ESTIMATION OF NUTATION TERMS USING GPS

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ABSTRACT

Satellite space-geodetic measurements have been used since a long time to determine UT1-UTC rates (or length of day values). The estimation of nutation rates (in longitude and obliquity), however, was thought to be reserved to VLBI and LLR. It can be shown, that there is no fundamental difference between the estimation of UT1-UTC rates and nutation rates. Significant contributions to nutation by GPS may be expected in the high frequency domain, i.e., for periods below about 20 days.

CODE, the Center for Orbit Determination in Europe, started to estimate nutation rates in March 1994 using the data of the global IGS network. By now, the series of nutation rates from 3-day solutions has a length of about 3.5 years. From this series corrections to the coefficients of 34 nutation periods between 4 and 16 days have been determined. The resulting coefficients show an agreement of 10 μas with the IERS 1996 nutation model. The GPS results are very consistent with the most recent model by Souchay and Kinoshita, too. GPS thus allows an independent verification of theoretical nutation models and results from VLBI and LLR. A thorough description and discussion of the estimation of nutation amplitudes using GPS may be found in [*Rothacher et al.*, 1998].

INTRODUCTION

CODE, a cooperation of the Astronomical Institute, University of Berne (Switzerland), the Swiss Federal Office of Topography, Wabern (Switzerland), the Bundesamt für Kartographie und Geodäsie, Frankfurt (Germany), and the Institut Géographique National, Paris (France), started to derive celestial pole offset parameters (nutation rates) in March 1994 in order to study whether GPS could be used to contribute to nutation theory.

From a mathematical point of view it can be shown [*Rothacher et al.*, 1998] that the estimation of nutation rates in obliquity $\Delta\epsilon$ and longitude $\Delta\psi$ is very similar to the estimation of UT1-UTC rates: the offsets in all three components ($\Delta\epsilon$, $\Delta\psi$, and UT1-UTC) are fully correlated with the orbital parameters describing the orientation of the orbital planes of the satellites (ascending node, inclination, and argument of latitude) and unmodeled orbit perturbations lead to systematic errors in the rate estimates. Major biases may be expected at a period of one revolution of the satellites or at annual and semi-annual periods (orientation of the orbital plane with respect to the sun) due to solar radiation pressure.

With a simple variance-covariance analysis it is possible to deduce in what frequency range corrections to nutation amplitudes may be computed with sufficient accuracy using GPS nutation rate estimates. Assuming a continuous nutation rate series of 1280 days and an RMS scatter of 0.27 mas/d for the nutation rate estimates — values taken from the actual GPS series produced at CODE — we find that the formal error $\sigma(A_T)$ of the nutation amplitude A_T at a nutation period T (in days) grows linearly with the period according to:

$$\sigma(A_T) \approx 0.0017 \cdot T \ mas \tag{1}$$

When estimating nutation amplitude corrections from nutation *offsets*, as in the case of VLBI and LLR, the formal errors of the amplitudes are constant over a wide range of periods (i.e., for periods much longer than the typical spacing of the series and much shorter than the time interval covered by the series considered). From the literature ([*Herring et al.*, 1991], [*Charlot et al.*, 1995], [*Souchay et al.*, 1995], [*Herring*, 1997]) we obtain the formal errors of nutation amplitudes when using VLBI and LLR data. These formal errors are shown in Figure 1 together with those expected from GPS.

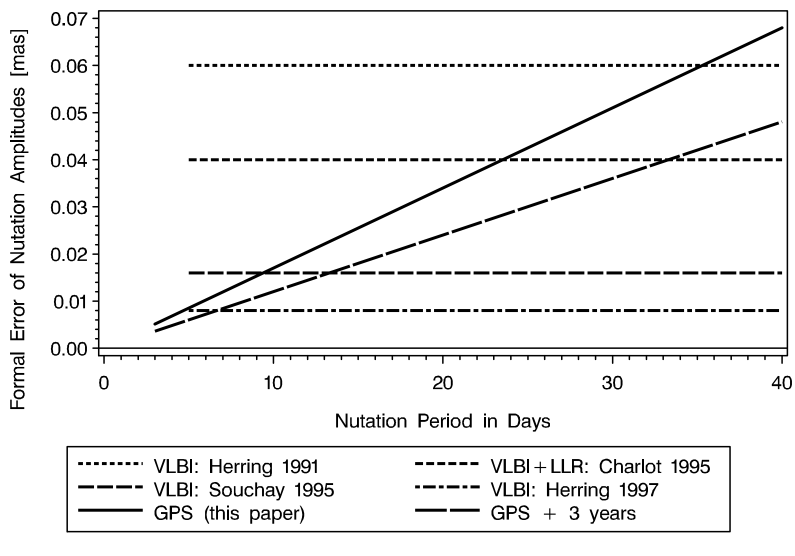


Figure 1. Precision of amplitude estimation from nutation offsets (VLBI and LLR) and from nutation rates (GPS) derived using a simple variance-covariance analysis.

