

Emil J. Gumbel's last course on the "Statistical theory of extreme values": a conversation with Tuncel M. Yegulalp

Lexuri Fernández¹ · Matthias Scherer¹

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Abstract GUMBEL. Eponym in mathematical statistics for the first type extreme value distribution and the copula that is both of extreme value and Archimedean kind. Hydrologists appreciate Emil J. Gumbel as a pioneer in promoting non-normal distributions in their field. Historians rank him among the most influential German intellectuals of the Weimar Republic. He disclosed secret societies that destabilized the Weimar Republic and used statistical methods to document political murders and to reveal a biased legal system. He was the first professor who lost his position for his political ideals and his stand against the national socialistic party, his books were banned and burned. Stripped of his nationality in 1933 he immigrated to France. In 1940 he escaped to the USA and settled in New York, where he was appointed Adjunct Professor at Columbia University, Department of Industrial Engineering, in 1953. We spoke with Tuncel M. Yegulalp –Professor emeritus of Mining Engineering at Columbia University– about Emil J. Gumbel's last course on the "Statistical Theory of Extreme Values" back in 1964.

Keywords Emil J. Gumbel · Extreme value theory · History of statistics

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✉ Matthias Scherer
scherer@tum.de
Lexuri Fernández
lexuri.fernandez@tum.de

¹ Lehrstuhl für Finanzmathematik, Technische Universität München, Parkring 11-13, 85748 Garching bei München, Germany

1 Emil J. Gumbel

“My research on extreme values started back in Germany in 1932, was developed in France until 1940, and has continued since then in the United States. Although my outward life suffered some discontinuities, the scientific work continued without break.” This self-deprecating description in Gumbel (1954, p. iii), a forerunner of Gumbel’s classical monograph (Gumbel 1958), is worth elaborating upon. In particular, the ironic understatement “*some discontinuities*” reveals a lot about Emil Gumbel’s personality.

Born in 1891 into a banker’s family in Munich, he grew up in the well-to-do district of Lehel, receiving a humanistic education at the traditional *Wilhelmsgymnasium*. From 1910 he studied mathematics and economics at Ludwig Maximilian University and graduated in 1913 with a diploma in actuarial science. One week before the outbreak of the First World War in 1914 he completed his doctoral thesis “Über die Interpolation des Bevölkerungszustandes” [translation: “On the Interpolation of the Population Level”] that was published two years later as a book, cf. (Gumbel 1914). He volunteered for military service, but was soon discharged because of health problems. The death of his brother and a cousin in war and the influence of his uncle Abraham Gumbel (see Gumbel 1931) made him a staunch pacifist. He moved to Berlin to study physics and statistics at Humboldt University. He attended lectures by Albert Einstein –with whom he shared his political ideals and collaborated in various pacifist organisations– and Ladislaus Bortkiewicz, who strongly influenced his academic career in mathematical statistics (Härdle and Vogt 2015) and who was among the pioneers in investigating extreme order statistics (von Bortkiewicz 1921).

Gumbel was a painstaking collector of newspaper clippings on political topics. His first political book “Zwei Jahre Mord” [translation: “Two Years of Murder”] (Gumbel 1921), which he later extended into “Vier Jahre politischer Mord” [translation: “Four Years of Political Murder”] (Gumbel 1922), contained a meticulously assembled summary of politically motivated murders during the early Weimar Republic, along with a statistical analysis of the resulting court proceedings. His summary statistic “354 political murders by the right; total punishment of 90 years, 2 months confinement, 730 Marks fines, and 1 life sentence” compared to “22 murders by the left; total punishment of 10 executions, 248 years and 9 months confinement, 3 life prison sentences” led to an official commission of inquiry (see Fig. 1), which ultimately confirmed Gumbel’s statistics, but to Gumbel’s amazement there were no political ramifications. In a subsequent publication entitled “Verschwörer” [translation: “Conspirators”] (Gumbel 1924) he reveals the development of nationalist secret societies that later became the nucleus of the National Socialist German Workers’ Party. Gumbel’s publications have in common that they list in plain text all names of perpetrators, victims, judges, politicians etc., regardless of the personal danger this brought to him. For instance, Gumbel only escaped summary execution by a military commando by chance in 1919; see Gumbel (1920). The objective and scientific style of his political writings and speeches is clearly noteworthy. He typically used scientific lines of argument as well as statistical and macro-economic explanations to discuss political matters. Gumbel believed that a sound statistical analysis and objective reports are well suited to convince the public about aberrations in

