Price-based Adaptive Scheduling in Multi-Loop Control Systems with Resource Constraints

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***Abstract*—Under many circumstances event-triggered scheduling outperforms time-triggered schemes for control applications when resources such as communication, computation, or energy are sparse. This article investigates another benefit of event-triggered control concerning the ability of adaptation that enables the implementation of distributed scheduling mechanisms. The system under consideration comprises multiple heterogeneous control systems that share a common resource for accomplishing their control tasks. Each subsystem is modelled as a discrete-time stochastic linear system. The design problem is formulated as an average cost Markov decision process (MDP) problem with unknown global system parameters that are to be estimated during execution. Techniques from distributed optimization and adaptive MDPs are used to develop distributed self-regulating event-triggers that adapt their request rate to accommodate a global resource constraint. Stability and convergence issues are addressed by using methods from stochastic stability for Markov chains and stochastic approximation. Numerical simulations show the effectiveness of the approach and illustrate the convergence properties.**

***Index Terms*—Resource-aware control, cyber-physical systems, event-triggered scheduling, stochastic optimal control**

I. Introduction

The increased interest in cyber-physical systems has led to various paradigm shifts in digital control design. The systems under consideration usually consist of a multitude of small-scale integrated entities that need to share common resources, like communication and computation. The efficient usage of these resources is a prerequisite for the successful operation of such control systems. This fact has stimulated researchers to look for advanced sampling schemes beyond the conventional periodic sampling scheme to reduce resource consumption. A large amount of literature [2]–[13] shows that event-triggered control schemes achieve the same control performance as time-triggered control schemes, while reducing the number of samples significantly. Event-triggered control has been applied successfully for example in control over communications [2]–[5], decentralized control in sensor actuator networks [8]–[10] and multi-agent systems [6], [7], and the control design in embedded real-time systems [11]–[13].

Besides the efficient use of resources, other non-functional requirements like self-configurability and adaptability need to be addressed within the design of cyber-physical systems. In the envisioned system, local entities, such as sensor nodes, are aware that a common resource is shared among them. Such awareness is reflected in the capability of adjusting the sampling strategy adaptively to reduce resource needs, while maintaining a certain amount of performance. This article focuses on the synthesis of adaptive event-triggered schedulers for multiple independent control systems, whose control loops are closed over a shared communication networks.

Related work is presented in [3], [5], [14]–[17] that analyze the stability and the performance of event-triggered control schemes in contention-based communication systems. The work in [3], [5] analyzes the performance of event-triggered control schemes using carrier sense multiple access (CSMA) models with priority or randomized arbitration. In [14], [15], event-triggered scheduling schemes have been analyzed in slotted and unslotted ALOHA transmission models. In [16], the performance of event-triggered control is analyzed for non-persistent and 1-persistent CSMA models, while a stability analysis for event-triggered control systems considering a p-persistent CSMA model is conducted in [17].

Throughout the aforementioned work, the design of the event-triggered control scheme is determined beforehand in a centralized fashion. This might be inconvenient due to its difficulty of implementation as the number of subsystems might be large, and it lacks of flexibility, as it needs to be rerun completely whenever changes in the system occur. In fact, the intrinsic ability of adaptation of event-triggered control systems has not been studied properly yet. This motivates us to address the problem of distributed scheduling design via adaptive event-triggered sampling for heterogeneous control subsystems closed over a common resource.

The contribution of this article is to develop a distributed scheduling algorithm for an event-triggered control system, where each subsystem adjusts its event-triggering mechanism to optimally meet a global resource constraint imposed by the communication network. Each subsystem is modelled as a discrete-time stochastic linear system with a quadratic control cost. The subsystems are coupled by a common resource that allows to close only a limited number of feedback loops in every time instant. A problem relaxation approach developed by the authors in [18] is used to formulate the design objective in the framework of constrained MDP with an average-cost criterion. Inspired by distributed optimization and adaptive MDPs [19], a distributed sample-path based algorithm is proposed. In contrast to previous work [18], a dual decomposition approach related to utility maximization of random access algorithms [20]–[23] is chosen to study the underlying scheduling problem. In our work, the dual price mechanism forces each subsystem to adjust their event-

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triggering thresholds according to the total transmission rate. By using a time-scale separation approach, stability and convergence properties of the distributed event-triggered scheme are established in terms of recurrence and almost-sure convergence, respectively. Numerical simulations are conducted to illustrate the effectiveness of the obtained algorithm.

The remaining part of this article is structured as follows. In Sec. II, we introduce the system model and describe the problem statement. Section III develops the adaptive event-triggered controller and discusses its properties. The efficiency of the proposed approach is illustrated in Sec. IV by numerical simulations.

Notation. In this article, the operator $(\cdot)^{\mathrm{T}}$ and $\mathrm{tr}[\cdot]$ denote the transpose and trace operator, respectively. The Euclidean norm is denoted by $\|\cdot\|_2$. The variable P denotes the probability measure on the abstract sample space denoted by Ω. The expression F, P −a.s. denotes that the event F occurs almost surely w.r.t. probability measure P. The expectation operator is denoted by $\mathsf{E}[\cdot]$ and the conditional expectation is denoted by $\mathsf{E}[\cdot|\cdot]$. The relation $x \sim \mathcal{N}(0, X)$ denotes a Gaussian random variable with zero-mean and covariance matrix X. The non-negative real line is denoted by $\mathbb{R}_{\geq 0}$. The operator $[\cdot]^+$ denotes the projection onto $\mathbb{R}_{\geq 0}$, i.e., $[\cdot]^+ = \max\{0, \cdot\}$. The operator $\mathbb{1}_{\{\cdot\}}$ denotes the indicator function. The sequence of a discrete-time signal x_k^i is denoted by $\{x_k^i\}_k = \{x_0^i, x_1^i, \ldots\}$. The truncated sequence up to time k of a signal $\{x_k^i\}_k$, is denoted by X_k^i, i.e., $X_k^i = \{x_0^i, \ldots, x_k^i\}$. Let the complement of a set A be denoted by A^c. If not otherwise stated, a variable with superscript i indicates that it belongs to subsystem i.

II. PROBLEM STATEMENT

We consider the cyber-physical control system illustrated in Fig. 1 with N independent control subsystems whose feedback loops are connected through a shared communication network. A control subsystem i consists of a process $\mathcal{P}^i$, a controller $\mathcal{C}^i$ that is implemented at the actuator and a scheduler $\mathcal{S}^i$ implemented at the sensor.

A. Process model

The process $\mathcal{P}^i$ to be controlled within each subsystem i is given by a stochastic linear system with state x_k^i taking values in $\mathbb{R}^{n_i}$ and evolving by the following difference equation

$$x_{k+1}^i = A^i x_k^i + B^i u_k^i + w_k^i, \tag{1}$$

where $A^i \in \mathbb{R}^{n_i \times n_i}, B^i \in \mathbb{R}^{n_i \times m_i}$. The control input u_k^i is taking values in $\mathbb{R}^{m_i}$. The system noise w_k^i takes values in $\mathbb{R}^{n_i}$ at each k and is i.i.d. with $w_k^i \sim \mathcal{N}(0, C^i)$. The initial states x_0^i, $i \in \{1, \ldots, N\}$, have a distribution whose density function is symmetric around its mean value $\mathsf{E}[x_0^i]$ and has finite second moment.

It is assumed that the sensor node is capable to acquire full state information x_k at time k. The control input may depend on the complete observation history of the received signal at the controller. Furthermore, the control inputs need not to be constant in between of successful transmissions, but are allowed to be time-varying.

B. Communication model

In this work, we use a generic communication model that is inspired by the idealized model for CSMA in [21], [24], [25].

At any time k, the scheduler $\mathcal{S}^i$ situated at the sensor decides, whether a transmission slot should be requested to transmit the current state of subsystem i to the controller $\mathcal{C}^i$. Due to bandwidth limitations the number of transmission slots per time step denoted by c is constrained and schedulers must be designed at the sensors that judge the importance of transmitting an update to the corresponding controller. In order to obtain a non-trivial problem setting, we assume that

$$1 \leq c \leq N.$$

We make the following simplifying assumptions on the CSMA model: (i) sensing the carrier is instantaneous, (ii) there are no hidden nodes, (iii) the backoff intervals are exponentially distributed with homogeneous backoff exponents, (iv) the mean backoff time is negligible with respect to the length of a transmission slot, (v) data packets are discarded after c retransmission trials.

It should be noted that due to the assumptions (i) and (ii), no packet collisions may occur [21]. The assumptions (iii)-(vi) are tailored to the discrete-time nature of the control process. In particular, assumption (v) reflects the idea of the try-once discard (TOD) protocol introduced in [26], which discards outdated state information. Though these assumptions yield an idealized communication model, they capture the main features of a generic contention-based transmission mechanism and enable the analysis of the aggregate cyber-physical control system. Though the focus of this article is on wireless communication, it is remarked that the presented model is also applicable for other shared resource constraints that go beyond bandwidth limitations.

The communication phase during time step k can be sketched as follows. At the beginning of this phase, all subsystems that request for transmission select a random backoff interval implying that the subsystem with the smallest value is granted to transmit. The remaining subsystems then wait until the transmission is accomplished and back off for another time. This procedure is repeated at most c times due to assumption (v). Due to the above assumptions, all requesting subsystems can transmit during time step k if the number of requests does not exceed c. On the other hand, if there are more requests than available transmission slots at time k, then the communication system selects c subsystems that may transmit information. All other subsystems are blocked and are informed instantaneously that their request has been rejected. As the backoff exponent is assumed to be homogeneous among subsystems, the communication model does not prioritize subsystems, i.e., the subsystems are chosen with identical probability. The request for a transmission of the ith subsystem at time k is indicated by the variable δ_k^i which takes the values

$$\delta_k^i = \begin{cases} 1 & \text{request for transmission} \\ 0 & \text{idle.} \end{cases}$$

The resolution of contention represented by the random variable q_k^i is defined as

$$q_k^i = \begin{cases} 1 & \text{allow transmission} \\ 0 & \text{block transmission.} \end{cases}$$

By taking the above assumptions for the CSMA scheme into account, the variable q_k^i can be modelled as a probability distribution conditioned on the requests δ_k^i and obeys

$$\mathsf{P}[q_k^i = 1 | \delta_k^i, i \in \{1, \ldots, N\}] = \begin{cases} 1 & \sum_{i=1}^N \delta_k^i \leq c \\ \frac{c}{\sum_{i=1}^N \delta_k^i} & \text{otherwise.} \end{cases} \quad (2)$$

for subsystem i with $\delta_k^i = 1$ and

$$q_k^1(\omega) + \cdots + q_k^N(\omega) = c$$

for all sample paths $\omega \in \Omega$ for which $\delta_k^1 + \cdots + \delta_k^N \geq c$ with Ω being the abstract sample space. In case of $\delta_k^i = 0$, q_k^i is set to 0. The received data at the controller $\mathcal{C}^i$ at time k is denoted by z_k^i and is defined by

$$z_k^i = \begin{cases} x_k^i & \delta_k^i = 1 \wedge q_k^i = 1 \\ \varnothing & \text{otherwise.} \end{cases} \quad (3)$$

Finally, we note that the scheduler $\mathcal{S}^i$ knows the value of q_{k-1}^i at time k due to the assumptions on the communication model that exclude packet collisions.

