Dual-Polarization Time Delay Estimation for Multipath Mitigation

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***Abstract*—Line-of-sight (LOS) delay estimation in multipath scenarios is a central problem in global navigation satellite systems (GNSS). Deterministic channel models can be used to describe the multipath environment, but this usually requires the estimation of several nuisance parameters. In order to avoid this effort, stochastic channel models can be used. In this case the multipath statistics have to be estimated. Besides tackling the problem of multipath mitigation by exploiting the spatial diversity of LOS and multipath, polarization diversity also offers opportunities to improve the estimation performance. State-of-the-art GNSS use right-hand-circular-polarization (RHCP) transmit antennas and GNSS receivers use RHCP receive antennas. Due to multipath reflections, the signals also contain left-hand-circular-polarization (LHCP) signals, and using the LHCP antenna output, can provide an additional performance gain if the channel is modeled properly. Thus, in this paper we model and discuss exploitation of the GNSS dual-polarization channel. Using a dual-polarization model that accounts for antenna cross-talk and signal reflections intensifies the problem of computational complexity for deterministic multipath models. Therefore, we propose a correlated path (CP) model that describes the temporal correlation between the LOS and multipath signals in a stochastic way. Besides providing a significant reduction in model complexity, the CP model avoids model order estimation and a decision on the actual LOS delay from the estimation results, which is a problem if the LOS and multipath signals are highly correlated.**

I. Introduction

Many signal processing applications rely on exact channel estimates. Multiple input multiple output (MIMO) communication systems use angle-of-arrival (AOA) estimates to apply beamforming, which improves the system performance. In mobile communication systems the channel response is measured for equalization. In GNSS LOS delay estimates are used to determine the position of a receiver.

GNSS exploit the fact that the propagation time between transmitter and receiver can be estimated by correlating the received signal with a local replica of the transmitted signal. As the propagation delay is proportional to the distance between transmitter and receiver, the receiver's position can be determined by triangulation. One severe problem in GNSS is the reception of different replicas additional to the LOS signal. These replicas occur from reflections and can lead to high performance losses if not considered in the estimation process. In the literature different solutions for the multipath mitigation problem have been discussed.

The space-alternating generalized expectation-maximization (SAGE) algorithm [1] is a well known algorithm for the estimation of LOS and multipath delays with spatially unstructured models. Unstructured models employ a general spatial signature that is independent of the AoA. If the AoA of the LOS is also of interest, a spatially structured model that includes the AoA dependency has to be used. An additional estimation of the AoA of the LOS and multipath signal can lead to a performance gain with respect to spatially unstructured models. In [2] an estimation algorithm for a spatially structured model is proposed, while [3] combines the SAGE algorithm with a spatially structured model. The latter method has the drawback of rather high computational complexity. [4] proposes a two-step approach for single-polarization antennas, which first estimates the delays in a spatially unstructured model with the SAGE algorithm and then uses the extended invariance principle (EXIP) to estimate the AoA and update the estimates of the time delay. In this case a two-dimensional search over the delay and AoA regime can be replaced by two one-dimensional searches which reduces the computational complexity.

All of these methods have in common that the multipath delays and AoA are modeled as deterministic parameters which have to be estimated. As the multipath parameters are often unimportant for the application, significant computation time is spent on the estimation of nuisance parameters. Therefore, other approaches model the multipath signal as a stochastic process which requires an estimation of the multipath statistics. In [5], [6] the multipath components are modeled as colored noise which is independent of the LOS. Nevertheless, especially for small delay differences between LOS and multipath signal, this assumption can be inaccurate.

The deterministic models mentioned above assess the problem of multipath mitigation by exploiting the spatial diversity of the LOS and multipath signal. However, especially in cases of highly spatially correlated LOS and multipath, spatial diversity methods are not able to separate the impinging signals. In these cases the signal polarization can be used to separate the signals. State-of-the-art GNSS use RHCP transmit antennas. Circular polarization signals can be produced by using two crossed linear polarization antennas which transmit the same signal with a 90° phase shift. For angles of reflection greater than the Brewster angle, the rotation of the circular polarization reverses, i.e. a RHCP signal becomes a LHCP signal and vice-versa. Therefore, for an odd number of reflections, the multipath signal is a LHCP signal and if the receiver has a RHCP antenna it rejects the LHCP multipath signal [7]. GNSS circular polarization

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receive antennas usually have only a RHCP output while the LHCP output is terminated with $50\,\Omega$ and not used. However, polarimetry can be used to achieve a polarization gain [8], [9] if both the RHCP and LHCP outputs are used. For this dual polarization processing the multipath signal model has to be supplemented by additional parameters. In [10] a model for the circular polarization multipath environment in GNSS has been derived, although the additional parameters increase the computational complexity for deterministic multipath models. Another problem is how to include the additional information into existing models [9], [11], especially if non-ideal receivers are used.

In the work at hand we propose a new signal model for the multipath propagation problem with a dual-polarization antenna array, which describes the LHCP signals as additional channels in the multi-antenna model. As the model exploits the correlation between LOS and multipath signals and performs a rank-one approximation of the multipath's spatial signatures [12, p. 601], it is called the correlated path (CP) model. The CP model combines the statistical and deterministic approaches mentioned above and leads to a significant reduction of parameters to estimate. In particular, the number of parameters to estimate is constant, even for a high number of multipath signals. In comparison to deterministic multipath estimators a model order estimation is not needed. Model order estimation, e.g. with the Akaike information criterion [13] or minimum description length criterion [14] can be inaccurate for highly correlated LOS and multipath signals. Moreover, common deterministic multipath estimation techniques do not separate the LOS from the multipath delays and determining the actual LOS delay can be a problem. On the other hand the CP model yields only the LOS delay. In order to reduce computational complexity, the CP model is estimated in a two step approach. This consists of a maximum-likelihood (ML) estimator followed by the EXIP, which further helps to reduce computational complexity.

The paper is organized as follows. In Section II a model for the dual-polarization channel is introduced that accounts for propagation effects and receiver characteristics. In Section III the CP model is introduced, in which the correlation between the LOS and multipath signals is described. In Section IV a two-step estimator for the LOS delay and AoA is derived for the CP model. The first step is a ML estimator which is similar to the one for the single path model, and in the second step the EXIP is used to refine the delay estimate and calculate the AoA. In Section V the estimation performance of dual-polarization estimation with the CP model is evaluated with the help of simulation results.

A. Notation

In this paper we define scalars, column vectors and matrices with lower case letters, lower case bold letters and upper case bold letters, respectively. The transposition and Hermitian (complex conjugation and transposition) of a matrix $\boldsymbol{A}$ are denoted $\boldsymbol{A}^{\mathrm{T}}$ and $\boldsymbol{A}^{\mathrm{H}}$, while the Moore-Penrose pseudo inverse is given by $\boldsymbol{A}^{+}$. $\operatorname{tr}(\boldsymbol{A})$ returns the trace of $\boldsymbol{A}$ while the Euclidian norm of a vector $\boldsymbol{a}$ is defined as $\|\boldsymbol{a}\|$.

