BEAT: NONLINEAR THERMOACOUSTIC INSTABILITY COUPLED WITH FLAME PULSATING IN A RIJKE TUBE

Fanglong Weng , Min Zhu*

Key Laboratory for Thermal Science and Power Engineering, Tsinghua University
Haidian District, 100084 Beijing, China
* Corresponding author: zhumin@tsinghua.edu.cn

In experimental investigations on thermoacoustic instability in a Rijke tube with premixed flame, a special kind oscillations are occasionally observed, which sounds like beat. Beat oscillations are characterized by the periodically modulation of oscillation amplitudes. In this paper, comparison between beat and regular limit cycle oscillations are carried out and the stability region when beat occurs within the unstable region is presented. It is found that beats only occur at low heat power and relatively low equivalence ratio in the unstable region. Heat release rate and instantaneous flame images are taken synchronously with acoustic pressures for the analysis of the relationship between the flame dynamic behavior and the heat release rate in a beat cycle. With basic understanding of nonlinear dynamics, it is suggested that beats is the coupling result of the flame pulsating instability and the subcritical instability of the thermoacoustic system.

1 INTRODUCTION

Lean premixed combustor are developed for reducing pollutant emissions especially NOx by lowering the flame temperature, while they are now facing the problem of liability to combustion instabilities. These phenomena are due to the coupling between acoustic pressure waves and unsteady heat release in a confined environment as Rayleigh explained [1], and they are also designated as thermoacoustic instabilities. Thermoacoustic instabilities are usually characterized by limit cycles, which means when acoustic pressure wave and heat release rate fluctuation are coupled to form a closed feedback loop, the pressure wave amplitude begins to grow exponentially and under the effect of nonlinearity of the system the amplitude will grow more and more slowly, then reach at a certain value and maintain relatively stable.

Analysis and prediction of combustion instabilities has been under investigation for decades, and it relies on the knowledge of acoustic wave propagation in combustion chamber and flame response to flow disturbances. Acoustic analysis of cylindrical and annular combustors account of effects of temperature gradient and mean flow has been taken Dowling & Stow [2, 3]. It has been shown that most of the acoustic characteristics can be analyzed by linearized equations. However the widely observed nonlinear features such as triggering and mode switching in combustion systems can not be predicted under the framework of linear analysis and it remains a problem to be further investigated. Nonlinear acoustic analysis can be found in [4, 5], and it is pointed out that only considering nonlinear acoustics is likely insufficient to interpret triggering behavior shown in pressure records and consideration of nonlinear flame dynamics is suggested. The flame heat release unsteadiness in a combustor can involve various physical and chemical processes [6], such as equivalence ratio fluctuations, vortex shedding, inlet velocity fluctuations, flame surface motion, which make the nonlinear flame response modeling difficult. In many experimental and theoretical investigations, the flame response is represented with a flame transfer function (FTF) [7]. But as a linear description, FTF cannot be used to predict mode switching, triggering and hysteresis observed in many combustors as well as the oscillation amplitude of the flow variables during unstable operation. Noiray et al. [8] have developed flame describing function (FDF) which uses a quasi-linear response function to define the nonlinearity associated with perturbation amplitudes by only considering the first harmonic component in the output, and they show that this method can provide satisfactory prediction of nonlinear combustion oscillations in the experiments.

Recently there has been considerable interest in the nonlinearity and non-normality of thermoacoustic systems. It is proved that linearized governing equations of the horizontal Rijke tube [9] and premixed combustion [10] in duct are non-normal and this non-normality may lead to linear transient growth even when all eigenvalues are stable. Wieczorek et al. [11] evaluate the non-normal effect in a combustion system using the $n-\tau$ model which means the heat release fluctuation is a linear function of the velocity fluctuation with a time delay, and calculates the maximum possible transient growth with variations of unsteady heat release strength n and mean flow Mach number. Transient growth is important in thermoacoustic instability because it may lead to finite amplitude oscillations where the nonlinearity of the system takes effect and finally towards limit cycle through subcritical bifurcation which have been verified in some investigations (see for examples [12, 13].

The nonlinearities in a thermoacoustic system can give rise to complex periodical or quasiperiodic oscillations and even chaos [14, 15]. Among them a special kind of oscillations distinguish itself by the periodical modulation of pressure fluctuation amplitudes, which was designated "intermittency" in [16] and "galloping limit cycle" in [17]. From the experiment result shown in the papers, it is obvious to see that the silent instants in the pressure oscillations do not appear randomly, and present a certain kind of periodicity which is the distinguishing feature of a beat oscillation. Generally, beat refers to the oscillation with periodical amplitude modulation, and for a simple case, when two waves with comparative amplitudes and slightly different frequency (which is called the beat frequency) are superimposed a beat can be heard. Based on this principle, Joos & Vortmeyer [18] believe that the beats of pressure oscillations found in their matrix combustor are caused by two simultaneously excited modes of the system and the frequency difference of the two modes is 167 Hz. Boudy et al. [19] have observed the similar beats in a combustor with perforated plate stabilized multi-flame and analyzed it as superimposing of two self-excited unstable modes with FDF. But it should be noticed that there are two categories of beat as mentioned in [19]. One can be explained as above which can be designated quasiperiodic oscillations in nonlinear dynamics, but for the other with very low beat frequency (compared to frequency of the dominant unstable mode), the mechanism of it remains unclear. The beats found by experiments in [16] and [17] belong to the second category. And in a gas turbine combustor, beating of pressure fluctuations with very low beat frequency (about 1.5Hz) is reports by Lieuwen [20]. The case shows that the beat oscillation occurs and lasts for several seconds before it develops into a limit cycle oscillation with larger amplitude, but no explanation of this phenomenon has been given. Matveev & Culick [21] found beats in a Rijke tube with electricity heated grids when the mass flow rate and the heat power are low.

