CTM Based Calculation of Number of Stops and Waiting Times

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Abstract

The Cell Transmission Model (CTM) is a numerical approximation of the shockwave theory. The CTM divides the links into discrete segments, known as cells, and updates the density of each cell (number of vehicles) after every second. Due to its simple formulations and high-speed execution, the CTM is considered advantageous for model-based Adaptive Traffic Control Systems (ATCSs). A CTM implementation was developed within Sitraffic Motion, the “Motion-CTM” [9]. The “Extended Motion-CTM” [11] calculates two performance indices for the online signal plan optimization in Sitraffic Motion: the number of stops and the waiting time. The Extended Motion-CTM introduces a new variable for every cell in the network, the Congestion Indicator (CI). The CI not only facilitates the calculation of the number of stops and the waiting time, but also assigns the calculated values to the appropriate signal. The significance of this novel idea is that it enables the calculation of signal related performance indices for the whole network in each second. The development of the CI algorithm in the form of a finite-state machine diagram is the core of this work. For the calculation of the number of stops, the CI is utilized to identify the cells at the end of the queue. Based on the outflow (vehicles per second) of these cells, the number of stops is derived. For the calculation of the waiting time, the CI is utilized to identify the cells that are in the queue. The waiting time is derived from the density of these cells and their outflow. The evaluation of the proposed algorithms shows that both the derived number of stops and waiting time are very close to the theoretical values obtained from the shockwave theory. Regarding the number of stops, the evaluation shows that the proposed algorithms perform much better than the approximation by Lin [10]. Regarding the waiting time, the evaluation shows that the delay calculation based on Almasri [6] and the waiting time in the Extended Motion-CTM give very similar results.

Introduction

The Cell Transmission Model (CTM) was introduced by Daganzo with a series of publications during the 90’s [1, 2, 3] as a numerical approximation of the shockwave theory. Daganzo’s CTM has been created originally for highway implementations. However, due to its simple formulations and high-speed execution, the CTM is considered advantageous for model-based Adaptive Traffic Control Systems (ATCSs). Daganzo [4] introduces the first adaptation of the CTM on signalized roads and Ziliaskopoulos and Lee [5] present the first application of the CTM on signalized intersections. Recently, the CTM appears in publications that deal with the online signal plan optimization in urban networks. Characteristic examples of this trend are the dissertations of Almasri [6], Rohde [7] and Pohlmann [8]. Zeng [9] extends the CTM in order to apply it into an urban network for Sitraffic Motion, which is a state of the art model-based ATCS developed by Siemens AG.

Sitraffic Motion uses vertical queuing models to calculate two performance indices in order to optimize the signal plans: the number of stops and the waiting time. However, it is decided to implement a horizontal queuing model in Sitraffic Motion, namely the CTM. The fundamental difference between these two approaches (vertical queuing-horizontal queuing) is that vertical queuing models do not take into account the physical length of the vehicles in the queue; the queue is modelled as a single point with certain capacity (point queue models). On the contrary, horizontal queuing models can model the physical spillback of the queue and therefore are considered to be more realistic (spatial queue models).

The Cell Transmission Model and its extensions for urban networks

The CTM divides the links into discrete segments, known as cells, and updates the density of each cell (number of vehicles) after every calculation interval (typically one second). The length of each cell is selected (by model definition) as the distance traveled by one vehicle in one time interval with free flow speed. Figure 1 illustrates the basic notions of the CTM. It has to be clearly stated that for the CTM cars are not distinguishable within the cells. The colored rectangles (left part of Figure 1) are used to support an intuitive understanding of the model. The right part of Figure 1 illustrates how the CTM decodes the presence of vehicles in a cell. Please note that the number of vehicles in a cell may not be an integer number.