C. Social cost optimization

In our system setting, we assume that each subsystem i with $i \in \{1, \ldots, N\}$ has an individual cost function J^i given by the linear quadratic average-cost criterion

$$J^i = \limsup_{T \to \infty} \frac{1}{T} \mathsf{E}\left[\sum_{k=0}^{T-1} (x_k^i)^{\mathrm{T}} Q_x^i x_k^i + (u_k^i)^{\mathrm{T}} Q_u^i u_k^i\right]. \quad (4)$$

The weighting matrix Q_x^i is semi-positive definite and Q_u^i is positive definite for each $i \in \{1, \ldots, N\}$. In the case of scalar subsystems, we also allow Q_u^i to be zero when $Q_x^i > 0$. We assume that the pair (A^i, B^i) is stabilizable and the pair $(A^i, Q_x^{i,\frac{1}{2}})$ is detectable with $Q_x^i = (Q_x^{i,\frac{1}{2}})^{\mathrm{T}} Q_x^{i,\frac{1}{2}}$.

It is assumed that the sensor and the controller of the ith subsystem merely have knowledge of the local system parameters, i.e. , A^i, B^i, C^i, the distribution of x_0, and Q_x^i, Q_u^i of (4).

As it will take a central role in the subsequent analysis, we define the individual request rate r^i of the ith subsystem by

$$r^i = \limsup_{T \to \infty} \frac{1}{T} \mathsf{E}[\sum_{k=0}^{T-1} \delta_k^i], \quad (5)$$

which also has the form of an average-cost criterion. With the definition of the individual request rate in (5), the total average request rate is defined as

$$y = \sum_{i=1}^N r^i. \quad (6)$$

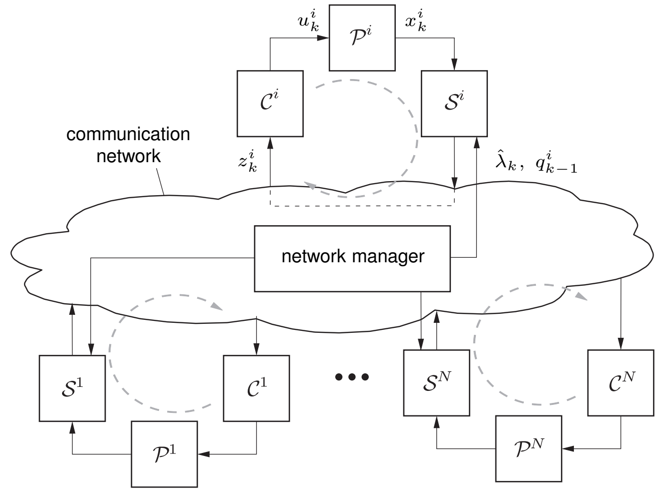

Fig. 1. System model of the cyber-physical system with N control systems closed over a shared communication network with processes $\mathcal{P}^1, \ldots, \mathcal{P}^N$, schedulers $\mathcal{S}^1, \ldots, \mathcal{S}^N$ and controllers $\mathcal{C}^1, \ldots, \mathcal{C}^N$. The network manager broadcasts the variable $\hat{\lambda}_k$ to all subsystems.

The control law $\gamma^i = \{\gamma_0^i, \gamma_1^i, \ldots\}$ reflecting the behavior of controller $\mathcal{C}^i$ is described by causal mappings γ_k^i of the past observations for each time step k, i.e.,

$$u_k^i = \gamma_k^i(Z_k^i),$$

where Z_k^i is the observation history until time k of subsystem i. We distinguish between two classes of schedulers $\pi^i = \{\pi_0^i, \pi_1^i, \ldots\}$ resulting from two types of network managers. In the first case, the network manager broadcasts a fixed parameter λ at the beginning and the static event-triggered scheduler is then given by

$$\delta_k^i = \pi_k^{i,\lambda}(X_k^i, Q_{k-1}^i),$$

where X_k^i is the state history of subsystem i and the variable $Q_{k-1}^i = \{q_0^i, \ldots, q_{k-1}^i\}$ is the sequence of received acknowledgements at time k. It should be remarked that we will usually omit λ for notational convenience. In the second case, the network parameter $\hat{\lambda}_k$ is allowed to change over time k and the scheduler adapts its law w.r.t. to $\hat{\lambda}_k$, i.e.,

$$\delta_k^i = \pi_k^{i,\hat{\lambda}_k}(X_k^i, Q_{k-1}^i).$$

The parameter $\hat{\lambda}_k$ itself is given by a causal mapping f_k of the past transmission history, i.e.,

$$\hat{\lambda}_k = \mathrm{f}_k(\delta_0^1, \ldots, \delta_0^N, \ldots, \delta_{k-1}^1, \ldots, \delta_{k-1}^N).$$

The mapping f_k represents the adaptation mechanism of the network manager to the current traffic and will be specified later in more detail. It should be noted that the network manager is implemented in a centralized manner, but it depends solely on the requests of the subsystems in the case of adaptive scheduling.

The design objective is to find the optimal control laws γ^i and optimal scheduling laws π^i, $i \in \{1, \ldots, N\}$ that minimize the social cost given as an average cost criterion. The social cost is defined by the sum of the individual costs J^i of each subsystem. Therefore, the optimization problem can be summarized as follows.

$$\min_{\substack{\gamma^1,\ldots,\gamma^N\\ \pi^1,\ldots,\pi^N}} \sum_{i=1}^{N} J^i. \tag{7}$$

It should be noted that optimization problems with an average-cost criterion as given above are underselective [27]. This is because it does not matter how well the policy works at the beginning. Only its stationary behavior determines its cost. Therefore, there may exist two policies, which differ completely with respect to their performance in the first k time steps, but eventually converge to the same stationary behavior and therefore yield the same average cost. Hence, the optimization problem does not distinguish between these two policies. For the purpose of our work, this feature turns out to be an advantage, as it allows us to design an adaptation mechanism for each subsystem that learns the optimal transmission rate over time.

III. ADAPTIVE EVENT-TRIGGERED SCHEDULING

This section focuses on the synthesis and analysis of the cyber-physical system that addresses the solution of the optimization problem (7). Section III-A introduces a problem relaxation to solve problem (7) and is followed by the characterization of optimal solutions of the relaxed problem in Sec. III-B. In Sec. III-C, ideas from dual decomposition and adaptive MDPs are then used to develop a distributed approach that solves the relaxed optimization problem introduced in Sec. III-A. A Lagrange approach is taken to formulate the dual problem of the relaxed problem, which enables us to derive a dual price exchange mechanism that broadcasts a price for the resource to each subsystem. An adaptive sample-path algorithm is proposed in Sec. III-D that estimates the average total transmission rate to approximate the pricing gradient. Section III-E and III-F address convergence and stability properties of the adaptive event-triggered scheme, respectively. The implementation of the algorithm is discussed in Sec. III-G.

A. Problem relaxation

Although the coupling between subsystems is solely caused by the resource limitation, determining the optimal event-based control system that solves (7) is a hard problem due to the distributed information pattern. It is shown in [28] that optimal stochastic control problems with distributed information pattern are generally hard to solve, even if linear dynamics and quadratic cost functions are considered. Besides the distributed information pattern, the impact of the bandwidth limitation is another complicating factor. In contrast to time-triggered scheduling schemes, distributed event-triggered scheduling is exposed to contention among subsystems that share the resource and it is in general not possible to guarantee that a request for the resource will be approved apriori.

Therefore, we introduce a problem relaxation in order to obtain a feasible method to find the event-triggered controllers that minimize the social cost in (7). This approach that is an approximation of the original problem will be the basis for our scheduling and control design. The motivation for such relaxation will be discussed at the end of the section. Moreover, the stability analysis of the resulting system is carried out by taking the impact of contention into account.

In the following, the hard constraint of a maximum of c transmissions at each time k is weakened and we require merely that the total average request rate is upper bounded by c. The relaxed optimization problem is then given by the following constrained MDP.

$$\min_{\substack{\gamma^1,\ldots,\gamma^N\\ \pi^1,\ldots,\pi^N}} \sum_{i=1}^{N} J^i \qquad \text{s.t.} \quad y \leq c. \tag{8}$$

As this approach does not consider contention among subsystems, the received signal at the controller in the design stage is assumed to be

$$z_k^i = \begin{cases} x_k^i & \delta_k^i = 1 \\ \varnothing & \text{otherwise.} \end{cases} \tag{9}$$

Aside from facilitating the optimization problem (7), the motivation for the above approach is two-fold.

First, the solution of the relaxed problem (8) is asymptotically optimal with regard to the optimization problem (7) when the number of subsystems N and the capacity c grow towards infinity. Because of the law of large numbers, it can be reasoned that the average rate r^i and the empirical mean within one time step coincide almost surely in the stationary regime as $N \to \infty$ while c/N is kept constant [18]. This implies that the hard transmission constraint considered in the original optimization problem (7) is satisfied in the limit. As the relaxed problem attains a lower bound with respect to the cost of problem (7), we conclude that the solution of (8) is asymptotically optimal.