Figure 1 clearly shows, that no major contributions to nutation theory may be expected from GPS for periods above about 20 days with the current orbit modeling. But GPS is in a good position to contribute at high frequencies (periods below 20 days). Let us mention that the VLBI formal errors will only slowly improve from now on. Another 13 years of VLBI data will be needed to reduce the formal errors by $\sqrt{2}$, whereas for GPS, a factor of $\sqrt{2}$ can be gained with another 3 years of data even without modeling improvements.

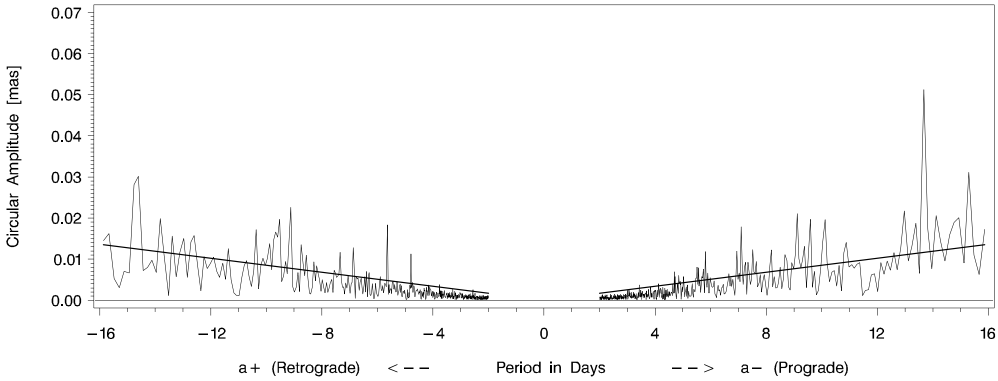
NUTATION RATE SERIES FROM GPS

The GPS nutation rate estimates were obtained from overlapping global 3-day solutions with 3-day satellite arcs using the data of up to 90 IGS sites. Over the three days one set of nutation rates was estimated in the two directions (obliquity and longitude) relative to the a priori nutation model, i.e. relative to the IAU 1980 Theory of Nutation (see [*McCarthy*, 1996]). It should be pointed out that rate estimates from 3-day solutions are more accurate than those from 1-day solution by about a factor of five. The reference frame was realized by heavily constraining 12 sites to their ITRF94 coordinates and velocities (see [*Boucher et al.*, 1996]). All other site coordinates were freely estimated. Troposphere zenith delays were determined for each site with 6-hour intervals. During the 3.5 years covered by the nutation rate series (from March 1994 to November 1997) two important modeling changes took place. First, starting in January 1995, the ambiguities for baselines with a length below 2000 km were fixed to integers (80-90%) and secondly, end of September 1996, the satellite orbit parameterization was changed from the "classical" radiation pressure model with two parameters (direct radiation pressure coefficient and y-bias) to the extended CODE orbit model [*Springer et al.*, 1998], where five parameters are routinely estimated in the 3-day solutions (constant radiation pressure coefficients in all three directions and periodic terms in X-direction). Both changes had an important effect on the nutation rate estimates. Whereas the ambiguity fixing improved the formal uncertainties of the rate estimates by almost a factor of three, the orbit model change deteriorated them by about the same factor. The worse formal uncertainties in the case of the new orbit parameterization is a consequence of the correlations between the nutation rates and the new radiation pressure parameters.

