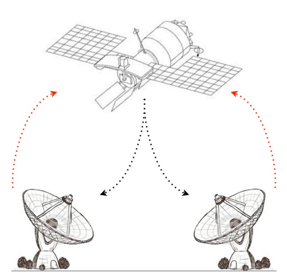
Broadcast Packet Erasure Channels with Feedback and Memory

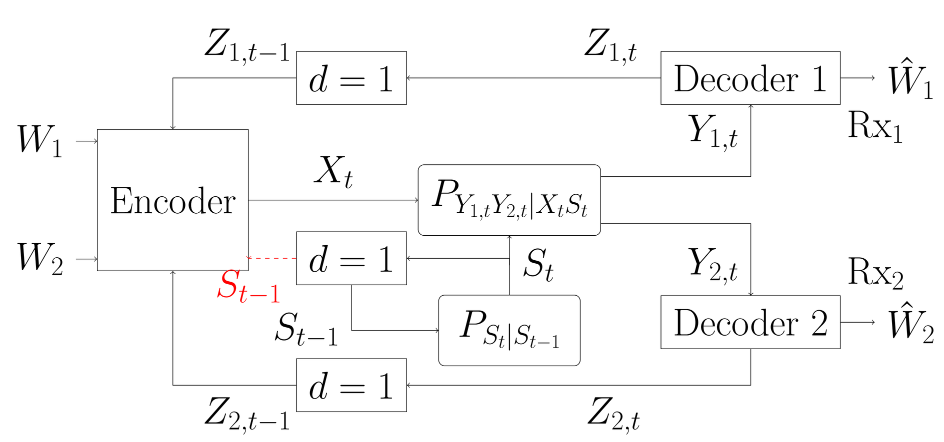
Michael Heindlmaier and Shirin Saeedi Bidokhti
Technical University of Munich, Stanford University

Motivation

- Two-User broadcast packet erasure channels (BPECs)
- Low cost (Ack/Nack) feedback
- Bursty nature of erasures in satellite communications



System Model

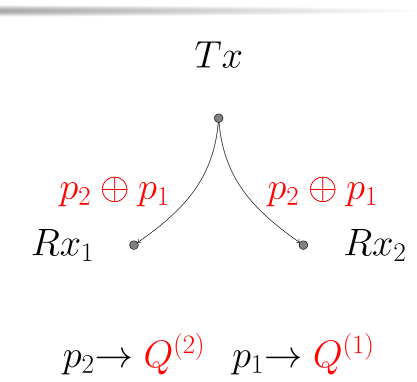


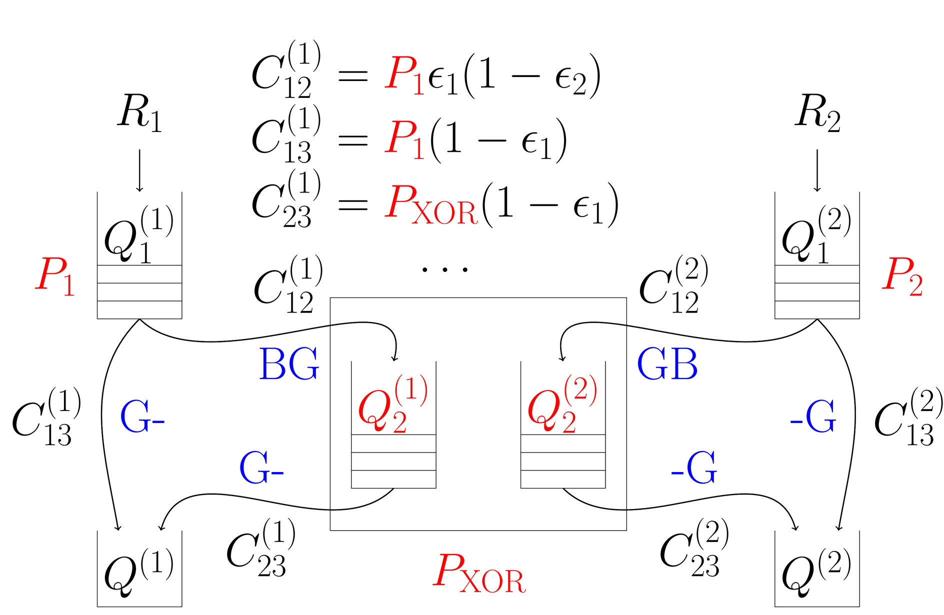
- W_1 message of rate R_1, W_2 message of rate R_2
- Received signals: $Y_{1,t}$, $Y_{2,t}$. $Y_{j,t}$ is either X_t, or completely erased
- Ack/Nack feedback after each transmission.
- S_t evolves according to a finite state machine with states $s \in \mathcal{S}$ (memory)