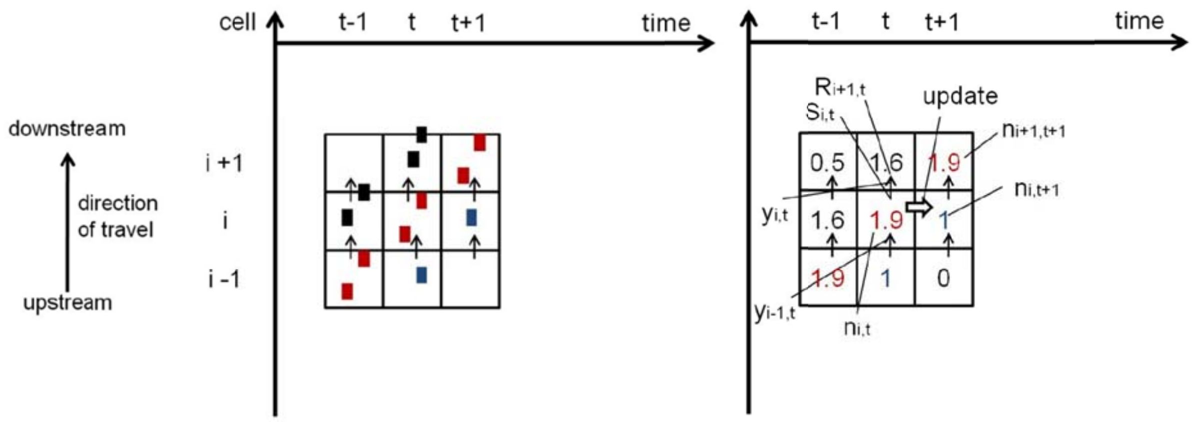


Figure 1. Basic notions of CTM

$n_{i,t}$: number of vehicles in cell i at time t (equivalent to the density of cell i at time t)

$y_{i,t}$: outflow of cell i at time t

$y_{i-1,t}$: outflow of cell i-1 at time t (inflow of cell i at time t)

$S_{i,t}$: sending ability of cell i (upstream) at time t

$R_{i+1,t}$: receiving ability of cell i+1 (downstream) at time t

The simulation in the CTM starts with a given state ($n_{i,t}$) of the cells and follows two simple steps. First the flows ($y_{i,t}, y_{i-1,t}$) between all cells are calculated using Equation (1) and then the number of vehicles ($n_{i,t+1}$) in each cell is updated using Equation (2).

$$y_{i,t} = \min\{S_{i,t}, R_{i+1,t}\} \tag{1}$$

$$n_{i,t+1} = n_{i,t} + y_{i-1,t} - y_{i,t} \tag{2}$$

The flow between two cells (Equation 1) is the minimum between how many vehicles the upstream cell can send ($S_{i,t}$) and how many vehicles the downstream cell can receive ($R_{i+1,t}$). The number of vehicles in cell i at time t+1 ($\mathrm{n_{i,t+1}}$) is equal to the number of vehicles in that cell at the previous step ($\mathrm{n_{i,t}}$), plus the inflow ($\mathrm{y_{i-1,t}}$), minus the outflow ($\mathrm{y_{i,t}}$) for that cell during the time interval t. Figure 2 illustrates the simplified trapezoid flow-density relationship that is used by Daganzo's CTM. Daganzo proved that the results of the approximation converge to those of the hydrodynamic model [2].

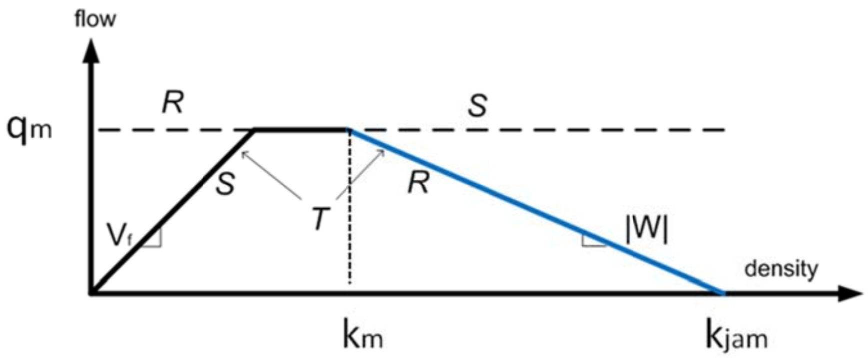


Figure 2. Flow-density relationship of the CTM

V_f : Free-flow speed

q_m : Maximum flow (saturation flow)

k_m : Saturation density

k_{jam} : Maximum density (jam density)

W : Backward shockwave speed (when traffic is congested)

S : Sending ability

R : Receiving ability

T : Function of flow-density ($y = T(k)$)

In order to simulate a whole network, Daganzo used three network topologies to describe road connections: The *Ordinary Links*, the *Merges* and the *Diverges*. Since then, many extensions have been proposed for a better simulation of urban networks, but the fundamental principles of the CTM remain the same (Equations 1 and 2). Zeng [9] developed the Motion-CTM as an implementation of the CTM in Sitraffic Motion (Figure 3). In addition to the ordinary cell connections (*Flow Model*), the Motion-CTM uses some special extensions to connect the links. These extensions are based on the *Merges* (*Merge Model*) and *Diverges* (*Diverge Model*) that Daganzo originally suggested but are proven to be more appropriate for urban applications. Furthermore, the Motion-CTM introduces the so called “virtual cells” (Figure 3). Virtual cells are added at both ends of each link and are used to simulate the connection between the links of the network. In addition, the virtual cells simulate the effect of the traffic lights by blocking the appropriate virtual cell. The virtual cells have no length and are added only for simulation and computational purposes.