In the paper, we are concerned about the second category of beats with low beat frequencies (~ 1 Hz) in a Rijke type combustor, which have long amplitude modulation periods (~ 1 s). In Part 2 of this paper, the experiment setup is briefly introduced. As this kind of oscillations have not been reported widely in thermoacoustic investigations, so comparisons between regular limit cycles and beats are taken for characterization of beat will be carried out in Part 3. In the studies of beats mentioned above, the stability regions of combustion systems with variation of flame equivalence ratio and heat power are seldom given, which are also to be presented. In an unstable combustion system, heat release rate fluctuations are also needed to be analyzed to understand the thermoacoustic coupling process, so they are also to be investigated with synchronous images of flame surface. This will be the topic of Part 4. In Part 5, a preliminary hypothesis about the mechanism of beat from the view of bifurcation analysis is proposed.

2 EXPERIMENT SETUP

The experimental setup used in the present work is sketched in Figure 1. A quartz tube length $L = 0.70$ m and diameter $D = 0.05$ m is adjustable in the vertical direction and it allows flame location x_g vary-

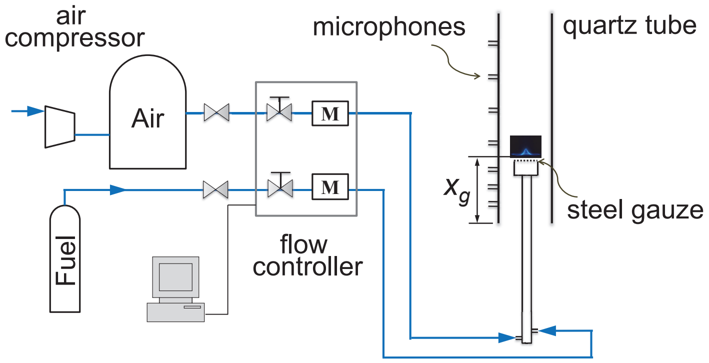

Figure 1: Layout of the Rijke-type combustor.

ing within 0 0.40 m from the bottom of quartz tube. Methane/air mixture flows through a fixed feeding manifold length L_m = 0.48 m and diameter D_m = 0.01 m and distributed in the flame holder diameter D_h = 0.026 m before combustion. The premixed flame is anchored on steel gauze to avoid flash back. Regimes of combustion instabilities are characterized by measurement of pressure fluctuations. Up to seven microphones can be used for pressure measurements along the tube and can be arranged at different branches of the quartz tube. To avoid influence of reflected wave, waveguides are used in the measurement. Velocity fluctuation can be calculated with multi-microphone method if concerned. Heat release rate fluctuation is estimated by measuring free radicals emissions, and in the present investigation, global OH* fluorescence is measured by a photodiode equipped with a photomultiplier tube centered at 310 nm. Acoustic pressure fluctuations and heat release rates data are obtained with a National Instruments DAQ system at a sampling rate of 10 kHz. Flame images are synchronously taken by a high speed camera. Gas flow rates are controlled by thermal mass flow controllers with measurement error less than 0.12×10^3 g/s. The fuel flow rate is set to vary within $3.4 \times 10^3 \sim 7.7 \times 10^3$ g/s, while the air flow rate is varied accordingly to get the equivalence ratios at which combustion instability occurs, and in experiments the equivalence ratio is mostly within the range $0.66 \sim 0.73$. The total thermal power of flame is between 170 W and 390 W. It can sustain pressure fluctuations up to 146 dB in sound pressure level (SPL) when combustion instability occurs.

3 CHARACTERIZATION OF BEAT

Beat oscillations are sometimes observed in the Rijke type combustor and can be distinguished from regular limit cycle oscillations since a tune can be heard clearly when it appears. Figures 2 gives the growth process of instability and formation of the beat oscillation, with fast Fourier transform (FFT) results of the acoustic pressure in a beat shown as well. For comparison, the results of a regular limit cycle are given in Figure 2. The two cases are under similar conditions with slightly difference in equivalence ratio. Under certain conditions, the combustion system can keep beating for more than 20 minutes without obvious change. When the quartz tube are moved away to stop the oscillations and then removed to the same place again, the same beat occurs. So beat seems not to be a stochastic phenomenon. FFT results show that the fundamental frequencies which correspond to the first longitudinal acoustic modes of the quartz tube are dominant for both cases. But in the case of beat the dominant frequency splits into three, which are very close to each other. It is not possible for the acoustic system to have three modes with so close frequencies. So the beats in self-excited oscillations cannot be interpreted as the cases in [18] and [19].