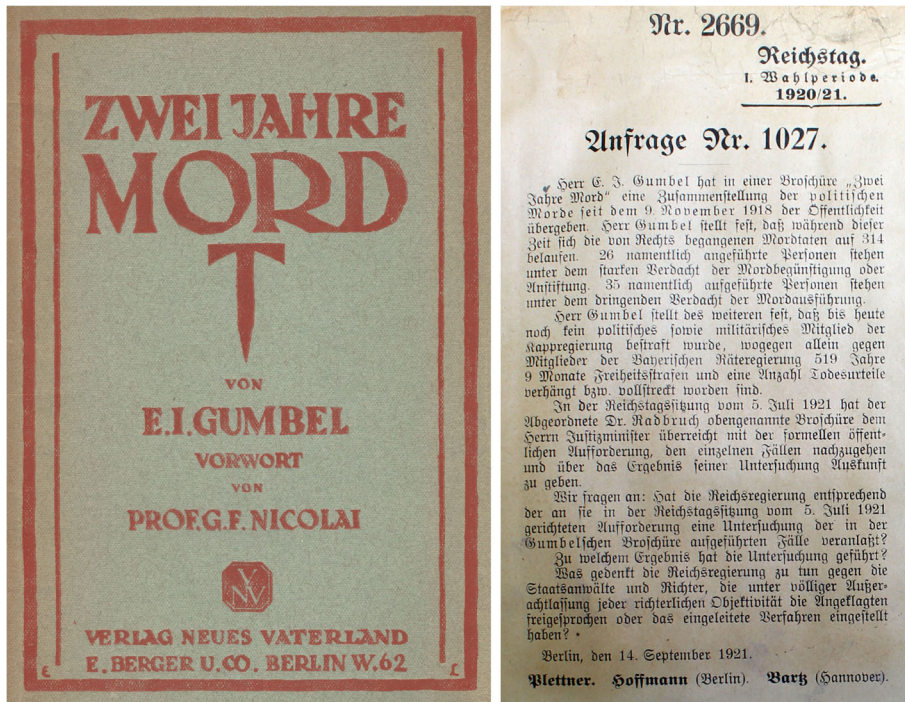


Fig. 1 *Left:* The cover of “Zwei Jahre Mord” [translation: “Two Years of Murder”] (Gumbel 1921). *Right:* The formal request to the Reichstag to investigate Gumbel’s charges; LBI, Gumbel collection, Box 149 OS Folder 1 (Eitelkeitsbuch (A.1))

politics. The echo of his ingenious speeches, articles, and books was not strong enough, however, to compete with the populist propaganda of his times.

In early 1923 he obtained his Habilitation at Ruprecht–Karl University in Heidelberg (see Fig. 2) and, starting in summer 1923, he taught mathematical statistics as a Privatdozent. His academic and teaching abilities were undisputed, but his numerous appearances as a passionate political speaker and his political articles in the left press made him an outsider in Heidelberg. Living with a divorced woman and descending from a Jewish family did not help much either, in provincial Heidelberg. Consequently, he preferred to spend his free time among left-wing intellectuals in Berlin (Lion Feuchtwanger, Ödön von Horváth, Heinrich Mann, Carl von Ossietzky etc.). Moreover, he acted as translator of Bertrand Russell and published (Russell 1922, 1923) in German, the forewords were written by Albert Einstein and David Hilbert, respectively. Scientifically, he worked on mortality tables and the statistical distribution of life distribution, among others, whilst in Heidelberg. The first public scandal—the “Gumbel case” that became nation-wide news—occurred in July 1924 when Gumbel was accused of denigrating the memory of fallen soldiers.¹ As a result, and

¹ After a speech in July 1924 to commemorate the Great War that started a decade ago, he concluded the meeting by encouraging the audience to stand silently for two minutes to honor the war dead, “die, ich will

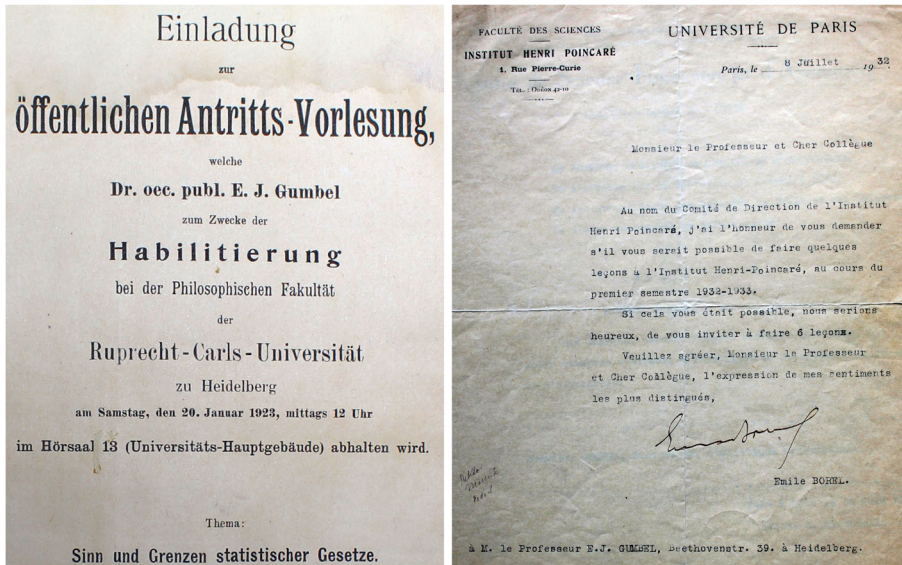


Fig. 2 *Left:* Invitation to Emil J. Gumbel’s habilitation lecture; LBI, Gumbel collection, Box OS 149 Folder 1 (Eitelkeitsbuch (A.1)) 1916, July – 1924, April (missing in the online catalogue). A related paper is Gumbel (1923). *Right:* Invitation by Emile Borel to provide a series of six lectures at the Henri Poincaré Institute in Paris during the first semester of 1932–1933; courtesy of Patricia Gumbel

