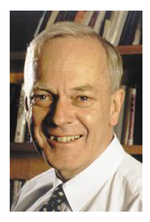
Distributed Controller Design for a Class for Sparse Singular Systems with Privacy Constraints


Frederik Deroo
fred.deroo@tum.de


Michael Ulbrich
mulbrich@ma.tum.de


Sandra Hirche
hirche@tum.de

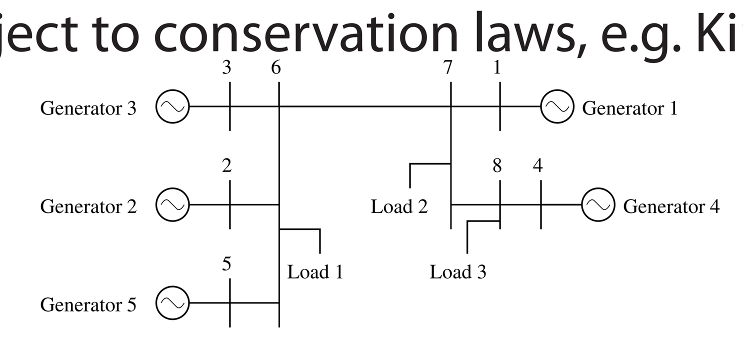

Brian D. O. Anderson
brian.anderson@anu.edu.au

Motivation

Controller **design** for large-scale systems should be done in distributed fashion with ***local*** model information:

- **Privacy**: Agents not willing to share their model with everybody or with a central designer
- **Model complexity**
- **Computational complexity** with regards to centralized model

Often neglected:
Many practical large-scale systems (e.g. power/water distribution systems) are subject to conservation laws, e.g. Kirchhoff laws



8-bus power system

Objective:

Design a distributed controller using local model information (exchange) for systems *with algebraic constraints*.
$\Longrightarrow$ Control design is structured **and** actual controller is structured

Problem Formulation

Consider the system:

$$\dot{x}_i = A_{xx,i}x_i + A_{xy,i}y_i + B_{x,i}u_i$$

$$0 = \sum_{j=1}^{N} A_{yx,ij}x_j + \sum_{j=1}^{N} A_{yy,ij}y_j + B_{y,i}u_i$$

where

A_{xx}, A_{xy} block-diagonal, A_{yy} invertible
A_{yx}, A_{yy} sparse matrices signifying the coupling

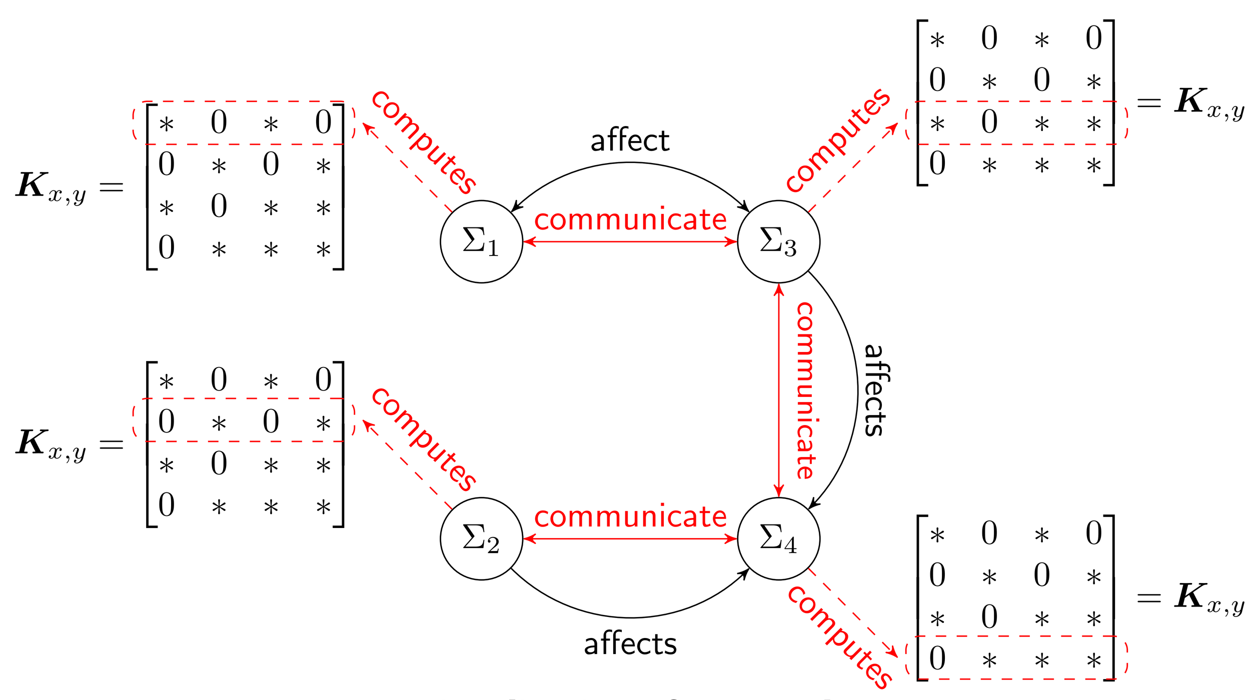
Set of neighborhood nodes: $\mathcal{N}_i = \{j | A_{yx,ij} \neq 0 \vee A_{yy,ij} \neq 0\}$