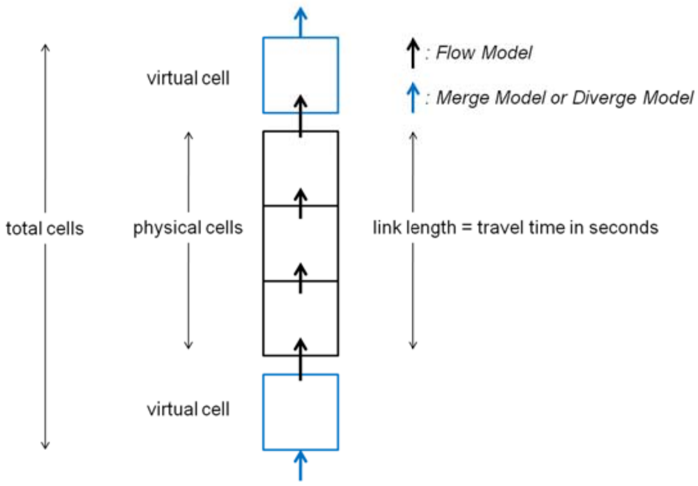


Figure 3. Motion-CTM: Introduction of Virtual Cells

Review of known CTM-based performance indices

Even though various extensions of the CTM have been introduced the last years, only two methods for calculation of CTM-based performance indices can be distinguished: The approximation of the number of stops on a link by Lin [10] and the estimation of the total delay on a link by Almasri [6].

Lin and Wang present in their work [10] a Mixed-Integer Linear Programming formulation based on the CTM and propose an approximation of the number of stops. For the estimation of the stopped vehicles, they use the flows of two consecutive time steps. They show that the number of stops for a link can be approximated with the formula:

$$Number\ of\ Stops_{link} = (0.5) \times \sum_{t} \sum_{i} |y_{i,t} - y_{i-1,t-1}| \tag{3}$$

In the above formula, $y_{i,t}$ indicates the outflow of cell i at time t and $y_{i-1,t-1}$ indicates the outflow of cell i-1 at time t-1.

Almasri [6] uses the definition of the total delay in cell i originally given by Daganzo [1]. Almasri presents a concrete approach to derive the total delay on a link with the CTM. This approach is also adopted by Pohlmann [8]. For a time interval of one second, the delay ($D_{i,t}$) in one cell during one simulation step is: $D_{i,t} = n_{i,t} - y_{i,t}$ .The total delay on a link that has n number of cells can be estimated by summing up all delays during the examined period:

$$Delay_{link} = \sum_{t} \sum_{i=1}^{n} D_{i,t} = \sum_{t} \sum_{i=1}^{n} (n_{i,t} - y_{i,t}) \tag{4}$$

In the above formula, $y_{i,t}$ indicates the outflow of cell i at time t and $n_{i,t}$ indicates the number of vehicles in cell i at time t. Since the outflow from a cell can never be bigger than the number of vehicles in the cell, the value in the parenthesis is never a negative number: $D_{i,t} = n_{i,t} - y_{i,t} \geq 0$.

Congestion Indicator (CI)

The Extended Motion-CTM [11] introduced algorithmic improvements and new extensions in order to calculate the necessary performance indices for traffic light optimization in Sitraffic Motion. A new cell variable is introduced: the so-called “Congestion Indicator” (CI). This extra variable is added to the cells of the Extended Motion-CTM and is updated in every time step. The main function of the CI, as the name suggests, is to identify which cells are congested. The CI stays active until the congestion dissolves. Furthermore, the CI facilitates the calculation of the number of stops and the waiting time. Additionally, the CI has one more important task in the Extended Motion-CTM. It identifies the source of the congestion. When the CI is active, it gets a value that represents the traffic signal that caused the congestion. Hence, it enables the calculation of signal-related performance indices.

Figure 4 presents an overview of the developed algorithm in the form of a simple finite-state machine. The algorithm for the CI works in three conditional steps: the *Generation*, the *Propagation* and the *Holding*. When the *Generation* condition is true, the CI variable is generated and it takes the value of the signal ID. When the *Propagation* condition is true the CI is propagated to the upstream cells. When the *Holding* condition is true, the CI is kept active for the next update. If the *Holding* condition is not true, the CI is reset to zero.

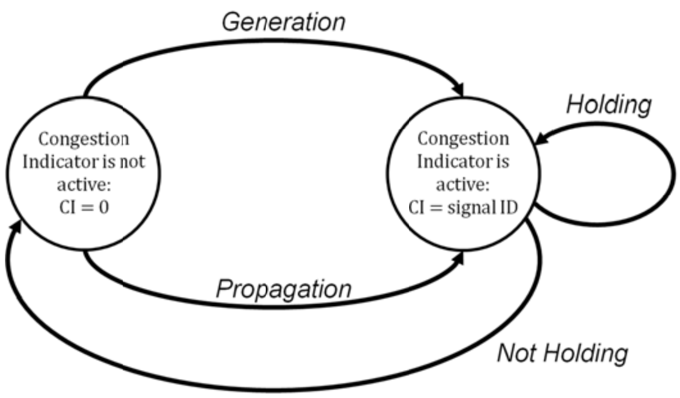


Figure 4. Finite-state machine of the Congestion Indicator algorithm

In terms of a graphical representation as in the shockwave theory, the CI is trying to identify the cells (yellow cells in figure 5) defined by the two shockwave lines as shown in figure 5, by exploiting the calculated densities of the cells in every time step.