1. Channel state is known causally at the encoder (visible state).
2. Channel state is NOT known at the encoder (hidden state).

Memoryless BPEC with Feedback [4]

- A Probabilistic Framework





- A max-flow (equiv. min-cut) analysis

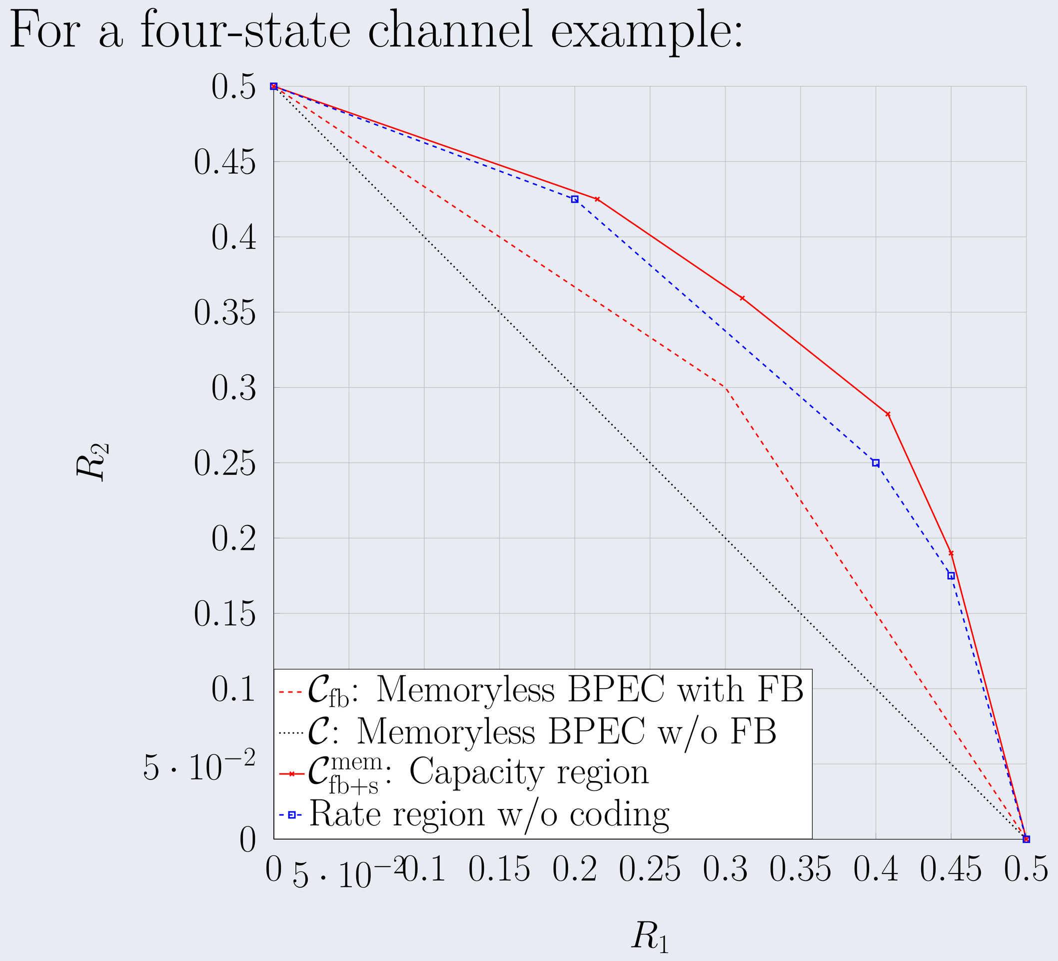
Capacity Region: Visible State [1][2]

