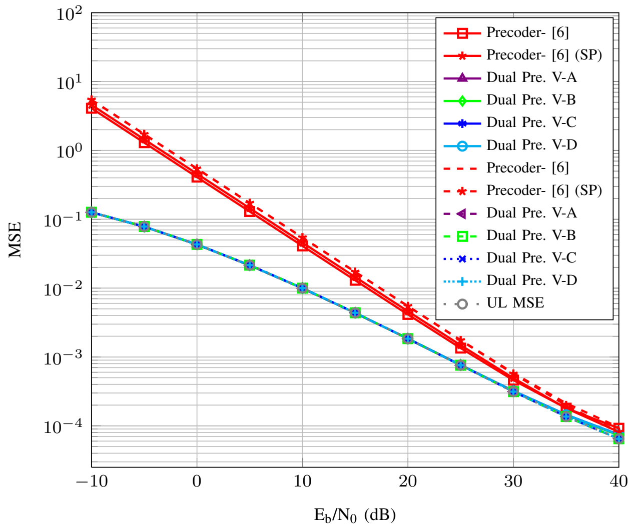

Figure 5. Analytical (Dotted) and Simulated (Solid) MSE of the Precoder- [6] and the four Dual Precoder designs for $N_t = 4$ and $U = 2$

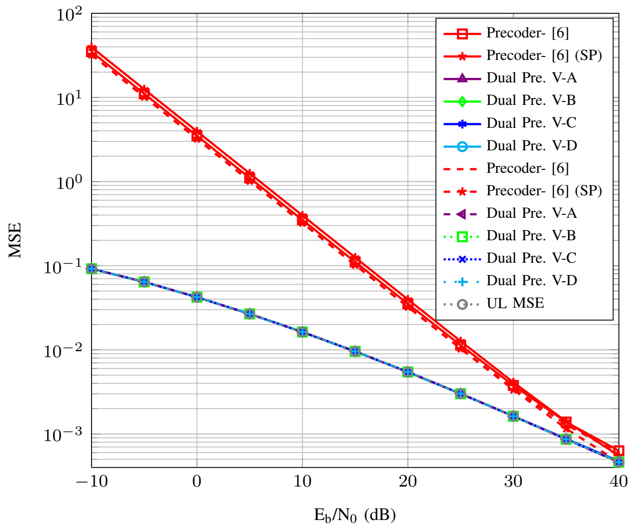

Figure 6. Analytical (Dotted) and Simulated (Solid) MSE of the Precoder- [6] and the four Dual Precoder designs for $N_t = 4$ and $U = 3$

and a dotted line to represent the analytical MSE from the formulas. We added the analytical MSE for the UL system to show that after the duality transformation the MSE has in fact remained unchanged.

Figure 3 shows the results of the MSE-duality versus the precoder- [6] design with two methods of transmit power normalization for $U = 1, 2$. The precoder- [6] design with a solid line used a user and sub-carrier wise normalization and the dotted plot used a *Sum-Power* (SP) normalization of the transmit power. Both the precoder- [6] design and the dual precoder designs benefited from a diversity gain. Furthermore, it should be noted that for $U = 1$ the dual precoder V-A was equivalent to the dual precoder V-B and the dual precoder V-C was equivalent to the dual precoder V-D since we summed over one user, therefore only one of each was plotted. In general, the dual precoders outperformed the precoder- [6] design throughout most of the E_b/N_0 regime. The systems where we used the V-A and the V-B transformation showed the most extreme performance improvements. This is due to the fact that these methods used a technique equivalent to inverse water filling and allocated more transmit power to those sub-carriers with poor channels. A similar behavior was observed in the precoder- [6] (SP) design where the transmit power was spread over all users and sub-carriers. As the number of users in the system was increased, a clear degradation of performance was observed in the gradient of the plots. Nevertheless, the dual precoders outperformed the precoder- [6] design with user and sub-carrier transmit power normalization. The precoder- [6] (SP) design showed improvements in higher E_b/N_0 values.

After the number of users was increased to $U = 3, 4$ in Figure 4 further degradation in system performance was observed as well as a smaller difference between the different dual precoders. The dual precoders outperformed the precoder- [6] design for both system assemblies and over almost the whole E_b/N_0 regime. This was probably due to the fact that the precoder- [6] design suffered from extra ICI since the precoders only minimized the *quasi*-MSE and the noise covariance matrix was not taken into account in the calculations. However, when the (SP) of the precoder- [6] design was normalized an improved BER for high E_b/N_0 values was observed. When the system was at full capacity in Figure 4, i.e. when $N_\mathrm{t} = U = 4$, the different dual transformations showed little difference in terms of BER performance.

The MSE plots, Figures 5 and 6, showed that the precoder- [6] design does not saturate for low E_b/N_0 values, unlike the dual precoders. This was due to the fact that the noise was not included in the precoder- [6] design calculations and therefore the MSE does not saturate for low values of E_b/N_0. We also observed that the more users in the system, the greater the difference in MSE between the dual precoders and the precoder- [6] design. Furthermore, it was observed in Figures 5 and 6 that both the analytical and simulated MSE results stayed equal between the UL and DL systems for all duality transformations.