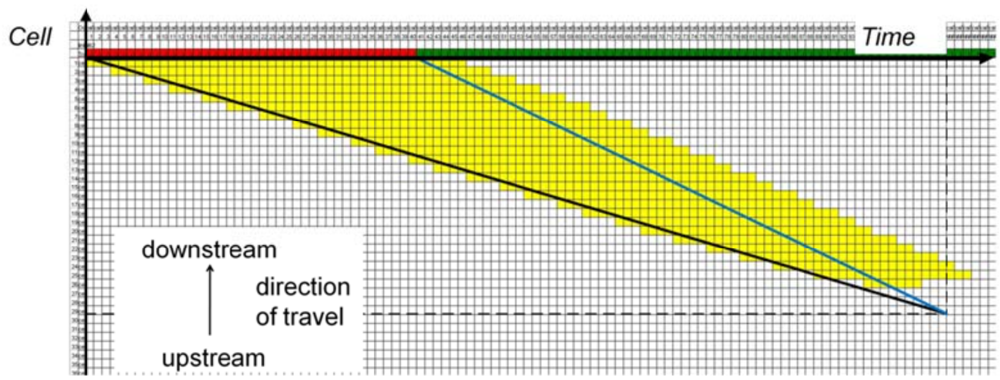
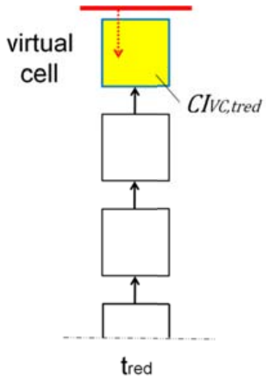


Figure 5. Congestion Indicator: Output of the Extended Motion-CTM and theoretical shockwaves

The following lines describe the algorithm and the aforementioned conditions (Figure 4). The Congestion Indicator algorithms, even though they are developed for Sitraffic Motion, are valid for any potential CTM implementation.

Generation:

This step simulates the start of the congestion caused by the red traffic signal. If the traffic light is red, the CI variable is generated according to the signal ID. The CI is passed to the virtual cell of the link in front of the traffic light.



Generation:

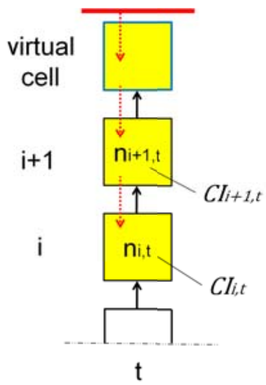
if traffic light is red,

then, $CI_{VC,tred} = signal\ ID$

else, $CI_{VC,tred} = 0$

Propagation:

This step simulates the propagation of the congestion in space against the traffic flow. If the number of vehicles in a cell ($n_{i,t}$) is bigger than the available space in the downstream cell ($N_{i+1,t} - n_{i+1,t}$), the congestion reaches the examined cell. In that case, the Congestion Indicator ($\mathrm{CI}_{i+1,t}$) from the downstream cell is passed to the upstream cell ($CI_{i,t}$). $N_{i+1,t}$ (maximum density of a cell by model definition) is the maximum allowed number of vehicles for the downstream cell.



Propagation:

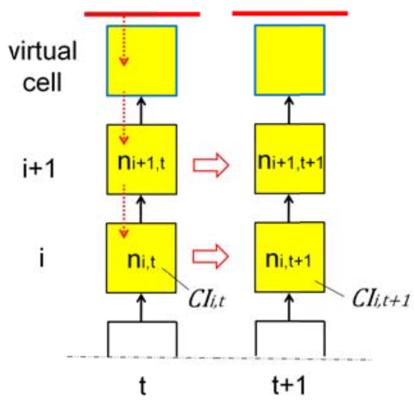
if $n_{i,t} > N_{i+1,t} - n_{i+1,t}$,

then, $CI_{i,t} = CI_{i+1,t}$

else, $CI_{i,t} = 0$

Holding:

This step simulates the duration of the congestion. If a cell has the Congestion Indicator ($CI_{i,t}$), it keeps it as long as the number of vehicles ($n_{i,t}$) in that cell is bigger than the saturation density (k_m).



