Efficiency of monolithic solvers for incompressible fluid-structure interaction problems

Matthias Mayr, Michael W. Gee

Mechanics & High Performance Computing Group, Technische Universität München, Germany

Motivation

- Real world problems require accurate predictions at reasonable computational costs (accuracy vs. numerical effort)
- Parallel scalability for very large problem sizes

Problem Definition & Discretization

- Solid domain $\Omega^{\mathcal{S}}$: nonlinear elastodynamics
- Fluid domain $\Omega^{\mathcal{F}}$: incompressible Navier-Stokes equations with ALE observer
- Fluid-Structure Interface Γ_{FSI}: weak enforcement of coupling conditions using Lagrange Multiplier field $\boldsymbol{\lambda} = \underline{\mathbf{h}}^{\mathcal{S}}_{\Gamma_{\mathrm{FSI}}} = -\underline{\mathbf{h}}^{\mathcal{F}}_{\Gamma_{\mathrm{FSI}}}$ [1]

$$\left(\delta\underline{\boldsymbol{\lambda}}, \underline{\mathbf{d}}^{\mathcal{S}}_{\Gamma_{\mathrm{FSI}}} - \underline{\mathbf{d}}^{\mathcal{G}}_{\Gamma_{\mathrm{FSI}}}\right)_{\Gamma_{\mathrm{FSI}}} = 0 \text{ on } \Gamma_{\mathrm{FSI}} \times (0, T)$$

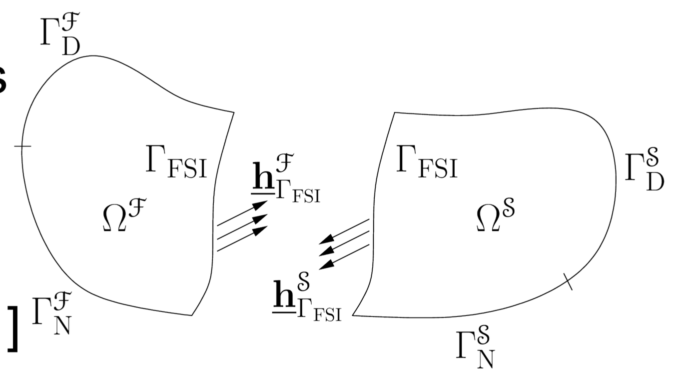


Fig. 1: Problem definition

- Spatial discretization of structure and fluid field with finite elements
- Dual Mortar method for Lagrange multipliers [2] allows for non-matching interface discretization and cheap condensation of Lagrange multipliers.
- Temporal discretization with single-step, single-stage, and fully implicit marching time integrators

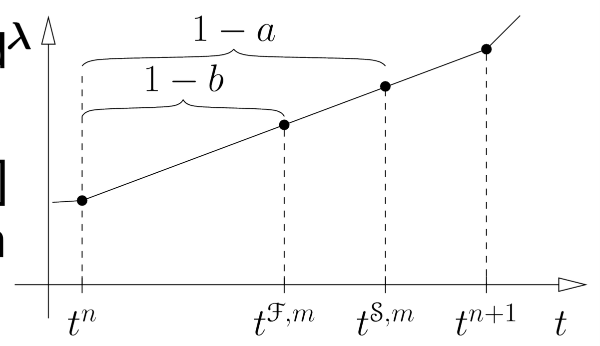


Fig. 2: Temporal interpolation of interface tractions [1]

Monolithic System of Equations

Linear System of Equations

- We exemplarily choose the structure field as master field → structure-handled interface motion

