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Intercontinental height datum connection with GOCE and GPS-levelling data

Research Article

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Abstract:
In this study an attempt is made to establish height system datum connections based upon a gravity field and steady-state ocean circulation explorer (GOCE) gravity field model and a set of global positioning system (GPS) and levelling data. The procedure applied in principle is straightforward. First local geoid heights are obtained point wise from GPS and levelling data. Then the mean of these geoid heights is computed for regions nominally referring to the same height datum. Subsequently, these local mean geoid heights are compared with a mean global geoid from GOCE for the same region. This way one can identify an offset of the local to the global geoid per region. This procedure is applied to a number of regions distributed worldwide. Results show that the vertical datum offset estimates strongly depend on the nature of the omission error, i.e. the signal not represented in the GOCE model. For a smooth gravity field the commission error of GOCE, the quality of the GPS and levelling data and the averaging control the accuracy of the vertical datum offset estimates. In case the omission error does not cancel out in the mean value computation, because of a sub-optimal point distribution or a characteristic behaviour of the omitted part of the geoid signal, one needs to estimate a correction for the omission error from other sources. For areas with dense and high quality ground observations the EGM2008 global model is a good choice to estimate the omission error correction in theses cases. Relative intercontinental height datum offsets are estimated by applying this procedure between the United State of America (USA), Australia and Germany. These are compared to historical values provided in the literature and computed with the same procedure. The results obtained in this study agree on a level of 10 cm to the historical results. The changes mainly can be attributed to the new global geoid information from GOCE, rather than to the ellipsoidal heights or the levelled heights. These historical levelling data are still in use in many countries. This conclusion is supported by other results on the validation of the GOCE models.

Keywords:
GOCE • GPS-levelling • Height systems • Vertical datum

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1. Introduction and Problem Definition.

The global connection of height datums represents one of the major goals of European Space Agency's (ESA) GOCE mission (Gravity field and steady-state Ocean Circulation Experiment) (Drinkwater et al. 2007, Arabelos and Tscherning, 2001). One of the main purposes is to achieve the relative connection between the different systems. This allows for example to connect tide gauges in a consistent system compatible with satellite altimetry for improving studies and modelling of coastal and even global sea level processes. GOCE aims to provide precise geoid information globally (apart from the polar gaps of the GOCE orbit) with 1-2 cm accuracy at a spatial resolution of about 100 km. This geoid is independent of terrestrial information, which by itself usually depends on a local height datum. In other words, by exploiting the GOCE geoid within the resolution supported by the mission a global height reference intrinsically becomes within reach. In order to identify how the global GOCE geoid provides valuable information for this pur-

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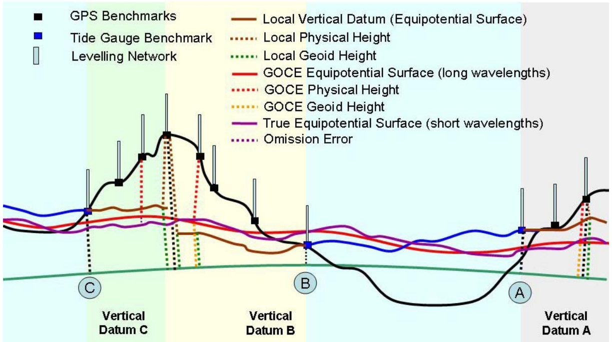

Figure 1. Overview of local height systems and related equipotential surfaces.

pose, one needs to review the commonly used procedures for the definition of local height datums.

In Fig. 1 the basic relations between local height datums and global geoids are shown. For the following general descriptions it is referred to (Heiskanen and Moritz 1967). Conventionally, local height datums are defined by local equipotential surfaces that pass through the tide gauge zero (throughout this paper this surface is named local geoid). Basically, these local equipotential surfaces are defined by long term observations of the local sea level and by some kind of averaging of the observed tide gauge heights. In Fig. 1 local height datums are denoted as vertical datums A, B and C, where the local equipotential surfaces are composed of the oceanic equipotential surface through the mean ocean surface at the tide gauge benchmarks (marked by the solid blue lines) and the related local equipotential surface inside the land masses (marked by the solid brown lines). Heights referring to one local datum can be transferred from the tide gauge to any other point on the Earth surface by means of spirit levelling and gravimetry. If one would have perfect measurements, one would get by this method local physical (orthometric) heights above the local equipotential surfaces (marked by dotted brown lines) as well as local geoid heights above a reference ellipsoid (dotted green lines). As it immediately becomes obvious from Fig. 1, local physical and local geoid heights are different for the same point on the Earth surface when transferring heights either from tide gauge B or tide gauge C (see benchmark point in the middle between tide gauge benchmarks B and C). This represents the problem of different vertical datum definitions per country, as it is the case for example in Europe between Germany and France with reference tide gauges at Amsterdam (for Germany) and Marseille (for France). By introducing the GOCE equipotential surface (solid red line in Fig. 1) and neglecting for a moment the omitted geoid signal beyond the GOCE resolution as well as the GOCE commission error, it would be straight forward possible to determine vertical datum offsets for each local equipotential surface and thus to connect all local height datums. Physical and geoid heights related to the GOCE equipotential surface (marked by the red and orange dotted lines in Fig. 1) could be determined for any point on the Earth surface. Ideally, by subtracting the GOCE geoid height from the ellipsoidal heights, as they are observed by Global Navigation Satellite System (black dotted lines in Fig. 1), one immediately gets physical heights referring to a global height system, without the need for spirit levelling and gravimetry. This method usually is called GPS-Levelling (nowadays GNSS- Levelling) and only works in case the global geoid would be perfectly known. As GOCE has limited sensitivity due to satellite height and instrument specifications, the mission aims to deliver a centimeter geoid with a spatial resolution of about 100 km. This implies that the fine structure of the global geoid could still play a role and needs to be taken into account in specific cases. In Fig. 1 this situation is shown by the true equipotential surface (solid purple line) and the deviations between the true and the GOCE equipotential surfaces, denoted as omission error (dotted purple lines). The omission error represents the impact of geoid variations with spatial resolutions smaller than 100 km on height estimates. It is strongly linked to the topography and the areas under investigation and could reach numbers between some millimetres and several metres in specific cases. This means that prior of using the GOCE geoid for height datum unification, one needs to know the impact of the omission error on the results.