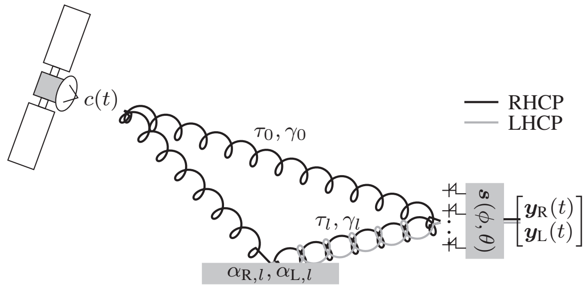

Figure 1. Channel model for the received signal

II. Dual-Polarization Channel Model

Consider the scenario illustrated in Figure 1. The satellite transmits a signal $c(t) \in \mathbb{R}$. A dual-polarization M-antenna receiver with RHCP output signal $\boldsymbol{y}_{\mathrm{R}}(t) \in \mathbb{C}^M$ and LHCP output signal $\boldsymbol{y}_{\mathrm{L}}(t) \in \mathbb{C}^M$ receives one LOS signal and L signals due to multipath reflections. The LOS signal has a time-delay $\tau_0 \in \mathbb{R}$ and the l-th multipath signal has a time-delay $\tau_l \in \mathbb{R}$. The base-band representation of the received signal can be denoted by

$$\begin{bmatrix}\boldsymbol{y}_{\mathrm{R}}(t)\\ \boldsymbol{y}_{\mathrm{L}}(t)\end{bmatrix} = \boldsymbol{y} = \boldsymbol{b}_0 c\,(t-\tau_0) + \sum_{l=1}^{L} \boldsymbol{b}_l c\,(t-\tau_l) + \boldsymbol{\eta}\,(t)\,, \quad (1)$$

where $\boldsymbol{b}_l \in \mathbb{C}^{2M}$ denotes the spatial signature of the l-th path and $\boldsymbol{\eta}(t)$ is temporally white additive Gaussian noise with spatial covariance matrix $\boldsymbol{C}_{\boldsymbol{\eta}}$. The parameters of (1) are collected together as

$$\boldsymbol{\xi}_{\text{MP,u}} = \left[\boldsymbol{\tau}^{\mathrm{T}}, \boldsymbol{b}_0^{\mathrm{T}}, \boldsymbol{b}_1^{\mathrm{T}} \ldots, \boldsymbol{b}_L^{\mathrm{T}}\right]^{\mathrm{T}} \in \mathbb{C}^{(L+1)(2M+1)}, \quad (2)$$

with $\boldsymbol{\tau} = [\tau_0, \ldots, \tau_L]^{\mathrm{T}} \in \mathbb{R}^{L+1}$. As the spatial signatures $\boldsymbol{b}_l$ are not described by physical or geometric quantities (1) will be referred to as the spatially unstructured signal model. However, in [4] it has been shown that a spatially structured signal model, which parametrizes the wavefront based on a model of the receive array response, is beneficial for the estimation of the LOS delay τ_0. The array response depends on the azimuth AoA $\phi_l \in \mathbb{R}$ and the elevation AoA $\theta_l \in \mathbb{R}$ of each received signal. In general, the array response is different for the RHCP and LHCP channel. Additionally, the isolation between RHCP and LHCP input channels (cross-polar isolation) is finite for real receivers. Therefore, the RHCP receive signal also contains LHCP signal power and vice-versa. In the following $\boldsymbol{s}_{\mathrm{R,c}}(\phi_l, \theta_l) \in \mathbb{C}^M$ and $\boldsymbol{s}_{\mathrm{L,c}}(\phi_l, \theta_l) \in \mathbb{C}^M$ are the vectors of the co-polar embedded patterns of the RHCP and LHCP antennas, which is the array steering vector $\boldsymbol{a}(\phi_l, \theta_l)$ for isotropic antennas. The vectors $\boldsymbol{s}_{\mathrm{R,x}}(\phi_l, \theta_l) \in \mathbb{C}^M$ and $\boldsymbol{s}_{\mathrm{L,x}}(\phi_l, \theta_l) \in \mathbb{C}^M$ correspond to the cross-polar embedded patterns of the RHCP and LHCP antennas. For isotropic antennas $\boldsymbol{s}_{\mathrm{R,x}}(\phi_l, \theta_l) = \boldsymbol{s}_{\mathrm{L,x}}(\phi_l, \theta_l) = s_x \boldsymbol{a}(\phi_l, \theta_l)$ with cross-talk coefficient s_x. The cross-talk is often assumed to be in the range of -20 dB and therefore negligible. Nevertheless, especially for low-elevation scenarios, the cross-talk is likely to be in the range of -5 dB or more and can have a significant influence on the receive signal and on the estimation performance.

In GNSS the satellite transmits only a RHCP signal. Therefore, the LOS receive signal contains only RHCP power. It is often assumed that the multipath signals contain only LHCP power as the multipath tends to change its polarization for reflection angles larger than the Brewster angle. However, in general

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the reflected signal will contain RHCP and LHCP power. The scalars $\alpha_{\mathrm{R},l} \in \mathbb{C}$ and $\alpha_{\mathrm{L},l} \in \mathbb{C}$ account for the magnitude and phase of the RHCP and LHCP signal of the l-th path after reflection. They depend on the angle of reflection, the reflector material and the number of reflections [10]. Using the above descriptions, the spatially structured dual polarization model can be described by

$$\boldsymbol{b}_l = \begin{cases} \gamma_0 \begin{bmatrix} \boldsymbol{s}_{\mathrm{R,c}}(\phi_0, \theta_0) \\ \boldsymbol{s}_{\mathrm{L,x}}(\phi_0, \theta_0) \end{bmatrix} & \text{for } l = 0 \\ \gamma_l \begin{bmatrix} \alpha_{\mathrm{R},l} \boldsymbol{s}_{\mathrm{R,c}}(\phi_l, \theta_l) + \alpha_{\mathrm{L},l} \boldsymbol{s}_{\mathrm{R,x}}(\phi_l, \theta_l) \\ \alpha_{\mathrm{L},l} \boldsymbol{s}_{\mathrm{L,c}}(\phi_l, \theta_l) + \alpha_{\mathrm{R},l} \boldsymbol{s}_{\mathrm{L,x}}(\phi_l, \theta_l) \end{bmatrix} & \text{for } l = 1 \dots L \end{cases}. \quad (3)$$

