Intercell Interference Blindness in Fairness Optimizations

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***Abstract*—In the downlink of a cellular network with channel state information on a limited number of interference channels, the *intercell interference* (ICI) variance at a mobile device can change unpredictably. This ICI blindness leads to outages or a waste of resources because the link rate adaptation is based on an assumed signal to interference plus noise ratio. Different approaches to deal with ICI blindness have been discussed and compared with sum rate maximizations. In this paper, we extend the handling of the ICI blindness problem to fairness optimizations. Fairness scheduling realizes fairness over many time slots, which includes Round Robin, ThroughPut Fairness and Proportional Fairness. Algorithms are developed to combine the ICI robustness methods and fairness criteria and the performance of the different combinations are compared.**

I. Introduction

Frequency reuse implies orthogonal frequency bands in neighbouring cells, which leads to small ICI. However, this technique is very expensive because only a fraction of the possible rate can be reached in each cell. Therefore, frequency reuse will not be employed in the forthcoming 5G standard and the ICI from the neighbouring cells will tremendously worsen the *signal to interference plus noise ratio* (SINR) at the receivers.

In the downlink of an ideal wireless cellular system with full cooperation, the ICI at each *mobile device* (MD) can be known and handled by a central processor. However, this requires perfect *channel state information* (CSI) about all channels including the interference channels. Measuring all these channels can occupy almost all air time in large systems, which leaves no time for data transmission. Therefore, the assumption of full cooperation is unrealistic. There is always some ICI variance remaining that has to be regarded as noise.

Even if the not measured interference channels stay constant, the ICI variance at a MD can change unpredictably whenever a *base station* (BS) in the network changes its beamforming. A BS doesn't know the actual ICI or the supported rates of the MD it is serving. Therefore, BSs use assumed ICI variances for the beamforming optimization and link rate adaptation. The mismatch between assumed ICI and actual ICI leads to the intercell interference blindness problem. When the actual ICI is larger than the assumed one, the channel is worse than assumed and the MD cannot decode the data. The transmission fails and the achieved rate becomes zero. When the actual ICI is smaller than the assumed one, the MD can only communicate with the assumed data rate and some resources are wasted.

The target is to make the system robust against random changes of the ICI variance. In [1], the gambling method is proposed to handle the ICI blindness problem. With the gambling method, the ICI mismatch is accepted and a conservative link rate adaptation is used to deal with the ICI uncertainty. A backoff factor β is introduced to lower the risk of a failed transmission. The BSs serve the MDs with modest rates.

It is assumed in the genie method that the ICI at each MD is simply known [1]. The BSs calculate the beamforming and the generated ICI iteratively until the resulting ICI converges. A possibility to make the ICI variance available at the BSs is to measure it with additional piloting. If the measurements are limited to a single additional pilot, the method is called the second pilot, where the BSs can serve the MDs with ICI-aware rates [2].

Dotzler et al. proposed the covariance shaping method in [3], where the uncertainty in the ICI is eliminated by imposing a shaping constraint on the sum transmit covariance. Then the ICI variances will not change even if the other BSs update their beamforming. Although the ICI blindness problem is solved, the shaping constraint reduces the region of achievable data rates.

The expected rate method operates on the expectation of the rates to optimize the beamforming and select the link rate adaptation [4]. Taking the expectation of the rate leads to a weighted rate in the optimizations. To implement the expected rate method, the statistics of the ICI at each MD need to be known, which can be approximated by long term measurements at the MDs.

Hybrid Automatic Repeat reQuest (HARQ) can also be used to treat the ICI blindness problem [5]. The ICI blindness problem can be relaxed such that a transmission is completed successfully if the data can be decoded with the combination of several retransmissions.

Up to now, the ICI blindness problem has only been addressed in sum rate maximizations. However, maximizing the sum throughput of the system usually ends up with serving only the MDs with high SINR. As the provider and users of a cellular network are typically interested in a fair resource distribution, we focus on the ICI blindness problem in fairness optimizations in this paper.

The fairness among the users can be realized with schedul-

ISBN 978-3-8007-3662-1

ing. With fairness scheduling, the beamforming is still selected to maximize a weighted sum rate, but the scheduler changes the weights of the MDs in each time slot according to a fairness criterion. *Round Robin* (RR), *ThroughPut Fairness Scheduling* (TPFS) and *Proportional Fairness Scheduling* (PFS) can be implemented using a fairness scheduler.

In RR, the MDs are served in turn. Each MD gets the same number of time slots assigned. Under the TPFS criterion, the MDs with the smallest historical throughput are scheduled, which leads to equal throughput for each user in the end. The goal of PFS is to find a balance point between the sum throughput maximization and equal throughput among all users [6].

The paper is constructed as follows. Section II describes the system setup and the assumption about the ICI variance. The interpretation of and the possible approaches to the ICI blindness problem are shown in Section III. Section IV specifies the ideas and the objective formulations for the different fairness criteria. The simulation results and corresponding analysis are provided in Section V. Last but not the least, Section VI includes the conclusion and the potential future work.

II. System Model

We consider the downlink (DL) of a cellular wireless network where BSs have multiple transmit antennas and each MD has a single receive antenna. Each cell is a multi-user multi-input single-output (MU-MISO) channel.

A. Single Cell with random ICI variance

In order to optimize the beamforming and compute the possible achievable rates for the link rate adaptation, the BSs need the information of the ICI variances at each MD. The received ICI variance θ at a MD depends on the interference channel vectors and the sum transmit covariance matrices of the interfering BSs. Since measuring all interference channels is unrealistic in a large scale system, the ICI is split into two parts—the ICI over known channels and over unknown channels. For the unknown interference channels, the interference can only be regarded as noise.