Chapter 2 provides some details about the procedure applied to estimate vertical datum offsets from GPS-Levelling data and the GOCE geoid. Following this, in Section 3 the characteristics of the available GPS-Levelling data and the GOCE gravity field models are investigated in more detail. Experimental results for height offset estimates as well as height system connections are presented in the Chapter 4 for different areas in the world. Special attention will be given to the impact of the omission error. Finally, in Chapter 5 results are discussed and conclusions are derived from the results obtained.

2. Procedure

In order to estimate local height datum offsets, local geoid heights are compared to GOCE geoid heights. This means that local geoid heights need to be determined by some method, while the GOCE geoid directly can be computed for any point worldwide from the gravity field spherical harmonic series. While the latter is straightforward and well described in the literature, the computation of local geoid heights is always connected to physical heights, which are derived from spirit levelling and gravimetry.

2.1. Local Geoid Heights

As already mentioned, physical heights traditionally are determined from spirit levelling and gravimetry. By this method we get either orthometric heights above the local geoid (Stokes theory) or normal heights above the local quasi-geoid (theory of Molodenskii). For details it is referred again to (Heiskanen and Moritz 1967). If simultaneously geometric heights are observed by GPS (GNSS) at these points, one simply can compute local geoid heights or height anomalies by subtracting orthometric or normal heights from the

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observed ellipsoidal heights, respectively. This quantity usually is called GPS-Levelling geoid height or height anomaly.

2.2. GOCE Geoid Heights

The GOCE gravity field models are available as gravitational potential spherical harmonic series. Geoid heights or height anomalies for any discrete point on the Earth are computed according to the procedure described in (Gruber et al. 2011) following the theory described in (Heiskanen and Moritz 1967). For completeness the main processing steps are repeated here:

1 Rescaling of geopotential spherical harmonic series to GM (gravity constant times Earth mass) and semi major axis of the reference ellipsoid used for the local geoid heights, i.e. the geometric reference ellipsoid applied for computing the local geoid heights at GPS- Levelling points.

2 Computation of geocentric spherical coordinates per point from ellipsoidal coordinates (including ellipsoidal height);

3 Evaluation of spherical harmonic series for the spherical coordinates on the Earth's surface in order to compute height anomalies or quasi-geoid heights, respectively;

4 Computation and addition of the geoid-quasigeoid-separation based on the procedure as described in (Rapp 1997). The latter processing step only has to be applied in case geoid heights have to be computed, because orthometric heights are available at GPS-Levelling stations. The correction term is computed using the Software (hsynth_WGS84.f) and the spherical harmonic expansion of the correction term (Zeta-to-N_to2160_egm2008.gz) provided by the EGM2008 development team. Compare (Pavlis et al. 2012) and http://earth-info.nga.mil/GandG/wgs84/gravitymod/egm2008/egm08_wgs84.html).

2.3. Comparison of local and GOCE Geoid Heights

Now one can compare the local geoid at the GPS-Levelling points with the GOCE geoid at the same points and in theory one would get one offset for each local vertical datum to which the spirit levelling refers. Results applying this simple approach already were published by (Rapp 1994), where, instead of GPS derived heights, geometric heights at Doppler stations were used. It should be emphasised that this relation only is correct for ideal situations assuming error-free spirit levelling and gravimetry, error-free GNSS as well as an error-free GOCE geoid with a negligible omission error. In order to have a complete picture about the procedure the following simple set of observation equations has to be applied for all points on the Earth surface belonging to the same local vertical datum.

$$N^{A}{}_{i} = h_i - H^{A}{}_{i} \quad (1)$$

$$\Delta N^{A}{}_{i} = N^{A}{}_{i} - \left(N^{GOCE}{}_{i} + N^{res}{}_{i}\right) \quad (2)$$

$$\overline{\Delta N}^{A} = \frac{\sum_A \Delta N^{A}{}_{i}}{n^{A}{}_{total}} \quad (3)$$

where:
h_i Ellpsoidal height for point i determined by GPS
$H^{A}{}_{i}$ Orthometric or normal height of point i from spirit levelling and gravimetry referring to vertical datum A
$N^{A}{}_{i}$ Observed geoid height referring to vertical datum A
$N^{GOCE}{}_{i}$ Computed geoid height/height anomaly from GOCE model
$N^{res}{}_{i}$ omission error (residual geoid height/height anomaly signal not represented by the GOCE model)
$\Delta N^{A}{}_{i}$ Local geoid offset of vertical datum A with respect to the GOCE geoid for point i
$n^{A}{}_{total}$ Number of GPS-Levelling geoid points referring to vertical datum A
$\overline{\Delta N}^{A}$ Mean offset of local vertical datum with respect to the GOCE geoid