under pressure from protesting, largely right-wing students, disciplinary proceedings were initiated against him. The case was dropped in May 1925, but the fact that the findings (including a vilifying psychological profile) of the committee were publicly sent to the national press and all other German universities was a real humiliation and a disaster for his academic career. Following this scandal he spent a sabbatical at the Marx–Engels Institute in Moscow, editing the mathematical writings of Karl Marx (Vogt 1996). Moreover, he wrote the book “Vom Russland der Gegenwart” [translation: “Russia today”] (Gumbel 1927), in which he describes, as objectively as possible, the status of Russia shortly after the revolution in October 1917. Back in Heidelberg, he faced massive conflicts with right-wing nationalist student groups, who openly (via press, speeches, and petitions) campaigned for Gumbel’s dismissal. His family was threatened repeatedly and he could not walk to university without guards. The university was not prepared (or willing) to withstand this political pressure any longer and withdrew his teaching license in 1932. Thus, Emil Gumbel became the first professor in the Weimar Republic to lose his position for his political ideals.

Shortly after the National Socialists seized power, a list of 33 top state enemies was published. All of them were stripped of their German citizenship and their assets were confiscated; see Pfeiffer and Rott (2016). This list included famous German

nicht sagen, auf dem Felde der Unehre gefallen sind, aber doch auf gräßliche Weise ums Leben kamen” [translation: “who, I shall not say, fell on the field of dishonor, but lost their lives in the most horrible way.”]; quote and translation were taken from Brenner (2001, Chapter 5, p. 90).

politicians, writers, and the scientist Gumbel – a fact he was very proud of in later years. Shortly thereafter, Gumbel's books were forbidden and burned. At that time, he was giving lectures in Paris (see Fig. 2), so he remained in French exile and built himself a new life in Lyon, where in 1934 he gained a position at the Institut de Science Financière et d'Assurances (ISFA) – supported by Jacques Hadamard and Maurice Fréchet, another founding-father of extreme value theory (Fréchet 1927). During this time, he was very active in mathematics and developed his theoretical grounding in extreme value theory (Gumbel 1935), formally proving the limiting law for exponential base distributions. Besides probabilistic investigations, he soon started to use extreme value statistics in applications, in particular to describe hydrological and meteorological observations, but also in actuarial science (Gumbel 1933). After German troops invaded France in 1940, he hurriedly fled to the USA (Gumbel 1941) to escape from the Gestapo. In New York, he initially lived on grants from the Rockefeller Foundation and small research commissions for the US government. He held various short-term positions at local colleges/universities before he was appointed at Columbia University in 1953.

The study of extremes became the primary topic of Gumbel's research in France and remained so in the USA. Inspired by applications in different fields, he developed estimation methods for the three limit distributions (also called extreme value distributions) and wrote survey articles as well as easy-to-use tutorials for engineers and other practitioners. In particular, he popularized the use of "probability paper" and the "return period".² He promoted the use of extreme value statistics in different disciplines by publishing articles (see the list of references in Gumbel 1958), by providing numerous talks at scientific and professional conferences (see Folder 11 in Box 7 of the Gumbel collection at the LBI), and by traveling the world to give guest lectures at universities. Being media conscious, he enjoyed giving interviews to the popular press, explaining for example how floods can be predicted using extremes (see Folder 14 in Box 7 of the Gumbel collection at the LBI). Equipped with the experience of 25 years of research on extremes, Gumbel finally published his monograph (Gumbel 1958), a book that he developed over almost a decade (see the introduction within the book). Gumbel's book became a scientific bestseller and was translated into different languages. It is an interesting anecdote that Boris Gnedenko, who ultimately proved the three cases theorem of Fisher and Tippett (1928) in Gnedenko (1943), wrote the foreword to the Russian translation.

There exist some biographies about Emil J. Gumbel, these particularly highlight his role as an active pacifist and a witness to German history during the Weimar Republic, e.g. Jansen (1991), Vogt (1991), and Brenner (2001). The Ph.D. thesis of Sébastien Hertz contains a mathematical biography of Emil J. Gumbel and a historical account on the theory of extremes (see Hertz 1997). Gumbel's legacy, the "Emil J. Gumbel (1891–1966) Collection," is stored at the Leo Baeck Institute (LBI) in New York and is accessible online.