For simplicity, we limit ourself to a scenario with no cooperation among the BSs. Nevertheless, the methods and the results in this paper can be applied to any type of cooperative scenario with interference coordination, where each MD is only served by one associated BS but the BSs try to mitigate the ICI over measured interference channels. Interference coordination includes techniques like interference alignment or interference temperatures [7].

In this paper, it is assumed that there are no measured interference channels and the transmit processing of the BSs are unknown to each other. We regard the ICI variance θ as noise, which means that the supported rate of a MD does not relate to the interfering BSs. The optimization problem over the whole network can be split up to individual optimization problems in each BS, respectively. Therefore, it is sufficient to look at the signal processing of a single cell.

The BS has N transmit antennas and serves $K = |\mathcal{K}|$ single-antenna MDs, where a MD is specified by the index k. The vector $\boldsymbol{h}_k \in \mathbb{C}^N$ is the channel between the antennas of the BS and MD k. $\boldsymbol{Q}$ is the sum transmit covariance matrix of the BS, which is the sum of the covariances of the individual beamforming vectors. The rate of user k can be expressed as

$$r_k = \log\left(1 + \frac{\boldsymbol{h}_k^{\mathrm{H}}\boldsymbol{Q}_k\boldsymbol{h}_k}{\sum_{\hat{k}>k}\boldsymbol{h}_k^{\mathrm{H}}\boldsymbol{Q}_{\hat{k}}\boldsymbol{h}_k + \theta_k + \sigma_k^2}\right), \qquad (1)$$

where $\boldsymbol{Q}_k$ is the covariance of the beamforming vector for user k and σ_k^2 is the variance of the thermal noise. $\sum_{\hat{k}>k}\boldsymbol{h}_k^{\mathrm{H}}\boldsymbol{Q}_{\hat{k}}\boldsymbol{h}_k$ is the variance of the intracell interference with dirty paper coding (DPC) [8]. θ_k is the ICI variance at MD k.

We assume perfect CSI, which means pilot contamination and other errors during the channel measurements are neglected. The transmit covariance matrix $\boldsymbol{Q}_k$ for user k can be updated at each time instance. Within the block-fading block length T_{block}, the channels stay constant while the ICI θ_k can vary at each time slot. It was shown in [4] that θ_k can be approximated by a gamma distribution $\theta_k \sim \Gamma(a_k, b_k)$, where a_k and b_k are derived from the 3GPP MIMO urban macro cell model. It is assumed that the statistics of the ICI measured in the past are available to the BS. In addition, the SINR at each MD during the pilot sequences is perfectly known to the BS.

B. ICI Blindness Problem

The actual interference θ_k^{actual} and the supported rate r_k^{actual} at the MD k during the transmission cannot be known in advance. An assumed ICI variance $\theta_k^{\text{assumed}}$ and the corresponding assumed rate r_k^{assumed} have to be used for the optimization of the transmit processing. The mismatch between the assumed ICI $\theta_k^{\text{assumed}}$ and the true ICI θ_k^{actual} leads to the ICI blindness problem.

After the optimization of the beamforming, the BS assigns the data rate r_k^{assumed} to user k. If the assumed rate is larger than the supported rate, MD k cannot decode the data and the transmission fails. In other words, when the actual ICI is larger than the assumed ICI, the channel is worse than assumed and the MD cannot decode the signals successfully. If the assumed ICI is smaller than the actual ICI, MD k can only communicate with the assumed rate r_k^{assumed}, but some resources will be wasted. If the transmission is successful, the achieved rate r_k^{achieved} does not depend on the actual ICI but on the assumed ICI. The ICI blindness problem can be formulated as

$$r_k^{\text{achieved}} = \begin{cases} r_k^{\text{assumed}}, & \text{for } \theta_k^{\text{assumed}} \geq \theta_k^{\text{actual}}, \\ 0, & \text{for } \theta_k^{\text{assumed}} < \theta_k^{\text{actual}}. \end{cases} \qquad (2)$$

III. ICI Robustness Methods

The general objective is to maximize the expectation of the weighted sum rate over the whole system

$$\max_{\boldsymbol{Q}_k} E\left[\sum_k w_k r_k^{\text{achieved}}\right] \quad \text{s.t. } \boldsymbol{Q}_k \succeq \boldsymbol{0},\ \mathrm{tr}\left(\sum_k \boldsymbol{Q}_k\right) \leq P. \qquad (3)$$

With fairness scheduling, the optimization problems can always be formulated as a maximum weighted sum rate

ISBN 978-3-8007-3662-1

(MWSR) problem, where the weights w_k are defined by different fairness criterion. Taking the ICI blindness problem into consideration, the performance of different methods is evaluated by comparing the expected value of the achieved rates r_k^{achieved}. Different ICI robustness methods use distinct ideas to deal with the expectation operator.