As already pointed out, this system of equations only is correct for the error-free case. It is important to remember this fact, when analysing the results presented in the subsequent chapters. The impact of the omission error is analysed separately for some areas under investigation and it is also presented further below.

3. Models and Data Sets used for the Study

As identified in the procedure description in Chapter 2, basically two data sets are required for estimating vertical datum offsets. First, a global gravity field model based on GOCE data is needed to compute the global GOCE geoid. Second, regional GPS-Levelling data for a set of discrete points in an area are required containing information about ellipsoidal heights, physical heights (orthometric, normal orthometric or normal heights) and the derived local geoid heights or height anomalies. In addition, it is necessary to know the datum zone to which each of the GPS-Levelling points refers. The following sub-chapters provide a detailed description of the data sets applied for this study.

3.1. GOCE Gravity Field Model

Until today all together 8 GOCE based models have been delivered by ESA, while in addition 5 models based on a combination of GOCE and the gravity field and climate experiment (GRACE) data were computed by some groups. An overview about the published GOCE based models is shown in Table 1. For this kind of analysis it is very important to use a model, which solely is based on satellite information in order to avoid leakage of height datum inconsistencies of terrestrial data into the solutions. For this reason either GOCE only or combined GOCE, GRACE and satellite laser ranging

Table 1. Overview of released GOCE based global gravity field models (m = months of data , y = years of data). All models can be accessed via the ICGEM Web Server (http://icgem.gfz-potsdam.de/ICGEM/ICGEM.html).

Model Name (Originator)	Maximum D/O	Data	Description	References
DIR1 (ESA)	240	GOCE 2m GRACE 6y CHAMP 6y Alt./Terr.	GOCE direct approach. EIGEN-5C was used as reference model.	Bruinsma et al. 2010 Pail et al. 2011
TIM1 (ESA)	224	GOCE 2m	GOCE time-wise approach. GOCE-only model.	Pail, Goiginger, Mayrhofer, et al. 2010 Pail et al. 2011
SPW1 (ESA)	210	GOCE 2m GRACE 5y	GOCE space-wise approach. GOCE-only plus GRACE for low degrees.	Migliaccio et al. 2010 Pail et al. 2011
DIR2 (ESA)	240	GOCE 6m GRACE 7y	GOCE direct approach. GRACE was used as reference model.	-
TIM2 (ESA)	250	GOCE 6m	GOCE time-wise approach. GOCE-only model.	-
SPW2 (ESA)	240	GOCE 6m	GOCE space-wise approach. GOCE-only model.	-
DIR3 (ESA)	240	GOCE 1y GRACE 6y LAGEOS 6y	GOCE direct approach. GRACE and SLR normal equations included.	-
TIM3 (ESA)	250	GOCE 1y	GOCE time-wise approach. GOCE-only model.	-
GOCO01S (GOCO Consortium)	224	GOCE 2m GRACE 7y	TIM1 model including ITG-GRACE2010S normal equations	Pail, Goiginger, Schuh, et al. 2010
GOCO02S (GOCO Consortium)	240	GOCE 6m GRACE 7y LAGEOS 5y	TIM2 model including ITG-GRACE2010S and LAGEOS SLR normal equations.	-
EIGEN-6S (GFZ/CNES)	240	GOCE 6m GRACE 7y LAGEOS 7y	GOCE direct approach including GRACE and LAGEOS normal equations.	-
EIGEN-6C (GFZ/CNES)	1420	GOCE 6m GRACE 7y LAGEOS 7y Alt./Terr.	EIGEN-6S including normal equations for EGM2008 gravity anomalies.	-
GOCO03S (GOCO Consortium)	250	GOCE 1y GRACE 7y LAGEOS 5y	TIM2 model including ITG-GRACE2010S and LAGEOS SLR normal equations.	-

(SLR) models shall only be taken into account. This implies that the DIR1 and the EIGEN-6C models are not applicable for this work. In a second step it shall be analysed, which of the available models contains the best high resolution signal, or to what degree and order the models represent the full gravity field signal. For this purpose signal degree variances are computed and inspected. Figure 2 shows the signal degree variances for all models under consideration and in addition the Tscherning-Rapp signal degree variance model (Tscherning and Rapp 1974) between degree 150 and 240, where the main differences occur. It shall be mentioned that for degrees below 150 all models show quasi identical behaviour. The first conclusion one can derive from this figure is that by using more GOCE data the signal content increases. This is well visible regarding only the three GOCE-only models (TIM1, TIM2 and TIM3), where the most recent TIM3 model based on one year of GOCE data exhibits largest signal content. Comparing the GOCE only models with the combined GOCE, GRACE and SLR models one can identify different behaviour depending on what additional constraints have been applied during computation. It becomes obvious that different strategies have been applied by the GOCO consortium (Kaula regularization) and the GFZ/CNES team (spherical cap regularization). So, one can assume, that the divergence of the signal degree variances of these models mostly is caused by the different regularization strategies, rather than by a better representation of the global gravity field signal. It also becomes obvious and it is well known, that by adding GRACE and SLR information the signal content in this frequency range cannot be improved (compare the TIM and GOCO model series). Taking all this into account and in order to identify the value of GOCE for height system unification, it was decided to use the TIM3 GOCE only model for the further analyses