Holding:

if $CI_{i,t} > 0$

AND $n_{i,t} > k_m$,

then, $CI_{i,t+1} = CI_{i,t}$

else, $CI_{i,t+1} = 0$

Number of stops

The calculation of the number of stops $(NoS_{i,t})$ in a cell i at time t utilizes the Congestion Indicator $(CI_{i,t})$ variable. Complete stops occur only at the end of the queue. In fact, the number of stops in one time interval is equal to the inflow at the end of the queue during that time interval. Consequently, if the end of the queue is identified, the flow of that cell can be derived and accordingly the number of stops. Figure 6 illustrates the result of the algorithm that recognizes the end of the queue cells. The grey cells (both dark grey and light gray cells) represent the end of the queue cells that are included in the $NoS_{i,t}$ calculations. To distinguish between the grey cells, the light grey cells are labelled simply as $EQ_{i,t}$ and the dark grey cells are labelled as $EQ'_{i,t}$. The $EQ'_{i,t}$ cells are distinguished from the $EQ_{i,t}$ cells in order to model the sudden transition (jump) of the end of the queue from a downstream cell (i+1) to the upstream cell i.

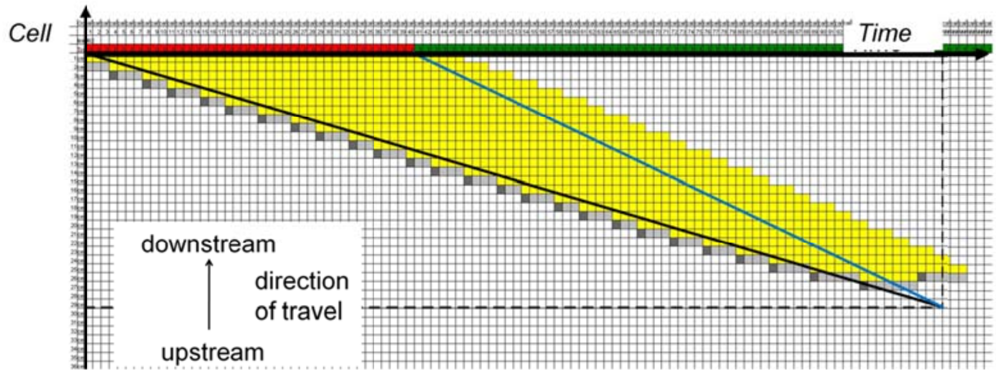
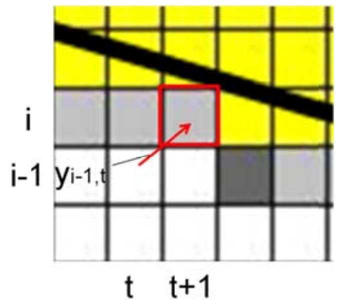
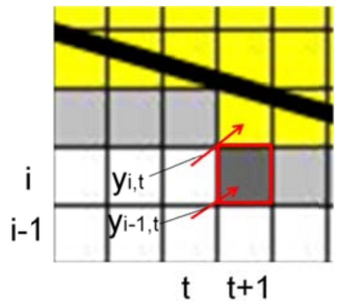


Figure 6. End of the queue cells

After the identification of the cells at the end of the queue is done, the calculation of the number of stops is performed.

<u>*Number of Stops*</u> *$(NoS_{i,t+1})$:*

	<u>*Number of Stops*</u> *$(NoS_{i,t+1})$:* if $EQ_{i,t+1} = active,$ then, $NS_{i,t+1} = y_{i-1,t}$
	<u>*Number of Stops*</u> *$(NoS_{i,t+1})$:* if $EQ'_{i,t+1} = active$ then, $NS_{i,t+1} = y_{i-1,t} + y_{i,t}$

Based on the above calculations, it is possible to compute the total number of stops in a cell i (NoS_i) for a complete period:

$$NoS_i = \sum_{0}^{t} NoS_{i,t+1}$$

If n is the number of physical cells in one link and m is the number of links of the whole network, the total number of stops in one link (NoS_{link}) and the total number of stops in the whole network$(NS_{network})$ are:

$$NoS_{link} = \sum_{i=1}^{n} NoS_i$$

$$NoS_{network} = \sum_{link=1}^{m} NoS_{link}$$

Finally, the number of stops can be assigned to the respective signals, with the help of the CI. Equation (5) gives the total number of stops for each signal in the network:

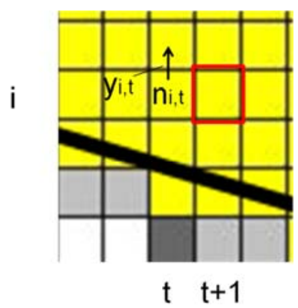
$$NoS_{signal} = \sum_{link=1}^{m} \sum_{i=1}^{n} \sum_{0}^{t} NoS_{i,t+1}^{signal}$$

$$with\ NoS_{i,t+1}^{signal} = \begin{cases} NoS_{i,t+1}^{signal}, if\ CI_{i,t+1} = signal\ ID \\ 0 \qquad , \qquad otherwise \end{cases} \tag{5}$$

