Information-theoretic results for phase noise channels

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Outline

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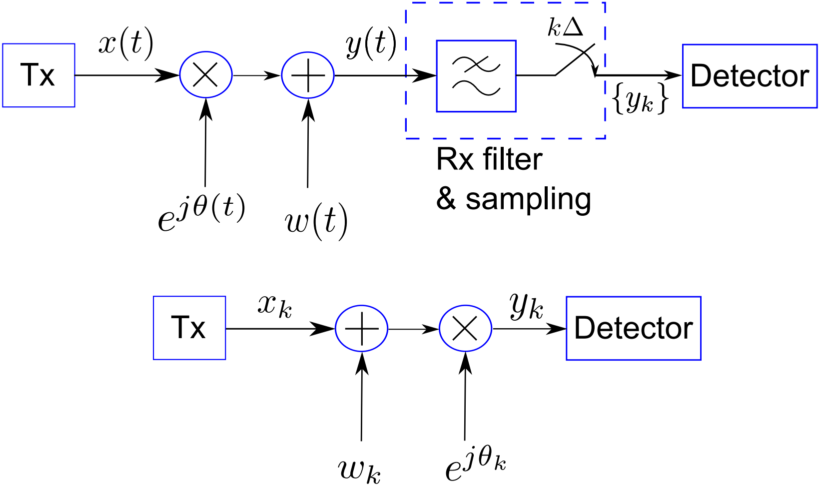
Phase noise in fiber-optic communication

- Laser phase noise
  - Lorentzian spectrum - Wiener process
  - f^{-n} spectrum - Autoregressive moving-average process
- Nonlinear Kerr-induced phase noise
  - Self phase modulation (SPM)
  - Cross phase modulation (XPM)

This talk mainly considers phase noise generated by lasers/oscillators.

Waveform model vs. discrete-time model



Information-theoretic limits

- What can be said about capacity?

$$\mathsf{C}\left(\mathsf{SNR}\right) = \lim_{n\to\infty} \sup_{\mathsf{E}[|X_k|^2]\leq \mathsf{SNR}\Delta} \frac{1}{n} I\left(X_1^n ; Y_1^{Ln}\right) \tag{1}$$

where $L = T_{\mathsf{symb}}/\Delta$ is the oversampling factor

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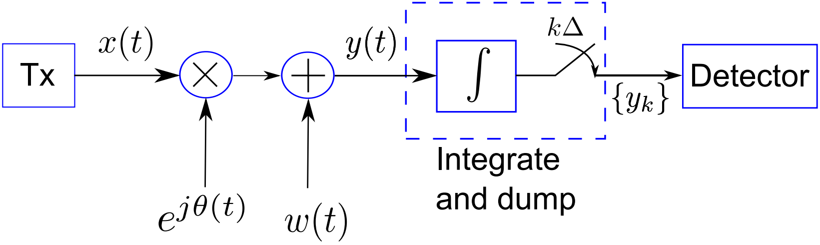
  where $L = T_{\mathsf{symb}}/\Delta$ is the oversampling factor
- It can be a challenging problem due to phase noise's memory
- We review capacity results for
  - Wiener phase noise
  - White phase noise

Outline

Waveform model with integrate & dump receiver



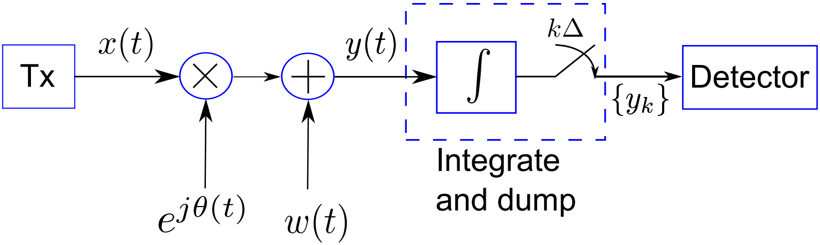
$$Y_k = \int_{(k-1)\Delta}^{k\Delta} Y(t)\,\mathrm{d}t = X_{\lceil k\Delta/T_{\mathsf{symb}}\rceil} \frac{1}{\Delta} \int_{(k-1)\Delta}^{k\Delta} e^{j\Theta(t)}\,\mathrm{d}t + W_k \quad (2)$$

Hypothesis: Tx uses a rectangular pulse shape in time domain

Δ: sampling time, T_{symb}: symbol time

$W_k \sim \mathcal{CN}(0,1)$, $\mathsf{E}\left[W_m W_n^\star\right] = 1_{\{m=n\}}$.

Waveform model with integrate & dump receiver



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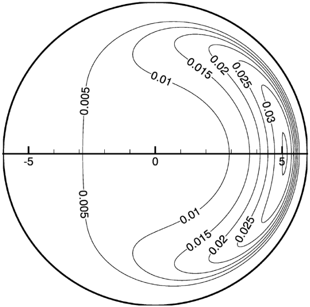
Phase noise and amplitude fading!