The capacity region $\mathcal{C}_{\text{fb+s}}^{\text{mem}}$ of the two-user BPEC with feedback and visible state is the closure of rate pairs (R_1, R_2) for which there exist x_s, y_s, $s \in \mathcal{S}$ s.t.

$$
\begin{aligned}
&0 \le x_s \le 1 \quad \forall s \in \mathcal{S}\\
&0 \le y_s \le 1 \quad \forall s \in \mathcal{S}\\
&R_1 \le \sum_{s\in\mathcal{S}} \pi_s(1-\epsilon_1(s))x_s\\
&R_1 \le \sum_{s\in\mathcal{S}} \pi_s(1-\epsilon_{12}(s))(1-y_s)\\
&R_2 \le \sum_{s\in\mathcal{S}} \pi_s(1-\epsilon_2(s))y_s\\
&R_2 \le \sum_{s\in\mathcal{S}} \pi_s(1-\epsilon_{12}(s))(1-x_s).
\end{aligned}
$$

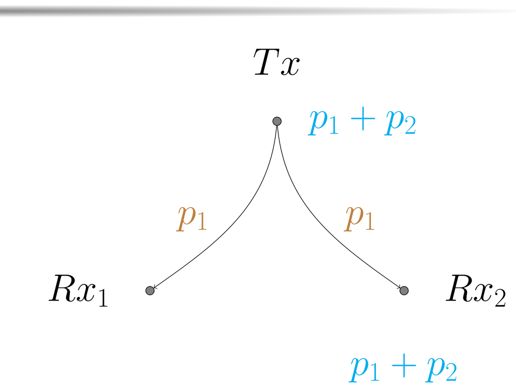
where $\epsilon_1(s)$, $\epsilon_2(s)$, and $\epsilon_{12}(s)$ are computed via distribution $P_{\underline{Z}_t|S_{t-1}}$

- For a four-state channel example:



Proactive Coding [2]

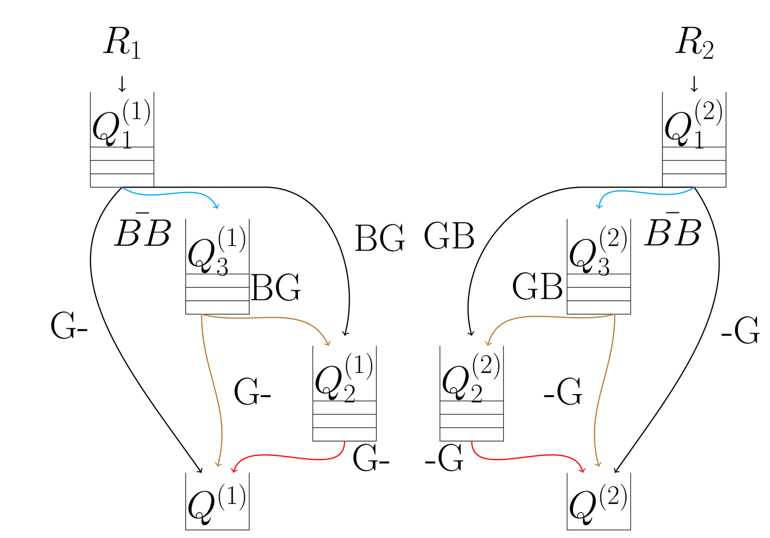
- poison $p_1 + p_2$ is sent.
- remedy $p_1 \rightarrow Q_3^{(1)}, Q_3^{(2)}$.
- Remedy packets are useful to both receivers.