²In Gumbel (1958, Chapter 1, p. 23), Gumbel defines the return period as follows: "If an event has probability p , we have to make, on the average, $1/p$ trials in order that the event happen[sic] once. The mean $T(x)$ will be called the *return period*."

2 Interview on Emil J. Gumbel's last course

Question: How did you develop your interest in extreme value theory? What motivated you to register for the course by Emil J. Gumbel?

Answer: It was in 1964 and I was still taking graduate courses. I was interested in statistics, although I was a mining engineering student, but my interest has always been in statistics and its applications. While I was searching for interesting courses, a colleague of mine told me that there was this course on extreme value statistics that I might be interested in. Out of curiosity I wanted to register, but they said “you can't register before you talk to the professor.” So I said “fine.” I made an appointment with Professor Gumbel and went to see him. His first question was “Why do you want to take this course?” Normally, professors are welcoming the class saying “I'll do this, I'll do that,” but Gumbel was not interested in advertising his course. Instead, he was interested in finding out why students want to take this course. I told him, “I'm a mining engineer and I'm dealing with statistical issues in explorations and extremes sounds very interesting to me.” He said “All right, what's your background, what did you take?” So, I told him that I did some graduate courses in statistics, probability theory, and statistical inference. He said “This is fine.” We did not extend this discussion and he said “You can take the course and come to the class.” So, that was how it started. We were just a couple of students and that was the beginning of Gumbel's last course. It was supposed to last for two semesters.

Question: What was your very first impression of Emil J. Gumbel?

Answer: He was quite serious and sort of scared me a little bit. I said “Well, I'm taking the chance to that great course.” He was not chatting and he was not making small talk. He was quite serious and sounded like a very busy man. In our first meeting, he just wanted to find out what I am taking his course for. And he was satisfied. So, that first interaction was simple and straightforward, our relationship developed later as time went on. I even think my relationship with him increased after his death. That is another interesting thing ...

Question: How was the style of his lecture?

Answer: He hardly got up. He came into class, sometimes went to the blackboard, but basically he was sitting down and lecturing as he was sitting; and we were taking notes. After some classes he began to ask us questions, because we were supposed to have a background in statistical inference. So, he was probing and he was making sure that we knew what we were talking about. Most of the initial notes were basic statistical concepts. We knew them, but took notes anyway. But the class style was turning into questions and answers. Some questions were like riddles, difficult to understand, but we had to figure them out. Essentially, he made us develop extreme value theory from scratch by answering questions. He followed parts of his book “Statistics of Extremes,” but I did not have the book. Later, I think in the middle of the semester, he said “look, you probably don't have much money.” At that time the

book costed \$15, which was expensive in 1964. And indeed, I did not have much money. And he said, "I'll give you a book." So he gave me my first copy of "Statistics of Extremes" from the first edition – I still have it. Later, I bought two or three more books, second edition and a paper back. I also bought copies and gave them to students in my class.

Question: Was this teaching style typical for him, or was it caused by the small group size?

Answer: I have never seen anybody else doing this. Of course, I did not know his typical class structure, because I did not know anyone who took the course before. Because of the small class size, I guess, nobody talked about that in general.

Question: Do you remember specific problems he gave you to work on?

Answer: He used to talk about the Gompertz formula,³ and then he was asking questions. After giving the Gompertz formula, he would say "Well, does mean age decrease with age?⁴ And can you show it right there using a piece of paper?" So you have to take the Gompertz formula, which he called the force of mortality, develop a distribution function, and compute its mean and so on. There were a couple of other questions, I remember that one of them had no answer. The next week I went to see him and said "I can't answer this question." He said "Well, if you did, you would have gotten a Ph.D." It was about the density function corresponding to a complex characteristic function. Of course, most questions he was asking were taken from his book. One of them was a long one, it took about six pages of rigorous algebraic work. It is about order statistics,⁵ you find it around page 70 in his book. He expresses the distribution of the number of exceedances over the median⁶ and then he says "this is a function of k only multiplied by a function of k and z ." Finally he says, "from Stirling's formula, it follows from a straightforward calculation that when the limit k goes to infinity, this converges to $c \exp(-z^2/2)$. That means that the number of exceedances over the m -th value in the neighbourhood of the median in a large sample size and in n future trials asymptotically follows the normal distribution." The idea was to algebraically develop this. He gave us Stirling's formula, but the actual work was terribly laborious. You had to make sure to make no mistakes, expanding all factorials, and so on. The book has a lot of questions like this, also some simpler

³The Gompertz distribution has its roots in life testing and is used in various applications in reliability theory. Benjamin Gompertz assumed that the force of mortality (beyond infant mortality) increases exponentially with the age at death x , i.e. $\mu(x) = \beta \exp(\gamma x)$. In Gumbel (1958, Chapter 6.3.5), Gumbel relates the resulting distribution function to the first asymptotic probability for smallest values.