Wiener phase noise

Define $\Theta_k = \Theta((k-1)\Delta)$ and $N_k \sim \mathcal{N}(0,1)$:

$$\Theta_k = \Theta_{k-1} + \gamma\sqrt{\Delta}N_k \tag{3}$$

$$Y_k = X_{\lceil k\Delta/T_{\text{symb}}\rceil} e^{j\Theta_k} \frac{1}{\Delta}\int_{(k-1)\Delta}^{k\Delta} e^{j(\Theta(t)-\Theta_k)}\,\mathrm{d}t + W_k \tag{4}$$



Contour plot of the unnormalized fading pdf for $\Delta = 6$ and $\gamma = 1$. (Y. Wang *et al.*, TCOM 2006, vol. 54, no. 5)

Symbol-spaced Wiener phase noise channel

$$\Theta_k = \Theta_{k-1} + \gamma\sqrt{T_{\mathsf{symb}}}N_k \quad (5)$$

$$Y_k = X_k e^{j\Theta_k} + W_k \quad (6)$$

- The phase noise is assumed constant in each symbol time, and varies symbol by symbol according to (5)

[1] Barletta *et al.*, J. Lightw. Technol., 2012, vol. 30, no. 12
[2] Barletta *et al.*, IEEE Photon. Technol. Lett., 2013, vol. 25, no. 13

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- The continuous state space $[0, 2\pi)$ is discretized into bins, and a low-complexity trellis-based detector is devised to lower-bound the mutual information [2]

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Symbol-spaced Wiener phase noise channel - Results

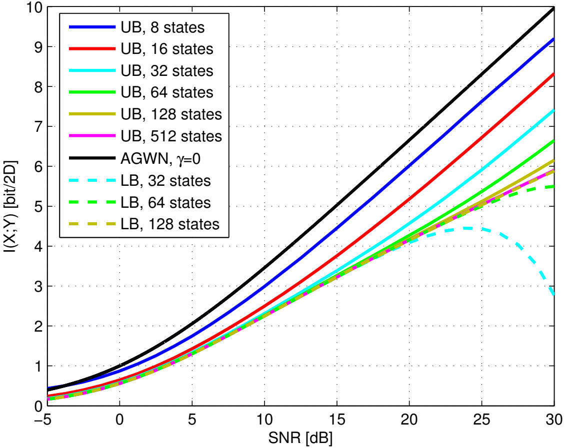

Figure 1 : Circularly symmetric Gaussian input distribution.

Symbol-spaced Wiener phase noise channel - Results

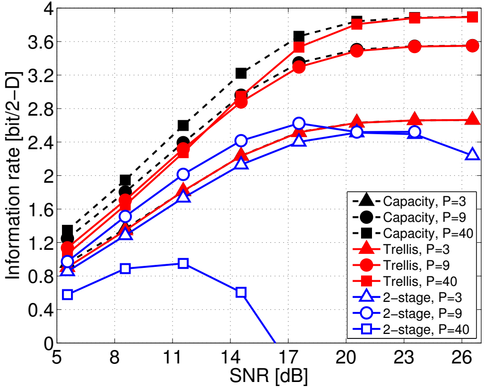

Figure 2 : Low-complexity trellis-based detector (in red), versus a two-stage carrier recovery proposed by Magarini *et al.*, IEEE PTL, 2012, vol. 24, no. 9.

Oversampled Wiener phase noise channel

$$\Theta_k = \Theta_{k-1} + \gamma\sqrt{\Delta}N_k \quad (7)$$

$$Y_k = X_{\lceil k\Delta/T_{\mathsf{symb}}\rceil} e^{j\Theta_k} + W_k \quad (8)$$

- The phase noise is assumed constant in each sample time, and varies sample by sample according to (7)

[3] Ghozlan/Kramer, ISIT, 2013 and 2014
[4] Barletta/Kramer, arXiv:1411.0390

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- An analytical capacity upper bound is found by a genie-aided decoder argument [4]

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High SNR analysis

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- We present analytical results on the so-called capacity *prelog*:

$$\lim_{\mathsf{SNR}\to\infty} \frac{\mathsf{C}\left(\mathsf{SNR}\right)}{\log(\mathsf{SNR})} \tag{9}$$

- Example: for an AWGN channel, $\mathsf{C}\left(\mathsf{SNR}\right) = \log(1 + \mathsf{SNR})$, therefore the prelog is 1

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- We let the sampling time Δ scale with the SNR as

$$\Delta = \frac{1}{\mathsf{SNR}^{\alpha}}, \qquad 0 < \alpha < 1 \tag{10}$$

Oversampled Wiener phase noise channel - Results

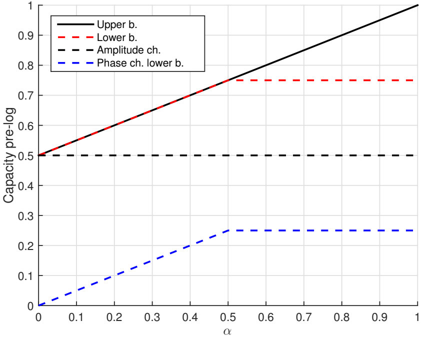

Figure 3 : Prelog upper bound and lower bound versus $\alpha = -\frac{\log(\Delta)}{\log(\text{SNR})}$.