An Optimal Probabilistic Scheme

1. Use feedback to update the queues
2.
   - w.p. $p(1|s)$ send an original packet for Rx$_1$ from $Q_1^{(1)}$
   - w.p. $p(2|s)$ send an original packet for Rx$_2$ from $Q_1^{(2)}$
   - w.p. $p(3|s)$ send a coded packet from $Q_2^{(1)}$ and $Q_2^{(2)}$
   - w.p. $p(4|s)$ send a poison packet from $Q_1^{(1)}$ and $Q_1^{(2)}$
   - w.p $p(5|s)$ send a remedy packet from $Q_3^{(1)}$ or $Q_3^{(2)}$

Achievable Rates



- Max-Flow Min-Cut analysis

Converse

- In previous works on BPEC with feedback:
  - Physically degrade the channels using a genie.
  - Feedback does not increase the capacity over physically degraded BCs [5].
  - The capacity region is known for degraded BPECs without feedback [6], [7].
- This technique is not directly applicable here.

Converse Proof from scratch:

$$
\begin{aligned}
nR_1 &\le I(W_1; Y_1^n)\\
&\le \sum_s \pi_s(1-\epsilon_1(s))\, \underline{I(U_{1,T}; X_T|T, S_{T-1}=s)}
\end{aligned}
$$

x_s

$$
\begin{aligned}
nR_2 &\le I(W_2; Y_1^n Y_2^n|W_1)\\
&\le \sum_s \pi_s(1-\epsilon_{12}(s)) I(U_{2,T}; X_T|U_{1,T}V_T T, S_{T-1}=s)
\end{aligned}
$$

where $U_{1,t} = (W_1 Y_1^{t-1} S^{t-1})$, $U_{2,t} = (W_2 Y_2^{t-1} S^{t-1})$, and $V_t = (Y_1^{t-1} Y_2^{t-1} S^{t-1})$.

Capacity Region: Hidden State [1]

The capacity region $\mathcal{C}_{\text{fb}}^{\text{mem}}$ of the two-user BPEC with feedback and hidden state is approximated by the closure of rate pairs (R_1, R_2) for which there exist variables $x(\underline{z}^L), y(\underline{z}^L)$, $\underline{z}^L \in \mathcal{Z}^L$ s.t.

$$
\begin{aligned}
&0 \le x(\underline{z}^L), y(\underline{z}^L) \le 1, \qquad \forall\, \underline{z}^L \in \mathcal{Z}^L\\
&R_1 \le \sum_{\underline{z}^L\in\mathcal{Z}^L} P_{\underline{Z}^L}(\underline{z}^L)(1-\epsilon_1(\underline{z}^L))x(\underline{z}^L) + C_L\\
&R_1 \le \sum_{\underline{z}^L\in\mathcal{Z}^L} P_{\underline{Z}^L}(\underline{z}^L)(1-\epsilon_{12}(\underline{z}^L))(1-y(\underline{z}^L)) + C_L\\
&R_2 \le \sum_{\underline{z}^L\in\mathcal{Z}^L} P_{\underline{Z}^L}(\underline{z}^L)(1-\epsilon_2(\underline{z}^L))y(\underline{z}^L) + C_L\\
&R_2 \le \sum_{\underline{z}^L\in\mathcal{Z}^L} P_{\underline{Z}^L}(\underline{z}^L)(1-\epsilon_{12}(\underline{z}^L))(1-x(\underline{z}^L)) + C_L,
\end{aligned}
$$

where $-2|\mathcal{S}|(1-\sigma)^L \le C_L \le 2|\mathcal{S}|(1-\sigma)^L$ and $\epsilon_j(\underline{z}^L)$, $\epsilon_{12}(\underline{z}^L)$ are computed via $P_{\underline{Z}_t|\underline{Z}_{t-L}^{t-1}}$.