⁴This is a direct quotation from Emil J. Gumbel in class. More formally one might say that in the Gompertz model the average remaining time until death is decreasing with age.

⁵The origin of this result traces back to Gumbel (1935), one of Gumbel's first and most technical publications on extremes.

⁶We thank the referee for pointing out that in modern language, this concerns the limit distribution of a central quantile. A comprehensive and early source is Smirnov (1949); a translation by M.M. Day was published by the American Mathematical Society in 1952.

ones, but getting the answers requires, especially for an engineering student, hard work. Those were the typical questions he used to ask in class. Most of them were not that difficult, they were just “develop this, develop that,” and we did that to make sure we exercised our knowledge in reaching a particular conclusion that he would mention in the book.

Question: So essentially, he was giving you problems to create your extreme value theory from scratch?

Answer: Yes, step by step! My notebook does not have much explanations, except for some notes here and there, so probably I gave up writing down everything. So, the book became the bible.

Question: Was he referring to one of the extreme value limits as the Gumbel distribution?

Answer: No, we did not say Gumbel distribution. Nothing like that, other people did.⁷ We always referred to it as the first asymptotic distribution, we never said Gumbel. And the second asymptotic and the third asymptotic distribution. Others mentioned Gumbel, because nobody knew more about the first asymptotic than him. So, when people talked about the Gumbel distribution, they always meant the first type, $\exp(-\exp(-z))$.

Question: Was there an exam?

Answer: The answer is very simple: No exam. The exam was actually our performance in class when we were solving the questions. He used to come around and look over our shoulder. Sometimes he made comments, sometimes encouraging, sometimes not so encouraging [laughing]. But it was really like a workshop. A quiet class, a little lecture was given, but then a question was asked and everybody went on solving the problem. He used to go around and see how we were doing. Having someone looking over your shoulder is really stressful while answering questions, you are sweating out the answer [laughing]. We were glad not to have an exam, because the exam probably would be full of riddles.

Question: At Brooklyn College, he once failed the entire class (cf. Brenner 2001, p. 185) ...

Answer: I do not know what he was teaching there, presumably not extreme value statistics. [Remark: It was elementary statistics.] He probably could not tolerate the

⁷Early references using the name Gumbel are Potter (1949) and van Elteren (1953). Gumbel’s name was particularly used by hydrologists in connection with the first-type limit distribution, but also relating to its estimation method. Sometimes, even Gumbel-paper was used synonymous for the corresponding probability paper.

level.⁸ That is why I think he was questioning "How much do you know about statistics?" before you came to his class.

Question: Did you keep material from the course?

Answer: I have a notebook that starts nicely, with meticulous notes and general material, and then it gets sketchy. Some things are in there which are valuable. For example, we developed the three asymptotic cases in class. Of course, the main reference was always the book. So, my notebook is there and I keep it as a souvenir.

Question: We suppose that Gumbel spoke about the history of extreme value theory? Do you recall that he mentioned other researchers and their contributions?

Answer: Those are also mentioned in the book and, yes, he talked about them. But I did not take notes on this except for the Gompertz formula which is there for some reason, I think because he was talking about mortality statistics. He also mentioned Weibull somewhere and those who actually gave rise to the basic ideas behind extremes, but I do not have them in my notebook. He talked about people who made contributions to the basic ideas by simply mentioning their names or, as we developed certain sections, he referred to them. That is what I remember after 52 years.

Question: Did he mention particular real-world examples where he applied extreme value theory – like infrastructure projects in hydrology?

Answer: He gave us copies of monographs he did for the government.⁹ They were almost like instructions on how to do extreme value statistics by hand. When we talked about applications, he mostly talked about plots. Examples are in the book anyway, so that was our first introduction to applications. He encouraged us to look for applications and I did my first exercise on a flood case. But that was not my focus, because I was not a hydrologist. So, I just showed him the material, he says "that's great, that means you know what you are doing." For the first type Gumbel distribution, estimating parameters in those days –of course, no computers– was done by

⁸This was indeed the case. In a letter to Maurice Fréchet from August 30, 1948 he wrote –this is quoted from Hertz 1997, Annexe 1, p. 44– "L'enseignement à Brooklyn était une grande déception : Le niveau intellectuel était très bas. Même ceux de 20 ans et plus n'avait pas encore l'âge intellectuel d'un français lycéen de 17. C'était peu agréable." [translation: "Teaching in Brooklyn was a big disappointment: The intellectual level was very low. Even those aged 20 and over did not yet have the intellectual age of a French high school student at the age of 17. It was very unpleasant."] In a private conversation, Patricia Gumbel told us that Emil J. Gumbel was desperate about the results in the exam and let his son Harold try the exam (unprepared). When Harold easily did it, Emil J. Gumbel decided that it is justified to fail the entire class.