FFT result only shows the global frequency components of a signal with limited length, but the time-varying features cannot be distinguished. For this consideration, joint time-frequency analysis (JTFA) of the beat signals was taken. Three commonly used methods for JTFA are short time Fourier transform, wavelet analysis and Wigner-Ville distribution (WVD). Short time Fourier transform is a method that breaks down the signal sequence into smaller segments which can have overlap with each other and

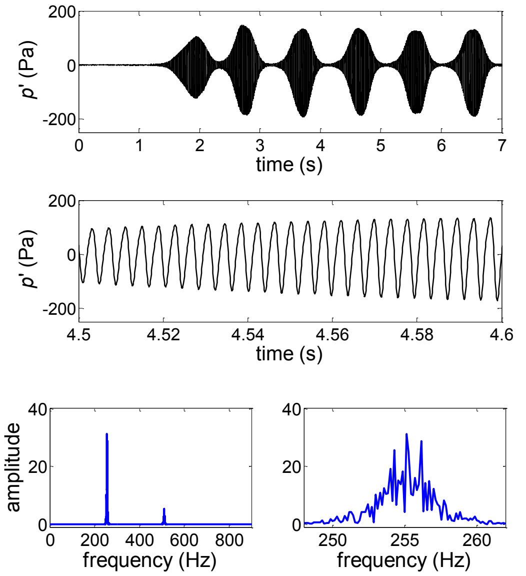

Figure 2: Acoustic pressure oscillation of a beat. Time history and FFT are both shown with their zoomed views. Combustion heat release power is 257 W and equivalence ratio is 0.683.

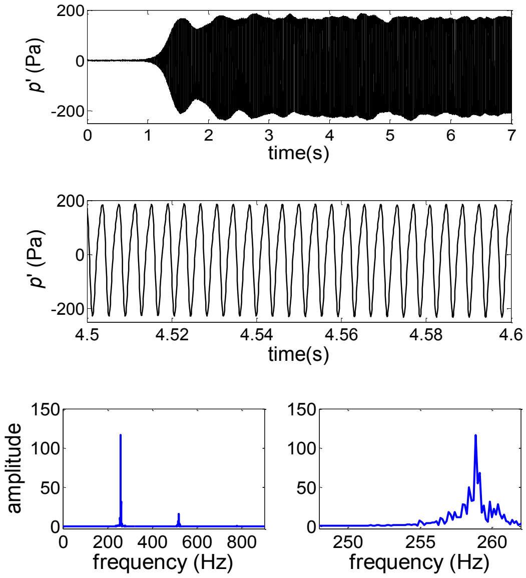

Figure 3: Acoustic pressure oscillation of a regular limit cycle oscillation. Time history and FFT are both shown with their zoomed views. Combustion heat release power is 257 W and equivalence ratio is 0.707.

then applies the conventional FFT to these segments. Following the Heisenberg principle, the resolution of STFT cannot be optimized in time domain and frequency domain at the same time. Wavelet transform use mother wavelet functions with translation and dilation/contraction at different frequency ranges, so the resolution varies at different frequency ranges, namely it can obtain high resolution in time domain at high frequencies and high resolution in frequency domain at low frequencies. But for the case of beat as above, the resolution needs to be optimized in both time and frequency domain and especially the frequency range should be near to 260Hz in this case, which is still difficult to be achieved for wavelet analysis. Wigner-Ville distribution takes Fourier transform of the instantaneous auto-correlation of a signal and have better resolution in the time and frequency domains. However, it may produce interference term between different components in the time-frequency representation. To eliminate this effect, pseudo-WVD [22] and smoothed WVD [23] has been used. Detailed description of the different time-frequency analysis methods and comparisons between them can be found in [24, 25]. In the present study, the smoothed WVD method is adopted and implemented using the Time-Frequency Toolbox for Matlab [26].

The result of joint time-frequency analysis of the measured acoustic pressure signals of a typical beat case is presented in Figure 4. The frequency axis is scaled up. It can be seen that the frequency decreases slowly during strong oscillation period in a beat and nearly silent intervals exist. Checking over the entire view of the time-frequency diagram with frequency range 0~5000Hz which is not shown here, no mode switching is observed. According to energy balance in a thermoacoustic system, acoustic energy grows only when the gain is larger than damping. In a limit cycle oscillation, the frequency remains constant, so the phase difference between acoustic pressure and heat release rate is also constant which sustains the balance of acoustic energy gain and damping. While in a beat, the gain and damping cannot keep balanced and trade-off between them is modulated periodically with frequency drift slightly, and this variation may lead to the phase difference change between the acoustic wave and heat release rate. For this consideration, the heat release rate needs to be examined carefully, which is to be discussed in the next part of this paper.