Second, we give another argument that justifies the absence of contention among subsystems assumed in the above approach. Because of the underselective nature of the average-cost criterion, we are primarily interested in the stationary behavior of the overall system. Assuming that the scheduling and control policies are stationary, the impact of contention on an individual subsystem i in the stationary regime can be aggregated as a Bernoulli-distributed q_k^i with a fixed blocking probability. It has been shown in [29] that the form of the optimal solution does not alter when introducing a Bernoulli-distributed blocking process. Furthermore, the work in [29] demonstrates quantitatively through numerical simulations that the optimal solution is comparatively insensitive with respect to the blocking probability when assuming a fixed price for using the feedback channel. The explanation for this observation is based on two compensating effects when introducing contention. On the one hand, contention lowers the control-related benefit of closing the feedback loop which effectively increases the control cost. On the other hand, the average transmission rate is increased, as it is more likely for another request after a blocked transmission than after a successful transmission.

In summary, these arguments suggest that the assumptions in the relaxed design problem given by the average rate constraint and the absence of contention serve as an adequate model for the original problem.

B. Characterization of optimal policies

The main advantage of the relaxed problem lies in the possibility to characterize the form of the optimal controller and the event-triggered scheduler. This characterization facilitates the optimization problem considerably and enables the development of computationally feasible design algorithms. In the following, we revise the main structural properties of the optimal control and triggering laws that have been derived in [18].

When seeking for the policies γ^i and π^i that are optimal with respect to (8), we observe that the main specifications needed for the relaxed problem are given by the transmission rate r^i and the cost J^i of subsystem i. It should be noted that these values solely depend on the particular choice of policies within subsystem i. We are therefore interested in analyzing the feasible region of pairs (J^i, r^i) with respect to dominating strategies in order to characterize Pareto optimal policies. Once a dominating class of strategies is found, we can restrict our attention to this set in the optimization problem (8). This is because both the cost and resource constraint are composed of the summation of J^i and r^i, respectively. In this way, we can narrow down the set of admissible policies without losing optimality.

It is shown in [30] that the pair of polices with a controller having the certainty equivalence property is a dominating class of policies. Therefore, the optimal control law can be assumed to take the form

$$u_k^i = \gamma_k^{i,*}(Z_k^i) = -L^i\,\mathsf{E}[x_k^i|Z_k^i], \tag{10}$$

where $L^i = ((B^i)^{\mathrm{T}}P^iB^i + Q_u^i)^{-1}(B^i)^{\mathrm{T}}P^iA^i$ and P^i is the solution of the algebraic Riccati equation

$$\begin{aligned}P^i =& (A^i)^{\mathrm{T}}(P^i - P^iB^i((B^i)^{\mathrm{T}}P^iB^i + Q_u^i)^{-1}(B^i)^{\mathrm{T}}P^i)A^i \\ &+ Q_x^i.\end{aligned}$$

The reason for this structural property of the optimal control law is mainly given by the nestedness property of the information pattern. The information pattern is nested, since the information available at the controller is a subset of the information available at the scheduler. Furthermore, the work in [31] and [32] proves that symmetric scheduling laws for first-order systems are optimal. Adopted from previous work on event-triggered estimation as in [33]–[35], we also assume that the optimal triggering rule is symmetric for higher-order systems. There are several indicators that this form is optimal for higher-order systems, such as the overall problem setup is symmetric, and the proof techniques for the first-order case carry over to special cases of higher-order systems, e.g. the multi-dimensional random walk with uncorrelated noise inputs [36]. Nevertheless, a conclusive proof for the general case is still lacking. It is remarked that if this assumption does not hold, the optimal estimator must be extended by an additive bias term, see [37], which has essentially no consequences on the subsequent analysis. If the triggering rule is assumed to be symmetric, then the optimal estimator $\mathsf{E}[x_k^i|Z_k^i]$ is identical with the optimal estimator when triggering occurs in a time-triggered fashion. Therefore, the controller does not obtain implicit information between transmissions. This implies that the optimal estimator is a linear predictor $\mathsf{E}[x_k^i|x_\tau^i]$ with τ being the last time step of a state transmission at subsystem i. The optimal estimator can then be written in recursive from as follows.

$$\mathsf{E}[x_k^i|Z_k^i] = \begin{cases} x_k^i & \delta_k^i = 1 \wedge q_k^i = 1 \\ (A^i - B^iL^i)\,\mathsf{E}[x_{k-1}^i|Z_{k-1}^i] & \text{otherwise.}\end{cases} \tag{11}$$

with $\mathsf{E}[x_0^i|Z_0^i] = \mathsf{E}[x_0^i]$ for $\delta_0^i = 0$. It should be noted that in the absence of contention, which applies in the design stage, the first case in above estimator must be replaced by $\delta_k^i = 1$. We observe that the optimal control law is independent of λ. Therefore, the control law can be fully implemented prior to execution without additional knowledge. Because of this description of the optimal control law, the individual cost J^i can be written as

$$J^i = \mathrm{tr}[P^iC^i] + \limsup_{T\to\infty}\frac{1}{T}\,\mathsf{E}\left[\sum_{k=0}^{T-1}(1-\delta_k^i)(e_k^i)^{\mathrm{T}}Q_e^ie_k^i\right]. \tag{12}$$

where $Q_e^i = (L^i)^{\mathrm{T}}(Q_u^i + (B^i)^{\mathrm{T}}P^iB^i)L^i$ and the one-step ahead estimation error

$$e_k^i = x_k^i - \mathsf{E}[x_k^i|Z_{k-1}^i].$$

The term $\mathrm{tr}[P^iC^i]$ in (12) relates to the optimal cost

The estimation error e_k^i evolves according to the δ_k^i-controlled Markov chain with the difference equation

$$e_{k+1}^i = (1 - q_k^i\delta_k^i)Ae_k^i + w_k \tag{13}$$

with initial condition $e_0 = x_0 - \mathsf{E}[x_0]$ and q_k^i is assumed to be 1 during the design stage. The remaining problem is to characterize the optimal scheduling law $\pi^{i,\lambda}$. This can be cast as a multi-objective MDP problem with state e_k^i and objective functions J^i and r^i. In [38], it has been shown that the set of feasible cost vectors resulting from the multi-objective MDP problem is intrinsically convex for average-cost formulations. Intuitively, two policies can be used to synthesize one policy that randomly chooses between these two policies yielding a cost vector that lies on the line segment with corner points defined by the costs of the two selected policies. This implies that the Pareto curve can be represented as a convex function denoted by $J^i(r^i)$ in the following. Subsequently, we also assume that $J^i(r^i)$ is twice continuously differentiable and its curvature is bounded away from zero on $(0,1)$ in order to facilitate out analysis. Furthermore, the work in [38] shows that the scalarization approach can be applied to obtain the Pareto region, which is given by

$$\min_{\pi^{i,\lambda}} J^i + \lambda r^i, \tag{14}$$

where λ takes values in $\mathbb{R}_{\geq 0}$. Subsequently, we introduce a technical assumption on the admissible triggering rules $\pi^{i,\lambda}$.

(A1) For any $\lambda \in \mathbb{R}_{\geq 0}$, the scheduling policy of subsystem i satisfies $\pi^{i,\lambda}(e_k^i) = 1$ for $\|e_k^i\|_2 > M^i$ for some arbitrary bound M^i, $i \in \{1, \ldots, N\}$, where M^i may depend on λ.

This assumption does not put severe restrictions on the admissible scheduling policies, as the bound M^i may be chosen arbitrarily large. Under Assumption **(A1)**, the above optimization problem can be solved by value iteration and optimal policies are stationary mappings π_k^i of the estimation error e_k^i [27].

Subsequently, we restrict our attention to the aggregate error state given by the individual errors e_k^i, which is defined as

$$e_k = [e_k^1, \ldots, e_k^N]^{\mathrm{T}}.$$

This is because the state x_k^i in subsystem i can be viewed as an isolated stable process controlled by e_k^i, when considering the closed-loop system.

For a first-order subsystem, the resulting optimal scheduling law is a symmetric threshold policy and takes the following form

$$\delta_k^i = \pi^{i,\lambda}(e_k^i) = \mathbb{1}_{\{|e_k^i| > d^i(\lambda)\}} \tag{15}$$

parameterized by the threshold d^i that depends on the price λ. This is because the value iteration outputs a sequence of even and radially non-decreasing value functions [36].

C. Dual decomposition

Opposed to the work in [18], we pursue the analysis of the dual problem of the relaxed optimization problem after having obtained the structural characterization in the previous section that allows us to compute $J^i(r^i)$ for each subsystem i. This will allow us to develop a distributed algorithm to determine the optimal operation point for the event-triggered scheduler.

In the following, we define the Lagrangian function by

$$\mathcal{L}(r^1, \ldots, r^N, \lambda) = \sum_{i=1}^{N} J^i(r^i) + \lambda(y - c)$$

By taking the structural properties from Sec. III-B into account, the optimization in (8) can be rewritten as the dual formulation of a network utility maximization problem [39] with a single link, i.e.,

$$\max_{\lambda \geq 0} \min_{r^i, i \in \{1,\ldots,N\}} \sum_{i=1}^{N} J^i(r^i) + \lambda(y - c). \tag{16}$$

The Lagrange multiplier λ can be interpreted as a penalty or price for the transmission rate. Therefore, we sometimes refer to λ as the communication penalty or price. It should be remarked that strong duality holds for the underlying problem, as the primal problem is a convex optimization problem and Slater's condition holds for $c > 0$. It is well known that the above problem has a unique solution for assigning the optimal transmission rates r^i, $i \in \{1, \ldots, N\}$. As there is only one link, it is also clear that there is a unique λ^* that solves (16). In the following, we aim at developing a distributed gradient method that finds the optimal λ^*. Let $g(\lambda)$ be defined as

$$g(\lambda) = \min_{r^i, i \in \{1,\ldots,N\}} \sum_{i=1}^{N} J^i(r^i) + \lambda(y - c).$$

In [39], it is shown that the derivative of $g(\lambda)$ with respect to λ is obtained by

$$\frac{\partial g(\lambda)}{\partial \lambda} = y - c$$

Therefore, the continuous-time gradient algorithm to solve the dual problem is given by

$$\dot{\lambda}(t) = [y(t) - c)]_{\lambda}^{+} \tag{17}$$

with an arbitrary initial value $\lambda(0)$. The projection guarantees that the penalty $\lambda(t)$ remains non-negative at all times t and is defined as

$$[\xi]_{\lambda}^{+} = \begin{cases} \xi & \lambda > 0 \\ \max(\xi, 0) & \text{otherwise.} \end{cases}$$

The total request rate $y(t)$ is defined as

$$y(t) = \sum_{i=1}^{N} r^i(t) \tag{18}$$

where $r^i(t)$ is the average request rate defined in (5) assuming that the price $\lambda(t)$, the controller $\gamma^{i,*}$ given by (10) and the scheduling law $\pi^{i,\lambda(t)}$ obtained from (14) are used. Taking into account the uniqueness of the optimal λ^* for the considered problem, it is shown in [39] that the differential equation (17) converges to λ^* for any initial condition $\lambda(0)$.