⁹In Gumbel (1954), the theory of extremes is applied to different areas, mainly focusing on hydrology, although Gumbel also mentions applications in climatological extremes, gusts, breaking strength of materials, extreme durations of human life, the intervals between radioactive emissions, and stock markets. He defines *floods* as annual *maxima* of the daily discharges and he underlines the importance of a practical method for estimating *flood* frequencies. For this purpose, he defines the concept of "the largest flood reached up to a certain time."

hand, including graphs. And parameter estimation was a difficult task. You had to do all calculations by hand so it was time consuming. But later, when I expressed my interest in extremal earthquakes, he said “oh, that’s interesting, if you work on it, maybe we can talk about it.” We did eventually talk about it. I started working on it with another colleague, a Geophysicist. We were interested in looking at the entire world, see region by region, to develop some statistical sense of extremal earthquakes. So, that was our first job, the paper was published in 1974 (cf. Yegulalp and Kuo 1974). I started it in 1965, on and off, left the US and came back, and finally did publish it. I had other things in mind which came out later, but not while I was taking the class or while he was alive.

Question: Did he mention infrastructure products like dams or bridges?

Answer: He did. It was going to be an important input for dam constructions, because dams should not be constructed on the average flow or the largest flow of a particular year, one should actually look into the future, n years from now, and see how do they do. My approach, in fact, looking at this and inspired from his drought paper, I said, “this is interesting, I should look at extremes of extremes, not just extremes.”

Question: Did he discuss moral aspects within such projects?

Answer: Not at all. I thought about it myself, because dams cannot last forever. They fill up with silt and dams are subjected to other forces rather than floods, earthquakes and so on. So, extremal flows, of course, force dams to collapse. There are things in the book where he is looking at the extreme, or the average, and tries to make some kind of parametric approach to see how the dam should be constructed. That is very difficult. He gave the seeds, but he never talked about specifics. All these things developed in my mind years later when I was teaching the same course. Also the first part of the book, the study of exceedances, could be applicable. Some people hardly mention this, but I consider it well done and always cover it in class, since it leads to everything. You cannot just study asymptotics.

Question: Did your field of study –Mining Engineering– play a role, e.g. with respect to examples and applications?

Answer: He did, of course, not know much about me, because I did the course in the fall of 1964 and spring of 1965 and he passed away the next year. He did not have an understanding of what mining might be. Earthquakes, he was related to. When I started working on this paper (Yegulalp and Kuo 1974), I had issues that the classical approach was not going to solve the problem and I went back to him. Our last meeting was at his home, still working on this topic.

Question: Why is his book “Statistics of Extremes” such a success?

Answer: From my point of view, the book Gumbel (1958) is, for one thing, difficult to read, but still it is a success. It is not an easy textbook, especially for those getting into extremes for the first time. It often says "it is obvious, that you can do this and that." It is not obvious when you just read it, and to get it obvious, you have to work another three hours to find out why it is obvious. Everybody that I know who wrote anything about extremes referred to the book, because there is always something in there you can go back to. Most of the lectures and books I have seen later expand on it rather than starting from scratch. It is based on thorough fundamentals and there is very little space wasted. Every sentence is important. Every line is important. You cannot read it, take the book and say "ok, let me edit out certain things." And it provides many references. I call it the bible of extremes, because you can start from there and you can expand.

Question: In his last years, Emil J. Gumbel worked on bivariate exponential distributions, the Gumbel copula bears witness to this (cf. Gumbel 1960). Did he mention this line of research in class?

Answer: That affected Chandan Kumar Mustafi¹⁰ more than anybody else. He published papers with Gumbel on this. But in class, I do not remember him talking about bivariate extremes.

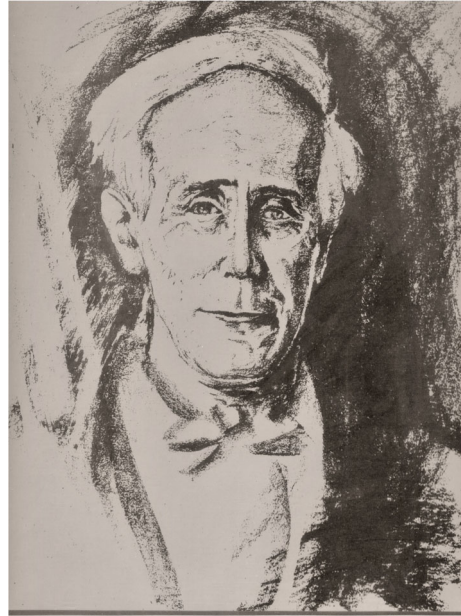
Question: When did you realize that he played a pivotal role in the pacifistic movement of the Weimar Republic?