Waiting time

The calculation of the waiting time $(WT_{i,t})$ in a cell i at time t utilizes again the Congestion Indicator $(CI_{i,t})$ variable. The waiting time is the number of vehicles in the queue multiplied by the time in the queue. In the case of the Extended Motion-CTM (as in the original CTM) the updates happen every second. That means that the waiting time is the number of vehicles that stay in the congested cell and they are not able to leave. Using the notions of CTM this number equals the number of vehicles that are in the cell in the previous time step $(n_{i,t})$ minus the vehicles that are able to leave the cell $(y_{i,t})$. If the vehicles that leave the cell $(y_{i,t})$ is zero, the waiting time in one time interval (1 second) equals the number of vehicles in the cell $(n_{i,t})$.

<u>Waiting Time</u> $(WT_{i,t+1})$:

	*<u>Waiting Time</u> $(WT_{i,t+1})$:* if $CI_{i,t+1} > 0$, then, $WT_{i,t+1} = n_{i,t} - y_{i,t}$

The waiting time $(WT_{i,t+1})$ can never be smaller than zero, since a cell cannot send more vehicles than it has. The total waiting time (WT_i) in a cell (i) is the sum of all waiting times $(WT_{i,t})$ for each time step:

$$WT_i = \sum_{t_{red}}^{t_{blocked}} WT_{i,t+1}$$

The time step when the traffic light turns red is noted as t_{red}. The time step when the Congestion Indicator becomes inactive again (CI is reset to zero) is noted as $t_{blocked}$. This number (WT_i) will grow as long as the cell remains congested, even though the maximum number of vehicles that the cell can hold is limited (N_i). The upper limit depends on how long the cell remains congested (how long the cell keeps the Congestion Indicator active).

If n is the number of physical cells in one link and m is the number of links of the whole network, the total waiting time of the link (WT_{link}) and the total waiting time in the whole network ($WT_{network}$) are:

$$WT_{link} = \sum_{i=1}^{n} WT_i$$

$$WT_{network} = \sum_{link=1}^{m} WT_{link}$$

Finally, the waiting time can be assigned to the respective signals, with the help of the CI. Equation (6) gives the total waiting time for each signal in the network:

$$WT_{signal} = \sum_{link=1}^{m} \sum_{i=1}^{n} \sum_{0}^{t_{blocked}} WT_{i,t+1}^{signal}$$ $$with\ WT_{i,t+1}^{signal} = \begin{cases} WT_{i,t+1}^{signal}, if\ CI_{i,t+1} = signal\ ID \\ 0 \qquad , \qquad otherwise \end{cases}$$	(6)

Evaluation

In order to evaluate the developed algorithms a test network from a city that has implemented the Sitraffic Motion is chosen, the city of Magdeburg. The evaluation compares the proposed methodology with the existing methods. The theoretical point of reference is the shockwave theory, since the CTM is a numerical approximation of the shockwave theory. The selection of the test area is made with the purpose of examining a real world critical area. The test area is part of the main corridor of the city (increased vehicle flows) and contains two signalized intersections that are relatively close (85m) to each other (Figure 7). The evaluation that follows focuses on the corridor from east to west on intersection K452 and especially on links 61 and 62. The fundamental diagram that is applied for the evaluation has a simplified triangular shape.

Figure 8 shows that for all inflows the performance of the new proposed algorithm is clearly better than the existing methodology. One more inherent advantage of the Extended Motion-CTM algorithm is that it has the ability to assign the calculated number of stops to the specific signal that originally caused the congestion and the subsequent stops. Furthermore, the performance of the algorithm is not affected by a potential big inflow that causes oversaturation. This is considered a big advantage of the new proposed formula, since these are the crucial cases for traffic light optimization.