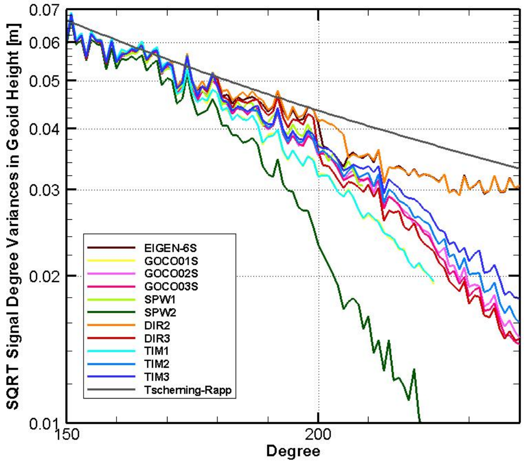

Figure 2. Square root of signal degree variances in terms of geoid heights for GOCE based satellite-only models between degree 150 and 240.

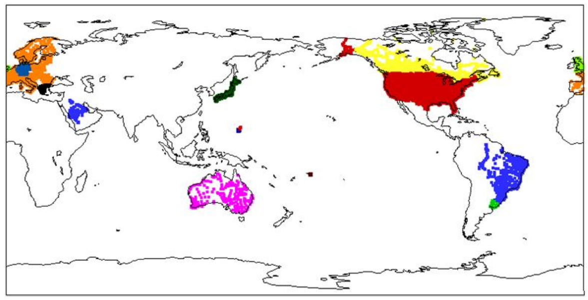

Figure 3. Overview of available GPS-Levelling data sets. Each colour represents one data set linked to a specific height datum. For UK and Germany national as well as the unified European data sets are available. Here the national data sets are colour coded separately.

up to degree and order 180. Degree 180 was chosen as at this point the signal degree variances are starting to diverge, which gives us some confidence that up to this degree the pure GOCE model contains the full gravity field signal.

3.2. GPS-Levelling Data Sets

A number of GPS-Levelling data sets from various areas in the world have been acquired during the recent years. These data refer to different vertical height datums usually connected to a national tide gauge station. In some cases (Europe) normal heights are available, while for most of the data sets orthometric or normal orthometric heights are provided. This implies that from the GOCE global models the correct quantity has to be computed (see Chapter 2 for more details). Figure 3 shows the geographical distribution of the data points, while Table 2 provides some details for each regional data set.

4. Experimental Results for Estimation of Height System Offsets and Height Datum Connection

4.1. Height System Offset Estimates and Omission Error

In a first attempt height system offsets between the TIM3 GOCE geoid (up to degree and order 180) and the local geoids for all data sets specified in Table 2 are computed by two approaches. The first approach just assumes that the omission error is negligible and only the GOCE model is used, while for the second approach we try to estimate the omission error, i.e. the signal not represented by the GOCE model above degree 180. It shall be mentioned that apart from the omission error still the global models commission error as well as errors of the GPS heights and particularly the errors in the levelled heights are present (e.g. Featherstone and Filmer, 2012; Wang et al, 2011). For the purpose of estimating the omission error the EGM2008 model (Pavlis et al. 2012) from degree and order 181 to 2190 is used and in addition, for the German data set, an RTM (Residual Terrain Model) derived residual geoid above the maximum resolution of EGM2008 is applied. For details it is referred to (Hirt et al, 2010, Gruber et al. 2011). Table 3 shows the results for both approaches applying equation 3 per regional data set, where a region can be an island or even a complete continent. As it can be identified the impact of the omission error differs significantly for the different data sets regardless which gravity field model is used. Compare for example the results obtained for the United Kingdom (UK) (height offset difference between both approaches is 3 mm) and for American Samoa (height offset difference is 2.525 m). It becomes obvious that the omission error impact is dependent on the roughness of the terrain, on the area size and on the distribution of the GPS-Levelling stations. For smooth terrains it can be expected that the omission error is not significant. However, also in rough terrain the omission error tends to cancel out if the region is large enough and/or the point distribution is homogeneous and dense enough. This is illustrated in Fig. 4 and Fig. 5, which show the omission error computed from EGM2008 between spherical harmonic degrees 181 and 2190. Figure 4 shows the omission error for the Pacific islands American Samoa, Guam and Northern Marianas, which all exhibit a strong impact of the omission error on the local geoid offset estimates. For all these islands there is a strong positive omission error, which does not cancel out by computing the mean over this area. In order to further investigate the impact of the omission error, the United States (US) East coast was taken as test area. Along the coast all together ten 2×2 degree blocks were defined and the local geoid offsets with and without the omission error were estimated as well as the omission error itself was computed per block from the EGM2008 model. The omission error impact on local geoid offset estimates for these blocks is in the range

Table 2. Overview of GPS-Levelling data sets used for this study.