In the following, we focus on the discrete-time version of (17) given by

$$\lambda_{k+1} = [\lambda_k + \beta_k(y_k - c)]^{+} \tag{19}$$

with an arbitrary initial value λ_0 and step size $\beta_k > 0$ for all k. Similar to the continuous-time case, the total request rate is defined as

$$y_k = \sum_{i=1}^{N} r_k^i, \tag{20}$$

where r_k^i is the average request rate defined in (5) assuming the controller $\gamma^{i,*}$ and the scheduling law π^{i,λ_k} are used.

Remark 1: The discrete-time algorithm (20) can be viewed as an approximation of the ordinary differential equation (ODE) given by (17).

The complete algorithm can be regarded as a dual price exchange mechanism: after broadcasting the price λ_k by a central network manager to all subsystems at time step k, each subsystem adjusts its scheduling policy according to the local optimization problem (14) with λ_k as the dual price. It is shown in [40] that the algorithm converges to the optimal price λ^* for sufficiently small β_k.

The above gradient method is completely decoupled from the actual dynamics of the subsystems. Hence, the optimal price λ^* can be calculated prior to execution and is spread to the subsystems that use the stationary event-trigger $\delta_k^i = \pi^{i,\lambda^*}(e_k^i)$ for $k \geq 0$.

D. Adaptive sample-based algorithm

The drawback of the gradient method in (19) is obvious. The total average transmission rate y_k is commonly not exactly known at time step k, as it is neither efficient to gather information about every individual transmission rate r^i from each subsystem at the central network manager, nor it is feasible to determine y_k through its empirical mean by letting $T \to \infty$. Instead, we consider an estimate $\hat{y}_k$ of the total request rate over a window length $T_{0,k}$ to approximate the gradient in (19). While estimating $\hat{y}_k$, the price remains constant. Hence, updates of the estimated price $\hat{\lambda}_k$ occur after an estimation period $T_{0,k}$ which may not be uniform. Therefore, the mapping f_k representing the network manager introduced in Sec. II is determined by

$$\hat{\lambda}_{k+T_{0,k}} = [\hat{\lambda}_k + \beta_k(\hat{y}_k - c)]^+ \tag{21}$$

with an initial value $\hat{\lambda}_0 \in \mathbb{R}_{\geq 0}$, and $k \in \mathcal{T}$, where the index set $\mathcal{T}$ defines the set of update times, i.e., $\mathcal{T} = \{\sum_{\ell=0}^{l} T_{0,\ell}\}_l$. In between updates, while $\hat{\lambda}_k$ remains constant, the total request rate y_k is estimated by its empirical mean during that period, i.e.,

$$\hat{y}_k = \frac{1}{T_{0,k}} \sum_{\ell=k}^{k+T_{0,k}-1} \sum_{i=1}^{N} \delta_\ell^i, \quad k \in \mathcal{T}.$$

We will consider two different choices for the step size β_k and the window length $T_{0,k}$ in the following. The next subsection assumes that the step size decreases, while the window length increases in time, whereas we assume in Sec. III-F that the step size and window length are constant. On the one hand, the former case allows us to prove convergence properties of the adaptive algorithm. On the other hand, the latter case is more practical in applications with frequent changes in the system settings, as the adaptive algorithm is capable to react adequately to these changes when having a constant step size and a constant window length.

Figure 2 summarizes the mechanism of the complete adaptive event-triggered control system by illustrating one particular subsystem and its interplay with the network manager. In contrast to the design mechanism described at the end of subsection III-C, the price is not determined prior to execution, but is continuously estimated within the network manager for every time step $k \in \mathcal{T}$.

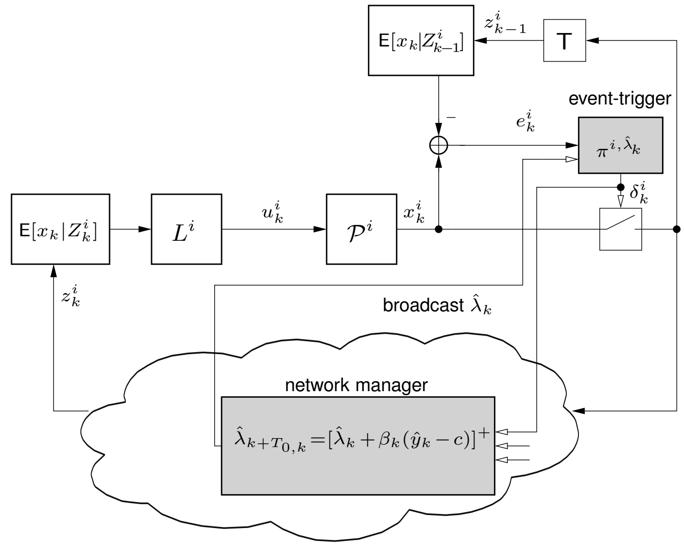

Fig. 2. Complete structure of the networked cyber-physical system for subsystem i with optimal event-triggered scheduler $\pi^{i,\hat{\lambda}_k}$ that adapts its law according to the price $\hat{\lambda}_k$. The price $\hat{\lambda}_k$ is broadcast to each subsystem from the central network manager. System block T denotes a 1-step delay element.

E. Convergence

This section is meant to validate the consistency of the adaptive sample-based algorithm with regard to the optimal solution of the relaxed problem. By supposing diminishing step sizes β_k and increasing window lengths $T_{0,k}$, we are able to show almost sure convergence of the process $\hat{\lambda}_k$ to the optimal solution of the relaxed problem (8) in the case without contention. The absence of contention implies that no transmissions are blocked and only an average resource constraint is considered. The main idea to show convergence comes from stochastic approximation [41] and relates the limiting behavior of the stochastic process $\{\hat{\lambda}_k\}_k$ to the ordinary differential equation given by (17). In the case with contention, we can not expect to obtain the same results, as there is no immediate deterministic description that relates to the evolution of $\{\hat{\lambda}_k\}_k$, when transmissions are blocked. However, the results in the absence of contention will serve as an indicator for the convergence of the contention-based case for increasing N.

The next assumption gives a condition on the step size β_k.

(A2) Assume that $\beta_k \to 0$ as $k \to \infty$ with $\sum_{k\in\mathcal{T}} \beta_k = \infty$ and $\sum_{k\in\mathcal{T}} \beta_k^2 < \infty$.

In order to show almost sure convergence $\{\hat{\lambda}_k\}_k$ in the absence of contention, the analysis is split into two parts. In the first part, we show stability of the process by establishing a recurrence property of $\{\hat{\lambda}_k\}_k$ with respect to any neighborhood set of the optimal price λ^*. A set is said to be *Harris recurrent*, if it is visited for infinitely many times by the stochastic process P-almost surely.

In the second part given by Theorem 1, a local analysis takes over and ODE methods from [41] are applied. In the local analysis, the process $\{\hat{\lambda}_k\}_k$ starts from a neighborhood set around the optimal solution λ^* and it is shown that while it enters the set infinitely many times due to the recurrence property, it leaves such neighborhood only finitely many times. This establishes the almost sure convergence result.

The first part is summarized by the following lemma.

Lemma 1: Let λ^* be the solution of the relaxed problem (8) and let β_k satisfy **(A2)** and $T_{0,k} \to \infty$. Under the absence of contention and the Assumption **(A1)**, the stochastic process $\{\hat{\lambda}_k\}_k$ evolving by (21) visits any small neighborhood of λ^* infinitely many times P-almost surely.

Proof: Consider the Lyapunov function

$$V(\hat{\lambda}_k) = (\hat{\lambda}_k - \lambda^*)^2.$$

By (21), we have for $k \in \mathcal{T}$

$$(\hat{\lambda}_{k+T_{0,k}} - \lambda^*)^2 \leq (\hat{\lambda}_k - \lambda^*)^2 + 2\beta(\hat{\lambda}_k - \lambda^*)(\hat{y}_k - c) + \beta^2 N^2$$

where the inequality follows from the fact that $[\cdot]_+$ is non-expansive [42].

Note that the T_0-sampled stochastic process $\{e_k, \hat{\lambda}_k\}_{k\in\mathcal{T}}$ is a time-inhomogeneous Markov chain. We define for $k \in \mathcal{T}$, the time-variant drift $\Delta_k V$ as

$$\Delta_k V = \mathsf{E}[V(\hat{\lambda}_{k+T_{0,k}})|e_k, \hat{\lambda}_k] - V(\hat{\lambda}_k).$$

With this definition, we have

$$\begin{aligned}\Delta_k V_k &\leq 2\beta(\hat{\lambda}_k - \lambda^*)\,\mathsf{E}[(\hat{y}_k - c)|e_k, \hat{\lambda}_k] + \beta^2 N^2 \\ &= 2\beta(\hat{\lambda}_k - \lambda^*)((y_k - c) + \\ &\quad + \mathsf{E}[(\mathsf{E}[\hat{y}_k|e_k, \hat{\lambda}_k] - y_k)|e_k, \hat{\lambda}_k]) + \beta^2 N^2, \quad (22)\end{aligned}$$

where y_k is defined in (20). Therefore, $y_k - c$ is the gradient $\frac{\partial g(\lambda)}{\partial \lambda}$ at $\lambda = \hat{\lambda}_k$. Based on this upper bound on Δ_k, we take the following super-Martingale lemma as a condition for recurrence from [20].