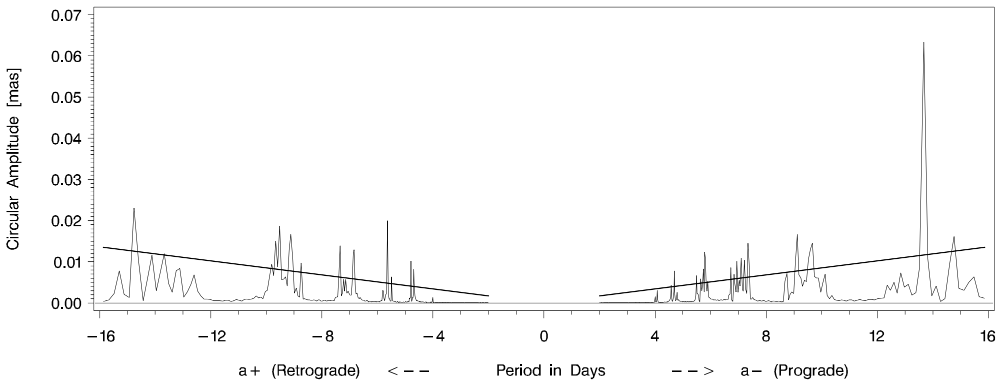
To give an impression of the type of signal contained in the GPS rate series, Figure 2a shows the high frequency spectrum derived from the nutation rate corrections relative to the IAU 1980 model. The *rate amplitudes* were thereby converted to actual amplitudes and transformed from $\Delta\epsilon$ and $\Delta\psi \cdot \sin\epsilon_0$ to amplitudes a^+ and a^- of *circular nutation* according to Eqn. (4) (see next section). For comparison the spectrum of the differences between the IAU 1980 and the IERS 1996 nutation model is depicted in Figure 2b. Many of the deficiencies of the IAU 1980 theory visible in Figure 2b, discovered by VLBI about a decade ago, are clearly seen by GPS, as well. The dashed lines in Figures 2 give the 1-σ uncertainties of the amplitudes estimates according to the Equation (1) (divided by a factor of 2 to account for the conversion to circular components of nutation).

ESTIMATION OF NUTATION AMPLITUDES

Starting with the GPS nutation rate corrections with respect to the IAU 1980 nutation model, a series of *total* nutation rates was generated by adding the rates given by the IAU 1980 model in order to obtain a series that is independent of the a priori model used.



(a) Spectrum of nutation corrections from GPS relative to the IAU80 model



(b) Spectrum of differences between the IERS96 and the IAU80 model

Figure 2. Spectrum of circular nutation amplitudes (see Eqn. (4) below) at low periods generated from (a) the GPS series of nutation rates converted to actual nutation amplitudes and (b) the differences between the IERS96 and the IAU80 model. The dashed lines indicate the 1-σ uncertainties of the amplitudes as expected according to Eqn. (1) (and (4)).

The nutation rate series was then used to estimate corrections to the nutation coefficients of a number of n=34 selected nutation periods relative to the more accurate IERS 1996 nutation model (IERS96) [*McCarthy*, 1996]. The corrections $\delta\Delta\epsilon$ and $\delta\Delta\psi$ in the nutation angles were thereby represented by

$$\delta\Delta\epsilon(t) = \sum_{j=1}^{n} (\delta\epsilon_{rj} \cos\theta_j(t) + \delta\epsilon_{ij} \sin\theta_j(t)) \qquad (2a)$$

$$\delta\Delta\psi(t) = \sum_{j=1}^{n} (\delta\psi_{rj} \sin\theta_j(t) + \delta\psi_{ij} \cos\theta_j(t)) \qquad (2b)$$

with θ_j denoting a combination of the fundamental nutation arguments, namely

$$\theta_j = \sum_{i=1}^{5} N_{ij} \cdot F_i \tag{3}$$

where N_{ij} are integer multipliers of the fundamental arguments $F_i \in \{l, l', F, D, \Omega\}$, also called Delaunay variables, and the angular frequency of the term j is given by $\omega := d\theta_j/dt$.

An alternative representation uses the circular components of nutation a^+_{rj}, a^-_{rj}, a^+_{ij}, and a^-_{ij}, which are related to the nutation coefficients in obliquity and longitude by

$$a^+_{rj} = -(\delta\epsilon_{rj} + \delta\psi_{rj} \sin\epsilon_0)/2 \tag{4a}$$

$$a^-_{rj} = -(\delta\epsilon_{rj} - \delta\psi_{rj} \sin\epsilon_0)/2 \tag{4b}$$

$$a^+_{ij} = -(\delta\epsilon_{ij} - \delta\psi_{ij} \sin\epsilon_0)/2 \tag{4c}$$

$$a^-_{ij} = +(\delta\epsilon_{ij} + \delta\psi_{ij} \sin\epsilon_0)/2 \tag{4d}$$

More details about the interpretation of the circular nutation components may be found in [*Herring et al.*, 1991].