Goal:

Distributedly determine K_x and K_y to solve

$$\begin{aligned}\min_{x,y,u} J(x,u) &= \int_0^{t_f} x^T(t)Q_x x(t) + y^T(t)Q_y y(t) + u^T(t)Ru(t)\mathrm{d}t \\ \text{s.t.} \quad \dot{x}_i &= A_{xx,i}x_i + A_{xy,i}y_i + B_{x,i}u_i \\ 0 &= \sum_{j=1}^{N} A_{yx,ij}x_j + \sum_{j=1}^{N} A_{yy,ij}y_j + B_{y,i}u_i \\ u(t) &= -[K_x\, K_y][x^T(t)\, y^T(t)]^T\end{aligned}$$

where $K_{x,ij} \neq 0$ and $K_{y,ij} \neq 0$ only if $j \in \mathcal{N}_i$,
where Q_x, Q_y and R block-diagonal



Visualization of approach

Control Synthesis Algorithm

Proposition

Feedback matrices can be determined iteratively using distributed gradient descent approach according to following algorithm.

For every agent:

1. Simulate states x_i , y_i for horizon t_f
2. Simulate adjoint states $\lambda_{x,i}$, $\lambda_{y,i}$ for t_f

$$\begin{aligned}\dot{\lambda}_x &= -A_{K,xx}^T\lambda_x + A_{K,yx}^T\lambda_y + 2(Q_x + K_x^T RK_x)x \\ &\quad + 2K_x^T RK_y y, \quad \lambda_x(t_f) = 0 \\ 0 &= -A_{K,xy}^T\lambda_x + A_{K,yy}^T\lambda_y + 2(Q_y + K_y^T RK_y)y \\ &\quad + 2K_y^T RK_x x\end{aligned}$$

3. Calculate respective entries of gradient of cost functional with respect to feedback matrices as

$$(\nabla_{K_x}J)_{ij} = \int_0^{t_f} -2R_i u_i x_j^T + (B_{x,i}^T\lambda_{x_i} - B_{y,i}^T\lambda_{y_i})x_j^T \mathrm{d}t$$

$$(\nabla_{K_y}J)_{ij} = \int_0^{t_f} -2R_i u_i y_j^T + (B_{x,i}^T\lambda_{x_i} - B_{y,i}^T\lambda_{y_i})y_j^T \mathrm{d}t$$

4. Step length γ_k is

$$\gamma_k = \frac{\langle \Delta\mathrm{vec}(K), \Delta\mathrm{vec}(K)\rangle}{\langle \Delta\mathrm{vec}(K), \Delta\mathrm{vec}(\nabla_K J)\rangle}$$