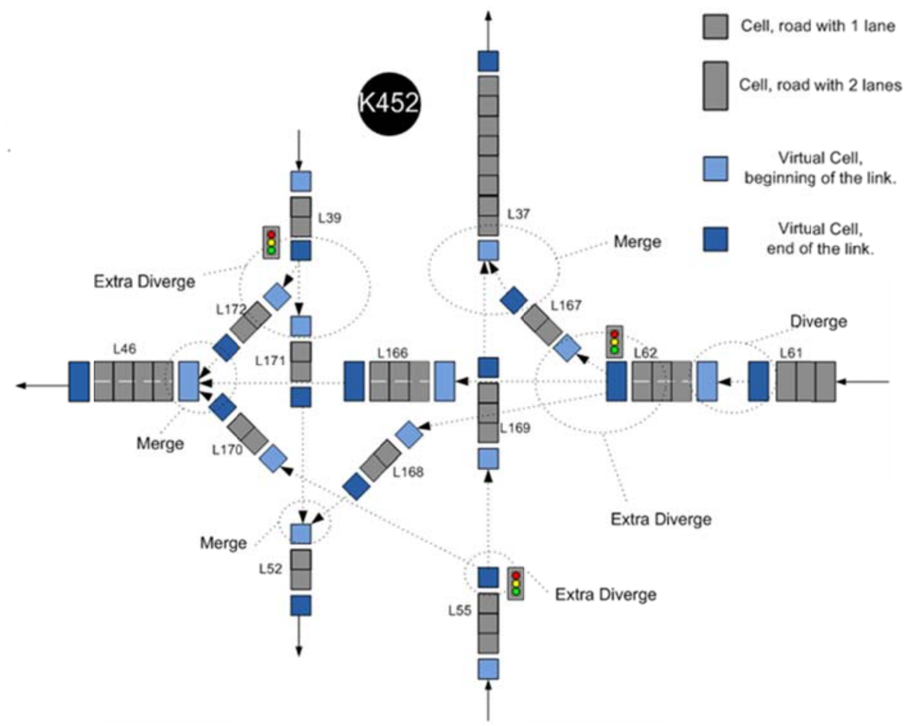


Figure 7. Test Area: Detailed plan of cells

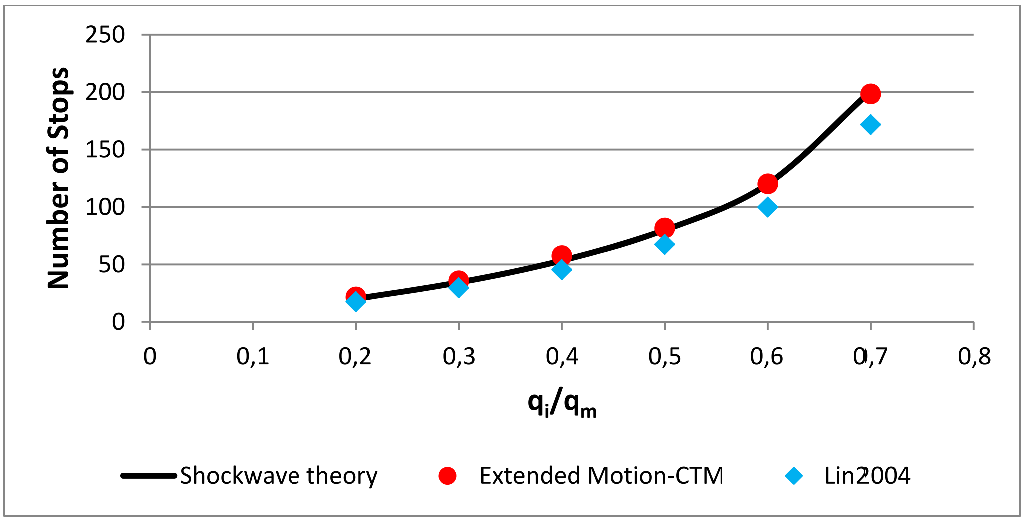


Figure 8. Number of Stops for various inflows

To get a better feeling of the importance of the precision for big inflows, the two extreme cases of Figure 8 can be beneficial. For a ratio of q_i/q_m equal to 0.2, the overestimation by the Extended Motion-CTM is less than two vehicles (1.6840 stopped vehicles). The respective underestimation by the Lin formula is 2.4 stopped vehicles. For the case of oversaturation (ratio of q_i/q_m equal to 0.7) the underestimation by the Extended Motion-CTM is just over two stopped vehicles (2.3514 stopped vehicles). The respective Lin underestimation is almost 30 vehicles (29.0584 stopped vehicles). This difference is solely for one oversaturated intersection. Consequently, this difference, in terms of missed stops, becomes extremely big for larger networks with many oversaturated intersections. Thus, the new algorithm that is proposed in this paper can be considered as very reliable even for big networks and oversaturated conditions.

Figures 9 and 10 give a more detailed look of the performance of the algorithms compared to the theoretical values derived from the shockwave theory for every second during two cycles. The continuous line illustrates the expected values from the shockwave theory. The points in Figure 9 give the results of the algorithm by Lin [10] and the points in Figure 10 give the results of the developed algorithms based on the Congestion Indicator [11].