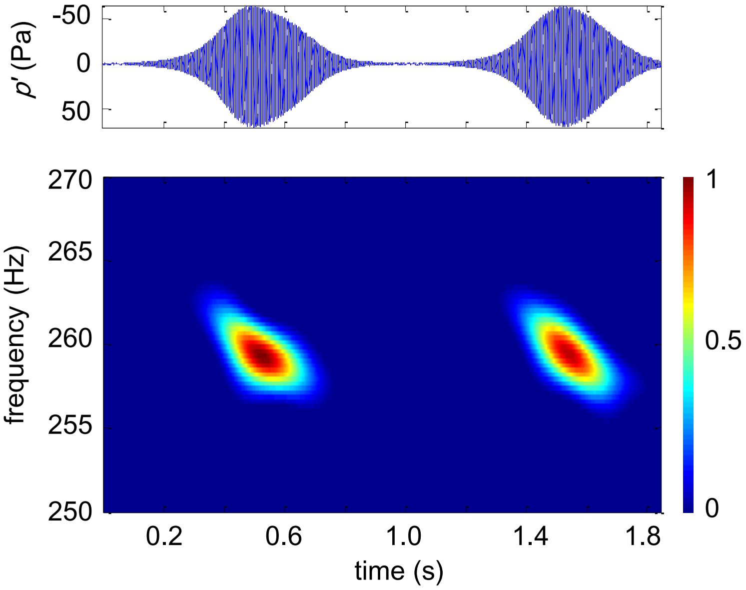

Figure 4: Time-frequency diagram of a typical beat case by smooth-pseudo-WVD method. Combustion heat release power is 214 W and equivalence ratio is 0.696.

Another important question about beat is what conditions are needed for beat to occur. Experiments with different fuel flow rates and equivalence ratios are carried out. As shown in Figure 5, beat instability region locates within small heat power and low equivalence ratio area. At large heat power conditions no beat phenomenon is observed whatever the equivalence is. As mentioned above, beat can be seen as the result of alternation of domination between energy gain and damping, and it is supposed at large heat powers the alternation cannot take place. This may provide some clues to further investigate the detailed mechanisms of beat. It should be mentioned that some cases are difficult to be classified as beat or limit cycle, as the oscillation amplitudes of them seem unstable but not strictly periodically varied. These cases often appear at the lower limit of equivalence ratio for a given heat power in Figure 5. And they are believed to contain some similar mechanisms of beat, so they are not specially marked out in the beat instability region.

4 FLAME PULSATING

For visualization of the flame movement during a beat cycle, flame images are taken at a sampling rate of 160 frames per second. At this rate the instantaneous flame surface movement forced by acoustic wave at the frequency about 260 Hz cannot been distinguished, while the flame pulsating at the beat frequency can be clearly observed. The heat release rate signals and flame images in a beat case are recorded synchronously, as shown in Figure 6 and Figure 7. For convenience of comparison, all the results in this part are based on the same case of beat which has a combustion heat release power of 214 W and equivalence ratio of 0.696.

With the length of premixed flame pulsating, the total flame heat release rate also pulsates at a very low frequency, which can be seen by taking the moving average of heat release rates, which is designated as $\bar{q}_t'(t)$ in Figure 6(c). As the fundamental frequency of the combustion system is influenced by the mean temperature downstream of the flame and the decreasing of $\bar{q}_t'(t)$ means the temperature tends to be lower, so the pulsating $\bar{q}_t'(t)$ is supposed to be the reason of frequency drift in Figure 4. To take a clear view of the oscillation amplitude variation of heat release rate and acoustic pressure at the frequency of the dominant acoustic mode, STFT method has been adopted to give the plots of $|\hat{p}'|_{f1}$ and $\left|\hat{q}_t'\right|_{f1}$

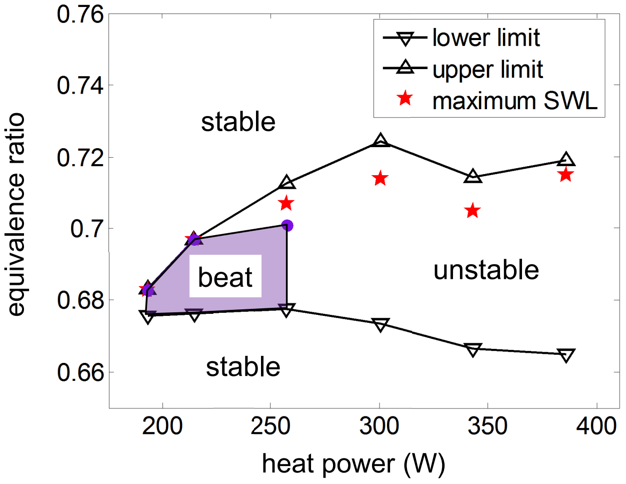

Figure 5: Stability diagram with beat region shaded when heat power and equivalence ratio of the flame are varied. The maximum sound power level (SWL) found at a certain heat power are marked with stars.