$$\begin{bmatrix} \mathcal{S}_{\mathrm{II}} & \mathcal{S}_{\mathrm{I}\Gamma} & & & & & \\ \mathcal{S}_{\Gamma\mathrm{I}} & \mathcal{S}_{\Gamma\Gamma} & & & & & -(1-a)\mathcal{M}^{\mathrm{T}} \\ & & \mathcal{F}_{\mathrm{II}} & \mathcal{F}_{\mathrm{I}\Gamma} & \mathcal{F}^{\mathcal{G}}_{\mathrm{II}} & \mathcal{F}^{\mathcal{G}}_{\mathrm{I}\Gamma} & \\ & & \mathcal{F}_{\Gamma\mathrm{I}} & \mathcal{F}_{\Gamma\Gamma} & \mathcal{F}^{\mathcal{G}}_{\Gamma\mathrm{I}} & \mathcal{F}^{\mathcal{G}}_{\Gamma\Gamma} & (1-b)\mathcal{D}^{\mathrm{T}} \\ & & & & \mathcal{A}_{\mathrm{II}} & \mathcal{A}_{\mathrm{I}\Gamma} & \\ & -\mathcal{M} & & \tau\mathcal{D} & & & \\ & \mathcal{M} & & & & -\mathcal{D} & \end{bmatrix}^{n+1}_{i} \begin{bmatrix} \Delta\mathbf{d}^{\mathcal{S}}_{\mathrm{I}} \\ \Delta\mathbf{d}^{\mathcal{S}}_{\Gamma} \\ \Delta\mathbf{u}^{\mathcal{F}}_{\mathrm{I}} \\ \Delta\mathbf{u}^{\mathcal{F}}_{\Gamma} \\ \Delta\mathbf{d}^{\mathcal{G}}_{\mathrm{I}} \\ \Delta\mathbf{d}^{\mathcal{G}}_{\Gamma} \\ \boldsymbol{\lambda} \end{bmatrix}^{n+1}_{i+1} = \begin{bmatrix} \mathbf{r}^{\mathcal{S}}_{\mathrm{I}} \\ \mathbf{r}^{\mathcal{S}}_{\Gamma} \\ \mathbf{r}^{\mathcal{F}}_{\mathrm{I}} \\ \mathbf{r}^{\mathcal{F}}_{\Gamma} \\ \mathbf{r}^{\mathcal{G}}_{\Gamma} \\ \mathbf{0} \\ \mathbf{0} \end{bmatrix}^{n+1}_{i} + \begin{bmatrix} \mathbf{0} \\ -a\mathcal{M}^{\mathrm{T}}\boldsymbol{\lambda}^n \\ \mathbf{0} \\ b\mathcal{D}^{\mathrm{T}}\boldsymbol{\lambda}^n \\ \mathbf{0} \\ \mathbf{0} \\ \mathbf{0} \end{bmatrix} + \delta_{i0} \begin{bmatrix} \mathbf{0} \\ \mathbf{0} \\ \mathbf{0} \\ \mathbf{0} \\ \mathbf{0} \\ \Delta t\, \mathcal{D}\, \mathbf{u}^{\mathcal{F},n}_{\Gamma} - \mathcal{M}\Delta\mathbf{d}^{\mathcal{S}}_{\Gamma,p} \\ \mathcal{M}\Delta\mathbf{d}^{\mathcal{S}}_{\Gamma,p} \end{bmatrix}$$

Condensation of Lagrange Multipliers

- Use balance of linear momentum of slave interface DOFs for condensation
- Dual Mortar method leads to diagonal form of Mortar matrix $\mathcal{D}$
  → Computationally cheap condensation of Lagrange multipliers and slave interface DOFs

Adaptive Time Stepping: Algorithm

- Individual error estimation in both fields via
  - Comparison to auxiliary explicit scheme (e.g. Adams-Bashforth 2)
  - Zienkiewicz & Xie [3] (structure only)
- Error norms: length-scaled L2-norms
- Deduce separate norm for FSI interface DOFs to account for central role of the interface
- Adapt time step size based on estimated errors

$$\kappa_{opt} = \left(\frac{tol}{error}\right)^{\frac{1}{p+1}}$$

- Compute Δt^{Ω}_{new} for every subset of DOFs

$$\Delta t^{\Omega}_{new} = \min\left\{\Delta t_{max}, \max\left\{\min\left\{\kappa_{max}, \max\left\{\kappa_{min}, \kappa_s\kappa_{opt}\right\}\right\}\Delta t_{old}, \Delta t_{min}\right\}\right\}$$

- Choose minimal time step size suggestion to guarantee accuracy everywhere in the domain

