Overdemodulation for High-Performance Receivers with Low-Resolution ADC

Manuel Stein, Sebastian Theiler and Josef A. Nossek

***Abstract*—The design of the analog demodulator for receivers with low-resolution analog-to-digital converters (ADCs) is investigated. For infinite ADC resolution, demodulation to baseband with $M = 2$ orthogonal sinusoidal functions (quadrature demodulation) is an optimum design choice with respect to system performance. For receivers which are restricted to ADCs with low amplitude resolution we show that this classical approach is suboptimal under an estimation and information theoretic perspective. To this end, we analyze the theoretical channel parameter estimation performance (Fisher information measure) under an ideal receive situation when forming $M > 2$ analog demodulation channels prior to low-complexity 1-bit ADCs. In order to demonstrate the impact of overdemodulation for communication problems, we also provide a brief discussion on the achievable transmission rates (Shannon information measure) with 1-bit quantization of $M > 2$ demodulation channels.**

***Index Terms*—1-bit ADC, demodulation, channel parameter estimation, Fisher information, Shannon information**

I. Introduction

For the design of future wireless receivers the design of the ADCs has been identified as one of the bottlenecks when aiming at an architecture which achieves an optimum trade-off between low cost, moderate energy consumption and high performance [1]. As the complexity and the power dissipation of an ADC scales exponentially $\mathcal{O}(2^b)$ with the bits b used for amplitude resolution, 1-bit hard-limiting receivers have recently gained growing attention [2]–[6]. While such an ADC concept is highly attractive with respect to complexity, it has a strong impact on the performance of the receiver. Interestingly, for applications with low signal-to-noise ratio (SNR) the relative performance gap between a 1-bit system and an ideal receiver with infinite resolution is moderate with $2/\pi$ (-1.96 dB) [7]. In contrast, for the medium to high SNR regime the loss is much more pronounced. Recently, the potential of different techniques, targeted on the reduction of the quantization-loss, is discussed in various works. The benefit of oversampling the analog receive signal for communication over a noisy channel is discussed in [4]–[6]. In [8] and [9] the authors analyze the adjustment of the quantization threshold. The work [10] observes that noise correlation can increase the capacity of multiple-input multiple-output (MIMO) communication channels with coarse quantization, while [11] shows that adjusting the analog pre-filter prior to a 1-bit ADC partially recovers the quantization-loss for estimation problems. Here we discuss the adjusted design of the demodulation device, prior to low-resolution ADCs. In order to demodulate the carrier signal to baseband, classical receivers use a demodulator with in-phase and quadrature channel. Within each channel the receive signal is multiplied with a sinusoid, oscillating at carrier frequency, where the two sinusoids are kept orthogonal by a phase offset of $\frac{\pi}{2}$ [12, p. 582ff.]. While for receivers with infinite ADC resolution this method induces no information-loss during the subsequent transition from the analog to the digital domain, here we show that $M > 2$ demodulation channels allow to significantly reduce the loss due to coarse signal quantization.

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II. System Model

For the discussion we assume a single transmitter sending

$$x'(t) = x_1'(t)\sqrt{2}\cos(\omega_c t) - x_2'(t)\sqrt{2}\sin(\omega_c t), \tag{1}$$

where $\omega_c \in \mathbb{R}$ is the carrier frequency and $x_{1/2}'(t) \in \mathbb{R}$ are two independent input signals. The analog receiver observes

$$\begin{aligned} y'(t) = \gamma x_1'(t-\tau)\sqrt{2}\cos(\omega_c t - \psi) - \\ - \gamma x_2'(t-\tau)\sqrt{2}\sin(\omega_c t - \psi) + \eta'(t), \end{aligned} \tag{2}$$

where $\gamma \in \mathbb{R}, \gamma \geq 0$ is the attenuation and $\tau \in \mathbb{R}$ a time-shift due to signal propagation. $\psi \in \mathbb{R}$ characterizes the channel phase offset and $\eta'(t) \in \mathbb{R}$ is white additive sensor noise. For the demodulation to baseband the receiver forms $m = 1, \ldots, M$ channel outputs by performing the multiplications