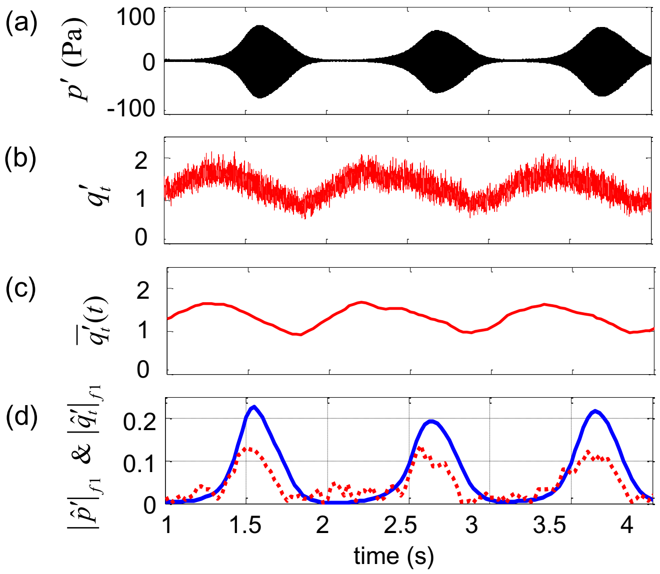

Figure 6: Acoustic pressure (a) and heat release rate (b) during beat oscillation. (c) is the transient average of heat release rate. (d) shows the time-varying oscillation amplitudes of heat release (dashed line) and acoustic pressure (solid line) at the frequency of the dominant acoustic mode. transient amplitude with time and is also given to show the pulsating flame influence. Combustion heat release power is 214 W and equivalence ratio is 0.696.

in Figure 6(d). Comparing Figure 6(c) and Figure 6(d), it is found that the acoustic pressure amplitude begins to grow near the peak of $\bar{q}'_t(t)$, and the strong oscillation period coincides with the decreasing of $\bar{q}'_t(t)$. It can also be noted in Figure 6(d) that there is obvious time delay of the decreasing of $\left|\hat{p}'\right|_{f1}$ when compared with $\left|\hat{q}'_t\right|_{f1}$. This is likely to be the damping effect of acoustics. And from these results, it is concluded that the amplitude modulation of acoustic pressure oscillation in a beat is dominated

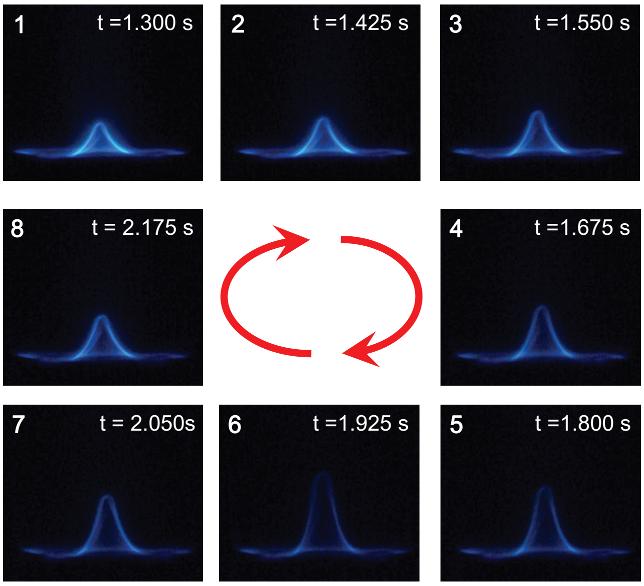

Figure 7: Flame images of one pulsating cycle corresponding to time 1.3 s ~ 2.3 s in Figure 6

by the heat release rate. Only when $\bar{q}'_t(t)$ reaches a certain value the coupling conditions between heat release rates and acoustic waves are satisfied, and when the $\bar{q}'_t(t)$ decreases to a lower value the coupling cannot be sustained and the $\left|\hat{q}'_t\right|_{f1}$ attenuates to nearly zero while $\left|\hat{p}'\right|_{f1}$ with some delay.

It is interesting to find that the maximum value of $\bar{q}'_t(t)$ appears when the premixed flame is very short, namely very small flame surface area. In many previous studies, the assumption that heat release rate is proportional to the flame surface area was taken for the modeling of premixed flames, while it seems not to hold in the case of beat here. When the flame length increases from photo 1 to photo 5 in Figure 7, the flame seems to be weaker and weaker and the $\bar{q}'_t(t)$ tends to decrease. After the acoustic waves are damped out, the flame restores to a short flame, as Picture 6 to Picture 8. But this process takes less time than the flame lengthening. For the laminar premixed flames in the experiment, flame lengthening reveals that the flame speed is decreasing. As we know, the flame speed can be influenced by the stretch rate which is strengthened when the flame is acoustically forced. For an adiabatic flame far from the lean limit, flame speed variation under acoustic force may be neglected, but for a weak flame with relatively lean equivalence ratio as in this case, the effect of stretch rate and heat loss should be carefully evaluated.

Flame pulsating instability is a kind of intrinsic instability due to the thermal and mass diffusivity unbalance, which can be observed in both premixed flame and nonpremixed flame [27, 28]. Usually a rather large Lewis number is needed for a premixed flame to present this instability, but when heat loss is considered the critical value of Lewis number can be much lower [29, 30]. Preliminary experimental investigation of intrinsic flame instabilities of the premixed flame without the quartz tube in the same system has been taken and it is found the flame pulsating instability occurs at some conditions, but they are far from the beat region in Figure 5. Further investigation based on detailed flame dynamics at different time scales is needed to recover the flame pulsating mechanism in a beat.