Answer: I knew Gumbel escaped from Germany, but I did not have any idea about his political involvement when I was taking the class. Much later, in the mid-seventies or early eighties, I learned about it through a colleague of him – Professor Sebastian Littauer.¹¹ He was the chair of the department and instrumental in getting Gumbel to Columbia. Sebastian retired from Columbia but stayed on Campus, he was a consultant to our department when I joined the faculty in 1972. There, we shared a room for a while and we always talked about these things. We all talked about Gumbel. And then he gave me Einstein's book, the one in which Einstein talks about Gumbel (cf. Einstein 1935, Chapter: "The Liberty of Doctrine – À Propos of the Gumbel Case"). That is how Littauer mentioned that Gumbel escaped from Germany. I think they were Jewish. In addition to that, obviously, he had political views, which I did not know much about. But Littauer mentioned that it was dangerous at that time writing political articles and he was going to be persecuted and escaped. He also gave me Gumbel's picture, a charcoal sketch (cf. Fig. 3). That was given to me, because he knew that I was the only person related to Gumbel. So, I kept the picture. It was hanging in my office while I was at Columbia and I brought it back home with me when I retired.

¹⁰Chandan Kumar Mustafi did his Ph.D. in the department of Mathematical Statistics at Columbia University. He attended the same class. He published two papers with Emil J. Gumbel (Gumbel and Mustafi 1966, 1967).

¹¹Sebastian Littauer studied at MIT, finishing his Ph.D. in 1930 under the supervision of Norbert Wiener.

Fig. 3 Charcoal sketch of Emil J. Gumbel, unknown artist.
Courtesy of Tuncel M. Yegulalp



Question: Did he ever speak about politics in class or his feelings towards Germany?

Answer: No, not a word.¹²

Question: Besides Statistics, what else did you learn from Emil J. Gumbel?

Answer: I can answer in three words. Number one is patience. You have to be patient, you have to work on it. The second thing I learned is that you have to do hard work. It is not easy, and we learned it during our one-year period [laughing]. And also trust. Trust yourself. Trust yourself and with confidence you can do it. Those three things I learned and they are really important to me because you cannot just quit. That is what I learned in that course. In other courses we just got lectures, learned the material, answered the questions, passed the course, and it was done. Some actually left something behind for you to take with you for applications, for addition, and development. But here, we were also learning discipline, which I appreciated very much.

Question: Was he well integrated into the faculty? Do you know of friends and collaborators?

Answer: The only one I knew in the department, not in terms of collaboration but in terms of closeness, is Littauer. When I interacted with Littauer later as a colleague,

¹²This is consistent to what Gumbel did in Heidelberg in the 1920s as well as at FU Berlin in the 1950s. Gumbel divided his activities, mathematics in class and politics outside university.

sharing the office, he always talked about Gumbel. He must have been the closest person. Concerning others, I do not know, because I was a student.

Question: How did you call him in class?

Answer: Just professor, or Sir. It was very formal in those days, I used to go to class with tie and jacket. Gumbel was always wearing a bow tie. Gumbel himself was quite formal. There was no specific pressure involvement, but in a sense, you respected the person who was giving you all this information. When you solved a question, you could see that he was pleased. Years later when I talked to Littauer, he said, "well, there was a discussion about you when you were considered for the department." Littauer said about me "I do not know him personally, but Gumbel did and he said he was good." That was his description of me to Littauer. Littauer mentioned that to the faculty and, maybe, that is one of the reasons why they hired me. I do not know. But being good, I guess, in Gumbel's standards must be very good.

Question: In your career, you successfully applied extreme value theory. What do you owe Emil J. Gumbel in retrospective?

Answer: Everything I did until, let us say 15 years ago, had something to do with extremes. I was interested in applications and I was encouraging my colleagues who worked on other fields to get involved. In one major project, I had the idea of working on extremes of extremes rather than just on extremes. That rose the idea of forecasting extremes in terms of "what is the largest earthquake for the next n years?" and "of what is it a function?" (cf. Yegulalp and Kuo 1974; Yegulalp 1974). That is actually an extreme value distribution to the power of n and you can develop its properties, particularly mean and upper and lower bounds for the next years. The first time we did this was for earthquakes in Alaska, which was interesting. And one can also adapt it, because every year one gets new data, so one can update the forecast. Particularly in case of earthquakes or floods, forecasting is more important than just knowing the expected extreme or largest or smallest extreme. The same idea kept coming back when I got involved in experimental mechanics. We were doing a lot of experiments on testing rocks in terms of strength and, of course, rock failure under stress – this is an interesting phenomenon.¹³ One can logically relate it to a distribution of the minimum. We did extensive experimental work and showed that the third type minima can actually be used to describe the test results. If I were younger, I would work on the instability of parameters due to the climate change. For example, Gumbel type flood statistics are questionable if the system is not in steady state conditions. I was looking at that in class when I was giving assignments to students. I used to tell them "you can take a section of past data, estimate parameters and compare this with new sections of data in the future. If there is a significant change in the parameters, you explore some relationships as a function of some other climate indices." That is done now by others, I am sure.

¹³cf. Yegulalp and Wane (1969), Yegulalp and Mahtab (1983), Yegulalp et al. (1986), Mahtab and Yegulalp (1986), and Yegulalp and Kim (1994).