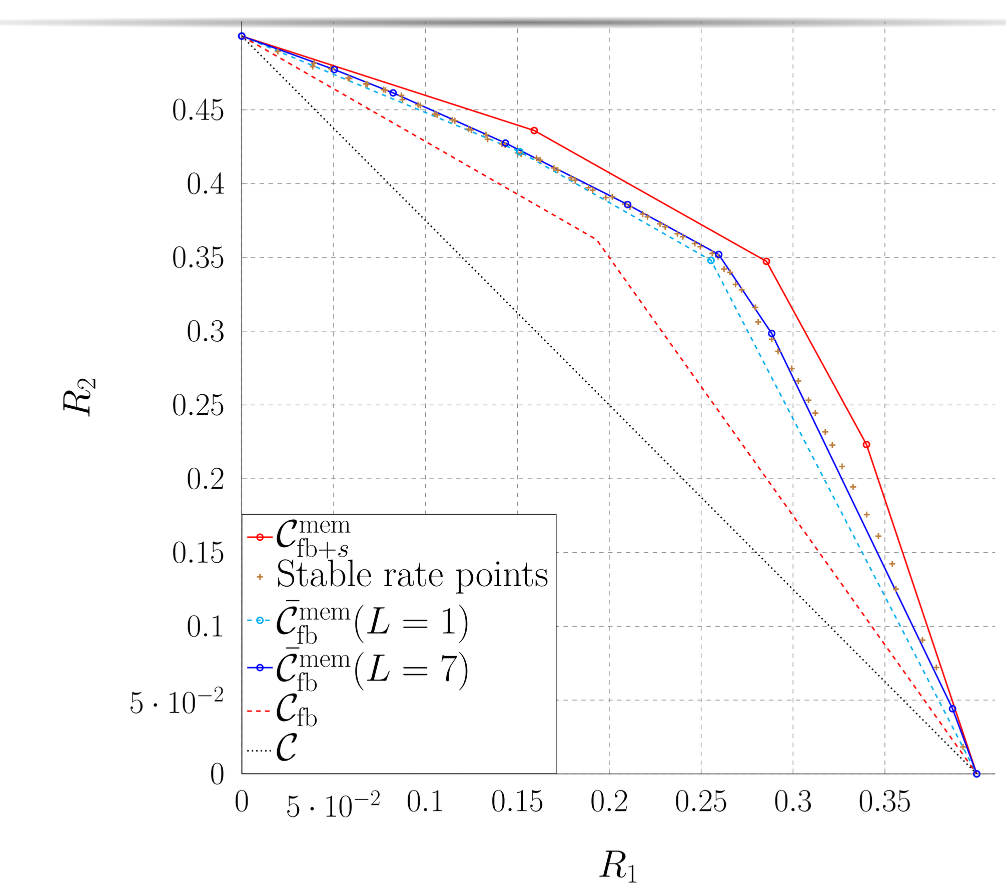
An Optimal Deterministic Scheme

At time t, the action with largest weight is chosen:

A_t	Weight depending on Q_t and $\underline{Z}^{t-1} = \underline{z}^{t-1}$
1	$[1-\epsilon_1(\underline{z}^{t-1})]Q_{1,t}^{(1)} + \epsilon_{1\bar{2}}(\underline{z}^{t-1})(Q_{1,t}^{(1)} - Q_{2,t}^{(1)})$
2	$[1-\epsilon_2(\underline{z}^{t-1})]Q_{1,t}^{(2)} + \epsilon_{\bar{1}2}(\underline{z}^{t-1})(Q_{1,t}^{(2)} - Q_{2,t}^{(2)})$
3	$[1-\epsilon_1(\underline{z}^{t-1})]Q_{2,t}^{(1)} + [1-\epsilon_2(\underline{z}^{t-1})]Q_{2,t}^{(2)}$
4	$[1-\epsilon_{12}(\underline{z}^{t-1})]\left(Q_{1,t}^{(1)} - Q_{3,t}^{(1)} + Q_{1,t}^{(2)} - Q_{3,t}^{(2)}\right)$
5	$\epsilon_{1\bar{2}}(\underline{z}^{t-1})(Q_{3,t}^{(1)} - Q_{2,t}^{(1)}) + [1-\epsilon_1(\underline{z}^{t-1})]Q_{3,t}^{(1)}$ $+\epsilon_{\bar{1}2}(\underline{z}^{t-1})(Q_{3,t}^{(2)} - Q_{2,t}^{(2)}) + [1-\epsilon_2(\underline{z}^{t-1})]Q_{3,t}^{(2)}$

- Scheme is shown to strongly stabilize all rates inside the capacity region (Lyapunov stability).

In a Picture...



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