To evaluate whether or not the coupling between heat release rate and acoustic wave occurs, it is often referred to Rayleigh index (RI). For a beat this index should be time-varying, so STFT based method is used to extract the single component of heat release rate which corresponds to the dominant acoustic mode in a moving time window. Then the transient RIs are calculated to plot Figure 8 (c). The moving window length is carefully chosen to reveal the pulsating feature of RI. The corresponding phase differences between heat release rate and acoustic pressure at the bottom of the flame are also calculated meanwhile, as plotted in Figure 8 (d). In Figure 8, it is found that the phase difference increases slow-

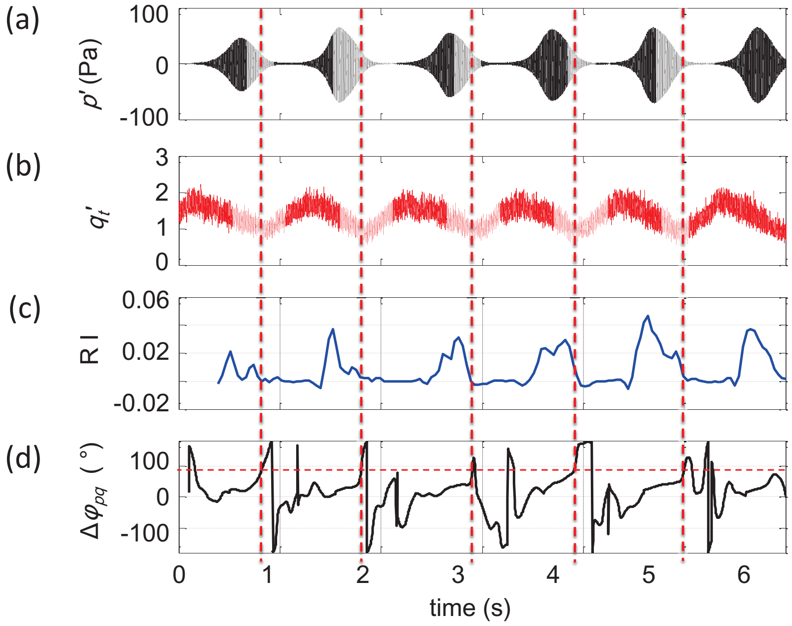

Figure 8: Time-varying Rayleigh index and phase difference between heat release rate and acoustic pressure at the frequency of the dominant acoustic mode.

ly during the oscillation period in a beat, then the increase rate suddenly becomes very large. At the moment when phase difference increases to 90°, the RI should be zero, which is the general feature of Figure 8 (d). Slight deviation at some points is due to the the resolution limit in joint time-frequency domain which is subjected to the Heisenberg principle. The time-varying RI agrees well with the acoustic pressure oscillation when comparing Figure 8 (a) and (c). It can also be seen that there are periods in which the phase difference is within ±90°, but RI remains zero. This reveals the nonlinearity in thermoacoustic instabilities. For the large amplitude oscillation to be triggered, it needs some time to establish the coupling between heat source and acoustic wave.

5 SUBCRITICAL INSTABILITY

As the results presented in Figure 2, beats usually occur near the stability boundary corresponding to equivalence ratio and total heat power, which inspires consideration of another kind of nonlinear phenomenon in thermoacoustic systems which also appears near the stability boundary, namely hysteresis, as shown in Figure 9. When varying equivalence ratio from different directions, two different critical values can be seen where the system stability suddenly changes (as A and B in Figure 10 (b)). And it is interest to find that hysteresis is obvious near the upper limit of equivalence ratio but hardly any exists near the lower limit.

For the understanding of hysteresis, it is often referred to Hopf bifurcations in nonlinear dynamics. There are two kinds of Hopf bifurcations as shown in Figure 10 (a) and Figure 10 (b). The solutions of some nonlinear systems can be stable or not stable, and it is assumed the systems have variable control parameters which influence the stability of solutions. Increasing the control parameter, the number of solutions and their stability vary in different ways. In the case of Figure 10 (a), the system has only one solution at each control parameter and the amplitude of limit cycle increases gradually after bifurcation point. In the case of Figure 10 (b), a range of control parameters exist at which the system has three solutions including a fixed point, an unstable limit cycle and a stable limit cycle, and the transitions point between them are designated Hopf bifurcation and fold bifurcation. Due to the unstable limit cycle, am-

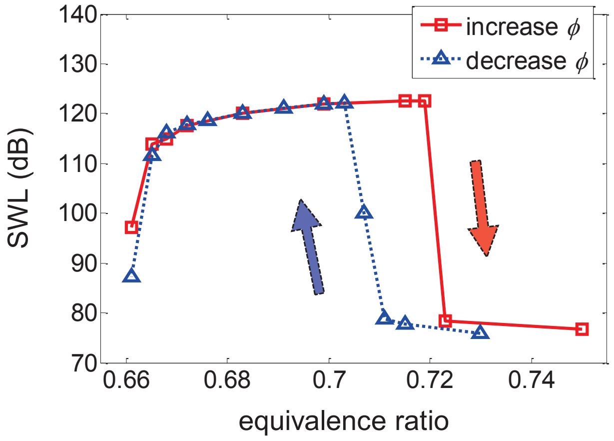

Figure 9: Hysteresis of the limit cycle oscillations as equivalence ratio varies in different directions. Combustion heat release power is 386 W.