Area	No. Points	Height System	Vertical Datum	Reference or Data Source
Europe	1233	Normal	EVRF2007 (European Vertical reference Frame 2007), European Vertical Network –Densification A.	(Kenyeres et al. 2007)
UK	177	Normal Orthometric	Newlyn Tide Gauge	(Ordnance Survey, UK)
Germany	675	Normal	Normaal Amsterdam Peil (NAP)	(Ihde and Sacher 2002)
Greece	1542	Orthometric	Helenic Vertical Datum	(Kotsakis and Katsambalos 2010)
Canada	2576	Orthometric	CGVD28: Canadian Geodetic Vertical Datum of 1928)	(NRCAN: Natural Resources Canada, 2007)
Continental USA	18398	Orthometric	NAVD88: North American Vertical Datum of 1988	(NGS: National Geodetic Survey, 2009)
Alaska (USA)	86	Orthometric	NAVD88: North American Vertical Datum of 1988	(NGS: National Geodetic Survey, 2009)
Guam (USA)	16	Orthometric	GUVD04: Guam Vertical Datum of 2004	(NGS: National Geodetic Survey, 2009)
American Samoa (USA)	22	Orthometric	ASVD02: American Samoa Vertical Datum of 2002	(NGS: National Geodetic Survey, 2009)
Northern Marianas (USA)	54	Orthometric	NMVD03: Northern Marianas Vertical Datum of 2003	(NGS: National Geodetic Survey, 2009)
Australia	197	Normal Orthometric	AHD71: Australian Height Datum of 1971	(GSI: Geoscience Australia, 2003)
Japan	837	Orthometric	Japanese Vertical Datum (MSL Tokyo Bay)	(Geospatial Information Authority of Japan, 2003)
Brazil	683	Orthometric	Imbituba, Santa Catarina State	(IBGE - Instituto Brasileiro de Geografia e Estatistica, 2012)
Uruguay	16	Orthometric	Montenvideo Port 1948	Universidade Federal do Paraná, 1998

Table 3. Local geoid offset with respect to TIM3 GOCE and EGM2008 geoid (up to degree and order 180) per region applying Eq. (3). TIM3+OM: TIM3 geoid including estimated omission error from EGM2008 (plus RTM for Germany); TIM3: pure TIM3 geoid without taking into account the omission error. EGM2008+OM: EGM2008 geoid from complete series; EGM2008(180): EGM2008 geoid truncated at degree 180. All mean differences in [m].

	Europe	UK	Germany	Greece	Canada	Cont. USA	Alaska
TIM3+OM	-0.294	+0.049	-0.332	-0.110	-0.015	-1.145	+1.098
TIM3 (180)	-0.286	+0.052	-0.347	-0.187	+0.191	-1.122	+1.503
EGM2008+OM	-0.297	+0.036	-0.324	-0.125	-0.025	-1.144	+1.100
EGM2008 (180)	-0.289	+0.039	-0.387	-0.202	+0.181	-1.121	+1.508
	Guam	**A. Samoa**	**Marianas**	**Australia**	**Japan**	**Brazil**	**Uruguay**
TIM3+OM	+0.458	-1.194	+0.485	-1.010	-0.722	-0.588	-0.694
TIM3	-1.361	-3.719	-0.520	-0.962	-0.694	-0.485	-0.787
EGM2008+OM	+0.385	-1.221	+0.455	-1.005	-0.713	-0.579	-0.674
EGM2008 (180)	-1.434	-3.746	-0.550	-0.956	-0.685	-0.476	-0.768

between 18.7 and 2.2 cm and is more or less randomly distributed. As an example Figure 5 shows two of these neighbouring blocks for the US states of North and South Carolina. For North Carolina the mean omission error computed over the 391 GPS-Levelling points is 2.5 cm, while for South Carolina it is 16.3 cm for 475 data points. Looking to Figure 5 one cannot immediately identify, for which of the two blocks the omission error is larger or smaller. Its impact on the local geoid offset estimate strongly depends on the data distribution of the GPS-Levelling points, the omission error signal at these locations. Concluding the investigations on local geoid offset and impact of omission error one can state, that it is required to estimate the omission error impact prior to computing the local geoid offset in order to identify, if it needs to be applied or not. It is not only sufficient to compute the omission error itself, but also the distribution of the GPS-Levelling points needs to be taken into account. Specifically, it has to be investigated beforehand, if the data

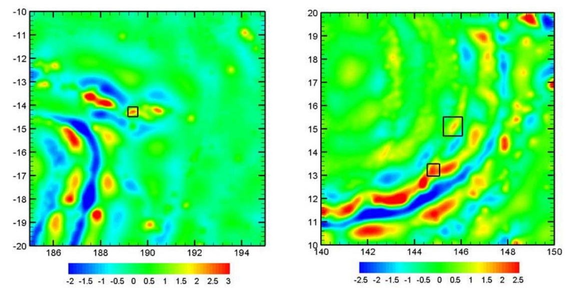

Figure 4. Omission error in terms of geoid height estimated from EGM2008 from degree and order 181 to 2190 in [m]. Left: American Samoa (see black box); Right: Guam (lower black box) and Northern Marianas (upper black box).

