Inverse Kinematics with Multiple Tasks and Multiple Task Definitions

Sang-ik An and Dongheui Lee

Abstract— **We discuss inverse kinematics methods focusing on multiple tasks and multiple task definitions.**

I. INTRODUCTION

A robot in the human friendly environment is required to execute tasks that are usually performed by human. A significant ability of humans is the variational dexterity, so human can assign more degrees of freedom (DOFs) of the body to the more important task, while executing multiple tasks on the same time. Also, human can easily change tasks in the middle of executions to adapt to the dynamic variation of the environments. This ability allows human to have complex and sophisticated behavior and it is demanded to build a control method for the robot that resembles humans'.

II. MULTIPLE TASKS

In the robot kinematic control, a task can be defined as a tuple $T \triangleq (\dot{\mathbf{x}}, \dot{\mathbf{x}}_d)$ where $\dot{\mathbf{x}} = \mathbf{J}(\mathbf{q})\dot{\mathbf{q}} \in \mathbb{R}^m$ is the task velocity and $\dot{\mathbf{x}}_d(\mathbf{q}, t) \in \mathbb{R}^m$ is the desired velocity of $\dot{\mathbf{x}}$. If there are multiple tasks $T_1, \ldots, T_k$ with $T_i \triangleq (\dot{\mathbf{x}}_i, \dot{\mathbf{x}}_{d,i})$, $\dot{\mathbf{x}}_i = \mathbf{J}_i(\mathbf{q})\dot{\mathbf{q}} \in \mathbb{R}^{m_i}$, and $\dot{\mathbf{x}}_{d,i}(\mathbf{q}, t) \in \mathbb{R}^{m_i}$, we can consider priority relations between tasks, so an unprioritized accumulation of tasks, $(T_1, \ldots, T_k)$, has equal priority relations between tasks and a prioritized accumulation of tasks, $[T_1, \ldots, T_k]$, has totally ordered priority relations in which T_i has higher priority than T_j if $i < j$. The unprioritized inverse kinematics (UIK) is to find the joint velocity $\dot{\mathbf{q}} \in \mathbb{R}^n$ that minimizes the total task error: $\dot{\mathbf{q}}^* = \arg\min_{\dot{\mathbf{q}}} \sum_{i=1}^{k} \|\dot{\mathbf{x}}_{d,i} - \dot{\mathbf{x}}_i\|^2 = \mathbf{J}^\dagger \dot{\mathbf{x}}_d$ where $\mathbf{J} \triangleq \begin{bmatrix}\mathbf{J}_1^T & \cdots & \mathbf{J}_k^T\end{bmatrix}^T$, $\dot{\mathbf{x}}_d \triangleq \begin{bmatrix}\dot{\mathbf{x}}_{d1}^T & \cdots & \dot{\mathbf{x}}_{d,i}^T\end{bmatrix}^T$, and $\mathbf{J}^\dagger$ is the Moore-Penrose pseudoinverse of $\mathbf{J}$. The prioritized inverse kinematics (PIK) is to find the joint velocity $\dot{\mathbf{q}}$ that minimizes a task error on the condition that task errors of the higher priority tasks are not changed: $\dot{\mathbf{q}}^* = \dot{\mathbf{q}}_k^*$, $\dot{\mathbf{q}}_0^* = \mathbf{0}$, and $\dot{\mathbf{q}}_i^* = \arg\min_{\dot{\mathbf{q}}_i} \|\dot{\mathbf{x}}_{d,i} - \dot{\mathbf{x}}_i\|^2$ subject to $\mathbf{J}_{i-1}^{\cup}\dot{\mathbf{q}}_i = \mathbf{J}_{i-1}^{\cup}\dot{\mathbf{q}}_{i-1}^*$ where $\mathbf{J}_{i-1}^{\cup} \triangleq \begin{bmatrix}\mathbf{J}_1^T & \cdots & \mathbf{J}_{i-1}^T\end{bmatrix}^T$. Recently, we have reformulated PIK solutions in both recursive and closed forms by using QR decomposition (QRD) and Cholesky decomposition (CLD) in order to resolve the imperfect orthogonalization problem of conventional methods [1]:

$$\dot{\mathbf{q}} = \tilde{\mathbf{R}}^{-1}\dot{\mathbf{q}}_k,\ \dot{\mathbf{q}}_i = \dot{\mathbf{q}}_{i-1} + \hat{\mathbf{J}}_i^T \mathbf{C}_{ii}^\dagger(\dot{\mathbf{x}}_i - \mathbf{J}_i\dot{\mathbf{q}}_{i-1}),\ \dot{\mathbf{q}}_0 = \mathbf{0}$$
$$\dot{\mathbf{q}} = \tilde{\mathbf{R}}^{-1}\hat{\mathbf{J}}^T(\mathbf{I}_m + \mathbf{C}_D^{\ddagger}\mathbf{C}_L)^{-1}\mathbf{C}_D^{\ddagger}\dot{\mathbf{x}}$$