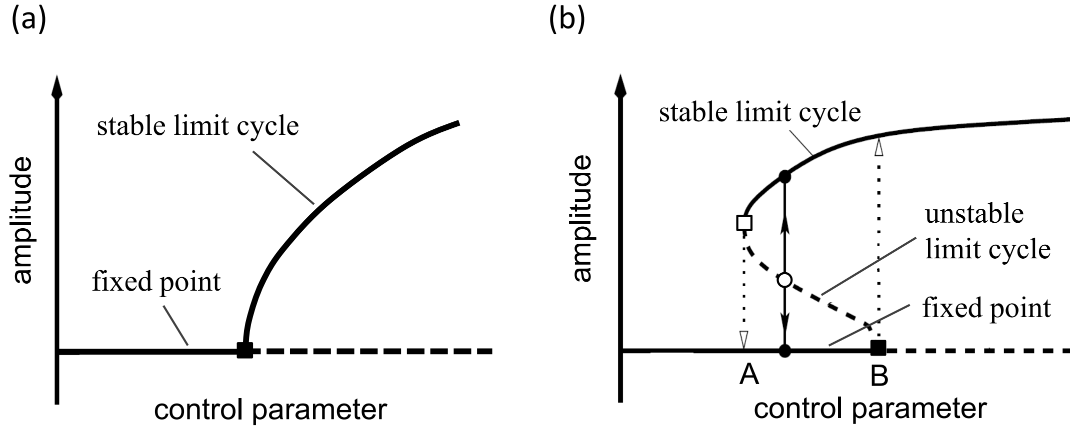

Figure 10: Nonlinear dynamic systems with supercritical instability (a) and subcritical instability (b). ■: Hopf bifurcation, □: fold bifurcation.

plitude of limit cycle suddenly jumps to a nonzero value after bifurcation point. The Hopf bifurcations in these two cases are termed supercritical bifurcation and subcritical bifurcation respectively. Hysteresis can be found in a nonlinear system with subcritical bifurcation. In Figure 10 (b), it is easy to obtain the hysteresis range determined by the fold bifurcation and the Hopf bifurcation. For a thermocaoutic system with subcritical instability, when the control parameters corresponding to the fold bifurcation and the Hopf bifurcation are very close to each other, the hysteresis can be difficult to be distinguished by experiment.

With these basic understandings of a nonlinear thermoacoustic system and the analysis results shown above, a hypothesis is proposed that when some control parameter corresponding to the pulsating flame is periodically varied near the hysteresis range, the growth rate of instability will also be periodically varied, then the beat will appear. It is important to find out the corresponding control parameters modified at the beat frequency, so the modeling of flame dynamics in a beat can be carried out. And this will be the ongoing work about the beat phenomenon in combustion instabilities.

6 CONCLUSIONS

The present work focuses on beat phenomenon observed in a Rijke tube combustor. By joint time-frequency analysis, the time-varying features of beat can be presented, such as the frequency drift in the oscillation periods. It is also found that beat only occurs at small heat release power and relatively low equivalence ratio within the unstable region of the thermoacoustic system. The heat release rate oscillations of beat instability have an evident component corresponding to beat frequency and this is supposed to be the key factor which causes the alternation of gain and damping of acoustic energy and finally leads to beat. With special attention to the hysteresis in the combustion system, a hypothesis to explain the mechanism of beat is proposed from the view of nonlinear dynamic systems.

This paper is the first work to take a detailed analysis of hear release rate in a beat oscillation, and suggest the flame pulsating instability coupled with thermoacoustic instability can be the underlying mechanism of beat. Since beat shows a organized patten of amplitude modulation, the instability growth rate and damping rate can be evaluated with high reliability and nonlinear characters of the system may stand out during the periodical alternation between stable and unstable state. With further understanding of its mechanism and special designed experiments, beat may provide a desirable platform for the experimental investigation of nonlinearities in a thermoacoustic system.

References

[1] B.J.W.S. Rayleigh. *The theory of sound*, volume 2. Macmillan, 1896.

[2] A.P. Dowling. The calculation of thermoacoustic oscillations. *Journal of Sound and Vibration*, 180(4):557–581, 1995.

[3] A.P. Dowling and S.R. Stow. Acoustic analysis of gas turbine combustors. *Journal of Propulsion and Power*, 19(5):751–764, 2003.

[4] V. Yang, S.I. Kim, and F.E.C. Culick. Triggering of longitudinal pressure oscillations in combustion chambers. i: Nonlinear gasdynamics. *Combustion Science and Technology*, 72(4-6):183–214, 1990.