A. Gambling

One method to deal with the ICI mismatch problem is conservative gambling [1]. With the gambling method, we just accept the ICI power mismatch and use a conservative link rate adaptation. We hope that the actual ICI θ_k^{actual} doesn't differ too much from the assumed ICI $\theta_k^{\text{ass.}}$:

$$\max_{\boldsymbol{Q}_k} \sum_k w_k r_k \Big|_{\theta_k=\theta_k^{\text{ass.}}} \quad \text{s.t. } \boldsymbol{Q}_k \succeq \boldsymbol{0},\ \text{tr}\left(\sum_k \boldsymbol{Q}_k\right) \leq P. \quad (4)$$

The assumed rate depends on the assumed ICI $\theta_k^{\text{assumed}}$ we choose. The assumed ICI power $\theta_k^{\text{assumed}}$ can be the expected value of the ICI, the ICI measured in the last time frame or some predefined value. To lower the risk of a failed transmission, a backoff β is introduced and the BSs serve the MDs with modest rates $(1-\beta)r_k^{\text{assumed}}$. The precoders $\boldsymbol{Q}_k$ are optimized based on the assumed ICI variance $\theta_k^{\text{assumed}}$ and the common backoff β is applied after the precoders $\boldsymbol{Q}_k$ are selected. The backoff β provides a new degree of freedom in the system. In our simulation, the backoff β is chosen such that the objective is maximized

$$\max_{\beta} \sum_k w_k r_k^{\text{achieved}} \Big|_{\theta_k=\theta_k^{\text{assumed}}} \qquad \text{s.t. } 0 \leq \beta \leq 1. \quad (5)$$

One disadvantage of the gambling method is that when the actual ICI is smaller than the assumed ICI, the MDs are only served with the assumed rates and cannot benefit from the extra resources. Besides, occasionally the conservative link rate adaptation fails completely and some users have zero rates.

Algorithm 1 describes the optimization with the gambling method. The backoff factor β_{opt} is given and used to calculate the achieved rates r_k^{achieved}. The calculation of the optimum precoder $\boldsymbol{Q}_k$ in line 2 depends on the utility function.

Algorithm 1 Gambling

Require: weights w_k, β_{opt}, a_k and b_k $\forall k$
1: Compute assumed ICI $\theta_k^{\text{assumed}} \leftarrow a_k \cdot b_k$
2: Compute optimum precoder $\boldsymbol{Q}_k$ and assumed rates r_k^{assumed} from (4)
3: $r_k^{\text{assumed}} \leftarrow (1-\beta)r_k^{\text{assumed}}$

B. Covariance Shaping

The uncertainty in the ICI can be eliminated with covariance shaping [3] (a generalization of 'Stabilization' in [1]). When the sum transmit covariance matrices of the interfering BSs are limited, the maximum received ICI at the MDs are known to the BS after the measurements of the SINR in the piloting phase. Therefore, the robustness to the unpredictable ICI can be increased by imposing a shaping constraint on the sum transmit covariance, $\sum_k \boldsymbol{Q}_k \preceq \frac{P}{N}\boldsymbol{I}$.

The optimization problem of (3) can be reformulated as:

$$\max_{\boldsymbol{Q}_k} \sum_k w_k r_k \qquad \text{s.t. } \boldsymbol{Q}_k \succeq \boldsymbol{0}, \quad \sum_k \boldsymbol{Q}_k \preceq \frac{P}{N}\boldsymbol{I}. \quad (6)$$

With the covariance shaping method, all BSs restrict the sum transmit covariance $\sum_k \boldsymbol{Q}_k$ to a scaled identity matrix, so the uncertainty of the ICI over unknown but fix channels is eliminated. During T_{block}, the ICI variances cannot be larger than the maximum interference $\theta_k^{\text{assumed}}$ when the other BSs update their beamforming $\boldsymbol{Q}_k$ while fulfilling $\sum_k \boldsymbol{Q}_k \preceq \frac{P}{N}\boldsymbol{I}$. The expectation operator in the performance measure (3) disappears as a result.

Although the shaping constraint reduces the set of feasible beamforming vectors, we still have the ability to serve the MDs with adaptive beamforming. Another disadvantage is that the shaping constraint forces some limits on the eigenvalues of the sum transmit covariance $\sum_k \boldsymbol{Q}_k$. For $\sum_k \boldsymbol{Q}_k$ with full rank N, the shaping constraints implies that each eigenvalue should be smaller or equal to $\frac{P}{N}$. If the BS serves only $K < N$ MDs, $\sum_k \boldsymbol{Q}_k$ will have rank K and only $\frac{K}{N}$ of the transmit power is used.

For the covariance shaping, the downlink maximization problem (6) can be solved by transforming it to an uplink minimax problem [3]. The uplink minimax problem can be solved using a joint water spilling algorithm similar to Algorithm 1 in [10].

C. Expected Rate

Most optimizations in the literature use the expectation of the ICI or one ICI realization from a previous step as the assumed ICI $\theta_k^{\text{assumed}}$. However, as the rate depends on the ICI variance with a log function, the expectation of the ICI variances does not lead to the expectation of the rate. To counteract this problem, the expected rate method operates on the expectation of the achieved rates [4]. The objective function is changed into

$$\max_{\boldsymbol{Q}_k, \theta_k^{\text{assumed}}} E_{\Theta_k}\left[\sum_k w_k r_k^{\text{achieved}}\right], \quad (7)$$
$$\text{s.t. } \theta_k^{\text{assumed}} \geq 0, \quad \boldsymbol{Q}_k \succeq \boldsymbol{0}, \quad \text{tr}\left(\sum_k \boldsymbol{Q}_k\right) \leq P.$$

The expected utility for MD k is

$$E_{\Theta_k}\left[w_k r_k^{\text{achieved}}\right] = w_k r_k^{\text{assumed}} F_{\Theta_k}(\theta_k^{\text{assumed}}), \quad (8)$$

where $F_{\Theta_k}(\theta_k)$ is the cumulative distribution function (CDF) of Θ_k at MD k. Taking the expectation leads to a maximum weighted sum rate problem with new weights $w_k^{\text{expected}} = w_k F_{\Theta_k}(\theta_k^{\text{assumed}})$.