$$\Delta t_{new} = \min\left(\Delta t^{\Omega}_{new}\right)$$

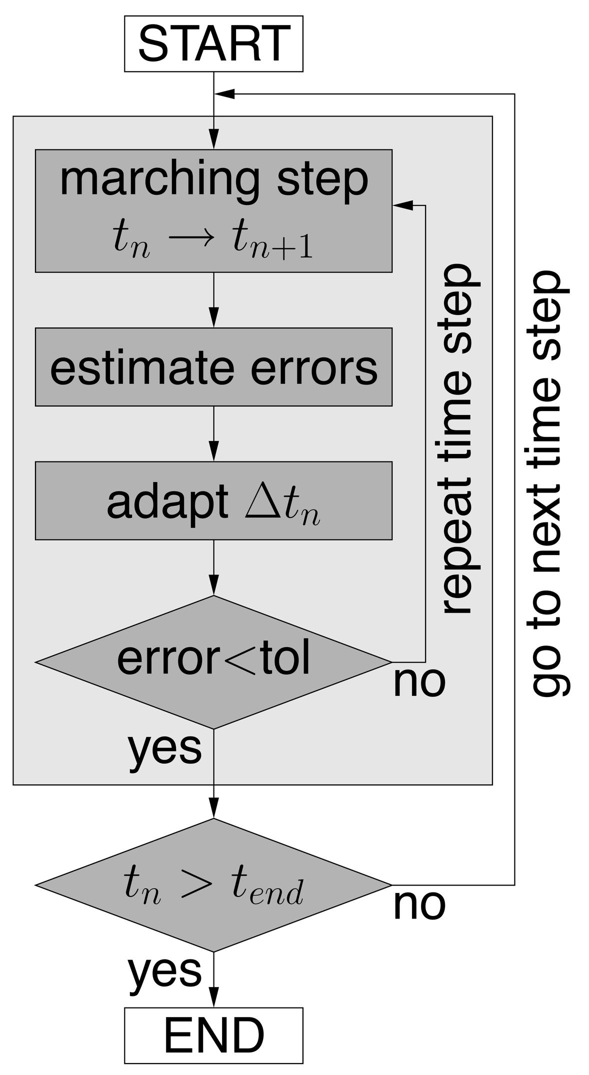
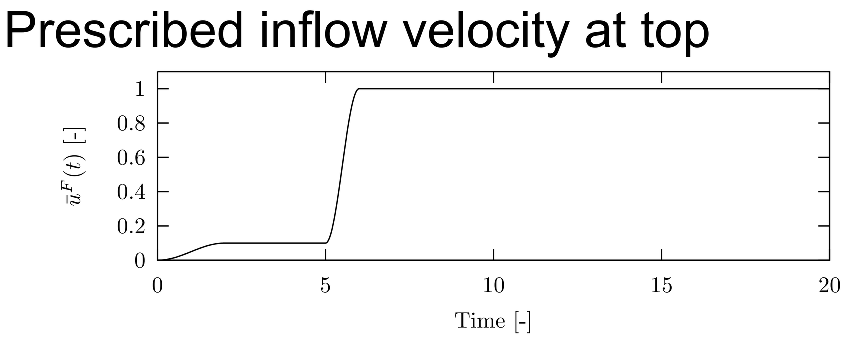


Fig. 3: Adaptive time stepping loop

Adaptive Time Stepping: Numerical Example

- Thin-walled spherical structure with fluid loading
- Prescribed inflow velocity at top



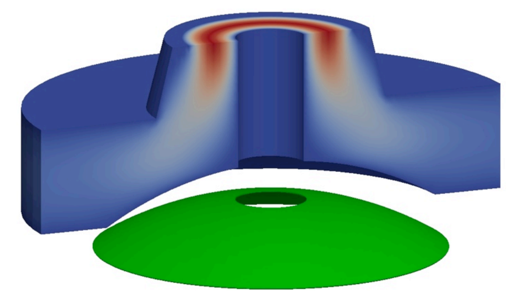
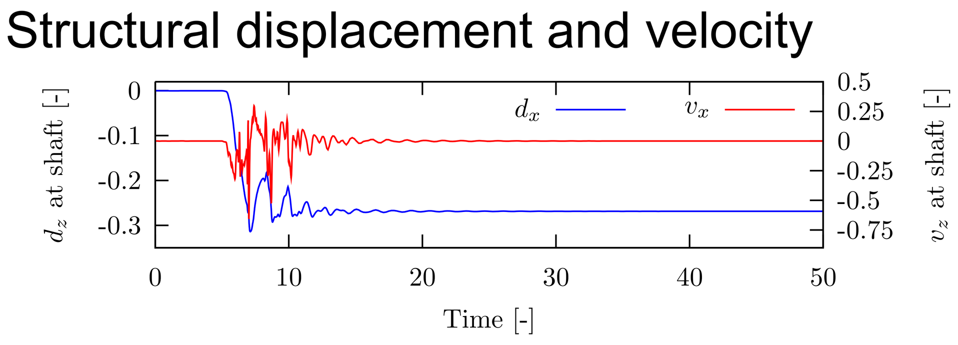