$$\begin{aligned} y_m'(t) &= y'(t) \cdot \sqrt{2}\cos(\omega_c t + \varphi_m) \\ &= \gamma x_1'(t-\tau)\big(\cos(2\omega_c t - \psi + \varphi_m) + \cos(\psi + \varphi_m)\big) - \\ &- \gamma x_2'(t-\tau)\big(\sin(2\omega_c t - \psi + \varphi_m) - \sin(\psi + \varphi_m)\big) + \\ &+ \eta'(t)\sqrt{2}\cos(\omega_c t + \varphi_m) \end{aligned} \tag{3}$$

with constant demodulation offsets φ_m. Behind a low-pass filter $h(t; B)$ of bandwidth B, the m-th output channel is

$$\begin{aligned} y_m(t) = \gamma x_1(t-\tau)\big(\cos(\psi)\cos(\varphi_m) - \sin(\psi)\sin(\varphi_m)\big) + \\ + \gamma x_2(t-\tau)\big(\sin(\psi)\cos(\varphi_m) + \cos(\psi)\sin(\varphi_m)\big) + \\ + \cos(\varphi_m)\eta_1(t) + \sin(\varphi_m)\eta_2(t), \end{aligned} \tag{4}$$

where

$$\begin{aligned} \eta_1(t) &= \sqrt{2}\cos(\omega_c t)\big(h(t;B) * \eta'(t)\big) \\ \eta_2(t) &= -\sqrt{2}\sin(\omega_c t)\big(h(t;B) * \eta'(t)\big) \end{aligned} \tag{5}$$

are independent random processes with unit power spectral density. The described demodulation operation is depicted in Fig. 1. Note that we use the notation $z(t) = h(t; B) * z'(t)$,

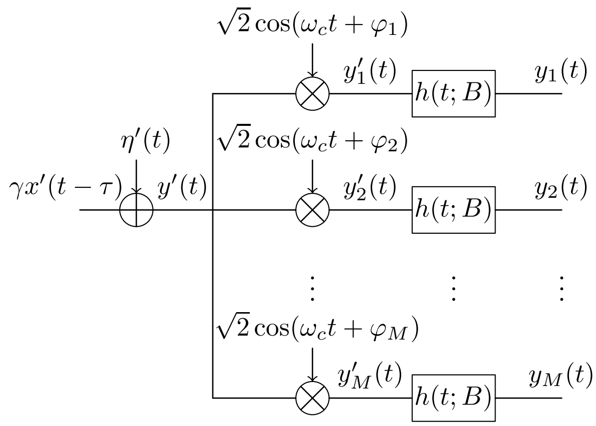

Fig. 1. Analog radio front-end with overdemodulation ($M>2$)

where $*$ is the convolution operator. Defining the demodulation offset vector

$$\boldsymbol{\varphi}=\begin{bmatrix}\varphi_1 & \varphi_2 & \dots & \varphi_M\end{bmatrix}^{\mathrm{T}}, \tag{6}$$

the signals of the M demodulation channels can be written as

$$\boldsymbol{y}(t)=\boldsymbol{A}(\boldsymbol{\varphi})\big(\gamma\boldsymbol{B}(\psi)\boldsymbol{x}(t-\tau)+\boldsymbol{\eta}(t)\big) \tag{7}$$

with the analog signals

$$\begin{aligned}\boldsymbol{y}(t)&=\begin{bmatrix}y_1(t) & y_2(t) & \dots & y_M(t)\end{bmatrix}^{\mathrm{T}}\\ \boldsymbol{x}(t-\tau)&=\begin{bmatrix}x_1(t-\tau) & x_2(t-\tau)\end{bmatrix}^{\mathrm{T}}\\ \boldsymbol{\eta}(t)&=\begin{bmatrix}\eta_1(t) & \eta_2(t)\end{bmatrix}^{\mathrm{T}}\end{aligned} \tag{8}$$

and the matrices

$$\begin{aligned}\boldsymbol{A}(\boldsymbol{\varphi})&=\begin{bmatrix}\cos(\varphi_1) & \sin(\varphi_1)\\ \cos(\varphi_2) & \sin(\varphi_2)\\ \vdots & \vdots\\ \cos(\varphi_M) & \sin(\varphi_M)\end{bmatrix}\\ \boldsymbol{B}(\psi)&=\begin{bmatrix}\cos(\psi) & \sin(\psi)\\ -\sin(\psi) & \cos(\psi)\end{bmatrix}.\end{aligned} \tag{9}$$