Lemma 2 ([20]): Suppose that there exists a set $\mathcal{D} \in \mathbb{R}_{\geq 0}$ such that for all $k \in \mathcal{T}$

$$\Delta_k V \leq -\beta_k \epsilon + v_k, \quad \lambda_k \notin \mathcal{D}, \tag{23}$$

where $\epsilon > 0$ β_k satisfies **(A2)** and $\sum_k |v_k| < \infty$ P-almost surely. Then $\mathcal{D}$ is recurrent with respect to the stochastic process $\{\hat{\lambda}_k\}_{k\in\mathcal{T}}$.
By identifying $v_k = \beta_k^2 N^2$ and taking **(A2)** into account, we observe that $\sum_k \in \mathcal{T}\beta^2 N^2 < \infty$. Since we have almost sure convergence of $\mathsf{E}[\hat{y}_k|e_k, \hat{\lambda}_k]$ to y_k as $T_{0,k} \to \infty$ due to **(A1)**, see [33] and using the fact that $0 \leq \hat{y}_k \leq N$, we have

$$\lim_{k\to\infty} \mathsf{E}[|\,\mathsf{E}[\hat{y}_k|e_k, \hat{\lambda}_k] - y_k)|\;|e_k, \hat{\lambda}_k] = 0. \tag{24}$$

By fixing the set $\mathcal{D}$, where λ^* is in the interior of $\mathcal{D}$, we have an appropriate $\epsilon_1 > 0$ such that $(\hat{\lambda}_k - \lambda^*)((y_k - c) < -\epsilon_1$ for each $\hat{\lambda}_k \notin \mathcal{D}$ due to the properties of the gradient $\frac{\partial g(\lambda)}{\partial \lambda}$. This implies together with (24) that an ϵ can be found that satisfies (23) for every neighborhood $\mathcal{D}$ of λ^*. This concludes the proof. ■

The subsequent theorem gives a statement on the limiting behavior of $\{\hat{\lambda}_k\}_k$ in terms of almost sure convergence.

Theorem 1: Let λ^* be the solution of the relaxed problem (8) and let β_k satisfy **(A2)** and $T_{0,k} \to \infty$. Under the absence of contention and the Assumption **(A1)**, the stochastic process $\{\hat{\lambda}_k\}_k$ evolving by (21) converges to λ^* for $k \to \infty$ P-almost surely.

The formal proof of Theorem 1 can be found in the appendix. The reasoning can be summarized as follows. With the recurrence result of Lemma 1, we can suppose that the process $\{\hat{\lambda}_k\}_k$ visits any neighborhood of λ^* infinitely many times. By constructing a continuous-time interpolation of $\{\hat{\lambda}_k\}_k$ related to the ODE in (17), it is shown that the process can exit any neighborhood of λ^* only finitely many times. Hence, we can conclude almost sure convergence to λ^*.

F. Stability

Until now, stability has been a minor issue, as no contention among subsystems was considered due to the relaxed problem formulation that made the problem tractable. In this section, we focus on stability properties of the aggregate system with the adaptive sample-based event-triggered system introduced in Sec. III-D in the presence of contention. We consider the adaptive sample-based algorithm (21) with constant step size β and constant window length T_0. Under this assumption, the resulting stochastic process can be viewed as a T_0-sampled time-homogeneous Markov chain. The stability notion used here is given by Harris recurrence. To prove Harris recurrence for time-homogeneous Markov chains with an uncountable state space, we use the notion of *petite* sets that take the form of compact sets in the underlying system and show that we can construct such a petite set that is Harris recurrent [43].

In order to simplify the following analysis, we add the following technical assumption.

(A3) The function $M^i(\lambda)$ introduced in Assumption **(A1)** grows asymptotically at most linear in λ, i.e., $M_i \in \mathcal{O}(\lambda)$.

This does not put severe restrictions on the analysis as it is possible to weaken Assumption **(A3)** to higher growth rates, while proving Theorem 3 follows along the same lines involving higher order terms.

The following theorem states conditions that guarantee ergodicity for the aggregated system for a static price λ.

Theorem 2 ([5]): Let λ be fixed and let $\gamma^{i,*}$ be the control law given by (10) and $\pi^{i,\lambda}$ be a feasible scheduling law which satisfies Assumption **(A1)**. If

$$\frac{c}{N} > 1 - \frac{1}{\|A^i\|_2^2} \tag{25}$$

is satisfied for all $i \in \{1,\ldots,N\}$, then the Markov chain $\{e_k\}_k$ is ergodic.
The proof of the above theorem uses drift criteria and the aperiodic ergodic theorem that can be found in [43] in order to derive the stability condition in (25) that relates the bandwidth limitations with the process dynamics of each subsystem. Theorem 2 will be helpful in the proof of the following theorem. The next statement gives us a means to analyze the stability of the stochastic process $\{e_k, \hat{\lambda}_k\}_k$ in terms of recurrence. The main idea is to find a sufficiently large window length T_0, such that a time-scale separation can be established between the dynamics within the subsystems and the dynamics of $\hat{\lambda}_k$.

Theorem 3: Let $\beta_k = \beta$ and $T_{0,k} = T_0$ be constant. If Assumptions **(A1)**, **(A3)**, and (25) hold, then there exists a sufficiently large $\bar{T}_0$, such that the T_0-sampled Markov chain $\{e_k, \hat{\lambda}_k\}_{k\in\mathcal{T}}$ with $T_0 \geq \bar{T}_0$ is a Harris recurrent Markov chain.

Proof: The sampled Markov chain evolves by (21) and the evolution of the estimation error e_k^i of subsystem i is given by

$$\begin{aligned}e_{k+T_0}^i &= (1 - q_k^i \pi^{i,\hat{\lambda}_k}(e_k^i)) \prod_{l=k+1}^{k+T_0-1} (1 - q_l^i \pi^{i,\hat{\lambda}_k}(e_l^i))A^i e_{k+1}^i + \\ &\quad + \sum_{l=k}^{k+T_0-1} \prod_{n=k+1}^{k+T_0-1-l} (q_n^i \pi^{i,\hat{\lambda}_k}(e_n^i))A^{k+T_0-1-l} w_l^i.\end{aligned}$$

The subsequent stability analysis is based on drift conditions developed for Markov chains with uncountable state spaces, see [43]. First, it can be seen that the chain is ψ-irreducible.

This is because of the absolute continuity of the Gaussian noise w_k^i and the fact that (21) can be viewed as a random walk on the half line with a probability of a negative drift greater than zero, see Proposition 4.3.1 in [43]. The drift operator Δ is defined as

$$\Delta V(e_k, \hat{\lambda}_k) = \mathsf{E}[V(e_{k+T_0}, \hat{\lambda}_{k+T_0})|e_k, \hat{\lambda}_k] - V(e_k, \lambda_k),$$

where V is a map from $\mathbb{R}^{n_1+\cdots+n_N} \times \mathbb{R}_{\geq 0}$ to $\mathbb{R}_{\geq 0}$.

We consider the following Lyapunov candidate

$$V(e_k, \hat{\lambda}_k) = b_1 \sum_{i=1}^{N} \|e_k^i\|_2^2 + b_2 \hat{\lambda}_k^3 \tag{26}$$

with $b_1, b_2 > 0$. It follows immediately that the drift for this choice of V is bounded within any compact set. Based on ψ-irreducibility of the Markov chain, we rely on the following drift criterion proposed in [43]. If the condition

$$\Delta V(e_k, \hat{\lambda}_k) \leq -\epsilon, \quad (e_k, \lambda_k) \in \mathbb{R}^{n_1+\cdots+n_N} \backslash \mathcal{D}, \tag{27}$$

where $\epsilon > 0$ and $\mathcal{D}$ is a compact set, is satisfied, then the T_0-sampled Markov chain $\{e_k, \hat{\lambda}_k\}_{\{k \in \{0, T_0, 2T_0, \ldots\}\}}$ is Harris recurrent. Due to linearity of the conditional expectation, we can split the drift into $N+1$ contributing terms given by

$$\begin{aligned}
&\Delta^i = \mathsf{E}[\|e_{k+T_0}^i\|_2^2 | e_k, \hat{\lambda}_k] - \|e_k^i\|_2^2, i \in \{1, \ldots, N\}, \\
&\Delta^\lambda = \mathsf{E}[\hat{\lambda}_{k+T_0}^3 | e_k, \hat{\lambda}_k] - \hat{\lambda}_k^3.
\end{aligned}$$

In a first step, fix a price $\hat{\lambda}_k$ and with it fix the individual bounds M^i in Assumption **(A1)**. Subsequently, we focus on determining upper bounds on Δ^i.