Fig. 4: Geometry and flow field before snap-through of structure

- Structural displacement and velocity



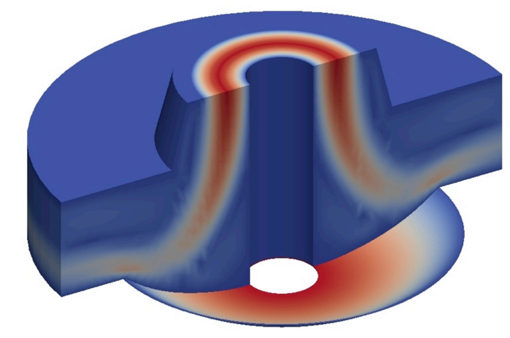
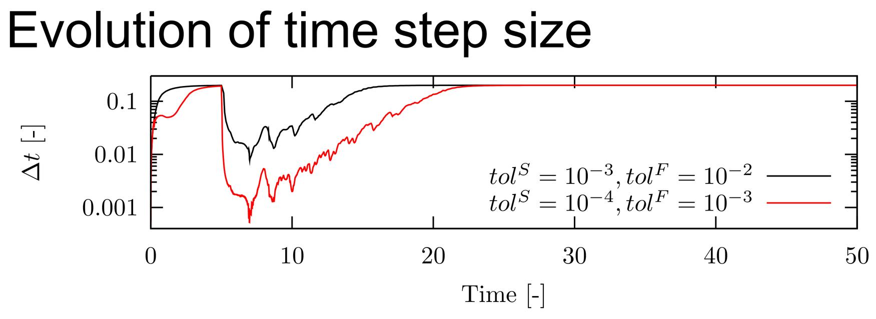


Fig. 5: Solution at end time after snap-through of structure

- Evolution of time step size



Main Results

- saving ca. 96% of number of computed time steps
- guaranteed level of accuracy

Newton-Krylov with FSI-specific Preconditioning [4]

Algorithm

- Algebraic coarse level problem

$$\mathbf{A}^{\mathrm{FSI}}_{k+1} = \begin{bmatrix} \mathbf{R}^{\mathrm{S}}_k & & \\ & \mathbf{R}^{\mathrm{G}}_k & \\ & & \mathbf{R}^{\mathrm{F}}_k \end{bmatrix} \begin{bmatrix} \mathbf{S} & & \mathbf{S}^{\mathrm{SF}} \\ & \mathbf{G} & \mathbf{G}^{\mathrm{GF}} \\ \mathbf{F}^{\mathrm{FS}} & \mathbf{F}^{\mathrm{FG}} & \mathbf{F} \end{bmatrix}_k \begin{bmatrix} \mathbf{P}^{\mathrm{S}}_k & & \\ & \mathbf{P}^{\mathrm{G}}_k & \\ & & \mathbf{P}^{\mathrm{F}}_k \end{bmatrix}$$

$$= \begin{bmatrix} \mathbf{R}^{\mathrm{S}}_k\mathbf{S}_k\mathbf{P}^{\mathrm{S}}_k & & \mathbf{R}^{\mathrm{S}}_k\mathbf{S}^{\mathrm{SF}}_k\mathbf{P}^{\mathrm{F}}_k \\ & \mathbf{R}^{\mathrm{G}}_k\mathbf{G}_k\mathbf{P}^{\mathrm{G}}_k & \mathbf{R}^{\mathrm{G}}_k\mathbf{G}^{\mathrm{GF}}_k\mathbf{P}^{\mathrm{F}}_k \\ \mathbf{R}^{\mathrm{F}}_k\mathbf{F}^{\mathrm{FS}}_k\mathbf{P}^{\mathrm{S}}_k & \mathbf{R}^{\mathrm{F}}_k\mathbf{F}^{\mathrm{FG}}_k\mathbf{P}^{\mathrm{G}}_k & \mathbf{R}^{\mathrm{F}}_k\mathbf{F}_k\mathbf{P}^{\mathrm{F}}_k \end{bmatrix}$$

- FSI coupling on all levels

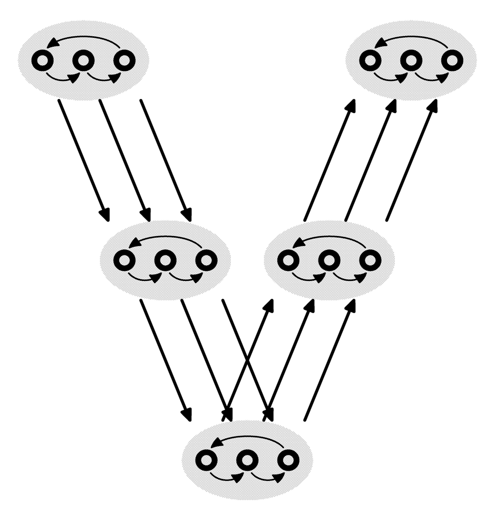


Fig. 6: FSI-AMG V-Cycle for AMG(BGS/Schur) [4]