Sampling each of the M output channels at a rate of $f_s=\frac{1}{T_s}=2B$ for the duration of $T=NT_s$ and defining the parameter vector $\boldsymbol{\theta}=\begin{bmatrix}\psi & \tau\end{bmatrix}^{\mathrm{T}}$, the digital receive signal is comprised of N temporally white snapshots $\boldsymbol{y}_n\in\mathbb{R}^M$ with

$$\begin{aligned}\boldsymbol{y}_n&=\gamma\boldsymbol{A}(\boldsymbol{\varphi})\boldsymbol{B}(\psi)\boldsymbol{x}_n(\tau)+\boldsymbol{A}(\boldsymbol{\varphi})\boldsymbol{\eta}_n\\ &=\gamma\boldsymbol{s}_n(\boldsymbol{\theta})+\boldsymbol{\zeta}_n.\end{aligned} \tag{10}$$

The individual digital samples are given by

$$\begin{aligned}\boldsymbol{y}_n&=\left[y_1\Big(\tfrac{(n-1)}{f_s}\Big) \quad y_2\Big(\tfrac{(n-1)}{f_s}\Big) \quad \dots \quad y_M\Big(\tfrac{(n-1)}{f_s}\Big)\right]^{\mathrm{T}}\\ \boldsymbol{x}_n(\tau)&=\left[x_1\Big(\tfrac{(n-1)}{f_s}-\tau\Big) \quad x_2\Big(\tfrac{(n-1)}{f_s}-\tau\Big)\right]^{\mathrm{T}}\\ \boldsymbol{\eta}_n&=\left[\eta_1\Big(\tfrac{(n-1)}{f_s}\Big) \quad \eta_2\Big(\tfrac{(n-1)}{f_s}\Big)\right]^{\mathrm{T}}.\end{aligned} \tag{11}$$

The sampled noise $\boldsymbol{\eta}_n$ is a zero-mean Gaussian variable with $\mathrm{E}\left[\boldsymbol{\eta}_n\boldsymbol{\eta}_n^{\mathrm{T}}\right]=\boldsymbol{I}_2$ while the snapshot noise covariance is

$$\boldsymbol{C}=\mathrm{E}\left[\boldsymbol{\zeta}_n\boldsymbol{\zeta}_n^{\mathrm{T}}\right]=\boldsymbol{A}(\boldsymbol{\varphi})\boldsymbol{A}^{\mathrm{T}}(\boldsymbol{\varphi}). \tag{12}$$

In the following, we assume that the ADC for each of the M output channels is a symmetric hard-limiter, such that the final digital receive data $\boldsymbol{r}_n\in\{-1,1\}^M$ is given by

$$\boldsymbol{r}_n=\mathrm{sign}\left(\boldsymbol{y}_n\right), \tag{13}$$

where $\mathrm{sign}(\cdot)$ is the element-wise signum-function.

III. Performance Analysis - Estimation

In order to discuss the benefits of using $M>2$ demodulation outputs, a channel estimation problem is considered. The receiver infers the deterministic but unknown parameters $\boldsymbol{\theta}$ by using the maximum-likelihood estimator (MLE)

$$\hat{\boldsymbol{\theta}}(\boldsymbol{r})=\arg\max_{\boldsymbol{\theta}\in\Theta}\ln p(\boldsymbol{r};\boldsymbol{\theta}), \tag{14}$$

where the receive signal with N snapshots has the form

$$\boldsymbol{r}=\begin{bmatrix}\boldsymbol{r}_1^{\mathrm{T}} & \boldsymbol{r}_2^{\mathrm{T}} & \dots & \boldsymbol{r}_N^{\mathrm{T}}\end{bmatrix}^{\mathrm{T}}. \tag{15}$$