Question: After your return to Columbia University in 1972, you started to teach “Gumbel’s course” on extreme value theory. How did this come to be?

Answer: When I arrived, I was hired by a different department and I gave lectures that had nothing to do with extremes. They knew my background, so I was also teaching some statistics for our undergraduates. Some people did not like it, because there were other departments doing this, but we were focusing on engineering applications rather than general theory. I was also teaching a class on operations research, applying it to mining problems. Gumbel’s class was still in the catalog as a two semester class, but not being offered. A couple of professors approached me in the department chair office and said “well, we know about your background and relation to Gumbel. Can you teach this course for us?” I was very much interested in doing so. My department chair said “no, you shouldn’t do it, because you will defocus and you will probably not get any recognition for tenure just teaching a course.” So, they canceled the course. Two or three years later, I said “well, nobody is doing it, but I’ll do it.” And I reconstructed the course, not in his style, of course. It was for one semester, for engineers and earth sciences only, theory as well as applications. We developed the theory in one semester, basically the theory of exceedances, asymptotic theory, and exact theory. We focused on tools for applications. I did not give exams either, but I gave long projects. Everybody had to come up with their own project. It was data based and the students had to come up with their own extremal data to do the work. I taught that course until I retired.

Question: How was your last meeting with Emil J. Gumbel?

Answer: I was working on extremes of earthquakes, not getting anywhere. The problem was censored data. In the past, smaller earthquakes have not been measured or studied. Everything below 4 or 5 on the Richter scale was not recorded. We were looking at data starting from around 1900. The smallest observations were 5 or 6, but for some years there were no observations. So, I could not talk about sample statistics, I could not take averages, I could not compute variances. Every time I tried to do something, it just did not work. And I did not want to change the time period either, because what is the meaning of having an earthquake largest of every two years? I was stuck with this censored extremal data, but when I plotted the data –I did this by hand– I had the sense that the third type extremal statistic might work. I’m doing this, I’m doing that, doing everything again. Gumbel’s patience, hard work, if it doesn’t work, try it again. I finally went to him, 1965 spring or summer, because he was not in his office. He was at home, I was told that he was not feeling well. And I called him and he said “come over.” So I went to see him. He was in his bed, sitting on one side of the bed and the lady who was taking care of his house was there.¹⁴ She just served him lunch. He made me sit next to him and said “let’s have a look at what you got.” I explained it and he asked “did you try this, did you try that?” I said “yes, but still there is missing data.” He gave me a few more ideas and I said “thank you” and

¹⁴Presumably, this was his wife’s sister Erna.

left. He was not really talking about anything else, no small talk. None of the ideas did actually work out and I ended up with something else later. But that was the last time I saw him. I still remember it like yesterday, how nice he was, how gentle he was. And I just thought that he had a cold or something, he will come back to work. That will remain with me forever.

Question: When you went to his place, was it easy to communicate with him?

Answer: Yes, he was very easy to communicate with. He was formal, but that did not mean he put up barriers. As soon as I talked to him on the phone, he said "why don't you come over and talk to me?" I never had difficulties talking to him. Whenever I needed to talk to him in his office, he was always available. He was serious, but he was always there to answer questions. He probably did not display many emotions. Maybe in his private life he was different, but we did not have any emotional situation. He was calm when he answered questions. He was polite. Gentleman. Old-fashioned, wearing a bow tie. He expected respect and he got respect from his students.

Question: Emil J. Gumbel passed away on September 10, 1966.

Answer: I heard it immediately, of course, because when a professor dies, everybody knows about it in the department. I went to his funeral¹⁵ with a friend of mine, but I think I was the only student. Some faculty members were there, of course. It was a small crowd of 30 people. It was in New York, seventy second street, in a funeral home. It was a short event.

3 Tuncel M. Yegulalp

Tuncel M. Yegulalp is Professor emeritus of Mining Engineering at Columbia University. He received his MS in Mining Engineering from Istanbul Technical University (Istanbul/Turkey) in 1961 and his EngScD from Columbia University in 1968. He worked as research engineer at Mobil Research & Development Corporation, Paulsboro, NJ from 1967 to 1969; as manager of iron ore feasibility studies' group at Mineral Research and Exploration Institute (Ankara/Turkey) from 1969 to 1971; as chief systems engineer at SISAG Ltd., (Ankara/Turkey) from 1971 to 1972, and joined Columbia University as assistant professor in 1972. Professor Yegulalp specializes in application of statistical methods and operations research methodology in areas related to minerals industry and geosciences, particularly the analysis and forecasting of extremal events, statistical modeling of fracture systems, application of statistical methods in rock mechanics for data analysis and probabilistic design. His recent research interests include modeling of zero emission power plants, carbon capture processes while making hydrogen from carbon based fuels.

¹⁵ A funeral speech was given by C. Harrison Layton, (Layton 1966).

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