To perform this optimization, the CDFs of the ICI at each MD need to be available at the associated BS. The CDFs can be approximated with long term measurements at the MDs.

ISBN 978-3-8007-3662-1

Now the sum rate is maximized over both the transmit covariances $\boldsymbol{Q}_k$ and the assumed ICI $\theta_k^{\text{assumed}}$. The objective (7) can be solved by an alternating optimization, which optimizes the precoders $\boldsymbol{Q}_k$ and assumed interference $\theta_k^{\text{assumed}}$ in turn [4]. After taking the average over multiple realizations, it shows that the expected rates $\boldsymbol{r}^{\text{expected}}$ converge to the achieved rates $\boldsymbol{r}^{\text{achieved}}$. The cost function for the selection of the beamforming equals to the performance measure during the data transmission by taking this expectation. The disadvantage of this method is that the statistics of the ICI have to be known to the BS.

IV. Fairness Scheduling

Up to now, the ICI robustness methods have only been discussed for sum rate maximizations. However, maximizing the sum throughput of the system usually ends up with serving only the MDs with high SINR. Consequently, the users with poor channels are starving. As the provider and users of a cellular network are typically interested in a fair resource distribution, we focus on the ICI blindness problem in fairness optimizations in this paper.

The fairness among the users can be realized by assigning different priority coefficients to the MDs in the optimizations. The selection of the user priority is based on a compromise between the maximum possible rates and a fair resource distribution. Fairness scheduling is implemented by using the maximum weighted sum rate (MWSR) framework, where the weights of the MDs are the priority coefficients. The user with a higher weight is preferred compared to the users with smaller weights. In our scenario, the beamforming is still selected to maximize a weighted sum rate, but the scheduler changes the weights of the MDs at each time slot according to a fairness criterion.

A. Optimization Framework

In the following, we will transform all problems to a MWSR problem, which can be represented by

$$\max_{\boldsymbol{Q}_k} \sum_k w_k r_k \quad \text{s.t. } \boldsymbol{Q}_k \succeq \boldsymbol{0},\ \operatorname{tr}\left(\sum_k \boldsymbol{Q}_k\right) \leq P. \tag{9}$$

The MWSR problem (9) can be solved by an extension of the covariance-based framework in [9]. Inside the covariance-based framework, the orthogonally projected scaled gradient descend is realized with the water-spilling algorithm [9].

According to [6], the general objective function of fairness scheduling is

$$\max_{\boldsymbol{Q}_k} E\left[\sum_{k\in\mathcal{K}^{(t)}} \frac{(r_k^{(t)})^\alpha}{(R_k^{(t)})^\beta}\right] \tag{10}$$

$$\text{s.t. } \boldsymbol{Q}_k \succeq \boldsymbol{0}, \quad \operatorname{tr}\left(\sum_k \boldsymbol{Q}_k\right) \leq P, \quad R_k^{(t)} = \frac{1}{t-1}\sum_{\tau=1}^{t-1} r_k^{(\tau)}.$$

The transmit covariance matrices have to fulfill the transmit power constraint $\operatorname{tr}\left(\sum_k \boldsymbol{Q}_k\right) \leq P$. $r_k^{(t)}$ is the data rate potentially achievable by user k at time t and $R_k^{(t)} = \frac{1}{t-1}\sum_{\tau=1}^{t-1} r_k^{(\tau)}$ is the historical average rate of user k. $\mathcal{K}^{(t)}$ is the set of users that are scheduled by the BS at time t. α and β are the factors that tune the fairness of the scheduler.

	Sum Rate Max	RR	TPFS	PFS
α	1	0	0	1
β	0	0	1	1

Table I: α and β original setup

	Sum Rate Max	RR	TPFS	PFS
α	1	1	1	1
β	0	0	1	1
$\mathcal{K}^{(t)}$	All MDs	$\mathcal{K}_{\text{RR}}^{(t)}$	$\mathcal{K}_{\text{TPFS}}^{(t)}$	All MDs

Table II: α and β with Multi-User Diversity

The values of α and β with the origin setup are shown in Table I. Typically, the scheduler doesn't care about the channel quality of the MDs and serves them in turn under RR. For TPFS, the scheduler ignores the channel quality and serves the MDs with the smallest throughput. However, since we want to exploit multi-user diversity to have larger data rates, the possible data rates $r_k^{(t)}$ are included in the objective function for RR and TPFS. The active user set $\mathcal{K}^{(t)}$ is employed to specify which two MDs are chosen at each time slot for RR and TPFS. RR, TPFS and PFS can be implemented with the selection of the coefficients α, β and the active user set $\mathcal{K}^{(t)}$ in Table II.

B. Round Robin

For round robin (RR), the scheduler assigns a fixed time block to different users and the MDs are served in turn. The goal is to achieve resource fairness, where the resource is time in this case.

Typically one user is served at each time for round robin. In our simulation, we use a heuristic approximation of round robin. Two users, rather than one, are served at each time slot and the multi-user diversity can still be exploited. One general example of the user rates is shown in Figure 1, where exactly two MDs have non-zero rates at each time instance.

The objective function is similar to that of the sum rate maximization, except that the active set $\mathcal{K}_{\text{RR}}^{(t)}$ is changing as shown in Table III.