Example: Pressure Wave

- Significant superiority in "difficult" FSI problems $\rho^{\mathcal{S}}/\rho^{\mathcal{F}} \approx 1$
  - 3 Newton steps per time step (avg.)
  - outperforms partitioned approaches by far
- Suitable for very large scale computations

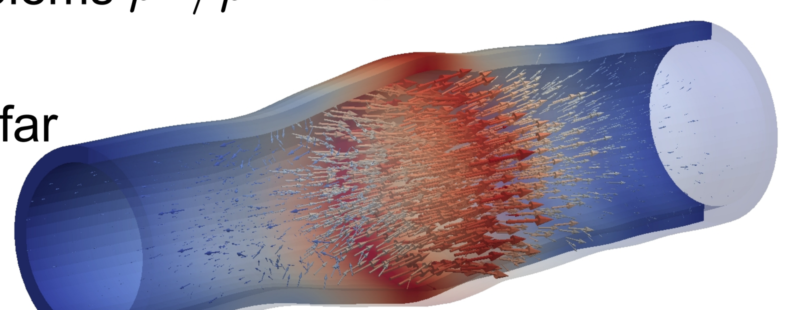


Fig. 7: Solution of pressure wave

AMG(BGS) characteristics averaged over timesteps

n_{ele}	Newton	GMRES	time	n_{proc}	$t \cdot n_{proc}/n_{dof}$
16	2.97	36.1	6.09	4	0.0013
32	2.98	32.8	27.1	8	0.0016
48	2.97	37.1	50.68	12	0.0016
64	2.78	42.7	121.68	12	0.0014

Degrees of freedom

n_{ele}	Structure	Fluid	ALE	Total
16	2,592	9,612	7,209	19,413
32	14,688	71,444	53,583	139,715
48	44,352	189,916	142,437	376,705
64	96,960	540,956	405,171	1,043,087

Nonlinear GMRES with AMG-FAS Preconditioning

- FAS: Full Approximation Scheme
- Variational residual evaluation

$$F_c = R^0_c F_0\left(P^c_0 x_c\right)$$

- Motivation
  - Timings: residual evaluation vs. linear solve
  - residual evaluation scales perfectly for fine grained parallelization
- Idea
  - nonlinear FSI-coupling on coarse levels using FSI-AMG hierarchy
  - acceleration by outer nonlinear Krylov-type solver [5]
- Multigrid library: Trilinos/MueLu [6]

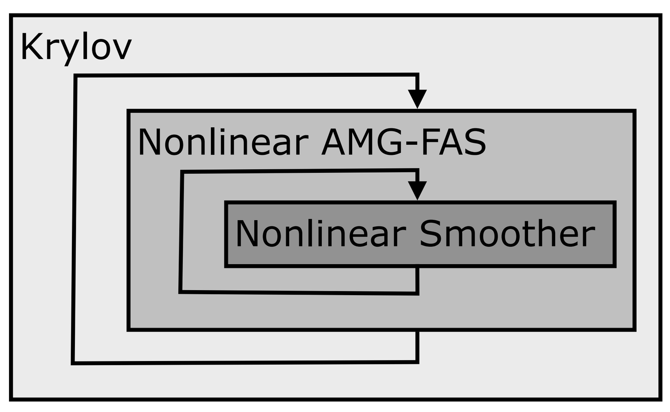


Fig. 8: Sketch of algorithm

References

[1] Mayr M, Klöppel T, Wall WA, Gee MW: A temporal consistent monolithic approach to fluid-structure interaction enabling single field predictors, SIAM J. Sci. Comput., 37(1):B30-B59, 2015

[2] Klöppel T, Popp A, Küttler U, Wall WA: Fluid-structure interaction for non-conforming interfaces based on a dual mortar formulation, Comput. Methods Appl. Mech. Engrg., 200(45-46):3111-3126, 2011

[3] Zienkiewicz OC, Xie YM: A simple error estimator and adaptive time stepping procedure for dynamic analysis, Earthquake Engng. Struct. Dyn., 20(9):871-887, 1991

[4] Gee MW, Küttler U, Wall WA: Truly monolithic algebraic multigrid for fluid-structure interaction, Int. J. Numer. Meth. Engng., 85(8): 987-1016, 2011

[5] De Sterck H: A Nonlinear GMRES Optimization Algorithm for Canonical Tensor Decomposition, SIAM J. Sci. Comput., 34(3):A1351-A1379, 2012

[6] Prokopenko A, Hu JJ, Wiesner TA, Siefert CM, Tuminaro RS: MueLu User's Guide 1.0, Techn. Report SAND2014-18874, Sandia National Laboratories, 2014