For higher E_b/N_0 values we observed ill-conditioned matrices when calculating the equalizer vectors (3) and scaling factors from Section V-B, we therefore heuristically investigated a threshold for the Moore-Penrose pseudo-inverse and a parameter for a regularization matrix [6]–[8]. For the results showed in this Section we took a threshold for the pseudo-inverse as in [6].

VII. Conclusions

In this contribution we proposed a new method to design dual precoders for a MU-MISO FBMC system from the equalizers of a MU-SIMO FBMC system. The techniques investigated were based on the MSE-duality transformation between an UL and a DL system, where we attempted to conserve either the *System-Wide Sum-MSE*, the *User-Wise Sum-MSE*, the *Sub-Carrier-Wise Sum-MSE* or the *User and Sub-Carrier-Wise MSE*. Throughout our simulations we observed that the *System-Wide Sum-MSE* performed the best over the whole E_b/N_0 regime, which could be explained by its equivalence to an inverse water-filling technique, i.e. the poorer channels are allocated more power to improve their performance. By first designing equalizer filters for the MU-SIMO FBMC system on a per sub-carrier basis to compensate for ISI, ICI and MUI and then applying the MSE-duality transformation, we eliminated the inter-user and inter-sub-carrier dependencies of the precoder filter from the design of the MMSE-based precoder filters. These interdependencies were problematic in the precoder- [6] design. The dual precoders all outperformed the original precoder- [6] design in terms of BER and MSE over a wide SNR range. Furthermore, the MSE values of the dual precoder designs all outperformed the precoder- [6] design and saturated where the precoder- [6] design did not.

Some extensions of this work include investigating non-linear *Decision Feedback Equalization* (DFE) and *Tomlinson Harashima Precoding* (THP), adding multi-tap equalizers in the receivers of the MU-MISO FBMC system and, finally, multi-streaming in a MU-MIMO FBMC system.

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References

[1] P. Siohan, C. Siclet, and N. Lacaille, "Analysis and design of OFDM/OQAM systems based on filterbank theory," *IEEE Trans. Signal Process.*, vol. 50, no. 5, pp. 1170–1183, 2002.

[2] L. G. Baltar, D. Waldhauser, and J. A. Nossek, "Out-of-band radiation in multicarrier systems: A comparison," in *6th International Workshop on Multi-Carrier Spread-Spectrum*, May 2007, Herrsching, Germany.

[3] L. G. Baltar and J. A. Nossek, "Multicarrier systems: a comparison between filter bank based and cyclic prefix based OFDM," in *OFDM 2012, Proceedings of 17th International OFDM Workshop 2012 (InOWo'12);*, 2012, pp. 1–5.

[4] L. G. Baltar, F. Schaich, M. Renfors, and J. A. Nossek, "Computational complexity analysis of advanced physical layers based on multicarrier modulation," in *Future Network Mobile Summit (FutureNetw), 2011*, June 2011, pp. 1–8.

[5] D. S. Waldhauser, L. G. Baltar, and J. A. Nossek, "MMSE subcarrier equalization for filter bank based multicarrier systems," in *Signal Processing Advances in Wireless Communications, 2008. SPAWC 2008. IEEE 9th Workshop on*, 2008, pp. 525–529.

[6] M. Newinger, L. G. Baltar, A. Swindlehurst, and J. A. Nossek, "MISO broadcasting FBMC system for highly frequency selective channels," in *Smart Antennas (WSA), 2014 18th International ITG Workshop on*, 2014, pp. 1–7.

[7] Y. Cheng, L. G. Baltar, M. Haardt, and J. A. Nossek, "Precoder and equalizer design for multi-user MIMO FBMC/OQAM with highly frequency selective channels," in *IEEE International Conference on Acoustics, Speech and Signal Processing (ICASSP) 2015, Brisbane, Australia*, 2015.

[8] Y. Cheng, M. Haardt, L. G. Baltar, and J. A. Nossek, "Downlink per-user multi-streaming for FBMC/OQAM based multi-user MIMO with highly frequency selective channels," in *International ITG Workshop on Smart Antennas, 2015, Ilmenau, Germany*, 2015.

[9] A. Mezghani, M. Joham, R. Hunger, and W. Utschick, "Transceiver design for multi-user MIMO systems," in *International ITG Workshop on Smart Antennas*, 2006, Ulm, Germany.

[10] R. Hunger, M. Joham, and W. Utschick, "On the MSE-duality of the broadcast channel and the multiple access channel," *WSA*, vol. 57, no. 2, pp. 698–713, 2009.

[11] L. Hentilä, P. Kyösti, M. Käske, M. Narandzic, and M. Alatossava. (2007, December) MATLAB implementation of the WINNER Phase II Channel Model ver1.1.

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