While the LOS signal is RHCP only, the multipath signal is a weighted sum of a RHCP and LHCP signal and both have to be considered in the multipath spatial signature. Equation (3) is parametrized by

$$\boldsymbol{\xi}_{\mathrm{MP,s}} = \left[\boldsymbol{\tau}^{\mathrm{T}}, \boldsymbol{\gamma}^{\mathrm{T}}, \boldsymbol{\phi}^{\mathrm{T}}, \boldsymbol{\theta}^{\mathrm{T}}, \boldsymbol{\alpha}_{\mathrm{R}}^{\mathrm{T}}, \boldsymbol{\alpha}_{\mathrm{L}}^{\mathrm{T}}\right]^{\mathrm{T}} \in \mathbb{C}^{(4(L+1)+2L)}, \quad (4)$$

where $\boldsymbol{\gamma} = [\gamma_0, \dots, \gamma_L]^{\mathrm{T}} \in \mathbb{C}^{L+1}$ contains the complex magnitudes which account for path loss and antenna gain and $\boldsymbol{\phi} = [\phi_0, \dots, \phi_L]^{\mathrm{T}} \in \mathbb{R}^{L+1}$ and $\boldsymbol{\theta} = [\theta_0, \dots, \theta_L]^{\mathrm{T}} \in \mathbb{R}^{L+1}$ contain the azimuth and elevation AoA. The vectors $\boldsymbol{\alpha}_{\mathrm{R}} = [\alpha_{\mathrm{R},1}, \dots, \alpha_{\mathrm{R},L}]^{\mathrm{T}} \in \mathbb{C}^L$ and $\boldsymbol{\alpha}_{\mathrm{L}} = [\alpha_{\mathrm{L},1}, \dots, \alpha_{\mathrm{L},L}]^{\mathrm{T}} \in \mathbb{C}^L$ contain the complex reflection coefficients.

III. Correlated Path Model

The optimum estimator for τ_0 in the structured multipath model is the maximum-likelihood (ML) estimator which estimates all parameters in (4). However, this estimator has to perform a model order estimation and must determine the actual LOS delay from all other multipath delays. Moreover, this estimator has to cope with a number of nuisance parameters. To avoid these problems, we propose the CP model in the following. The CP model describes the temporal correlation between the LOS and multipath signal as

$$\rho_i = \mathrm{E}\left[c(t-\tau_0)\, c(t-\tau_i)\right] \quad (5)$$

to divide the multipath into a part correlated and a part uncorrelated with the LOS. Additionally, we assume

$$\mathrm{E}\left[c(t-\tau_1)\right] = \mathrm{E}\left[c(t-\tau_0)\right] \quad (6)$$
$$\mathrm{E}\left[c(t-\tau_1)\, c(t-\tau_1)\right] = \mathrm{E}\left[c(t-\tau_0)\, c(t-\tau_0)\right] = 1. \quad (7)$$

In the case of only one multipath, i.e. $L = 1$, the multipath signal can be decomposed:

$$\boldsymbol{b}_1 c(t-\tau_1) = \rho_1 \boldsymbol{b}_1 c(t-\tau_0) + \sqrt{1-\rho_1}\boldsymbol{b}_1 u_1(t), \quad (8)$$

where $u_1(t)$ is an arbitrary stochastic process with zero mean and variance one. In the case of many multipaths, i.e. $L > 1$, the multipath signal can be decomposed as

$$\sum_{l=1}^{L} \boldsymbol{b}_l c(t-\tau_l) = \sum_{l=1}^{L} \rho_l \boldsymbol{b}_l c(t-\tau_0) + \sum_{l=1}^{L} \sqrt{1-\rho_l^2}\boldsymbol{b}_l u_l(t). \quad (9)$$

In the following we assume that the multipath components are closely spaced in time and therefore are highly correlated, i.e. $\rho \approx \rho_1 \approx \dots \approx \rho_L$. Therefore the multipath signal can be expressed by the rank-one approximation

$$\sum_{l=1}^{L} \boldsymbol{b}_l c(t-\tau_l) \approx \rho \boldsymbol{b}_{\mathrm{CP}} c(t-\tau_0) + \sqrt{1-\rho^2}\boldsymbol{b}_{\mathrm{CP}} u(t). \quad (10)$$

(10) is the unstructured CP model with parametrization

$$\boldsymbol{\xi}_{\mathrm{CP},u} = \left[\tau_0, \boldsymbol{b}_0^{\mathrm{T}}, \boldsymbol{b}_{\mathrm{CP}}^{\mathrm{T}}, \rho, \sigma_{\boldsymbol{\eta}}^2\right]^{\mathrm{T}} \in \mathbb{C}^{(3+4M)}, \quad (11)$$

where $\sigma_{\boldsymbol{\eta}}^2$ is the spatial noise power and the received signal is

$$\boldsymbol{y} = (\boldsymbol{b}_0 + \rho \boldsymbol{b}_{\mathrm{CP}})\, c(t-\tau_0) + \sqrt{1-\rho^2}\boldsymbol{b}_{\mathrm{CP}} u(t) + \sigma_{\boldsymbol{\eta}} \boldsymbol{\eta}(t). \quad (12)$$

Using (3), the structured CP model parametrization is

$$\boldsymbol{\xi}_{\mathrm{CP},s} = \left[\tau_0, \gamma_0, \phi_0, \theta_0, \boldsymbol{b}_{\mathrm{CP}}^{\mathrm{T}}, \rho, \sigma_{\boldsymbol{\eta}}^2\right]^{\mathrm{T}} \in \mathbb{C}^{(6+2M)}. \quad (13)$$

Note, that (13) does not incorporate the structured multipath parameters ϕ_l, θ_l,$\boldsymbol{\alpha}_{\mathrm{R}}$ and $\boldsymbol{\alpha}_{\mathrm{L}}$.

IV. Parameter Estimation

In the following, we assume that the received signal is sampled with a sampling rate $f_{\mathrm{s}} = 2B$, where B is the receiver bandwidth, and N samples are collected in a matrix such that

$$\boldsymbol{Y} = \left[\boldsymbol{y}[T_s], \boldsymbol{y}[2T_s], \dots, \boldsymbol{y}[NT_s]\right] \in \mathbb{C}^{2M \times N}. \quad (14)$$

A. Estimation of the Unstructured Model

Without loss of generality, we assume that the receiver noise is spatially i.i.d. Gaussian noise and therefore $\boldsymbol{C}_{\boldsymbol{\eta}} = \sigma^2 \boldsymbol{I}_{2M}$. As we assume that $\boldsymbol{C}_{\boldsymbol{\eta}}$ is known by measurement this can be achieved by pre-whitening [15, p. 44]. The optimum estimator for (11) is the ML estimator which maximizes the probability density function (pdf) $p(\boldsymbol{Y}|\boldsymbol{\xi}_{\mathrm{CP,u}})$. In the following we assume $u(t)$ as temporally white Gaussian as in GNSS applications $\sigma_{\boldsymbol{\eta}} \gg \sqrt{1-\rho^2}$ and therefore the influence of $u(t)$ is small in comparison to $\boldsymbol{\eta}(t)$. Additionally, this allows us to employ the simple ML estimator

$$\hat{\boldsymbol{\xi}}_{\mathrm{CP},u} = \arg\max_{\boldsymbol{\xi}_{\mathrm{CP,u}}} \frac{1}{\pi^{MN} \det\left(\boldsymbol{C}_{\mathrm{CP}}(\boldsymbol{\xi}_{\mathrm{CP,u}})\right)^N} \cdot \exp\left(-\mathrm{tr}\left((\boldsymbol{Y}-\boldsymbol{M}_{\mathrm{u}}(\boldsymbol{\xi}_{\mathrm{CP,u}}))^{\mathrm{H}} \boldsymbol{C}_{\mathrm{CP}}^{-1}(\boldsymbol{\xi}_{\mathrm{CP,u}})\,(\boldsymbol{Y}-\boldsymbol{M}_{\mathrm{u}}(\boldsymbol{\xi}_{\mathrm{CP,u}}))\right)\right) \quad (15)$$