The coefficients $\delta\epsilon_{rj}$, $\delta\epsilon_{ij}$, $\delta\psi_{rj}$, and $\delta\psi_{ij}$ were determined with a least squares algorithm using the nutation rates from the GPS analysis as pseudo-observations. Figure 3 shows the differences between the nutation coefficients estimated from the GPS nutation rates and the coefficients of the IERS96 nutation model. The shaded area represents the 2-σ error bars of the coefficients derived from GPS and shows the increase of these uncertainties with the nutation period.

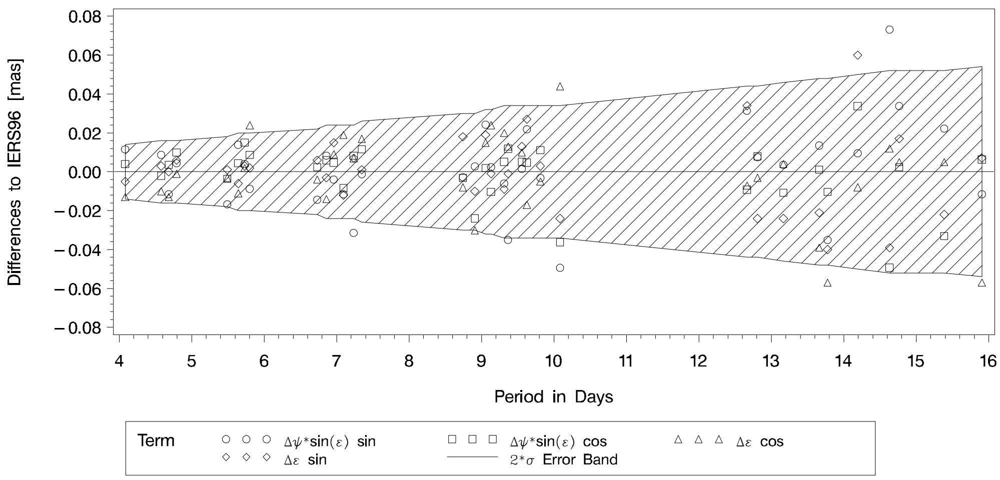


Figure 3. Nutation corrections relative to the IERS96 model for 34 periods estimated from the rate series obtained from GPS data. The shaded area represents the 95% confidence interval (2σ).

With a few exceptions all the nutation coefficients agree with the IERS96 nutation model at the level of twice the formal uncertainties (shaded area). The median agreement between the GPS results and the IERS96 model over all 136 coefficients amounts to about 10 μas. No major deviations from the IERS96 model can be detected by GPS. The actual values of the nutation coefficients from GPS for the 34 periods may be found in [*Rothacher et al.*, 1998].

A more detailed comparison of various VLBI and LLR results given in the literature and the GPS results with the most recent model by Souchay and Kinoshita (SKV972) [*Herring*, 1997] may be seen in Figure 4.