where $\Delta\mathrm{vec}(X) = \mathrm{vec}(X^{(k)}) - \mathrm{vec}(X^{(k-1)})$

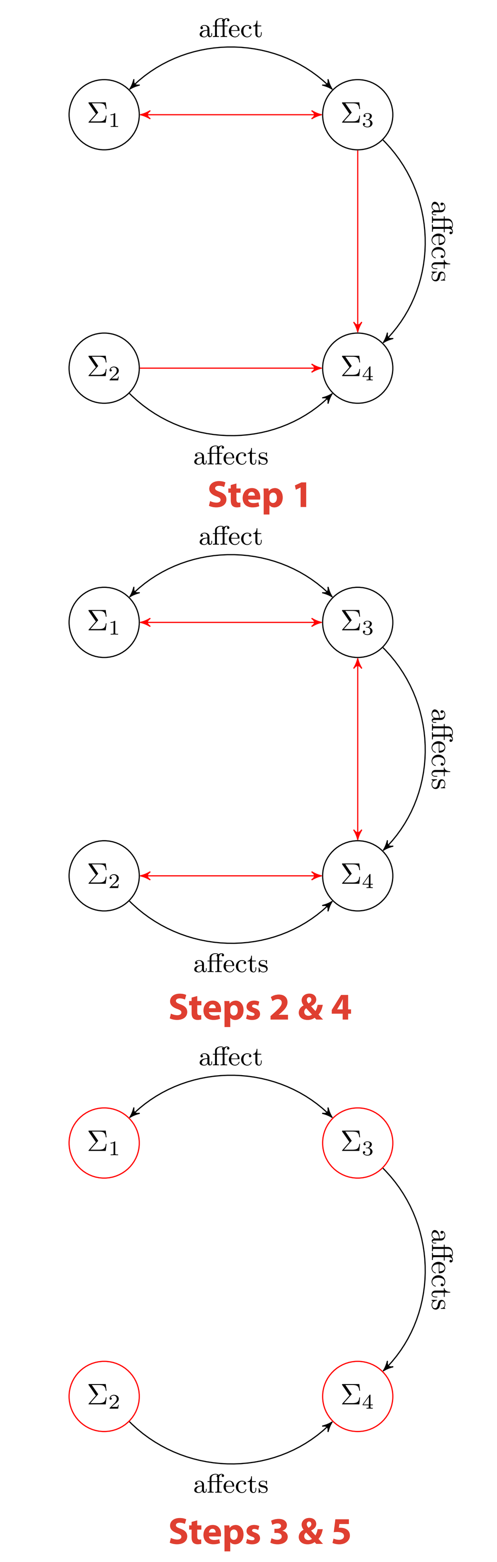
5. For each neighboring agent j, update

$$K_{x,ij}^{(k+1)} = K_{x,ij}^{(k)} - \gamma_k(\nabla_{K_x}J)_{ij}^{(k)},$$

$$K_{y,ij}^{(k+1)} = K_{y,ij}^{(k)} - \gamma_k(\nabla_{K_y}J)_{ij}^{(k)}.$$

6. If all $\|(\nabla_K J)_{ij}^{(k)}\| < \epsilon$, stop. Otherwise increase k and go back to 1.

Information exchange

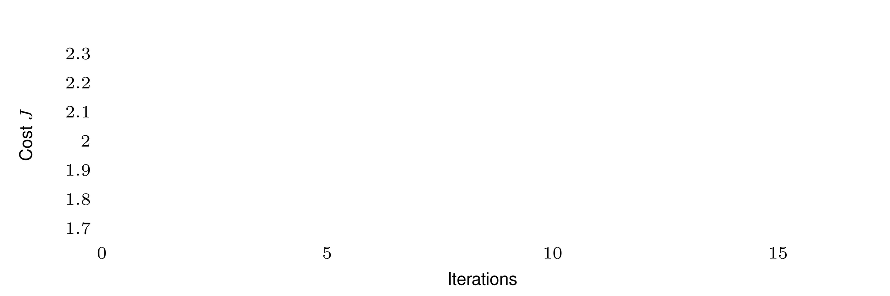


Using averaging over x_0 , independence of the state initial condition is achieved.

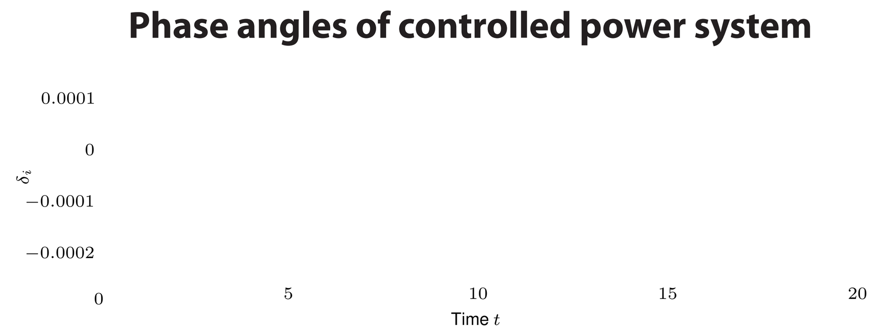
Numerical investigations

- Comparison with non-sparse controller:
  only 0.41% cost increase for sparse controller (N=4,T=5)
- Averaging over x_0 better overall results:
  lower cost (>10%), fewer iterations (average 20 instead of 42)
- Application to small (8 Bus) power system requires 60 iterations

Validation of cost decrease



Phase angles of controlled power system



Conclusions

- Design method allows computation of a feedback matrix **respecting the distributed structure** with only **local model information** and information exchange, thus ensuring **privacy** and no centralized knowledge.
- Trade-off between communication effort and model knowledge: higher with local model knowledge, but design is done entirely offline.

References

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2. K. Martensson, A. Rantzer, (2012), A scalable method for continuous-time distributed control synthesis, *Proc. American Control Conference*