with mean and covariance matrix

$$\boldsymbol{M}_{\mathrm{u}}(\boldsymbol{\xi}_{\mathrm{CP,u}}) = (\boldsymbol{b}_0 + \rho \boldsymbol{b}_{\mathrm{CP}})\, \boldsymbol{c}^{\mathrm{T}}(\tau_0) \quad (16)$$
$$\boldsymbol{C}_{\mathrm{CP}}(\boldsymbol{\xi}_{\mathrm{CP,u}}) = \left(1-\rho^2\right) \boldsymbol{b}_{\mathrm{CP}} \boldsymbol{b}_{\mathrm{CP}}^{\mathrm{H}} + \sigma_{\boldsymbol{\eta}}^2 \boldsymbol{I}_{2M}, \quad (17)$$

where $\boldsymbol{c}(\tau_0) = \left[c(T_{\mathrm{s}}-\tau_0), \dots, c(NT_{\mathrm{s}}-\tau_0)\right]^{\mathrm{T}}$.

1) Derivation of the ML-Estimator: An equivalent problem to (15) is given by the maximization of the log-likelihood function $l(\boldsymbol{Y}|\boldsymbol{\xi}_{\mathrm{CP,u}}) = \log(p(\boldsymbol{Y}|\boldsymbol{\xi}_{\mathrm{CP,u}}))$:

$$\hat{\boldsymbol{\xi}}_{\mathrm{CP,u}} = \arg\max_{\boldsymbol{\xi}_{\mathrm{CP,u}}} -N \log\det\left(\boldsymbol{C}_{\mathrm{CP}}(\boldsymbol{\xi}_{\mathrm{CP,u}})\right) - \mathrm{tr}\left((\boldsymbol{Y}-\boldsymbol{M}_{\mathrm{u}}(\boldsymbol{\xi}_{\mathrm{CP,u}}))^{\mathrm{H}} \boldsymbol{C}_{\mathrm{CP}}^{-1}(\boldsymbol{\xi}_{\mathrm{CP,u}})\,(\boldsymbol{Y}-\boldsymbol{M}_{\mathrm{u}}(\boldsymbol{\xi}_{\mathrm{CP,u}}))\right). \quad (18)$$

For the spatial signatures $\boldsymbol{b}_0$ and $\boldsymbol{b}_{\mathrm{CP}}$, (18) can be solved using the derivative of the log-likelihood function [16, p.563]. Setting the derivative of (18) with respect to $\boldsymbol{b}_0$ to zero

$$\frac{\partial l(\boldsymbol{\xi}_{\mathrm{CP,u}}|\boldsymbol{Y})}{\partial \boldsymbol{b}_0} = 2\boldsymbol{C}_{\mathrm{CP}}^{-1}(\boldsymbol{\xi}_{\mathrm{CP,u}})\left(\boldsymbol{Y} - (\boldsymbol{b}_0 - \rho \boldsymbol{b}_{\mathrm{CP}})\, \boldsymbol{c}^{\mathrm{T}}(\tau_0)\right) \boldsymbol{c}(\tau_0) = 0 \quad (19)$$

and solving for $\boldsymbol{b}_0$ yields the estimate

$$\hat{\boldsymbol{b}}_0 = \frac{\boldsymbol{Y}\boldsymbol{c}(\tau_0)}{\|\boldsymbol{c}(\tau_0)\|^2} - \rho \boldsymbol{b}_{\mathrm{CP}}. \quad (20)$$

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Inserting (20) into (18) and taking the derivative with respect to $\boldsymbol{b}_{\text{CP}}$ yields

$$\frac{\partial l\left(\boldsymbol{\xi}_{\text{CP,u}}|\boldsymbol{Y},\right)|_{\boldsymbol{b}_0=\hat{\boldsymbol{b}}_0}}{\partial \boldsymbol{b}_{\text{CP}}} = -2N\left(1-\rho^2\right)\boldsymbol{C}_{\text{CP}}^{-1}\left(\boldsymbol{\xi}_{\text{CP,u}}\right)\boldsymbol{b}_{\text{CP}} + 2\left(1-\rho^2\right)\boldsymbol{C}_{\text{CP}}^{-1}\left(\boldsymbol{\xi}_{\text{CP,u}}\right)\boldsymbol{B}(\tau_0)\boldsymbol{C}_{\text{CP}}^{-1}\left(\boldsymbol{\xi}_{\text{CP,u}}\right)\boldsymbol{b}_{\text{CP}}, \quad (21)$$

where the matrix

$$\boldsymbol{B}\left(\tau_0\right) = \left(\boldsymbol{Y} - \frac{\boldsymbol{Y}\boldsymbol{c}(\tau_0)\boldsymbol{c}^{\text{T}}(\tau_0)}{\|\boldsymbol{c}(\tau_0)\|^2}\right)\left(\boldsymbol{Y} - \frac{\boldsymbol{Y}\boldsymbol{c}(\tau_0)\boldsymbol{c}^{\text{T}}(\tau_0)}{\|\boldsymbol{c}(\tau_0)\|^2}\right)^{\text{H}} \quad (22)$$

is equivalent to the ML estimate of the noise covariance matrix $\boldsymbol{C}_{\text{CP,u}}$. Setting (21) to zero and inserting (17) we get

$$\boldsymbol{b}_{\text{CP}}\boldsymbol{b}_{\text{CP}}^{\text{H}} = \frac{\frac{\boldsymbol{B}(\tau_0)}{N} - \sigma_{\boldsymbol{\eta}}^2\boldsymbol{I}_{2M}}{1-\rho^2}. \quad (23)$$

A reasonable solution for $\boldsymbol{b}_{\text{CP}}$ can be found by an eigenvalue decomposition in which $\lambda_{\max}$ denotes the maximum eigenvalue of $\frac{\frac{\boldsymbol{B}(\tau_0)}{N} - \sigma_{\boldsymbol{\eta}}^2\boldsymbol{I}_{2M}}{1-\rho^2}$. The estimate of $\boldsymbol{b}_{\text{CP}}$ is therefore given by

$$\hat{\boldsymbol{b}}_{\text{CP}}\left(\rho, \sigma_{\boldsymbol{\eta}}^2\right) = \boldsymbol{v}_{\max}. \quad (24)$$