	Time					
user number	$\mathcal{K}_{\text{RR}}^{(1)}$	$\mathcal{K}_{\text{RR}}^{(2)}$	$\mathcal{K}_{\text{RR}}^{(3)}$	$\mathcal{K}_{\text{RR}}^{(4)}$	$\mathcal{K}_{\text{RR}}^{(5)}$	...
1	1	0	0	1	1	
2	1	1	0	0	1	
3	0	1	1	0	0	
4	0	0	1	1	0	

Table III: Active user set $\mathcal{K}_{\text{RR}}^{(t)}$

To use the MWSR framework, the scheduling weights of the MDs are

$$w_k^{\text{RR}} = \begin{cases} 1, & \text{for } k \in \mathcal{K}_{\text{RR}}^{(t)} \\ 0, & \text{else.} \end{cases} \tag{11}$$

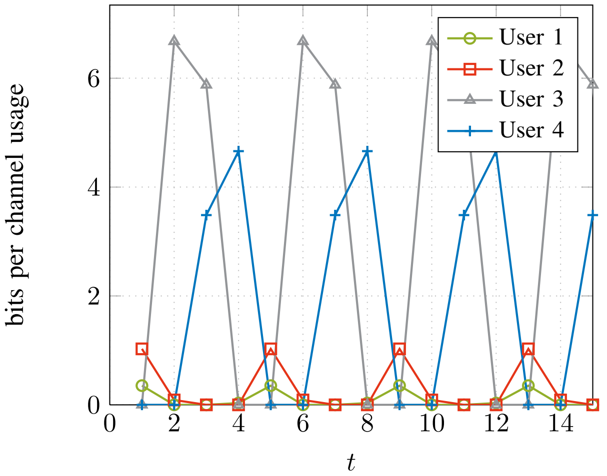

Figure 1: Instantaneous rates of the users with round robin

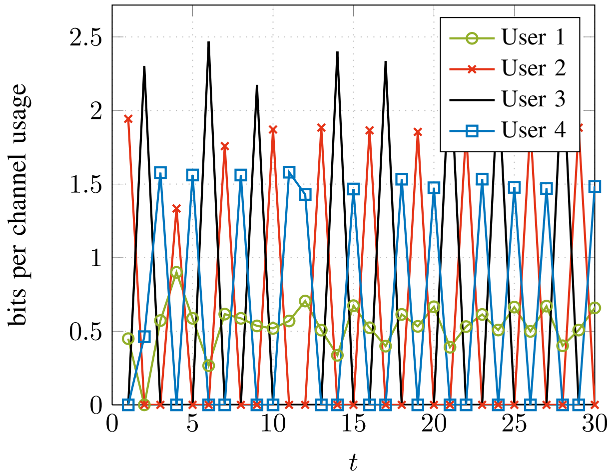

Figure 2: Instantaneous rates of the users with TPFS

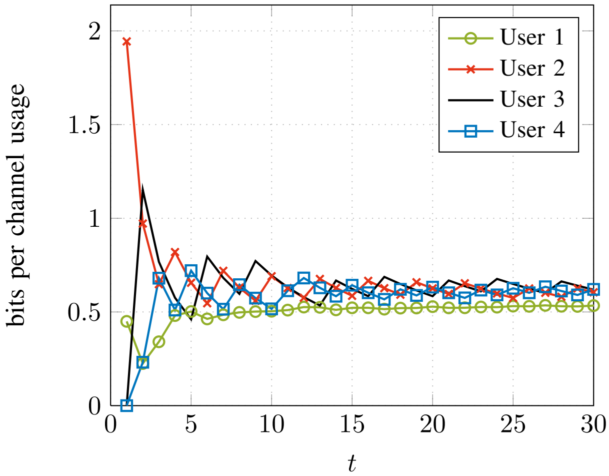

Figure 3: Averaged rates of the users with TPFS

C. Throughput Fairness

For throughput fairness, the scheduler only serves the users with the smallest historical throughput. In the end of T_{block}, all MDs have the same throughput and the MDs with poor channels are served more often. Multi-user diversity is still kept by serving two MDs at each time slot and $\mathcal{K}^{(t)}_{\text{TPFS}}$ is the set of the two users with the smallest $R_k^{(t)}$.

The weights for the MWSR problem can be written as

$$w_k^{\text{TPFS}} = \begin{cases} \frac{1}{R_k^{(t)}}, & \text{if } k \in \mathcal{K}^{(t)}_{\text{TPFS}}, \\ 0, & \text{else}, \end{cases} \tag{12}$$

$$\mathcal{K}^{(t)}_{\text{TPFS}} = \left\{ k_1 = \underset{k\in\mathcal{K}}{\text{argmin}} \left(R_k^{(t)}\right),\ k_2 = \underset{k\in\mathcal{K}\setminus k_1}{\text{argmin}} \left(R_k^{(t)}\right) \right\}.$$

The rates in Figure 2 and 3 are qualitative results to give an insight to throughput fairness. For traditional TPFS, one MD is scheduled each time and the MDs in the cell should have exactly the same throughput in the end. However, in our scenario, the two users with the smallest $R_k^{(t)}$ are served in each time instance, so the MD with the worst channel always has a smaller throughput than the rest. The multi-user diversity is influencing the fairness of the throughput, but it increases the sum rate and the robustness to the case that all users are starving because one user is not able to communicate at all.

D. Proportional Fairness

The goal of proportional fairness is to find a balance between maximum system throughput and equal throughput for all users.

For PFS, $\mathcal{K}$ is the whole set of users all the time. In the end of T_{block}, MDs with poor channels have smaller throughput compared to TPFS, but they are served more frequently than in the case of maximum sum rate. The scheduling weights in the MWSR problem are

$$w_k^{\text{PFS}} = \frac{1}{R_k} \quad \forall k \tag{13}$$

With the same parameters as for TPFS in Figure 3, the performance of PFS is shown in Figure 4. Several time slots after the initialization, the system converges to the stable state where the four MDs in the cell are served with the optimum weights $w_k = \frac{1}{R_k}$ which maximize $\sum_k w_k r_k$. It should be mentioned that the instantaneous rates of the MDs may oscillate, jumping among several optimum points of the objective function. This can be solved with time sharing and the averaged rates still converge.