For sufficiently large N, the MLE is unbiased and efficient, such that its MSE matrix can be characterized analytically through the Cramér-Rao lower bound [14], [15], which is given by the inverse of the Fisher information matrix (FIM)

$$\mathrm{E}\left[(\hat{\boldsymbol{\theta}}(\boldsymbol{r})-\boldsymbol{\theta})(\hat{\boldsymbol{\theta}}(\boldsymbol{r})-\boldsymbol{\theta})^{\mathrm{T}}\right]=\boldsymbol{F}^{-1}(\boldsymbol{\theta}). \tag{16}$$

The FIM is defined by

$$\boldsymbol{F}(\boldsymbol{\theta})=\int_{\mathcal{R}}p(\boldsymbol{r};\boldsymbol{\theta})\left(\frac{\partial\ln p(\boldsymbol{r};\boldsymbol{\theta})}{\partial\boldsymbol{\theta}}\right)^{\mathrm{T}}\frac{\partial\ln p(\boldsymbol{r};\boldsymbol{\theta})}{\partial\boldsymbol{\theta}}\mathrm{d}\boldsymbol{r}, \tag{17}$$

where $\mathcal{R}$ is the support of the receive vector $\boldsymbol{r}$. For temporally white samples $\boldsymbol{r}_n$, the FIM $\boldsymbol{F}(\boldsymbol{\theta})$ exhibits an additive property

$$\begin{aligned}\boldsymbol{F}(\boldsymbol{\theta})&=\sum_{n=1}^{N}\boldsymbol{F}_n(\boldsymbol{\theta})\\ \boldsymbol{F}_n(\boldsymbol{\theta})&=\int_{\mathcal{R}_n}p(\boldsymbol{r}_n;\boldsymbol{\theta})\left(\frac{\partial\ln p(\boldsymbol{r}_n;\boldsymbol{\theta})}{\partial\boldsymbol{\theta}}\right)^2\mathrm{d}\boldsymbol{r}_n.\end{aligned} \tag{18}$$

A. Estimation Performance - Pessimistic Characterization

As (18) requires in general the calculation of an M-fold integral, the analytic description of $\boldsymbol{F}_n(\boldsymbol{\theta})$ is difficult, especially if M is large. In order to circumvent this problem, we use an approximation $\tilde{\boldsymbol{F}}_n(\boldsymbol{\theta})$ of the FIM which exhibits the property

$$\boldsymbol{F}_n(\boldsymbol{\theta})\succeq\tilde{\boldsymbol{F}}_n(\boldsymbol{\theta}). \tag{19}$$

This guarantees that $\tilde{\boldsymbol{F}}_n(\boldsymbol{\theta})$ is a pessimistic characterization of the performance measure $\boldsymbol{F}_n(\boldsymbol{\theta})$. With the moments

$$\begin{aligned}\boldsymbol{\mu}_n(\boldsymbol{\theta})&=\int_{\mathcal{R}_n}\boldsymbol{r}_n p(\boldsymbol{r}_n;\boldsymbol{\theta})\mathrm{d}\boldsymbol{r}_n\\ \boldsymbol{R}_n(\boldsymbol{\theta})&=\int_{\mathcal{R}_n}\big(\boldsymbol{r}_n-\boldsymbol{\mu}_n(\boldsymbol{\theta})\big)\big(\boldsymbol{r}_n-\boldsymbol{\mu}_n(\boldsymbol{\theta})\big)^{\mathrm{T}}p(\boldsymbol{r}_n;\boldsymbol{\theta})\mathrm{d}\boldsymbol{r}_n,\end{aligned} \tag{20}$$

such a pessimistic version of the FIM is given by [13]

$$\tilde{\boldsymbol{F}}_n(\boldsymbol{\theta})=\left(\frac{\partial\boldsymbol{\mu}_n(\boldsymbol{\theta})}{\partial\boldsymbol{\theta}}\right)^{\mathrm{T}}\boldsymbol{R}_n^{-1}(\boldsymbol{\theta})\left(\frac{\partial\boldsymbol{\mu}_n(\boldsymbol{\theta})}{\partial\boldsymbol{\theta}}\right). \tag{21}$$