Inserting (24) into (18) and transforming into a minimization problem allows to formulate the final optimization problem

$$\left[\hat{\tau}_0, \hat{\rho}, \hat{\sigma}_{\boldsymbol{\eta}}^2\right] = \quad (25)$$
$$\arg\min_{\tau,\rho,\sigma_{\boldsymbol{\eta}}^2} N\log\det\left(\left(\sigma_{\boldsymbol{\eta}}^2\boldsymbol{I}_{2M} + \left(1-\rho^2\right)\hat{\boldsymbol{b}}_{\text{CP}}\left(\rho,\sigma_{\boldsymbol{\eta}}^2\right)\hat{\boldsymbol{b}}_{\text{CP}}\left(\rho,\sigma_{\boldsymbol{\eta}}^2\right)^{\text{H}}\right)\right) + \text{tr}\left(\boldsymbol{B}(\tau_0)\left(\sigma_{\boldsymbol{\eta}}^2\boldsymbol{I}_{2M} + \left(1-\rho^2\right)\hat{\boldsymbol{b}}_{\text{CP}}\left(\rho,\sigma_{\boldsymbol{\eta}}^2\right)\hat{\boldsymbol{b}}_{\text{CP}}\left(\rho,\sigma_{\boldsymbol{\eta}}^2\right)^{\text{H}}\right)^{-1}\right),$$

which has to be solved jointly for $\hat{\tau}_0$, $\hat{\rho}$ and $\hat{\sigma}_{\boldsymbol{\eta}}^2$. In the following a closed form solution for the case that all eigenvalues of the receiver noise covariance matrix $\boldsymbol{C}_{\boldsymbol{\eta}}$ are equal is presented.

2) Solution for Uniform Eigenvalues of $\boldsymbol{C}_{\boldsymbol{\eta}}$: In the case of uniform eigenvalues $\lambda_{\boldsymbol{\eta}}$ for $\boldsymbol{C}_{\boldsymbol{\eta}}$ before pre-whitening and $\|\boldsymbol{b}_{\text{CP}}\|^2 = 1$, the largest eigenvalue of (23) is $\lambda_{\boldsymbol{\eta}}^{-1}$ while all other eigenvalues are zero in the ideal case. This allows a closed form expression for $\sigma_{\boldsymbol{\eta}}^2$ and ρ to be determined. Considering the matrix (23) we notice that it is of the form $\boldsymbol{X} = \boldsymbol{Z} - \alpha\boldsymbol{I}_M$, where

$$\boldsymbol{Z} = \frac{\boldsymbol{B}\left(\tau_0\right)}{N\left(1-\rho^2\right)} \quad (26)$$

$$\alpha = \frac{\sigma_{\boldsymbol{\eta}}^2}{1-\rho^2}. \quad (27)$$

The eigenvalues of $\boldsymbol{X}$ are in general given by [15, p. 31]

$$\lambda_{\boldsymbol{X},i} = \lambda_{\boldsymbol{Z},i} - \alpha. \quad (28)$$

Therefore, the following system of equations holds

$$\lambda_{\boldsymbol{\eta}}^{-1} = \frac{\lambda_{\boldsymbol{B},1}}{N\left(1-\rho^2\right)} - \frac{\sigma_{\boldsymbol{\eta}}^2}{1-\rho^2}$$
$$0 = \frac{\lambda_{\boldsymbol{B},i}}{N\left(1-\rho^2\right)} - \frac{\sigma_{\boldsymbol{\eta}}^2}{1-\rho^2} \quad \text{for } i = 2\ldots 2M. \quad (29)$$

Let $\lambda_{\boldsymbol{B},1} \leq \lambda_{\boldsymbol{B},2} \leq \ldots \leq \lambda_{\boldsymbol{B},2M}$ be the sorted eigenvalues of $\boldsymbol{B}(\hat{\tau}_0)$. (29) can be solved by

$$\begin{bmatrix}\hat{\sigma}_{\boldsymbol{\eta}}^2 \\ 1-\hat{\rho}^2\end{bmatrix} = \begin{bmatrix} N & N\lambda_{\boldsymbol{\eta}}^{-1} \\ N & 0 \\ \vdots & \vdots \\ N & 0\end{bmatrix}^{+}\begin{bmatrix}\lambda_{\boldsymbol{B},1} \\ \lambda_{\boldsymbol{B},2} \\ \vdots \\ \lambda_{\boldsymbol{B},2M}\end{bmatrix} \quad (30)$$

in the minimum mean-squared error (MMSE) sense. Inserting (30) into (25) yields the solution for $\hat{\tau}_0$

$$\hat{\tau}_0 = \arg\min_{\tau_0}\left(N\log\det\left(\left(\hat{\sigma}_{\boldsymbol{\eta}}(\hat{\tau}_0)^2\boldsymbol{I}_{2M} + \left(1-\hat{\rho}(\hat{\tau}_0)^2\right)\cdot\hat{\boldsymbol{b}}_{\text{CP}}\left(\hat{\rho}(\hat{\tau}_0),\hat{\sigma}_{\boldsymbol{\eta}}(\hat{\tau}_0)^2\right)\hat{\boldsymbol{b}}_{\text{CP}}\left(\hat{\rho}(\hat{\tau}_0),\hat{\sigma}_{\boldsymbol{\eta}}(\hat{\tau}_0)^2\right)^{\text{H}}\right)\right) + \text{tr}\left(\boldsymbol{B}(\tau_0)\left(\hat{\sigma}_{\boldsymbol{\eta}}(\hat{\tau}_0)^2\boldsymbol{I}_{2M} + \left(1-\hat{\rho}(\hat{\tau}_0)^2\right)\hat{\boldsymbol{b}}_{\text{CP}}\left(\hat{\rho}(\hat{\tau}_0),\hat{\sigma}_{\boldsymbol{\eta}}(\hat{\tau}_0)^2\right)\hat{\boldsymbol{b}}_{\text{CP}}\left(\hat{\rho}(\hat{\tau}_0),\hat{\sigma}_{\boldsymbol{\eta}}(\hat{\tau}_0)^2\right)^{\text{H}}\right)^{-1}\right)\right) \quad (31)$$

which can for example be found with a line-search algorithm.

B. Estimation of the Structured Model

While improving the estimation performance for τ_0, a structured model also yields the AoA of the LOS. The parameters of the structured model (13) can be calculated using EXIP [4], which allows the structured parameters $\boldsymbol{\xi}_{\text{CP,s}}$ to be derived from the unstructured estimates $\hat{\boldsymbol{\xi}}_{\text{CP,u}}$:

$$\hat{\boldsymbol{\xi}}_{\text{CP,s}} = \arg\min_{\boldsymbol{\xi}_{\text{CP,s}}}\left[\hat{\boldsymbol{\xi}}_{\text{CP,u}} - \boldsymbol{f}\left(\boldsymbol{\xi}_{\text{CP,s}}\right)\right]^{\text{T}}\boldsymbol{W}\left[\hat{\boldsymbol{\xi}}_{\text{CP,u}} - \boldsymbol{f}\left(\boldsymbol{\xi}_{\text{CP,s}}\right)\right], \quad (32)$$

where $\boldsymbol{W}$ is the Fisher information matrix (FIM) of (12)

$$\left[\boldsymbol{I}\left(\boldsymbol{\xi}_{\text{CP,u}}\right)\right]_{ij} = \text{E}\left[\frac{\partial^2 l\left(\boldsymbol{Y}|\boldsymbol{\xi}_{\text{CP,u}}\right)}{\partial\boldsymbol{\xi}_{\text{CP,u}}\partial\boldsymbol{\xi}_{\text{CP,u}}^{\text{T}}}\right]. \quad (33)$$

EXIP performs a Taylor approximation of $\boldsymbol{\xi}_{\text{CP,s}}$ around $\hat{\boldsymbol{\xi}}_{\text{CP,u}}$.