[5] F.E.C. Culick. Some recent results for nonlinear acoustics in combustion chambers. *AIAA journal*, 32(1):146–169, 1994.

[6] T. Lieuwen. Modeling premixed combustion-acoustic wave interactions: A review. *Journal of Propulsion and Power*, 19(5):765–781, 2003.

[7] D. You, Y. Huang, and V. Yang. A generalized model of acoustic response of turbulent premixed flame and its application to gas-turbine combustion instability analysis. *Combustion Science and Technology*, 177(5-6):1109–1150, 2005.

[8] N. Noiray, D. Durox, T. Schuller, and S. Candel. A unified framework for nonlinear combustion instability analysis based on the flame describing function. *Journal of Fluid Mechanics*, 615(1):139–167, 2008.

[9] K. Balasubramanian and R.I. Sujith. Thermoacoustic instability in a Rijke tube: Non-normality and nonlinearity. *Physics of Fluids*, 20(4):44103–44103, 2008.

[10] P. Subramanian and R.I. Sujith. Non-normality and internal flame dynamics in premixed flame–acoustic interaction. *Journal of Fluid Mechanics*, 679(1):315–342, 2011.

[11] K. Wieczorek, C. Sensiau, W. Polifke, and F. Nicoud. Assessing non-normal effects in thermoacoustic systems with mean flow. *Physics of Fluids*, 23:107103, 2011.

[12] K. Kashinath, S. Hemchandra, and M.P. Juniper. Nonlinear phenomena in thermoacoustic systems with premixed flames. In *Proceedings of ASME Turbo Expo*, volume GT2012-68726, 2012.

[13] K.T. Kim and S. Hochgreb. Measurements of triggering and transient growth in a model lean-premixed gas turbine combustor. *Combustion and Flame*, 159(3):1215–1227, 2012.

[14] L. Kabiraj, R.I. Sujith, and P. Wahi. Bifurcations of self-excited ducted laminar premixed flames. *Journal of engineering for gas turbines and power*, 134(3), 2012.

[15] S. Lei and A. Turan. Nonlinear/chaotic behaviour in thermo-acoustic instability. *Combustion Theory and Modelling*, 13(3):541–557, 2009.

[16] L. Kabiraj and R.I. Sujith. Nonlinear self-excited thermoacoustic oscillations: intermittency and flame blowout. *Journal of Fluid Mechanics*, 713:376–397, 2012.

[17] F. Boudy, D. Durox, T. Schuller, and S. Candel. Nonlinear flame describing function analysis of galloping limit cycles featuring chaotic states in premixed combustors. In *Proceedings of ASME Turbo Expo*, pages GT2012–68998, 2012.

[18] F. Joos and D. Vortmeyer. Self-excited oscillations in combustion chambers with premixed flames and several frequencies. *Combustion and flame*, 65(3):253–262, 1986.

[19] F. Boudy, D. Durox, T. Schuller, and S. Candel. Analysis of limit cycles sustained by two modes in the flame describing function framework. *Comptes Rendus Mécanique*, 341(1):181–190, 2013.

[20] T.C. Lieuwen. Experimental investigation of limit-cycle oscillations in an unstable gas turbine combustor. *Journal of Propulsion and Power*, 18(1):61–67, 2002.

[21] K.I. Matveev and F.E.C. Culick. Limit-cycle properties of a Rijke tube. *Technical acoustics*, 12:1–13, 2003.

[22] T.A.C.M. Classen and W.F.G. Mecklenbrauker. The Wigner distribution-a tool for time-frequency signal analysis: Part I. continuous time signals. *Philips J. Res*, 35(3):217–250, 1980.

[23] M-C Pan, P. Sas, and H. Van Brussel. Nonstationary time-frequency analysis for machine condition monitoring. In *Proceedings of the IEEE-SP International Symposium on*, pages 477–480. IEEE, 1996.

[24] L. Cohen. Time-frequency distributions-a review. *Proceedings of the IEEE*, 77(7):941–981, 1989.

[25] S. Qian and D. Chen. Joint time-frequency analysis. *Signal Processing Magazine, IEEE*, 16(2):52–67, 1999.

[26] F. Auger, P. Flandrin, P. Gonçalvès, and O. Lemoine. Time-frequency toolbox. *CNRS France-Rice University*, http://tftb.nongnu.org/.

[27] P. Pelce and P. Clavin. Influence of hydrodynamics and diffusion upon the stability limits of laminar premixed flames. *Journal of Fluid Mechanics*, 124(1):219–237, 1982.

[28] E. Robert and P. A. Monkewitz. Thermal-diffusive instabilities in unstretched, planar diffusion flames. *Combustion and Flame*, 159(3):1228–1238, 2012.

[29] J. Buckmaster. Stability of the porous plug burner flame. *SIAM Journal on Applied Mathematics*, 43(6):1335–1349, 1983.

[30] V. N. Kurdyumov and M. Matalon. The porous-plug burner: Flame stabilization, onset of oscillation, and restabilization. *Combustion and Flame*, 153(1):105–118, 2008.