The first moment can be calculated element-wise by

$$
\begin{aligned}
[\boldsymbol{\mu}_n(\boldsymbol{\theta})]_m &= p\big([\boldsymbol{r}_n]_m = 1; \boldsymbol{\theta}\big) - p\big([\boldsymbol{r}_n]_m = -1; \boldsymbol{\theta}\big) \\
&= 1 - 2\,\mathrm{Q}\left(\frac{\gamma[\boldsymbol{s}_n(\boldsymbol{\theta})]_m}{\sqrt{[\boldsymbol{C}]_{mm}}}\right),
\end{aligned} \tag{22}
$$

where $\mathrm{Q}(\cdot)$ is the Q-function and $[\cdot]_{mk}$ indicates the matrix entry in the m-th row and k-th column. The second moment

$$
[\boldsymbol{R}_n(\boldsymbol{\theta})]_{mm} = 1 - [\boldsymbol{\mu}_n(\boldsymbol{\theta})]_m^2 \tag{23}
$$

with off-diagonal entries is given by

$$
[\boldsymbol{R}_n(\boldsymbol{\theta})]_{mk} = 4\Psi_{mk}(\boldsymbol{\theta}) - \big(1 - [\boldsymbol{\mu}_n(\boldsymbol{\theta})]_m\big)\big(1 - [\boldsymbol{\mu}_n(\boldsymbol{\theta})]_k\big), \tag{24}
$$

where $\Psi_{mk}(\boldsymbol{\theta})$ is the cumulative density function (CDF) of the bivariate Gaussian distribution

$$
p\big([\boldsymbol{\zeta}_n]_m, [\boldsymbol{\zeta}_n]_k\big) = \mathcal{N}\left(\begin{bmatrix}0\\0\end{bmatrix}, \begin{bmatrix}[\boldsymbol{C}]_{mm} & [\boldsymbol{C}]_{mk}\\ [\boldsymbol{C}]_{km} & [\boldsymbol{C}]_{kk}\end{bmatrix}\right) \tag{25}
$$

with upper integration boarder $\big[-\gamma[\boldsymbol{s}_n(\boldsymbol{\theta})]_m \;\; -\gamma[\boldsymbol{s}_n(\boldsymbol{\theta})]_k\big]^{\mathrm{T}}$. The derivative of the first moment is found element-wise by

$$
\left[\frac{\partial \boldsymbol{\mu}_n(\boldsymbol{\theta})}{\partial \boldsymbol{\theta}}\right]_{mk} = \frac{2\gamma \mathrm{e}^{-\frac{\gamma^2[\boldsymbol{s}_n(\boldsymbol{\theta})]_m^2}{2[\boldsymbol{C}]_{mm}}}}{\sqrt{2\pi[\boldsymbol{C}]_{mm}}}\left[\frac{\partial \boldsymbol{s}_n(\boldsymbol{\theta})}{\partial \boldsymbol{\theta}}\right]_{mk}, \tag{26}
$$

with

$$
\begin{aligned}
\frac{\partial \boldsymbol{s}_n(\boldsymbol{\theta})}{\partial \boldsymbol{\theta}} &= \begin{bmatrix}\frac{\partial \boldsymbol{s}_n(\boldsymbol{\theta})}{\partial \psi} & \frac{\partial \boldsymbol{s}_n(\boldsymbol{\theta})}{\partial \tau}\end{bmatrix} \\
&= \begin{bmatrix}\boldsymbol{A}(\boldsymbol{\varphi})\frac{\partial \boldsymbol{B}(\psi)}{\partial \psi}\boldsymbol{x}_n(\tau) & \boldsymbol{A}(\boldsymbol{\varphi})\boldsymbol{B}(\psi)\frac{\partial \boldsymbol{x}_n(\tau)}{\partial \tau}\end{bmatrix},
\end{aligned} \tag{27}
$$

where

$$
\begin{aligned}
\frac{\partial \boldsymbol{B}(\psi)}{\partial \psi} &= \begin{bmatrix}-\sin(\psi) & \cos(\psi)\\ -\cos(\psi) & -\sin(\psi)\end{bmatrix} \\
\frac{\partial \boldsymbol{x}_n(\tau)}{\partial \tau} &= -\begin{bmatrix}\frac{\mathrm{d}x_1(t)}{\mathrm{d}t} & \frac{\mathrm{d}x_2(t)}{\mathrm{d}t}\end{bmatrix}^{\mathrm{T}}\Bigg|_{t=\left(\frac{(n-1)}{f_s}-\tau\right)}.
\end{aligned} \tag{28}
$$