E. Combination with ICI blindness

The combination of fairness scheduling and different ICI robustness methods is done by changing the weights outside of the MWSR problems. The scheduling for RR, TPFS or PFS can be applied directly as wrap-around of the MWSR algorithms with different ICI robustness solutions. The general idea is shown in Algorithm 2.

The weights $w_k^{\text{orig.}}$ are the origin priority of MD k. The weights assigned by the fairness scheduler $w_k^{\text{sched.}}$ can be w_k^{RR}, w_k^{TPFS} or w_k^{PFS} according to which fairness criterion is adopted.

ISBN 978-3-8007-3662-1

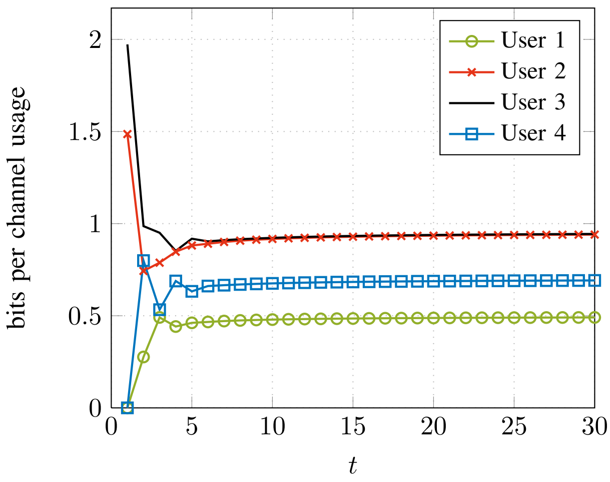

Figure 4: Averaged rates of the users with PFS

Algorithm 2 Combine fairness and ICI robustness methods

Require: original weights $w_k^{\text{orig.}}$ $\forall k$, T_{block}

$R_k \leftarrow 1\ \forall k$ ▷ initialize R_k
$t \leftarrow 1$ ▷ initialize time counter
repeat
  compute $w_k^{\text{sched.}}$ ▷ (11), (12) or (13)
  $w_k \leftarrow w_k^{\text{sched.}} \cdot w_k^{\text{orig.}}$ ▷ combine fairness
  Compute r_k^{assumed} $\forall k$ ▷ (4), (6) or (7)
  $R_k \leftarrow \frac{1}{t-1} \sum_{\tau=1}^{t-1} r_k^{(\tau)}$ ▷ update R_k
  $t \leftarrow t+1$ ▷ time counter
until $t > T_{\text{block}}$

After computing the weights, the optimal covariances Q_k and the assumed rates r_k^{assumed} are calculated by solving the MWSR problem with different ICI robustness methods (4), (6) or (7).

The time-varying property of fairness scheduling is shown by the time counter t and the update of the historical average rate R_k. Fairness scheduling can only realize fairness over many time slots.

V. Simulation Results

In our simulation, the BS is serving $K = 4$ single-antenna MDs with an $N = 4$ transmit antenna array. The origin weights of the MDs are all ones, which means there is no predefined preference in serving the MDs. In one realization, $T_{\text{block}} = 50$ time slots are simulated, during which the channel stays constant while the actual ICI θ_k^{actual} can be different in each time slot. The power limit of the BS is $P_T = 83$ Watt and the thermal noise at the MD is $\sigma_n^2 = 8.3 \times 10^{-12}$ Watt.

The channel realizations come from the 3GPP MIMO urban macro cell model [11]. The center frequency is 2 GHz and the MDs are uniformly distributed in the cell. The random actual ICI θ_k^{actual} is generated from a gamma distribution $\theta_k \sim \Gamma(a_k, b_k)$, where a_k and b_k are derived from the same channel model [4].

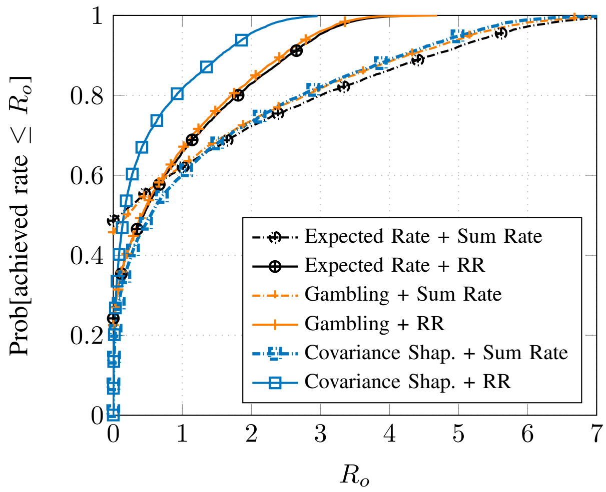

Figure 5: CDF of achieved rates with RR

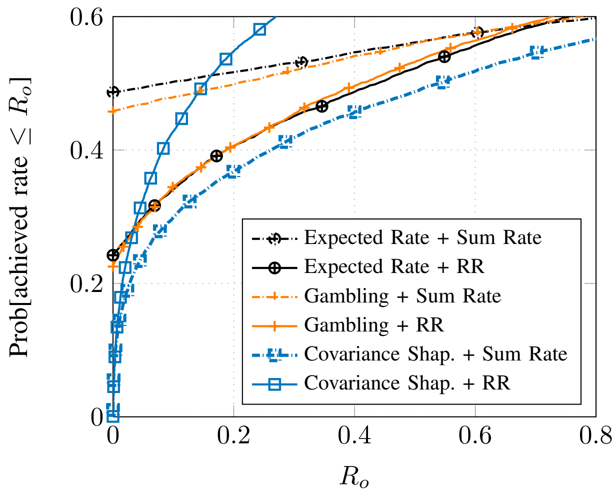

Figure 6: CDF of achieved rates with RR (small)

A. Fairness Scheduling

The combinations of different ICI robustness methods and different fairness scheduling are compared. The CDF of the average achieved rates of the MDs are plotted. Curves with a flat slope have a wider range of rates while a steep slope can be regarded as fair.