C. Real-valued Parametrization

We transform the parametrizations of (11) and (13) to their real-valued equivalents by applying

$$\boldsymbol{a} \in \mathbb{C}^{2M\times 1} \quad \rightarrow \quad \begin{bmatrix}\text{Re}\left(\boldsymbol{a}\right) \\ \text{Im}\left(\boldsymbol{a}\right)\end{bmatrix} \in \mathbb{R}^{4M\times 1}, \quad (34)$$

i.e. each complex number is split into its real and imaginary part. $\sigma_{\boldsymbol{\eta}}^2$ and τ_0 are decoupled in the FIM and therefore $\sigma_{\boldsymbol{\eta}}^2$ has no impact on the EXIP estimate of τ_0. The resulting parametrizations for EXIP are given by

$$\bar{\boldsymbol{\xi}}_{\text{CP,s}} = \left[\tau_0, \text{Re}\left(\gamma_0\right), \text{Im}\left(\gamma_0\right), \phi_0, \theta_0, \text{Re}\left(\boldsymbol{b}_{\text{CP}}\right)^{\text{T}}, \text{Im}\left(\boldsymbol{b}_{\text{CP}}\right)^{\text{T}}, \rho, \sigma_{\boldsymbol{\eta}}^2\right]^{\text{T}} \in \mathbb{R}^{(7+4M)\times 1} \quad (35)$$

$$\bar{\boldsymbol{\xi}}_{\text{CP,u}} = \left[\tau_0, \text{Re}\left(\boldsymbol{b}_0\right)^{\text{T}}, \text{Im}\left(\boldsymbol{b}_0\right)^{\text{T}}, \text{Re}\left(\boldsymbol{b}_{\text{CP}}\right)^{\text{T}}, \text{Im}\left(\boldsymbol{b}_{\text{CP}}\right)^{\text{T}}, \rho, \sigma_{\boldsymbol{\eta}}^2\right]^{\text{T}} \in \mathbb{R}^{(3+8M)\times 1}. \quad (36)$$

The mapping function from the structured to the unstructured parametrization $\bar{\boldsymbol{\xi}}_{\text{CP,u}} = f\left(\bar{\boldsymbol{\xi}}_{\text{CP,s}}\right)$ is

$$\bar{\boldsymbol{\xi}}_{\text{CP,u}} = f\left(\bar{\boldsymbol{\xi}}_{\text{CP,s}}\right) \quad (37)$$
$$= \left[\tau_0, \rho, \text{Re}\left(\boldsymbol{b}_{\text{CP}}\right)^{\text{T}}, \text{Im}\left(\boldsymbol{b}_{\text{CP}}\right)^{\text{T}}, \left(\begin{bmatrix}\text{Re}\left(\boldsymbol{s}\left(\theta_0,\phi_0\right)\right) & -\text{Im}\left(\boldsymbol{s}\left(\theta_0,\phi_0\right)\right) \\ \text{Im}\left(\boldsymbol{s}\left(\theta_0,\phi_0\right)\right) & \text{Re}\left(\boldsymbol{s}\left(\theta_0,\phi_0\right)\right)\end{bmatrix}\begin{bmatrix}\text{Re}\left(\gamma_0\right) \\ \text{Im}\left(\gamma_0\right)\end{bmatrix}\right)^{\text{T}}\right]^{\text{T}}.$$

In order to simplify the notation, parameters which are mapped one-to-one are denoted

$$\bar{\boldsymbol{\xi}}_{\text{u},1} = \bar{\boldsymbol{\xi}}_{\text{s},1} = \left[\tau_0, \rho, \text{Re}\left(\boldsymbol{b}_{\text{CP}}\right)^{\text{T}}, \text{Im}\left(\boldsymbol{b}_{\text{CP}}\right)^{\text{T}}\right]^{\text{T}}, \quad (38)$$

while the parameters which are mapped with a function are

$$\bar{\boldsymbol{\xi}}_{\text{u},2} = f\left(\bar{\boldsymbol{\xi}}_{\text{s},2}\right) = \begin{bmatrix}\text{Re}\left(\boldsymbol{s}\left(\theta_0,\phi_0\right)\right) & -\text{Im}\left(\boldsymbol{s}\left(\theta_0,\phi_0\right)\right) \\ \text{Im}\left(\boldsymbol{s}\left(\theta_0,\phi_0\right)\right) & \text{Re}\left(\boldsymbol{s}\left(\theta_0,\phi_0\right)\right)\end{bmatrix}\begin{bmatrix}\text{Re}\left(\gamma_0\right) \\ \text{Im}\left(\gamma_0\right)\end{bmatrix} = \boldsymbol{S}\begin{bmatrix}\text{Re}\left(\gamma_0\right) \\ \text{Im}\left(\gamma_0\right)\end{bmatrix}, \quad (39)$$

i.e. $\bar{\boldsymbol{\xi}}_{\text{s},2} = \left[\text{Re}\left(\gamma_0\right), \text{Im}\left(\gamma_0\right), \phi_0, \theta_0\right]^{\text{T}}$ in the following. Additionally, $\boldsymbol{W}_{\hat{\bar{\boldsymbol{\xi}}}_{\text{u,i}},\hat{\bar{\boldsymbol{\xi}}}_{\text{u,j}}} = \left[\boldsymbol{I}_{\text{u}}\left(\boldsymbol{\xi}_{\text{u}}\right)\right]_{\bar{\boldsymbol{\xi}}_{\text{u,i}},\bar{\boldsymbol{\xi}}_{\text{u,j}}}\Big|_{\hat{\bar{\boldsymbol{\xi}}}_{\text{u,i}},\hat{\bar{\boldsymbol{\xi}}}_{\text{u,j}}}$ denotes the respective sub-matrices of the weighting matrix $\boldsymbol{W}$.