B. Results - Channel Estimation

For visualization of the possible performance gain we use an example where the transmitter sends pilot signals

$$
x_{1/2}(t) = \sum_{k=-\infty}^{\infty} [\boldsymbol{b}_{1/2}]_{\mathrm{mod}\,(k,K)}\, g(t - kT_b). \tag{29}
$$

$\boldsymbol{b}_{1/2} \in \{-1, 1\}^K$ are binary vectors with $K = 1023$ symbols, each of duration $T_b = 977.52$ ns, $g(t)$ is a rectangular transmit pulse and $\mathrm{mod}\,(\cdot)$ is the modulo operator. The receiver band-limits the signal to $B = 1.023$ MHz and samples at a rate of $f_s = 2B$ in order to attain temporally white snapshots. After one signal period $T = 1$ ms, the receiver has available $N = 2046$ samples for the estimation task. The unknown channel parameters are assumed to be $\boldsymbol{\theta} = \begin{bmatrix}\frac{\pi}{8} & 0\end{bmatrix}^{\mathrm{T}}$. The demodulation offsets are equally spaced $[\boldsymbol{\varphi}]_m = \frac{\pi}{M}(m-1)$ and the performance is normalized with respect to an ideal reference with infinite ADC resolution and $M = 2$

$$
\chi_{\psi/\tau}(\boldsymbol{\theta}) = \frac{[\tilde{\boldsymbol{F}}^{-1}(\boldsymbol{\theta})]_{11/22}}{[\boldsymbol{F}_{\infty}^{-1}(\boldsymbol{\theta})]_{11/22}}, \tag{30}
$$

where the FIM of the reference system is

$$
\boldsymbol{F}_{\infty}(\boldsymbol{\theta}) = \gamma^2 \sum_{n=1}^{N}\left(\frac{\partial \boldsymbol{s}_n(\boldsymbol{\theta})}{\partial \boldsymbol{\theta}}\right)^{\mathrm{T}}\left(\frac{\partial \boldsymbol{s}_n(\boldsymbol{\theta})}{\partial \boldsymbol{\theta}}\right). \tag{31}
$$

Note that for $M = 2$ the noise in both channels is independent. Under this condition it holds that the approximated FIM with hard-limiting is exact [13], i.e. $\tilde{\boldsymbol{F}}(\boldsymbol{\theta}) = \boldsymbol{F}(\boldsymbol{\theta})$. Therefore, $\chi_{\psi/\tau}(\boldsymbol{\theta})\big|_{M=2}$ characterizes the 1-bit performance loss with classical I/Q demodulation precisely. For the case $M > 2$ the ratio $\chi_{\psi/\tau}(\boldsymbol{\theta})$ provides a pessimistic approximation, i.e. the quantization-loss might even be smaller. Fig. 2 and 3 show the estimation performance $\chi_{\psi}(\boldsymbol{\theta})$ and $\chi_{\tau}(\boldsymbol{\theta})$ for different choices of M versus SNR. For both parameters $M = 16$ allows to diminish the quantization-loss at SNR $= -15.0$ dB from $\chi_{\psi/\tau}(\boldsymbol{\theta}) = -1.99$ dB to $\chi_{\psi/\tau}(\boldsymbol{\theta}) = -1.07$ dB. For high SNR (e.g. SNR $= +10.0$ dB, $M = 16$), the gain is much more pronounced. The loss for phase estimation can be reduced from $\chi_{\psi}(\boldsymbol{\theta}) = -7.92$ dB to $\chi_{\psi}(\boldsymbol{\theta}) = -0.51$ dB . For the delay parameter τ, the 1-bit loss changes from $\chi_{\tau}(\boldsymbol{\theta}) = -6.45$ dB to $\chi_{\tau}(\boldsymbol{\theta}) = -3.18$ dB.