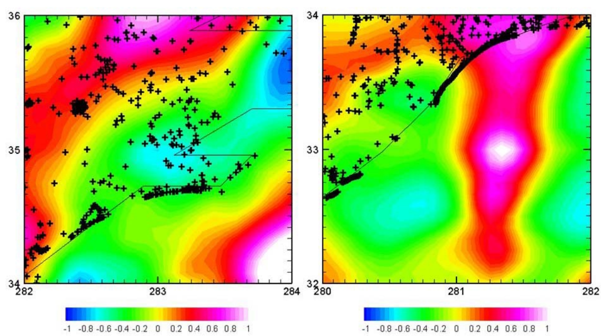

Figure 5. Omission error in terms of geoid height estimated from EGM2008 from degree and order 181 to 2190 in [m] and distribution of GPS-Levelling points in the area. Left: Block at USA East coast in North Carolina (Latitude 34°N to 36°N, Longitude 282°W to 284°W) with 391 GPS-Levelling points. Right: Block at USA East coast in South Carolina (Latitude 32°N to 34°N, Longitude 280°W to 282°W) with 475 GPS-Levelling points. Black line indicates the approximate coastlines.

distribution is good enough, such that the mean omission error cancels out. Obviously this seldom is true for small islands near the limit of the resolution of GOCE, especially when these are mountainous, but it could be true for specific areas. In case the omission error has a strong impact, it can be computed for example from an ultra high resolution global gravity field model like EGM2008. If such a model is used it has to be made sure that it is built on surface data of good quality for the area under investigation. This is true for well-observed areas like North America, Europe, Australia and Japan, but it could be not good enough for other areas in Africa, South America and Asia. In less surveyed areas one has to rely on the global geoid model and possibly get a feelling for the omission part from a topography model (if needed a satellite derived one) and consequently accept larger errors for the height datum connection. This is not further investigated in this study.

4.2. Experimental Results for Height Datum Connection

Now an attempt is made to connect height systems of Europe and North America over the North Atlantic as well as height systems of Japan/Australia and North America over the Pacific applying the GOCE geoid (degree and order 0-180) and the remaining signal (omission error) estimated from EGM2008 (degree and order 181-2190). Let us first look to the North Atlantic. For Europe those national sub-data sets of the unified European vertical network are applied, which have a connection to the North Atlantic. This is true for the following countries: Norway, Denmark, Germany, The Netherlands, Belgium, France and Spain. It needs to be mentioned that the UK data set was not used for this purpose as it exhibits a significantly different local geoid offset as compared to all other countries (for the unified network as well as for the national network tied to the Newlyn tide gauge). For the other countries mentioned above, the local offset is taken and a weighted mean (according to the number of GPS-Levelling stations available) is computed, which results in an offset for the European North Atlantic coast to the GOCE/EGM2008 geoid of -0.324 m.

Now a similar computation was done for each US East coast state. As it can be identified in Fig. 6, there is a steady change of the mean local geoid offsets with respect to the GOCE/EGM2008 geoid from North to South. One can classify the different East coast states according to their mean offsets more or less into three groups representing the states from Maine to Virginia in group 1, from North Carolina to Georgia in group 2 and Florida in group 3. Again a weighted mean based on the number of stations is computed and finally compared to the European weighted mean value. As a result, for the three North American groups we get a height datum offset between Europe and USA of -1.120 m for the Northern states (group 1), -1.305 m for the central states (group 2) and -1.773 m for the Southern state (group 3) (see Fig. 6). So the height datum connection over the North Atlantic applying the method of comparing GPS-Levelling data to the GOCE/EGM2008 geoid results in different offset estimates strongly linked to the data used along the US East coast. One should ask why these results are so different? As GPS-Levelling geoid heights are determined from geometric GPS and levelled heights, these differences can only originate from one of these quantities. As geometric GPS heights never can exhibit such a strong regional dependency, the differences can only be caused by errors in the levelled networks and consequently in the orthometric heights. This observation is confirmed by the investigations shown by (Wang et al. 2011). Figure 2 of Wang's et al paper shows the long wavelength errors of the US vertical datum. According to their results the error is roughly -0.36 m for group 1 states, -0.24 m for group 2 states and about 0.00 m for group 3 (Florida) (values have been taken from Fig. 2 in Wang et al. (2011)). Applying these errors as corrections to the above derived offsets we get height datum offsets between Europe and USA of -1.48 m, -1.55 m and -1,77 m for the three groups, respectively. In conclusion one can state that by applying such a surface as a corrector one could try to improve the intercontinental datum offset estimates based upon

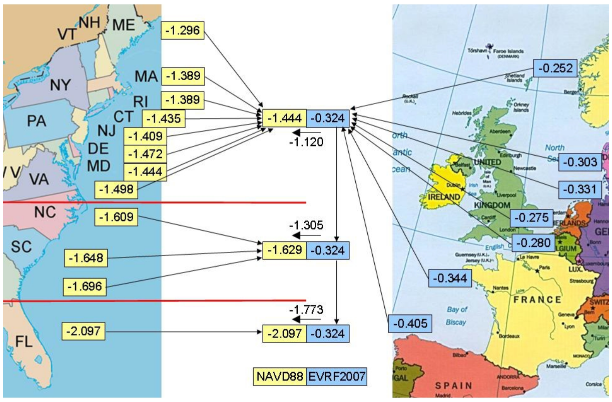

Figure 6. Height system offset estimates between Europe and North America, i.e. between the European Vertical Reference Frame 2007 (EVRF2007) and the North American Vertical Datum 1988 (NAVD88). Omission error from EGM2008 (d/o 181 to 2190) is taken into account for all estimated regional mean offsets.