1) Solution of the EXIP Equation: The real-valued optimization problem is given by

$$\hat{\hat{\bar{\xi}}}_{\mathrm{CP,s}} = \arg\min_{\bar{\xi}_{\mathrm{s},1}\bar{\xi}_{\mathrm{s},2}} \left[\hat{\hat{\bar{\xi}}}_{\mathrm{CP,u}} - \boldsymbol{f}\left(\bar{\xi}_{\mathrm{CP,s}}\right)\right]^{\mathrm{T}} \boldsymbol{W} \left[\hat{\hat{\bar{\xi}}}_{\mathrm{CP,u}} - \boldsymbol{f}\left(\bar{\xi}_{\mathrm{CP,s}}\right)\right]. \quad (40)$$

After solving (40) with respect to $\hat{\hat{\bar{\xi}}}_{\mathrm{s},1}$ we obtain

$$\begin{aligned}\hat{\hat{\bar{\xi}}}_{\mathrm{s},1} &= \left[\hat{\hat{\tau}}_0, \hat{\rho}, \mathrm{Re}\left(\hat{\hat{\boldsymbol{b}}}_{\mathrm{CP}}\right)^{\mathrm{T}}, \mathrm{Im}\left(\hat{\hat{\boldsymbol{b}}}_{\mathrm{CP}}\right)^{\mathrm{T}}\right]^{\mathrm{T}} \\ &= \hat{\hat{\bar{\xi}}}_{\mathrm{u},1} + \left(\hat{\hat{\bar{\xi}}}_{\mathrm{u},2} - \boldsymbol{S}\left(\theta_0, \phi_0\right)\begin{bmatrix}\mathrm{Re}\left(\gamma_0\right) \\ \mathrm{Im}\left(\gamma_0\right)\end{bmatrix}\right)^{\mathrm{T}} \boldsymbol{W}_{\hat{\bar{\xi}}_{\mathrm{u},2},\hat{\bar{\xi}}_{\mathrm{u},1}} \boldsymbol{W}^{-1}_{\hat{\bar{\xi}}_{\mathrm{u},1},\hat{\bar{\xi}}_{\mathrm{u},1}}. \quad (41)\end{aligned}$$

Note, that the second part of (41) behaves like a correction term for the unstructured estimate $\hat{\hat{\bar{\xi}}}_{\mathrm{u},1}$. Inserting (41) into (40) and solving with respect to $\left[\mathrm{Re}\left(\gamma_0\right), \mathrm{Im}\left(\gamma_0\right)\right]^{\mathrm{T}}$ yields

$$\begin{bmatrix}\mathrm{Re}\left(\hat{\gamma}_0\right) \\ \mathrm{Im}\left(\hat{\gamma}_0\right)\end{bmatrix} = \left(\boldsymbol{S}^{\mathrm{T}}\left(\theta_0, \phi_0\right) \boldsymbol{\Pi} \boldsymbol{S}\left(\theta_0, \phi_0\right)\right)^{-1} \boldsymbol{S}^{\mathrm{T}}\left(\theta_0, \phi_0\right) \boldsymbol{\Pi} \hat{\hat{\bar{\xi}}}_{\mathrm{u},2}, \quad (42)$$

where the matrix $\boldsymbol{\Pi}$ denotes the Schur-complement of $\boldsymbol{W}$

$$\boldsymbol{\Pi} = \boldsymbol{W}_{\hat{\bar{\xi}}_{\mathrm{u},1},\hat{\bar{\xi}}_{\mathrm{u},1}} - \boldsymbol{W}_{\hat{\bar{\xi}}_{\mathrm{u},1},\hat{\bar{\xi}}_{\mathrm{u},2}} \boldsymbol{W}^{-1}_{\hat{\bar{\xi}}_{\mathrm{u},2},\hat{\bar{\xi}}_{\mathrm{u},2}} \boldsymbol{W}^{\mathrm{T}}_{\hat{\bar{\xi}}_{\mathrm{u},2},\hat{\bar{\xi}}_{\mathrm{u},1}}. \quad (43)$$

Inserting (41) and (42) into (32), we can formulate the following final optimization problem

$$\begin{aligned}\left[\hat{\phi}_0, \hat{\theta}_0\right]^{\mathrm{T}} = \arg\min_{\phi_0,\theta_0,} \hat{\bar{\xi}}^{\mathrm{T}}_{\mathrm{u},2} \Big(\boldsymbol{\Pi} - \boldsymbol{\Pi}^{\mathrm{T}} \boldsymbol{S}\left(\theta_0, \phi_0\right) \quad (44) \\ \cdot \left(\boldsymbol{S}\left(\theta_0, \phi_0\right)^{\mathrm{T}} \boldsymbol{\Pi} \boldsymbol{S}\left(\theta_0, \phi_0\right)\right)^{-1} \boldsymbol{S}\left(\theta_0, \phi_0\right)^{\mathrm{T}} \boldsymbol{\Pi}\Big) \hat{\bar{\xi}}_{\mathrm{u},2}.\end{aligned}$$

Equation (44) is a non-linear optimization problem which provides the estimates $\left[\hat{\phi}_0, \hat{\theta}_0\right]^{\mathrm{T}}$. Inserting these into (42) results in the estimate for the amplitudes $\left[\mathrm{Re}\left(\gamma_0\right), \mathrm{Im}\left(\gamma_0\right)\right]^{\mathrm{T}}$. Inserting (44) and (42) into (41) yields the estimate $\hat{\hat{\bar{\xi}}}_{\mathrm{s},1}$ and therefore a refined estimate for the LOS delay τ_0 and the other parameters of the spatially unstructured model. With EXIP we get a closed form solution for all structured parameters except $\left[\phi_0, \theta_0\right]^{\mathrm{T}}$. In order to determine the azimuth and elevation angle, a search over (44) has to be performed. One possible method is to use a two-dimensional line-search algorithm over the domain of $\left[\phi_0, \theta_0\right]^{\mathrm{T}}$. A method that includes a closed form solution for centro-symmetric ULAs is given in [4].

V. Simulation Results

We assume a GPS C/A code with chip duration $T_{\mathrm{c}} = 997.52\,\mathrm{ns}$, bandwidth $B = 1.023\,\mathrm{MHz}$ and $N_{\mathrm{d}} = 1023$ chips per code period as transmit signal $c(t)$. The receive array is a 2×2 URA with $\lambda/2$ spacing, illustrated in Figure 2. It has a 10 dB separation between RHCP and LHCP channel, i.e. $s_x = 0.1$. The noise is spatially white Gaussian, i.e. $\boldsymbol{C}_{\boldsymbol{\eta}} = \boldsymbol{I}_{2M}$ and the RHCP channel signal-to-noise ratio (SNR) is

$$\mathrm{SNR} = \mathrm{C/N_0} - 10\log_{10}\left(2B\right) + 10\log_{10}\left(N_{\mathrm{c}}\right), \quad (45)$$

with carrier-to-noise density $\mathrm{C/N_0} = 46\,\mathrm{dB\text{-}Hz}$ and number of observed code periods $N_{\mathrm{c}} = 400$. During the observation interval, the channel parameters are assumed to be constant and the received signal is generated by applying (2) and (3). In the following, the estimation performance of the CP model is evaluated in different scenarios. In all scenarios the LOS azimuth AoA is $\phi_0 = 108°$ while the elevation is $\theta_0 = 72°$ from the zenith. As a measure for the estimation performance of the CP model we employ the RMSE of the estimate τ_0