The conditional expectation in Δ^i can be rewritten as

$$\begin{aligned}
\mathsf{E}[\|e_{k+T_0}^i\|_2^2 | e_k, \hat{\lambda}_k] &= \mathsf{E}[\mathsf{E}[\|e_{k+T_0}^i\|_2^2 | e_{k+T_0-1}, e_k, \hat{\lambda}_k] | e_k, \hat{\lambda}_k] \\
&= \mathsf{E}[\mathsf{E}[\|e_{k+T_0}^i\|_2^2 | e_{k+T_0-1}^i, \hat{\lambda}_k] | e_k, \hat{\lambda}_k],
\end{aligned}$$

where the first equality is due to the tower property of the conditional expectation and the second because of the Markov property of e_k between the updates of $\hat{\lambda}_k$. The statistical independence of w_k^i with respect to q_k^i and e_k^i and the fact that $w_k^i \sim \mathcal{N}(0, C^i)$ allows the following simplification.

$$\begin{aligned}
&\mathsf{E}[\|e_{k+T_0}^i\|_2^2 | e_{k+T_0-1}^i, \hat{\lambda}_k] \leq \\
&\leq \mathsf{E}[1 - q_k^i | e_k, \hat{\lambda}_k] \|A^i\|_2^2 \|e_{k+T_0-1}^i\|_2^2 + \operatorname{tr}[C^i].
\end{aligned}$$

Therefore, we have the following upper bounds

$$\mathsf{E}[\|e_{k+T_0}^i\|_2^2 | e_k, \hat{\lambda}_k] \leq \begin{cases}
\||A^i\|_2^2 |(M^i)^2 + \operatorname{tr}[C^i] \\
\qquad\qquad \text{for } \|e_{k+T_0-1}^i\|_2 \leq M^i \\
(1 - \frac{c}{N}) \|A^i\|_2^2 \, \mathsf{E}[\|e_{k+T_0-1}^i\|_2^2 | e_k, \hat{\lambda}_k] \\
+ \operatorname{tr}[C^i] \qquad \text{for } \|e_{k+T_0-1}^i\|_2 > M^i.
\end{cases}$$

In the second bound, we have used that $\mathsf{E}[1 - q_k^i | e_k, \hat{\lambda}_k]$ describes the probability that a request of subsystem i is blocked and is upper bounded by $1 - \frac{c}{N}$ because of (2). For notational convenience, let $\alpha = (1 - \frac{c}{N}) \|A^i\|_2^2$. Proceeding inductively, we obtain the following $T_0 + 1$ bounds on the drift Δ^i as

$$\Delta^i \leq \alpha^{t_0} \||A^i\|_2^2 |(M^i)^2 + \left(\sum_{n=0}^{t_0} \alpha^n \right) \operatorname{tr}[C^i] - \|e_k^i\|_2^2, \tag{28}$$

$$0 \leq t_0 \leq T_0 - 1, e_k^i \in \mathbb{R}_i^n,$$

$$\Delta^i \leq (\alpha^{T_0} - 1) \|e_k^i\|_2^2 + \left(\sum_{n=0}^{T_0 - 1} \alpha^n \right) \operatorname{tr}[C^i], \qquad \|e_k^i\|_2 > M^i. \tag{29}$$

The condition in (25) guarantees that we can find an appropriate ϵ and a sufficiently large compact set $\mathcal{E}$ such that the drift term $\sum_i \Delta^i \leq -h$ all $e_k \notin \mathcal{E}$, where h can be made arbitrarily large.

For the subsequent analysis, it should be noted that the drift term $\sum_i \Delta^i$ can be uniformly bounded from above as a function of M^i for a fixed $\hat{\lambda}_k$. As a next step, we aim at finding an upper bound on the drift Δ^λ. We have from (21)

$$\begin{aligned}
\hat{\lambda}_{k+T_0}^3 &= \left([\hat{\lambda}_k + \beta(\hat{y}_k - c)]_+ \right)^3 \\
&\leq \left| \hat{\lambda}_k + \beta(\hat{y}_k - c) \right|^3 \\
&= \hat{\lambda}_k^3 + 3\hat{\lambda}_k^2 \beta(\hat{y}_k - c) + 3\hat{\lambda}_k \beta^2 (\hat{y}_k - c)^2 + \beta^3 |\hat{y}_k - c|^3 \\
&\leq \hat{\lambda}_k^3 + 3\hat{\lambda}_k^2 \beta(\hat{y}_k - c) + 3\hat{\lambda}_k \beta^2 N^2 + \beta^3 N^3.
\end{aligned}$$

The first inequality is because $[\cdot]_+$ is non-expansive [42]. Therefore, the drift term Δ^λ can be bounded by

$$\Delta^\lambda \leq 3\hat{\lambda}_k^2 \beta \, \mathsf{E}[(\hat{y}_k - c) | e_k, \hat{\lambda}_k] + 3\hat{\lambda}_k \beta^2 N^2 + \beta^3 N^3. \tag{30}$$

What remains to be analyzed is the estimation of the gradient given by $\mathsf{E}[(\hat{y}_k - c) | e_k, \hat{\lambda}_k]$. Because of the ergodicity of the process $\{e_k\}$ for a fixed $\hat{\lambda}_k$ due to Theorem 2, the empirical mean of the total request rate converges to a biased estimate of y_k, i.e, the empirical request rate is given by

$$\lim_{T_0 \to \infty} \frac{1}{T_0} \mathsf{E}[\sum_{l=k}^{k+T_0-1} \delta_l^i | e_k, \hat{\lambda}_k] = \eta^i r_k^i, \quad \mathsf{P}\text{-a.s.},$$

where $\eta^i \geq 0$ denotes the deviation from the request rate r_k^i resulting in the absence of contention among subsystems. In the following, we give an upper bound on η^i. For that reason we define ρ_n^i that counts the number of subsequent requests of subsystem i until successful transmission and is then reset to 0 again. Formally, we define the increasing sequences

$$\begin{aligned}
&\{t_{1,n}\}_n = \{k | q_k^i \delta_k^i = 1\}, \\
&\quad t_{2,n} = \min(k | \delta_k^i = 1 \wedge k \in (t_n, t_{n+1}]), n \geq 1
\end{aligned}$$

with $t_{2,0} = 0$. Then by setting $\rho_n^i = \sum_{k=t_{2,n}}^{t_{1,n}} \delta_k^i$, we have

$$\begin{aligned}
\lim_{T_0\to\infty} \frac{1}{T_0} \mathsf{E}[\sum_{l=k}^{k+T_0-1} \delta_l^i | e_k, \hat{\lambda}_k] &= \lim_{T_0\to\infty} \frac{1}{T_0} \mathsf{E}[\sum_n \rho_n^i | e_k, \hat{\lambda}_k] \\
&= \lim_{T_0\to\infty} \frac{1}{T_0} \mathsf{E}[\sum_n \mathsf{E}[\rho_n^i | e_k, \hat{\lambda}_k] | e_k, \hat{\lambda}_k] \\
&\leq \lim_{T_0\to\infty} \frac{1}{T_0} \mathsf{E}[\sum_n \frac{N}{c} | e_k, \hat{\lambda}_k] \\
&= r_k^i \frac{N}{c}
\end{aligned}$$

where the last inequality is due to the fact that the probability that a request of subsystem i is granted is lower bounded by $\frac{c}{N}$ because of (2). The last equality is because the number of elements of the sum is related to the delayed renewal process given by the requests assuming no contention whose rate is r_k^i. Therefore, we have $\eta^i \leq \frac{N}{c}$. This implies for the empirical total request rate $\hat{y}_k$

$$\lim_{T_0\to\infty} \mathsf{E}[\hat{y}_k | e_k, \hat{\lambda}_k] = \eta y_k, \quad \eta \leq \frac{N}{c}, \quad \mathsf{P}\text{-a.s.}$$

As $\hat{y}_k \leq N$ for every $\omega \in \Omega$, almost sure convergence implies $\mathcal{L}_1$ convergence [44], i.e.,

$$\lim_{T_0\to\infty} \mathsf{E}[| \mathsf{E}[\hat{y}_k | e_k, \hat{\lambda}_k] - \eta y_k | | e_k, \hat{\lambda}_k] = 0.$$

Continuing our analysis of Δ^λ from (30), we obtain

$$\begin{aligned}
\Delta^\lambda \leq & 3\hat{\lambda}_k^2 \beta \left((\eta y_k - c) + \mathsf{E}[| \mathsf{E}[\hat{y}_k | e_k, \hat{\lambda}_k] - \eta y_k | | e_k, \hat{\lambda}_k] \right) + \\
& + 3\hat{\lambda}_k \beta^2 N^2 + \beta^3 N^3.
\end{aligned}$$

By choosing $\hat{\lambda}_k$ and T_0 sufficiently large, y_k and $\mathsf{E}[| \mathsf{E}[\hat{y}_k | e_k, \hat{\lambda}_k] - \eta y_k | | e_k, \hat{\lambda}_k]$ become arbitrarily small. Due to Assumption **(A3)** and $\Delta^i \leq \mathcal{O}((M^i)^2)$, we conclude that by choosing b_1, b_2 accordingly, we can show that there exists a $\bar{\lambda} > 0$ such that for any $\hat{\lambda}_k > \bar{\lambda}$ and any e_k, we have $\Delta V(e_k, \hat{\lambda}_k) \leq -\epsilon$, where $\epsilon > 0$. On the other hand, for $\hat{\lambda}_k \leq \bar{\lambda}$, we can find a sufficiently large compact set $\mathcal{B}$ such that for every $e_k \notin \mathcal{B}$, we also have $\Delta V(e_k, \hat{\lambda}_k) \leq -\epsilon$. By setting $\mathcal{D} = \mathcal{B} \times [0, \bar{\lambda}]$, the drift condition in (27) is satisfied which concludes the proof. ∎

Remark 2: By the definition of Harris recurrence, we can conclude that Harris recurrence of the sampled Markov chain also implies that the stochastic process $\{e_k, \hat{\lambda}_k\}_k$ is recurrent.

Remark 3: An explicit determination of a sufficiently large $\bar{T}_0$ that yields stability can be circumvented by letting $T_{0,k}$ grow towards infinity for $k \to \infty$, as assumed in the previous subsection. This is possible because the upper bounds on the drifts determined in the above proof can be chosen to be uniform in $T_0 \geq \bar{T}_0$ when taking into account the hypothesis of Theorem 3.

G. Implementation and discussion

The implementation of the overall system is accomplished in two phases. In the first phase, which is performed before execution and locally in each subsystem, the optimal controller $\gamma^{i,*}$ given by (10) is calculated and the mapping from price λ to the optimal event-trigger $\pi^{i,\lambda}$ by solving (14) through value iteration. In the second phase, the network manager adjusts the price accordingly to the empirical total transmission rate $\hat{y}_k$, which serves as an estimate to approximate the gradient $\frac{\partial g}{\partial \lambda}$. Unlike broadcasting the price $\hat{\lambda}_k$ by a network manager, a complete decentralized adaptation mechanism can be realized when each subsystem is able to sense the amount of requests directly. Then, the calculation of $\hat{\lambda}_k$ can be performed locally in each subsystem.

In contrast to a time-triggered scheduling mechanism, which needs a global exhaustive search at runtime to find the optimal scheduling sequence, the event-triggered scheme allows therefore a tractable implementation. Apart from the fact that the adaptation mechanism enables the distributed architecture, the local event-triggers are capable to adjust their thresholds according to runtime changes that are often found in real applications. These are for example given by adding or removing control loops during execution, changes in the resource constraint, or changes in the local system parameters. In such cases, the step size and window length of the adaptive sample-based algorithm either need to constant or must be reset in the occasion of a change in order to react appropriately.