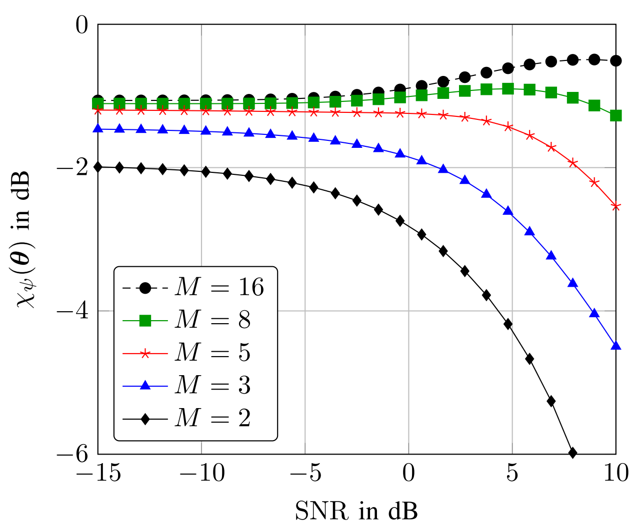

Fig. 2. $\chi_{\psi}(\boldsymbol{\theta})$ vs. signal-to-noise ratio (SNR)

IV. Performance Analysis - Communication

In the context of communication theory, our setup can be interpreted as a real-valued multiple-input and multiple-output (MIMO) channel with two inputs and M channel outputs

$$
\begin{aligned}
\boldsymbol{y} &= \boldsymbol{A}(\boldsymbol{\varphi})\boldsymbol{B}(\psi)\boldsymbol{x} + \boldsymbol{A}(\boldsymbol{\varphi})\boldsymbol{\eta} \\
&= \boldsymbol{H}\boldsymbol{x} + \boldsymbol{\zeta}
\end{aligned} \tag{32}
$$

followed by an element-wise hard-limiter $\boldsymbol{r} = \mathrm{sign}(\boldsymbol{y})$.

A. Transmission Rate - Pessimistic Characterization

In [10], it was shown that the capacity C_M can be bounded

$$
\begin{aligned}
C_M &= \max_{p(\boldsymbol{x})} I_M(\boldsymbol{x}; \boldsymbol{r}) \\
&\geq \frac{1}{2}\log_2 \det\left(\boldsymbol{1}_M + \boldsymbol{R}_{\zeta'\zeta'}^{-1}\boldsymbol{H}'\boldsymbol{R}_{xx}\boldsymbol{H}'^{\mathrm{T}}\right) \\
&= \tilde{I}_M(\boldsymbol{x}; \boldsymbol{r}),
\end{aligned} \tag{33}
$$

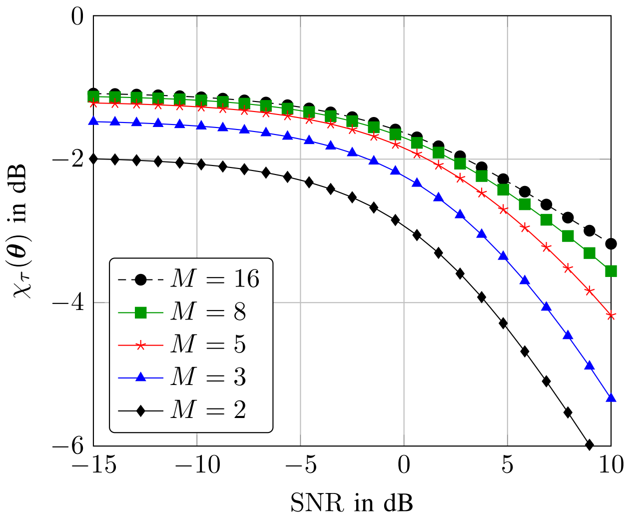

Fig. 3. $\chi_\tau(\boldsymbol{\theta})$ vs. signal-to-noise ratio (SNR)