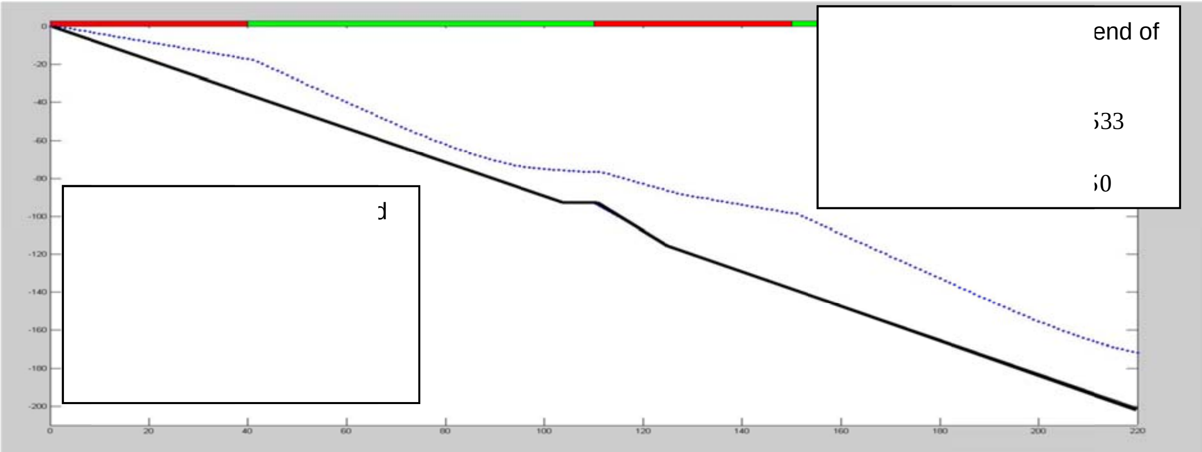


Figure 9. Number of stops comparison: Lin [10] and shockwave theory

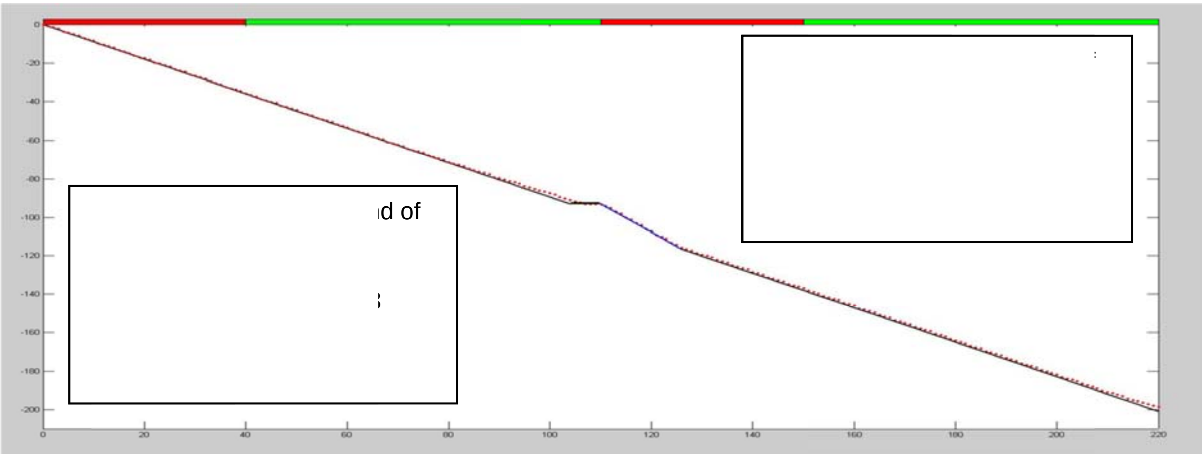


Figure 10. Number of stops comparison: Extended Motion-CTM [11] and shockwave theory

The evaluation of the waiting time calculation on the same corridor shows that the developed algorithms are very accurate (Table I).

	Waiting time (end of 2nd cycle)				
q_i/q_m	Shockwave theory	Extended Motion-CTM	Almasri2006 (Delay)	Extended Motion-CTM vs. Shockwave theory	Almasri2006 (Delay) Vs Shockwave theory
0.2	400.0000	398.0085	400.0000	- 0.4979 %	0.0000 %
0.4	1066.7000	1062.2129	1066.8000	- 0.4207 %	+ 0.0094 %
0.7 (oversaturation)	4637.5636	4572.7693	4612.4885	- 1.3972 %	- 0.5407 %

Table I. Waiting time for various inflows

Conclusions

This paper presents a new methodology for the calculation of the number of stops and the waiting time based on the Cell Transmission Model (CTM). With the development of the "Extended Motion-CTM" [11], a new variable for every cell in the network was introduced, the so called "Congestion Indicator" (CI). The CI identifies the congested area defined by the shockwave generation. It is modeled as a state for each cell and is defined using a finite-state machine approach. The CI faciliates the calculation of the number of stops and the waiting time. In addition, the developed methodology allows the association of the calculated values to the responsible traffic signal. The evaluation on a real world critical corridor shows very promising results. However, a complete network evaluation is needed to establish the value of the methodology for online signal plan optimization for urban networks. Comparison with various traffic models (microscopic and macroscopic) is considered to be the next research step.

Acknowledgements

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