Upper bound: Barletta/Kramer, arXiv:1411.0390

Lower bound: Ghozlan/Kramer, ISIT, 2013 and 2014

A capacity achieving scheme for $0 \leq \alpha \leq 1/2$

- Choose a uniform pdf for $\angle X_k$ and

$$p_{|X_k|^2}(x) = \frac{\Delta}{\mathsf{SNR}\Delta^2 - 1} \exp\left(-\frac{\Delta x - 1}{\mathsf{SNR}\Delta^2 - 1}\right), \qquad x \geq 1/\Delta \tag{11}$$

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- Phase modulation. Use the statistic

$$\angle \widetilde{Y}_k = \angle \left(Y_{(k-1)L+1} \left(\frac{Y_{(k-1)L}}{X_{k-1}} \right)^{\star} \right) \tag{12}$$

to detect $\angle X_k$

Complete model

$$\Theta_k = \Theta_{k-1} + \gamma\sqrt{\Delta}N_k \tag{13}$$

$$Y_k = X_{\lceil k\Delta/T_{\mathsf{symb}}\rceil} e^{j\Theta_k} \frac{1}{\Delta}\int_{(k-1)\Delta}^{k\Delta} e^{j(\Theta(t)-\Theta_k)}\,\mathrm{d}t + W_k \tag{14}$$

- Discrete-time Wiener phase noise and memoryless fading impair the transmission

[5] Ghozlan/Kramer, Globecom, 2013

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- Discrete-time Wiener phase noise and memoryless fading impair the transmission
- Lower bounds on information rates are evaluated by using Bayesian tracking techniques [5]
- The information rate increases compared to the simplified model

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Complete model - Results

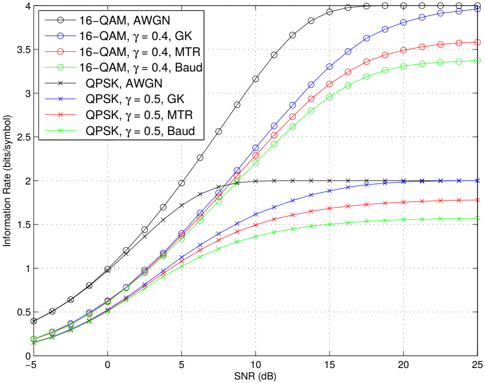

Figure 4 : Oversampled model (in blue) versus the simplified symbol-spaced model (in green).

Complete model - Results

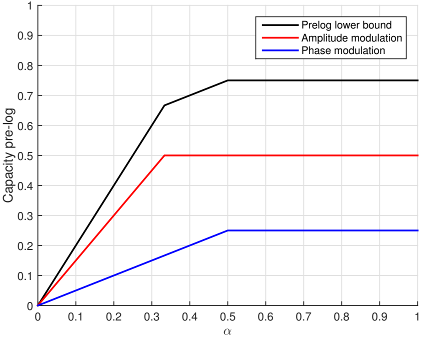

Figure 5 : Prelog lower bound versus $\alpha = -\frac{\log(\Delta)}{\log(\mathsf{SNR})}$.

For $1/3 \leq \alpha < 1$: Ghozlan, PhD Thesis, 2014. (Amplitude modulation only)

For $0 < \alpha < 1$: Barletta, unpublished. (Amplitude and phase modulation)

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White phase noise model

- Consider a phase noise process where the samples $\{e^{j\Theta(t)}\}$ are uncorrelated

[6] Barletta/Kramer, ISIT, 2014
Barletta/Kramer, CROWNCOM, 2014

White phase noise model

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- Also, consider a stationary average $\mathrm{E}\left[e^{j\Theta(t)}\right] = \mu_\Theta$

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White phase noise model

- Consider a phase noise process where the samples $\{e^{j\Theta(t)}\}$ are uncorrelated
- Also, consider a stationary average $\mathsf{E}\left[e^{j\Theta(t)}\right] = \mu_\Theta$
- It can be shown[6] that the output of the sampled matched filter $\{Y_k\}$ is a sufficient statistic for data detection:

$$Y_k = \mu_\Theta X_k + W_k \tag{15}$$

where $|\mu_\Theta| \leq 1$ represents an SNR loss

[6] Barletta/Kramer, ISIT, 2014
Barletta/Kramer, CROWNCOM, 2014

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Conclusions

- For Wiener phase noise channels, oversampling is needed to increase information rates
- Bayesian tracking techniques are good for designing quasi-optimal detectors
- White phase noise channels are equivalent to AWGN channels with an SNR penalty