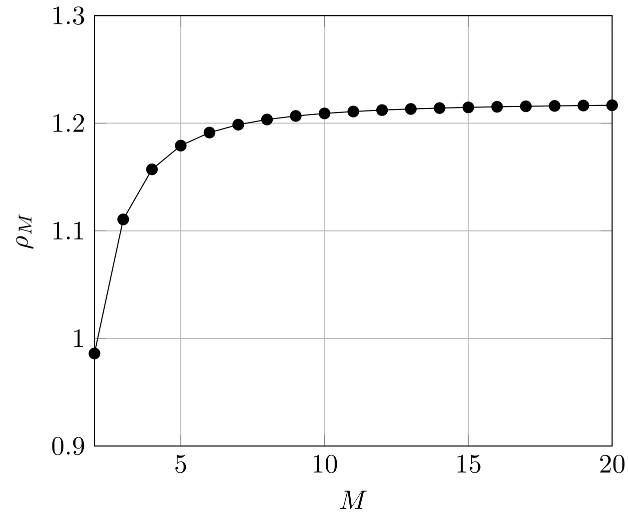

Fig. 4. ρ_M vs. demodulation channels M (SNR $= -15.0$ dB)

where $\boldsymbol{R}_{xx}$ is the second moment of the channel input $\boldsymbol{x}$ and

$$\begin{aligned}\boldsymbol{R}_{\zeta'\zeta'} &= \frac{2}{\pi}\Big(\arcsin\big(\operatorname{diag}(\boldsymbol{R}_{yy})^{-\frac{1}{2}}\boldsymbol{R}_{yy}\operatorname{diag}(\boldsymbol{R}_{yy})^{-\frac{1}{2}}\big)\Big)\\ &\quad-\frac{2}{\pi}\operatorname{diag}(\boldsymbol{R}_{yy})^{-\frac{1}{2}}\boldsymbol{R}_{yy}\operatorname{diag}(\boldsymbol{R}_{yy})^{-\frac{1}{2}}\\ &\quad+\frac{2}{\pi}\operatorname{diag}(\boldsymbol{R}_{yy})^{-\frac{1}{2}}\boldsymbol{R}_{\zeta\zeta}\operatorname{diag}(\boldsymbol{R}_{yy})^{-\frac{1}{2}}\\ \boldsymbol{H}' &= \sqrt{\frac{2}{\pi}}\operatorname{diag}(\boldsymbol{R}_{yy})^{-\frac{1}{2}}\boldsymbol{H}. \end{aligned} \tag{34}$$

The expression $\tilde{I}_M(\boldsymbol{x};\boldsymbol{r})$ can be interpreted as the mutual information of an equivalent Gaussian channel with Gaussian input. Note that for the case $M = 2$ and 1-bit quantization the exact capacity of the considered transmission line is [3]

$$C_2 = 2\Big(1-\beta\Big(\mathrm{Q}\left(\sqrt{\mathrm{SNR}}\right)\Big)\Big) \tag{35}$$

with $\beta(z) = -z\log_2(z) - (1-z)\log_2(1-z)$.

B. Results - Noisy Channel Communication

For visualization, we assume independent channel inputs with zero-mean and covariance $\boldsymbol{R}_{xx} = \mathrm{SNR}\cdot\boldsymbol{I}_2$. Fig. 4 shows the achievable relative gain in transmission rate

$$\rho_M = \frac{\tilde{I}_M(\boldsymbol{x};\boldsymbol{r})}{C_2} \tag{36}$$

with 1-bit ADC at the receiver and different numbers of demodulation channels M. It is observed that classical demodulation ($M = 2$) is suboptimal as with overdemodulation ($M = 20$) it is possible to increase the transmission rate by 22% in a low SNR scenario with $\mathrm{SNR} = -15.0$ dB.

V. Conclusion

A receiver which uses $M > 2$ demodulation channels to map the analog carrier signal to baseband has been analyzed. While with high ADC resolution this approach leads to redundant data, here it was shown by an estimation and information theoretic investigation, that for receivers which are restricted to low ADC resolution significant performance improvements can be achieved if more than two demodulation channels are used. During system design this opens the possibility to trade off the ADC resolution (exponential complexity) against the number of demodulation channels (linear complexity).

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