GPS and levelling data. Nevertheless it also becomes obvious, that the errors in the levelling networks play a significant role for such investigations. This is clearly visible from recent ocean levelling results where levelling data were excluded in the comparison at the tide gauges as presented in (Woodworth et al. 2012). These results confirm the findings about systematic distortions in the US levelling network as shown by Wang et al. (2011).

A similar attempt was made to connect the Australian and Japanese height datum to the North American one. For this purpose the US West coast states were analyzed separately and again a weighted mean has been computed. Similar as for the East coast a strong North-South change of local offsets is observed, which again can be attributed to long wavelength errors of the North American levelling network. Disregarding this, we get height system offsets between the US West coast and Japan of -0.555 m and between the US West coast and Australia of -0.843 m. Let us now apply again a correction value for each West coast state offset, which is extracted from Fig. 2 of (Wang et al. 2011). The mean correction value per state is -1.08 m for Washington, -1.04 m for Oregon, and -0.68 m for California. Applying these correction values, the mean local geoid offsets change to -0.82 m, -0.93 m and –1.07 m respectively, which results in a weighted mean of -0.99 m. Consequently the related height system offsets between Japan, Australia and the US West coast also change to +0.27 m for Japan and to -0.02 m for Australia. These results again show the importance of good quality levelling networks for this kind of work.

Finally, an attempt is made to repeat the investigations performed by (Rapp 1994) on intercontinental height datum offset determination. At that time, the global geoid was computed by a combination of JGM-2 (Nerem et al. 1994) and OSU91A for the omission error computation (Rapp, Wang, and Pavlis 1991), while the local geoid was determined from the difference of geometric heights as observed at Doppler satellite stations and levelled orthometric heights. Table 4 shows the historical values as well as the results obtained from this study, applying now JGM-2/OSU91A and GOCE/EGM2008 combinations for the global geoid, GPS observations for the ellipsoidal heights and levelled orthometric heights. Apart from the levelled heights, for which there is good reason to assume that these are based on the same levelling campaigns, now GPS and the new global gravity field models have been used. Differences in the results mainly shall be attributed to improvements obtained with GPS and a better global geoid. As already pointed out, one can assume that the same levelling networks have been used for both studies, which implies that the levelling errors are identical or are at similar level. There are good reasons to assume that geometric height observations could be improved with the systematic use of GPS. The impact of the GPS-Levelling versus the Doppler/GPS stations becomes visible when comparing case 1 and 2 of Table 4, where the same geoid was used for both analyses. The height datum offset estimates differ by a decimeter or so, except for the one from Germany to USA. If this agreement is regarded as an exception one could state that a decimeter improvement can be addressed to the GPS stations. The impact of the GOCE geoid can be identified by comparing the results obtained for case 2 and 4 and case 5 and 7. When using a pre GRACE gravity field model (JGM-2) the impact of the GOCE geoid on datum offset results is estimated to be at a level between 5 and 10 cm (compare case 2 and 4). When using a GRACE based model incorporating in the areas under investigation high quality terrestrial information (EGM2008 to full extension) offset differences at the level of several mm are resulting (compare case 5 and 7). Truncating the GRACE/terrestrial model (EGM2008 up to degree 180) and comparing the height datum offsets with those computed from the pure GOCE model (TIM3 up to degree 180) differences at a level of up to 5 cm show up (compare case 6 and 8).

5. Discussion and Conclusions

The goal of this study was to identify the value of GOCE for intercontinental height datum connection using GPS-Levelling data. For this purpose the global GOCE geoid is compared to local geoid heights derived from GPS-Levelling data. This is done for specific regions, which could be a part of a country, a complete country or sometimes even an entire continent. Height offset estimates between the global and local geoids are computed by taking the mean value of these differences over the area under investigation. From the results obtained in this study one can draw the following conclusions.

- GPS-Levelling data are useful for this kind of analysis as they provide information about the height datum used for a specific area. The height datum is related to physical heights derived from spirit levelling and gravimetry at GPS-Levelling points. Together with ellipsoidal heights, local

Table 4. Intercontinental height datum offsets between Australia, Germany and USA from (Rapp 1994) and this study (d/o means that the global model has been used up to this degree).