What enables the tractable design of event-triggered schedulers in which great parts of the optimization can be run locally relies on the fact that the subsystems are physically decoupled. A first step towards the incorporation of physical coupling is to model the interaction between subsystems as a joint source of disturbance assuming a weak coupling that may not destabilize the system. This modeling retains the possibility to apply the obtained methods for decoupled subsystems.

IV. NUMERICAL RESULTS

Suppose the system comprises of N subsystems with two differing system parameters $A^1 = 1.25$, $B^1 = 1$, $Q_x^1 = 1$, $Q_u^1 = 0$ and $A^2 = 0.75$, $B^2 = 1$, $Q_x^2 = 1$, $Q_u^2 = 0$. The systems start with $x_0^1 = \cdots = x_0^N = 0$ and the system noise is given by $w_k^i \sim \mathcal{N}(0, 1)$. The subsystems appear in equal proportions, while the communication constraint is set to $c = N/2$. The optimal control gain in (10) is given by $L^i = A^i$ for each subsystem $i \in \{1, \ldots, N\}$, which corresponds to a deadbeat control strategy. The optimal scheduling laws for a fixed λ of each subsystem is a symmetric threshold policy defined in (15) with threshold d^i, $i \in \{1, \ldots, N\}$. The optimal threshold for various fixed Lagrange multipliers λ is obtained by value iteration for the average cost problem in (14). Figure 3 shows the mapping from λ to the optimal thresholds of both subsystems. It should be noted that the determination of the mapping shown in Fig. 3 can be performed offline and locally for each subsystem. Therefore, the computationally intensive part for the determination of the optimal threshold can be accomplished before runtime.

Remark 4: It can be seen that the growth rate of d^i, $i \in \{1, 2\}$ is at most linear in Fig. 3 which supports Assumption **(A3)**.

Assume an increasing window length $T_{0,\cdot} = \{2, 4, 6, \ldots\}$ and a decreasing step size $\{\beta_k\}_{k\in\mathcal{T}} = \{2, 1, \frac{2}{3}, \frac{2}{4}, \frac{2}{5}, \ldots\}$ for the adaptive event-triggered scheduler with the adaptation mechanism for $\hat{\lambda}_k$ given by (21). Note that β_k satisfies **(A2)**.

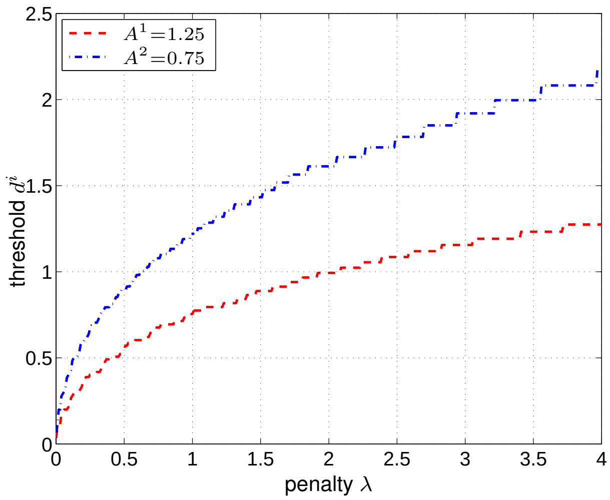

Fig. 3. Mapping from price λ to the optimal symmetric scheduling law described by the threshold d^i for subsystem class 1 with $A^1 = 1.25$ and class 2 with $A^2 = 0.75$.

Figure 4 compares the social cost for the adaptive event-triggered scheduler, the static event-triggered scheduler, the time-triggered case, and the optimal solution without contention, which corresponds to the minimum of the relaxed problem in (8). Different N are considered, where it is assumed that the ratio between subsystem classes remains equal, and the capacity c grows with the number of subsystems, i.e., $c/N = 0.5$. The static event-triggered scheme determines the optimal λ for the relaxed problem setting beforehand by the iterative method in (19) and takes the stationary event-triggered schedulers $\pi^{i,\lambda}$, $i = \{1, 2\}$. The time-triggered scheduler is given by $\{\delta_k^1\}_k = \{1, 1, 0, 1, 1, 0, \dots\}$ and $\{\delta_k^2\}_k = \{0, 0, 1, 0, 0, 1, \dots\}$, where the brute-force search for the optimal time-triggered scheme is constrained to have a period of 3 in order to remain numerical tractable [18]. It can be observed that the event-triggered schemes outperform the time-triggered scheme. The social cost of the adaptive scheme is slightly below the static event-triggered scheme. The cost obtained without contention takes a value of 1.07 and can be considered as a lower bound for the optimal scheduling policy with contention. This lower bound is slowly approached by the event-triggered schemes for increasing N. The fact that the social cost of the event-triggered scheme approaches the relaxed problem setting is also observed in [18]. As already mentioned in Sec. III-A, it can be reasoned by its convergence to a deterministic flow limit of the aggregate system when N approaches infinity.

Figure 5 compares the obtained price $\hat{\lambda}_k$ in the limit in the presence and in the absence of contention for the adaptive event-triggered scheduler, and with the optimal Lagrange multiplier λ^* for the relaxed problem setting in (8). Obviously, the optimal λ^* taking a value of 0.48 remains constant for different N, as $c/N = 0.5$. We observe that in the case of the adaptive event-triggered scheduler without contention, the price $\hat{\lambda}_k$ converges to the optimal λ^*. In the presence of contention, the price $\hat{\lambda}_k$ converges to a value strictly above the optimal price λ^*. It can also be observed that its deviation from λ^* becomes smaller when N increases. This can be interpreted by the fact that the average constraint in the relaxed problem (8) yields a better approximation for the original problem as N approaches infinity.

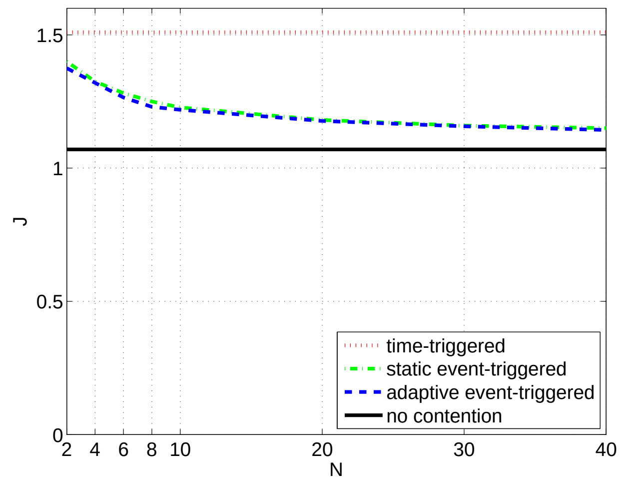

Fig. 4. Performance comparison for various number of subsystems and $c/N = 0.5$.

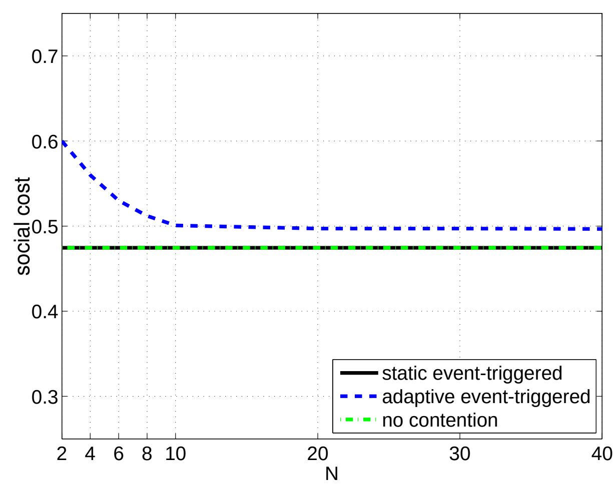

Fig. 5. Limit price $\hat{\lambda}_k$ for $k \to \infty$ for various number of subsystems and $c/N = 0.5$.

Figures 6, 7, and 8 show the sample-path behavior of the adaptive algorithm over a time-horizon of 2000 for the communication penalty $\hat{\lambda}_k$ and the normalized empirical total request rate $\tilde{y}_k$ for the case without contention, $N = 2$, and $N = 20$, respectively. The normalized request rate $\tilde{y}_k$ is defined as

$$\tilde{y}_k = \frac{1}{kc} \sum_{\ell=0}^{k-1} \sum_{i=1}^{N} \delta_\ell^i.$$

Figures 7 and 8 show in addition the normalized empirical throughput $\tilde{s}_k$ defined as

$$\tilde{s}_k = \frac{1}{kc} \sum_{\ell=0}^{k-1} \sum_{i=1}^{N} q_\ell^i \delta_\ell^i.$$

In the absence of contention, we observe that the price $\hat{\lambda}_k$ converges to $\lambda^* = 0.48$, while the normalized total request rate converges to 1 which is in accordance with Theorem 1. The normalized request rate and throughput converge to 1.05 and 0.8 for $N = 2$ and they converge towards 1.01 and 0.92 for $N = 20$.

As a concluding remark, it is interesting to observe that although the adaptive event-triggered scheduling schemes have a significant gap to the maximal throughput, they outperform the time-triggered scheme, which is throughput optimal, significantly with respect to the control performance as shown in Fig. 4.

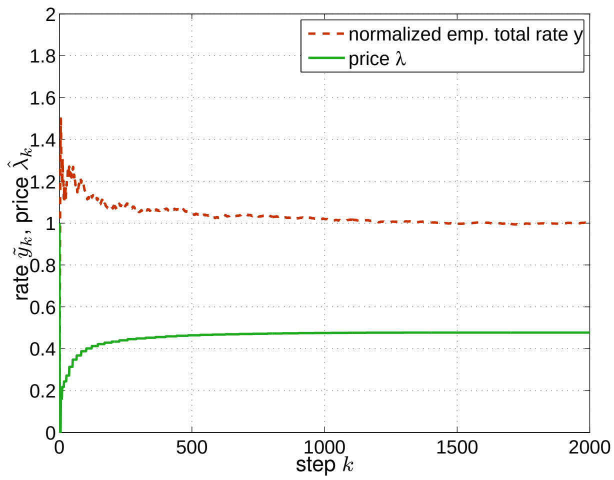

Fig. 6. Sample path of the normalized empirical total request rate $\hat{y}_k$ and of the price $\hat{\lambda}_k$.