where $\mathbf{W} \triangleq \mathbf{J}^T\mathbf{J} + \delta^2\mathbf{I}_n = \tilde{\mathbf{R}}^T\tilde{\mathbf{R}}$ is the CLD, $\mathbf{J}_R \triangleq \mathbf{J}\tilde{\mathbf{R}}^{-1} = \mathbf{C}\hat{\mathbf{J}}$ is the reduced QRD of $\mathbf{J}_R^T$, $\mathbf{C}_{ij} \in \mathbb{R}^{m_i \times m_j}$ is the (i, j)-th block of $\mathbf{C}$, $\hat{\mathbf{J}}_i \in \mathbb{R}^{m_i \times n}$ is the i-th block of $\hat{\mathbf{J}}$, $\mathbf{C}_D \triangleq \mathrm{diag}(\mathbf{C}_{ii})$, $\mathbf{C}_D^{\ddagger} \triangleq \mathrm{diag}(\mathbf{C}_{ii}^{\dagger})$, and $\mathbf{C}_L \triangleq \mathbf{C} - \mathbf{C}_D$.

S. An and D. Lee are with the Chair of Automatic Control Engineering, Technische Universität München, D-80333 Munich, Germany. sangik.an@tum.de and dhlee@tum.de

III. MULTIPLE TASK DEFINITIONS

In a complicated task scenario, the definition of tasks can be dependent to the condition of the environment and a robot needs to change tasks in the middle of task executions. We define a basic task $T \triangleq (T_1, \ldots, T_k)$ as an unprioritized task of basic subtasks T_j that does not change its definition during whole operation time. Then, we can construct (induced) tasks $T^i \triangleq [T_1^i, \ldots, T_{k^i}^i]$ by accumulating a subset of basic subtasks with the priority relation between (induced) subtasks T_j^i. For every task T^i, we can always find a PIK solution $\dot{\mathbf{q}}^i = \mathrm{PIK}(T^i)$ by using the aforementioned PIK methods. If we introduce a null task $\emptyset$ along with $\dot{\mathbf{q}}_\emptyset = \mathrm{PIK}(\emptyset) \triangleq \mathbf{0}$, then we can define an (induced) task set $\mathcal{T} \triangleq \{ T^1, \ldots, T^l \}$ that contains all tasks considered and an inverse solution set $\mathcal{Q} \triangleq \{ \dot{\mathbf{q}}^1, \ldots, \dot{\mathbf{q}}^l \}$ that contains all PIK solutions of $\mathcal{T}$. The mapping $\mathrm{PIK} : \mathcal{T} \to \mathcal{Q}$ is surjective, so we can work with inverse solutions to find joint velocities that allow task transitions among $\mathcal{T}$, instead of directly working with tasks. Lately, we have proposed the task transition control (TTC) that provides smooth, arbitrary, and consecutive task transitions within $\mathcal{T}$ by using barycentric coordinates and linear dynamical systems [2]:

$$\dot{\mathbf{q}} = \sum_{i=1}^{l} w^i \dot{\mathbf{q}}^i, \quad \mathbf{w}^{(s+1)} = -\sum_{j=1}^{s} k_j \mathbf{w}^{(j)} + k_0(\mathbf{w}_d - \mathbf{w})$$

where $\mathbf{w} \triangleq [w^1 \ \cdots \ w^l]^T \in \mathbb{R}^l$, $\mathbf{w}^{(j)} \triangleq d^j\mathbf{w}/dt^j$, $s \in \mathbb{N}^{\geq 0}$, $\mathbf{w}_d(\mathbf{q}, t) \in \{\hat{\mathbf{e}}^1, \cdots, \hat{\mathbf{e}}^l\} \subset \mathbb{R}^l$, $\mathbf{w}(t_0) \in \{\mathbf{a} \in \mathbb{R}^n : \mathbf{1}^T\mathbf{a} = 1,\ \mathbf{a} \geq \mathbf{0}\}$, $\mathbf{w}^{(j)}(t_0) = \mathbf{0}$, $\mathbf{1} \triangleq [1 \ \cdots \ 1]^T \in \mathbb{R}^l$, $\{\hat{\mathbf{e}}^1, \cdots, \hat{\mathbf{e}}^l\}$ is a set of the standard basis in $\mathbb{R}^l$, and $\{k_0, \cdots, k_s\} \subset \mathbb{R}$ are stabilizing control gains that do not generate overshoots of the $(s+1)$-th order linear dynamical system.

IV. CONCLUSIONS

A general kinematic control framework that is capable of multiple tasks and multiple task definitions is proposed. The method is expected to be used to build a sophisticated robot behavior in the human friendly environment.

ACKNOWLEDGMENT

This work is supported partially by Technische Universität München - Institute for Advanced Study, funded by the German Excellence Initiative.

REFERENCES

[1] S.-i. An and D. Lee, "Prioritized Inverse Kinematics using QR and Cholesky Decompositions," in *IEEE International Conference on Robotics and Automation*, vol. 1, pp. 5062–5069, 2014.

[2] S.-i. An and D. Lee, "Prioritized Inverse Kinematics with Multiple Task Definitions," in *IEEE International Conference on Robotics and Automation*, pp. 1423–1430, 2015.