The curves of sum rate maximization are used as reference lines. For sum rate maximization, the expected rate method performs better than the other two at $R_o \geq 1$ while the covariance shaping method has a much steeper slope at the rate region $R_o \leq 1$. With the covariance shaping method, the ICI is always known and no MDs have zero rates.

For RR, the expected rate method has better performance in system throughput than the gambling and the covariance shaping methods in Figure 5. The curves of the gambling method and the expected rate method under RR in Figure 6 starts at points around (0, 0.23) because only the time slots are assigned and there is no special scheduling for MDs with poor channels. In comparison, the curve of the covariance shaping method with RR is very steep and touches the origin point,

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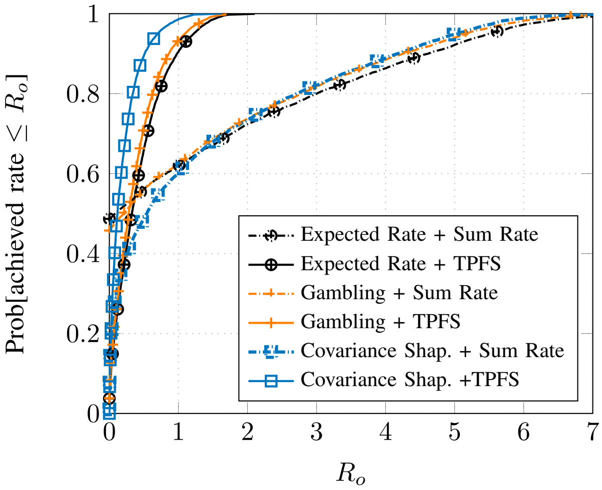

Figure 7: CDF of achieved rates with TPFS

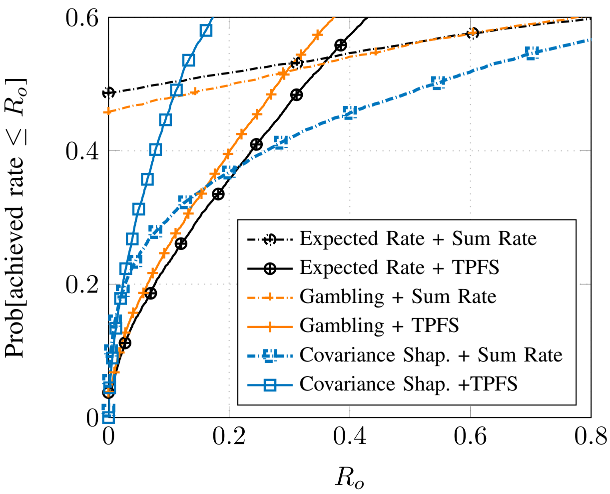

Figure 8: CDF of achieved rates with TPFS (small)

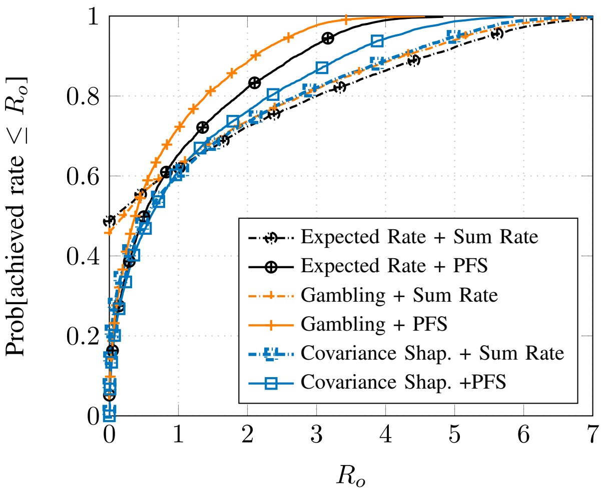

Figure 9: CDF of achieved rates with PFS

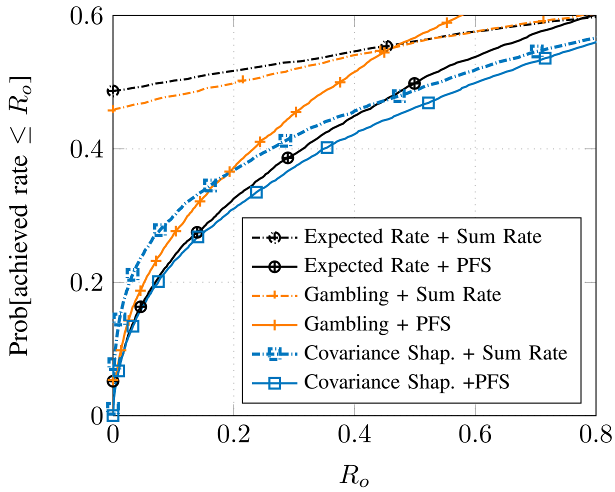

Figure 10: CDF of achieved rates with PFS (small)

which is the most fair.