	Ellipsoidal Heights & Levelling	Global Gravity Field Model	Australia to Germany [cm]	Germany to USA [cm]	USA to Australia [cm]
1	Doppler/GPS Stations (Rapp, 1994)	JGM-2(d/o 70) & OSU91A (d/o 71-360)	+72	-76	+4
2	GPS-Levelling (Table 2)	JGM-2(d/o 70) & OSU91A (d/o 71-360)	+57.8	-76.1	+18.3
3	GPS-Levelling (Table 2)	OSU91A (d/o 360)	+12.7	-40.4	+27.7
4	GPS-Levelling (Table 2)	TIM3 (d/o 70) & OSU91A (71-360)	+65.9	-78.3	+12.4
5	GPS-Levelling (Table 2)	TIM3 (d/o 180) & EGM2008 (181-2190)	+67.8	-81.3	+13.5
6	GPS-Levelling (Table 2)	TIM3 (d/o 180)	+61.5	-77.5	+16.0
7	GPS-Levelling (Table 2)	EGM2008 (d/o 2190)	+68.1	-82.0	+13.9
8	GPS-Levelling (Table 2)	EGM2008 (d/o 180)	+56.9	-73.4	+16.5

geoid heights at these points can be determined and compared to geoid heights derived from GOCE.

- GOCE provides for the first time the opportunity to observe the global geoid with an accuracy of some few cm at a spatial resolution of 100 km independent of any terrestrial data. This was shown by analysing the signal degree variances of a number of GOCE based models. In order to identify the impact of GOCE it was decided to use a pure GOCE gravity field model for this study (TIM3 model) up to degree and order 180.

- It is important to identify how good the GOCE geoid represents the real geoid. In other words, the omission error representing the signal not observable by GOCE needs to be quantified. A full quantification of the omission error would require perfect knowledge of the geoid. As this information is in many cases not available, we make use of the EGM2008 global model, which incorporates terrestrial and altimetric gravity field information and has a resolution of about 8 km. In well-observed areas one can assume that EGM2008 is a good representation of the high resolution geoid and can be used to estimate the omission error. For this reason, in this study a hybrid global geoid based on a pure GOCE model from degree 0 to 180 (TIM3) and EGM2008 from degree 181 to 2190 was used.

- From the analyses performed for some areas, it can be concluded, that the omission error plays a significant role in case its mean value estimated at the GPS-Levelling points does not vanish. In extreme cases, like on islands, the omission error has a huge impact (more than 1 metre) and has to be taken into account. For some other areas the omission error sometimes is on the level of a few cm only. This is strongly dependent on the roughness of the terrain, i.e. the local structure of the omission error, and the distribution of the GPS-Levelling points. In any case it is necessary to quantify the omission error beforehand.

- For intercontinental height datum connection it is important to select GPS-Levelling data of good quality, as any error in the levelling networks completely maps into the offset estimates. This is nicely shown by an attempt to connect the European with the US height datum as well as by connecting the US with the Japanese and Australian vertical datums. For this purpose US East and West coast states have been treated separately and offsets per state have been computed. It becomes obvious that there exists a relatively strong North-South trend for the offset estimates at both coasts. This trend can be linked to results shown by other investigations about the long wavelength error of the US levelling network. It became obvious that intercontinental height datum connection strongly depends on the quality of the national levelling networks.

- It was tried to quantify the impact of GOCE for intercontinental height datum connection by comparing newly obtained results from this study with those from an earlier study based on geometric heights on Doppler stations and an older global geoid solution. Results for vertical offsets between Australia, USA and Germany presented by Rapp (1994) and from this study differ by up to 10 cm. These differences can be addressed to improvements of the global gravity field model and the ellipsoidal heights. At this point it can not be stated, that these changes also can be regarded as improvements of the height offset estimates, as no reference values are available.

- By comparing vertical offset estimates based on the full EGM2008 and the hybrid TIM3/EGM2008 models one can identify variations of several mm. As the comparisons have been done in well-observed areas one can assume that the full EGM2008 solution already provides a high precision geoid and that GOCE can improve this geoid only marginally. On the other hand one may also conclude from the analysis of vertical datum offsets based on truncated gravity field models that the GOCE impact is at a level of up to 5 cm on height offset estimates. From both results it can be concluded that the quality of the omission error somehow dominates the accuracy of the datum offsets estimated by this method.

- While the results of this study are based on mean offset values over the areas of investigation, one could separate datum connection into two tasks: (1) Apply the same procedure as used in this study not to mean values of all GPS-Levelling points per datum zone, but to the actual datum point only. This provides the actual datum offset between different datum zones. (2) Extending the offset computation to all remaining GPS- Levelling points can then be regarded as an investigation on the inner quality of the different quantities per datum zone, especially the quality of the levelling network which suffers from large systematic errors. The internal quality of individual networks inside one datum zone is thereby separated from the question of datum offset to other datum zones or global datum unification. It might be a challenge however, to do the necessary computations for datum zones which are based on more than one datum point because these do in general not refer to the same equipotential surface (or local geoid).

As a final conclusion one could state, that GOCE supports height datum unification by providing a better knowledge about the medium frequency global geoid. For the current TIM3 GOCE solution improvements are visible for the frequency range between degree 120 and 180, for the final GOCE model it is expected that the global geoid will be improved with respect to any a- priori knowledge up to degree 200 and beyond.). But, it also has to be stated that the mean high frequency geoid signal is in many cases not negligible and that it needs to be quantified as good as possible from other sources of information. The total error budget for intercontinental height system connection using mean offsets over the areas of investigation is driven by systematic errors in national levelling networks, which need to be identified beforehand in order to derive the right conclusions.

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