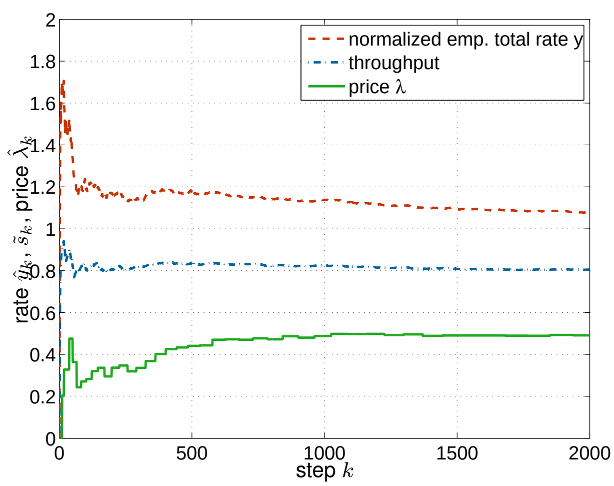

Fig. 7. Sample path of the normalized empirical total request rate $\hat{y}_k$, normalized empirical throughput $\tilde{s}_k$, and of the price $\hat{\lambda}_k$.

V. CONCLUSIONS

This article develops a novel framework for the synthesis of distributed event-triggered control systems that share a common resource using a price-based adaptation mechanism. It demonstrates the capability of adaptive event-triggered scheduling for the distributed design in resource-constrained

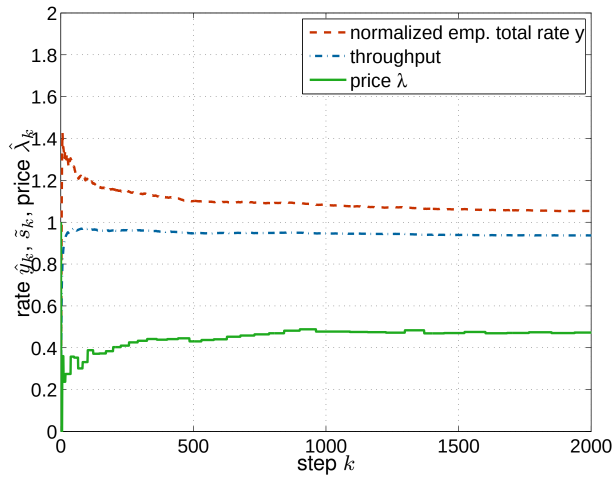

Fig. 8. Sample path of the normalized empirical total request rate $\hat{y}_k$, normalized empirical throughput $\tilde{s}_k$,and of the price $\hat{\lambda}_k$.

cyber-physical systems. By considering a previously developed relaxation approach for the event-trigger design, it is shown that convexity properties of the relaxed problem enable the application of dual formulations related to network utility maximization. The distributed design is realized by an adaptive event-trigger that adjusts its threshold according to the estimated price for the resource. The use of a time-scale separation technique allows to establish stability and convergence properties of the aggregate adaptive event-triggered scheme in terms of recurrence.

Future work includes the consideration of physical coupling among subsystems with multiple resource constraints and application-specific communication models.

APPENDIX

The proof of convergence relies on the ODE approach developed in the field of stochastic approximation and is built upon ideas from Section 5.4 of [41]. Before proving Theorem 1, we give the following definitions for the local analysis of the process $\{\hat{\lambda}_k\}_{k\in\mathcal{T}}$.

With slight abuse of notation, we redefine the index in the evolution of $\{\hat{\lambda}_k\}_{k\in\mathcal{T}}$ by

$$\hat{\lambda}_{n+1} = [\hat{\lambda}_n + \beta_n(\hat{y}_n - c)]^+, \tag{31}$$

where at each time n corresponds to a update time $k \in \mathcal{T}$.

Because of the recurrence property due to Lemma 1, the ϵ_1-ball with center λ^* defined as $\mathcal{B}_{\epsilon_1}(\lambda^*)$ is visited infinitely many times with probability 1. Moreover, we consider another neighborhood $\mathcal{B}_{\epsilon_2}(\lambda^*)$ with $\epsilon_2 > 2\epsilon_1$. As $\lambda^* > 0$, we further assume that the closed ball $\mathcal{B}_{\epsilon_2}(\lambda^*)$ does not intersect with the origin. Subsequently, we consider the family of sequences that start within $\mathcal{B}_{\epsilon_1}(\lambda^*)$ until they leave $\mathcal{B}_{\epsilon_2}(\lambda^*)$. As the gradient $\hat{y}_n - c$ is bounded within $[-N, N]$ and we are interested in the limiting behavior for large n, the projection operator in (21) can be omitted. Furthermore, due to the fact that $\mathcal{B}_{\epsilon_2}(\lambda^*)$ is a positively invariant set of the ODE (17), the projection operator can be discarded from the subsequent analysis. Then, we rewrite the difference equation in (21) as

$$\hat{\lambda}_{n+1} = \hat{\lambda}_n + \beta_n(y_n - c) + \beta_n \xi_n \tag{32}$$

where ξ_n is the error when estimating y_n by the empirical mean $\hat{y}_n$, i.e.,

$$\xi_n = \hat{y}_n - y_n. \tag{33}$$

Because of ergodicity of the overall system without contention assuming **(A1)**, we have $\lim_{n\to\infty} \xi_n = 0$ P-almost surely. In order to give a continuous-time interpolation of the discrete-time process (32) evolving within $\mathcal{B}_{\epsilon_2}(\lambda^*)$, we define the data points of the interpolation at times

$$t_n = \sum_{\ell=1}^{n-1} \beta_n.$$

with $t_0 = 0$. For $t \geq 0$, define $m(t)$ to be the unique index $n = m(t)$ such that $t_n \leq t < t_{n+1}$. Define the continuous-time interpolation $\hat{\lambda}^0$ of $\{\lambda_n\}_n$ by

$$\hat{\lambda}^0(t) = \hat{\lambda}_n, \quad t \in (t_n, t_{n+1})$$

and a shifted version of the interpolation by

$$\hat{\lambda}^n(t) = \hat{\lambda}^0(t_n + t).$$

We are now ready to prove Theorem 1 by the so-called ODE approach [41].

Proof of Theorem 1: Let Ω_0 be the null set on which ξ_n defined in 33 does not converge to zero or on which the process does not return to $\mathcal{B}_{\epsilon_1}(\lambda^*)$ infinitely often. Fix $\omega \in \Omega\backslash\Omega_0$.

Suppose that there are infinitely many escapes of $\{\hat{\lambda}_n(\omega)\}_n$ from $\mathcal{B}_{\epsilon_1}(\lambda^*)$ to $\mathcal{B}^c_{\epsilon_2}(\lambda^*)$. Then, there is an increasing sequence $n_k(\omega) \to \infty$ such that n_k is the last index at which $\hat{\lambda}_{n_k} \in \mathcal{B}^c_{2\epsilon_1}(\lambda^*)$ before leaving $\mathcal{B}^c_{\epsilon_2}(\lambda^*)$.

For $s \geq 0$, we have the following piecewise constant interpolation of the discrete-time process given by (32)

$$\hat{\lambda}^{n_k}(s) = \hat{\lambda}_{n_k}(\omega) + \sum_{l=n_k}^{m(t_{n_k}+s)-1} \beta_l(y_l - c) + B^{n_k}(s)$$

with $B^{n_k}(s) = \sum_{l=n_k}^{m(t_{n_k}+s)-1} \beta_l \xi_l$. Due to the boundedness of the stochastic gradient $\hat{y}_n - c$, we have $\beta_n(\hat{y}_n - c) \to 0$, which implies that the sequence $\{\hat{\lambda}_{n_k}\}_k$ converges to a point on the boundary $\partial\mathcal{B}^c_{2\epsilon_1}(\lambda^*)$. It also implies that there is a $T > 0$ such that we have $\hat{\lambda}^{n_k}(\omega, t) \in \mathcal{B}_{\epsilon_2}(\lambda^*)$, $t \leq T$ for sufficiently large k. As $\xi_n(\omega) \to 0$, we also have $B^{n_k}(\omega, t) \to 0$ for all $t \leq T$. Due to these results, it can be stated that the family of sequences $\{\hat{\lambda}^{n_k}(\omega, \cdot), 0 \leq t \leq T\}$ is equicontinuous and bounded. By the Arzelà-Ascoli theorem [45], we conclude that there exists a convergent subsequence of $\{\hat{\lambda}^{n_k}(\omega, \cdot), 0 \leq t \leq T\}$.

Let $\lambda(\omega, \cdot)$ be the limit of such convergent subsequence and let

$$\rho^{n_k}(\omega, t) = \int_0^t (y(s) - c) ds - \sum_{l=n_k}^{m(t_{n_k}+t)-1} \beta_l(y_l - c), \quad t \leq T.$$

Because the sequences $\rho^n(\omega, t)$ and $B^n(\omega, t)$ vanish over finite time intervals for $n \to \infty$, we conclude that the convergent sequence $\lambda(\omega, \cdot)$ satisfies the ODE given by (17) for $t \leq T$ with initial condition $\lambda(\omega, 0) \in \partial\mathcal{B}^c_{2\epsilon_1}(\lambda^*)$ and $|\hat{\lambda}(\omega, t) - \lambda^*| \geq 2\epsilon_1$ for $t \leq T$.

However, the right-hand side of the ODE in (17) points in the interior of $\mathcal{B}^c_{2\epsilon_1}(\lambda^*)$) at is boundary $\partial\mathcal{B}^c_{2\epsilon_1}(\lambda^*)$. Hence, $\lambda(\omega, \cdot)$ converges towards λ^*, which contradicts with the previous statement.

Therefore, our initial assertion does not hold and there are only finitely many excursions from $\mathcal{B}_{2\epsilon_1}(\lambda^*)$. As ϵ_1 can be chosen arbitrarily small, almost sure convergence to λ^* is guaranteed. This concludes the proof. ∎

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