Under TPFS, the expected rate method is the best considering the achieved rates. Since the MDs are scheduled to have the same throughput in the end and the MDs with bad channels are served very frequently, the three curves in Figure 7 have steep slopes. On the one hand, all three methods reach the left border at points close to the origin in Figure 8, which means most MDs have non-zero rates under TPFS. On the other hand, the curves of TPFS touch the top border at rates $R_o < 2$ because the MDs with smallest throughput are scheduled all the time.

Under the PFS criterion, all MDs in the cell are served all the time and the optimum weights $w_k = \frac{1}{R_k}\ \forall k$ are different for the three robustness methods. The covariance shaping method outperforms the other methods in achieving higher rates in Figure 9. Figure 10 shows that the MDs can always receive some data with all three methods and few MDs have zero rates.

Mean	Sum Rate	RR	TPFS	PFS
Expected Rate	1.3530	0.8702	0.4231	0.9580
Gambling	1.2688	0.8367	0.3725	0.7406
Covariance Shap.	1.3387	0.4729	0.2019	1.1900

Median	Sum Rate	RR	TPFS	PFS
Expected Rate	0.0892	0.4382	0.3262	0.5036
Gambling	0.2050	0.4098	0.2806	0.3765
Covariance Shap.	0.5355	0.1525	0.1169	0.6033

Table IV: Mean and Median achieved rate

Table IV shows the statistics, the mean and median rate of the MDs of different combinations. On the one hand, if we fix the PFS criterion, the covariance shaping method will outperform the other two methods in handling the ICI uncertainty. If RR or TPFS criterion is chosen, the expected rate method will have the larger statistics. On the other hand, when either the covariance shaping or the expected rate method is employed, PFS achieves the highest rates among the three fairness criteria.

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B. Fairness Comparison

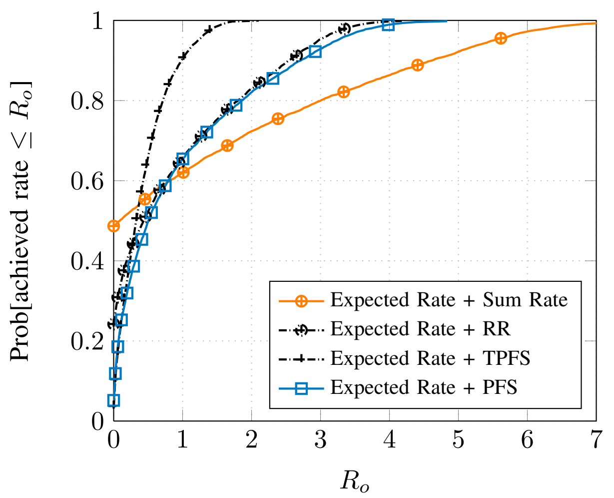

Figure 11: CDF of achieved rates with the expected rate method

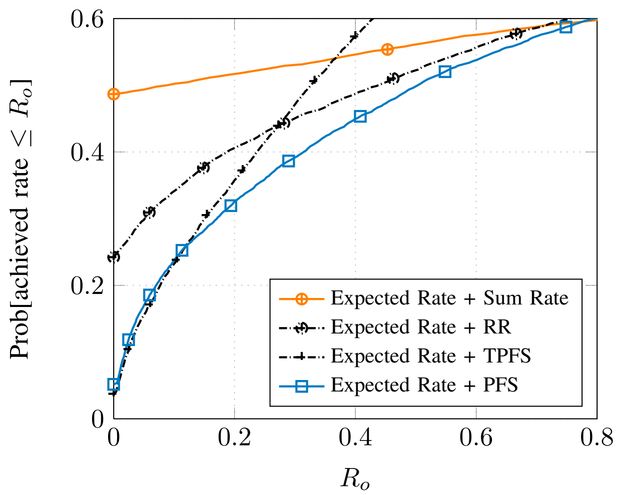

Figure 12: CDF of achieved rates with the expected rate method (small)

Fixing the expected rate method to improve the system robustness to ICI uncertainty, the qualitative property of different fairness criteria can be compared in Figure 11 and Figure 12. Figure 12 shows the details at rates $R_o \leq 0.8$.

In Figure 11, the sum rate maximization has the flattest slope while the TPFS has the steepest slope. As a balance, the cdf curve of PFS and RR have median gradient between TPFS and sum rate maximization.

Figure 12 illustrates the percentage of MDs who receive almost no data over the time block. For sum rate maximization, the starting point at $(0, 0.4871)$ implies there are more than 48% MDs have zero rates all the time. Under RR, around 25% MDs in the cell have no service. In contrast, only around 5% MDs are dropped using TPFS or PFS criterion, which improves dramatically from sum rate maximization. For TPFS and PFS, the fairness is guaranteed while achieved rates are smaller as a compromise.

VI. Conclusion

Simulation results show the performance of different combinations of robustness methods and fairness criteria. If the ICI robustness approach is fixed, PFS will outperform other fairness criteria making a good balance between fairness and system throughput. When the PFS criterion is preferred, the covariance shaping method leads to the highest rates among the three ICI robustness methods. When the RR or TPFS criterion is chosen, the expected rate method performs better in handling the ICI blindness problem.

The future work can be a further exploration of the ICI robustness methods. The combination of HARQ and the three ICI robustness methods can perform better in dealing with the ICI uncertainty. It is shown that the combination of the expected rate method with a less restrictive shaping constraint improves the data rates [12]. Besides, the optimizations where the fairness is achieved with the beamforming directly are not discussed yet.

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ISBN 978-3-8007-3662-1