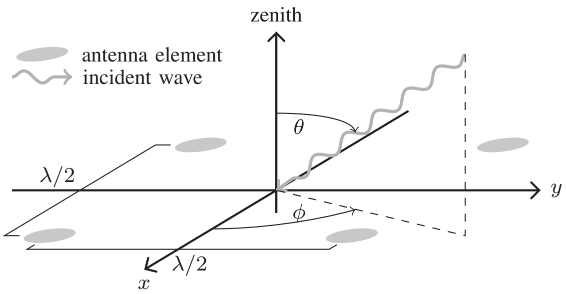

Figure 2. Antenna array orientation

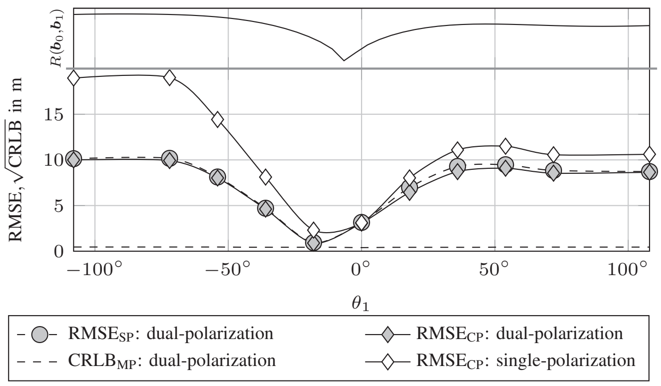

Figure 3. Performance in dependency of the multipath elevation angle θ_1

$$\mathrm{RMSE} = \sqrt{\frac{1}{N_{\mathrm{MC}}} \sum_{i=1}^{N_{\mathrm{MC}}} \left(\tau_0 - \hat{\tau}_0\right)^2}, \quad (46)$$

after applying EXIP. For comparison we also consider the RMSE if a single path (SP) model [5] is used for parameter estimation. This is equivalent to $\rho = 0$. Therefore, the noise covariance matrix $\mathrm{E}\left[\boldsymbol{\eta}_{\mathrm{sp}}\boldsymbol{\eta}^{\mathrm{H}}_{\mathrm{sp}}\right] = \boldsymbol{C}_{\boldsymbol{\eta}_{\mathrm{sp}}}$ has to be estimated. Additionally, the Cramér Rao lower bound (CRLB) of a hypothetical approach that estimates the multipath (MP) model (4) is shown.

A. Spatial Correlation Dependency

First, we evaluate the influence of the spatial correlation between the LOS and one multipath signal

$$R\left(\boldsymbol{b}_0, \boldsymbol{b}_1\right) = \boldsymbol{b}^{\mathrm{H}}_0 \boldsymbol{b}_1 \quad (47)$$

on the estimation performance. Figure 3 shows the RMSE and CRLB of the LOS delay estimate $\hat{\tau}_0$ over the multipath elevation angle θ_1 for $\Delta\tau = \tau_1 - \tau_0 = 0.6\,T_{\mathrm{c}}$. The estimation performance for all estimators follows the spatial correlation $R\left(\boldsymbol{b}_0, \boldsymbol{b}_1\right)$. In the case of low elevation, i.e. θ_1 approximately $90°$ or $-90°$, using dual polarization in combination with the CP model is most beneficial for the estimation performance. This is due to the fact, that for a high spatial correlation, the LOS and multipath signal are not spatially separable. Dual polarization here introduces an additional dimension which helps to separate the LOS and multipath signal and therefore improves the estimation performance.

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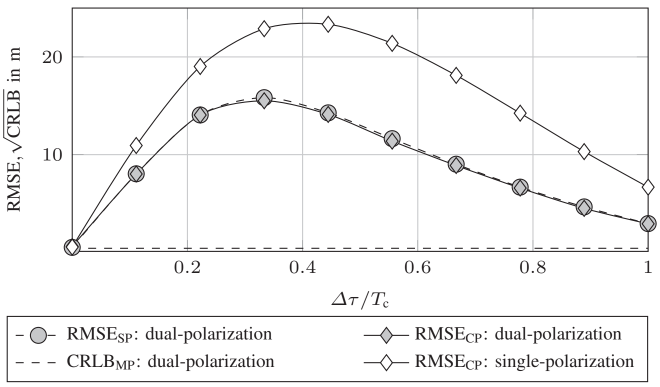

Figure 4. Performance in dependency of the delay difference $\Delta\tau$

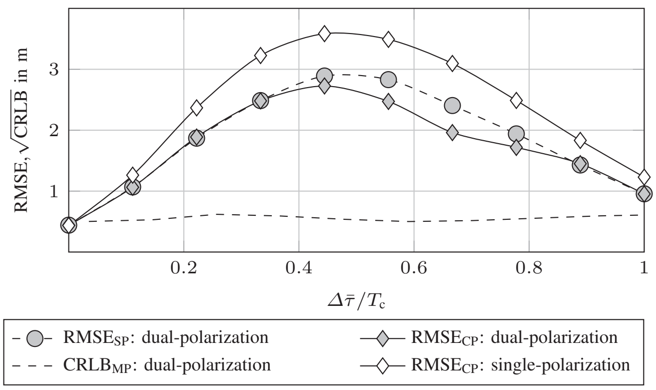

Figure 5. Performance in dependence of the mean delay difference $\Delta\bar{\tau}$

B. Delay Difference Dependency

Figure 4 shows the RMSE and CRLB for the estimate of τ_0 for different delay differences $\Delta\tau$ for the case of one multipath signal with $\phi_1 = 72°$ and $\theta_1 = -84°$, i.e. where the LOS and multipath are highly spatially correlated. Again, dual polarization estimation is in general beneficial for the estimation of τ_0. Estimation with the CP model has a slightly better performance than estimation with the SP model in the case of $\Delta\tau \approx 0.5\,T_c$.
Figure 5 shows the RMSE and CRLB of the estimate $\hat{\tau}_0$ for $L = 6$ multipath signals over the mean delay difference $\Delta\bar{\tau} = \frac{1}{L}\sum_{l=1}^{L}\tau_l - \tau_0$. For the multipath delays $\tau_l - \tau_{l+1} = 0.01\,T_c\,\forall\,l$ holds. The mulitpath azimuth AoAs are $\phi_1 = 72°$ while the elevation AoAs θ_l are approximately $90°$ or $-90°$. Therefore, the LOS and multipath are highly spatially correlated. In this scenario dual-polarization in general has better performance than single-polarization estimation. Especially for $\Delta\bar{\tau} \approx 0.5\,T_c$ the CP model outperforms the SP model.

VI. Conclusion

We have assessed the problem of multipath mitigation with dual polarization estimation. We have introduced a dual polarization multipath model which comprises polarized wave propagation effects and dual polarization receive array properties. In order to reduce the number of model parameters for dual polarization signal processing, the CP model was introduced. The CP model describes the temporal correlation between the LOS and multipath signal and can efficiently be estimated in a two-step approach. We have shown that dual polarization estimation can outperform single polarization estimation in GNSS scenarios. Especially in cases where LOS and multipath signal are highly spatially correlated, dual polarization estimation can add an additional degree of freedom which allows one to separate the LOS from the multipath signal. In the case of several multipath signals the CP model achieves a higher estimation performance than the SP model. In order to get closer to the multipath CRLB, an estimation of the full multipath model can be performed. However, in this case different problems arise. As in single-polarization scenarios, the model order has to be estimated and the LOS delay has to be determined from all estimated delays. Due to the dual-polarization model, the reflection coefficients of every multipath also have to be estimated, an issue that is avoided with the CP model. Due to these problems a real multipath estimator will also not always achieve the multipath CRLB.

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