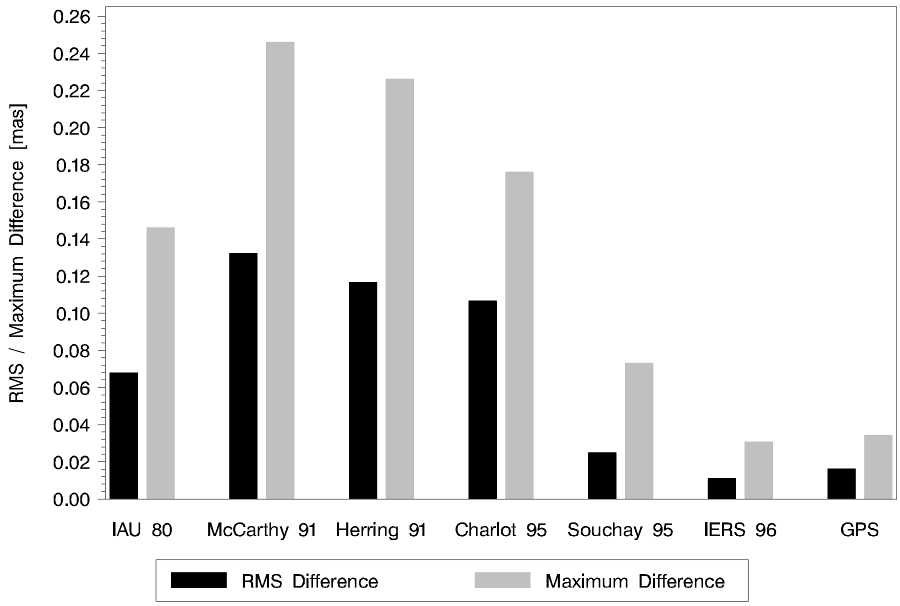


Figure 4. Rms difference and maximum difference over all terms of 4 major nutation periods, namely 13.66, 9.13, 14.77, 9.56 days, relative to the most recent model by Souchay/Kinoshita 1997.2 .

Apart from the IAU 1980 (IAU 80), the IERS96 model and the GPS results, the comparison involves the results from [*McCarthy and Luzum*, 1991] (combined analysis of 10 years of VLBI and about 20 years of LLR data), [*Herring et al.*, 1991] (9 years of VLBI data), [*Charlot et al.*, 1995] (16 years of VLBI and 24 years of LLR data), and [*Souchay et al.*, 1995] (14 years of VLBI data). Figure 4 depicts the rms differences as well as the maximum differences between these various results and SKV972 over all coefficients of the four major nutation periods at 13.66, 9.13, 14.77, 9.56 days (a total of 16 coefficients). We clearly see that the GPS results are in better agreement with the SKV972 model than most of the VLBI/LLR results.

A similar picture emerges when looking in detail at the coefficients of the 13.66 day period (see Figure 5), which is of special interest to geophysicists because of its large amplitudes. Again, the GPS results are very consistent with the results of the most recent model by Souchay and Kinoshita.

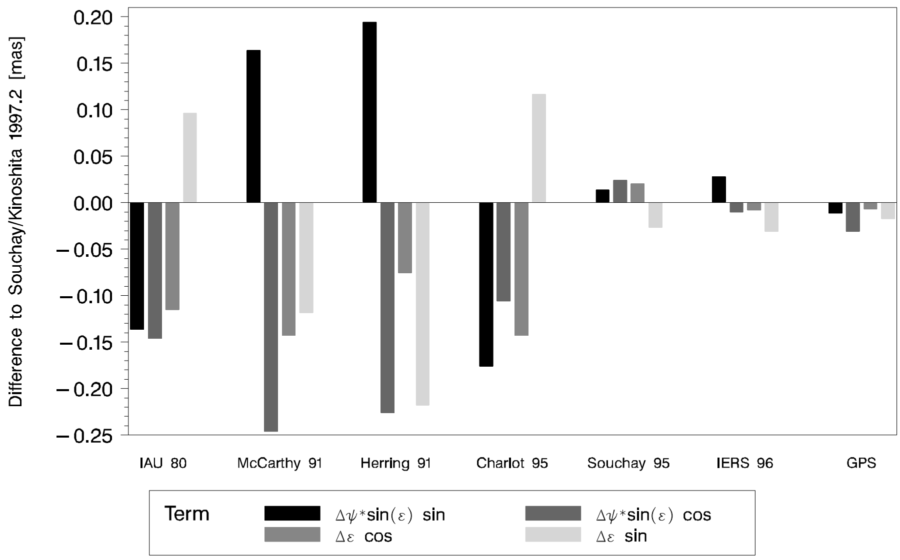


Figure 5. Comparison of the 13.66-day nutation coefficients from different sources with the most recent model by Souchay/Kinoshita 1997.2 .

CONCLUSIONS

From the above results we conclude that GPS may give a significant contribution to nutation in the high frequency range of the spectrum (periods below 20 days). The long term behavior is, however, reserved to VLBI and LLR. The nutation coefficients estimated from the GPS rate series show an overall agreement (median) of about 10 μas with the most recent nutation models by Souchay and Kinoshita. Using more refined orbit modeling techniques, carefully taking into consideration the correlations between the nutation rates and the orbital parameters, there is certainly much room for improvements. But already now GPS allows an independent check of present-day theoretical nutation models and VLBI/LLR results at the high frequency end of the spectrum. In future a combined analysis of VLBI, LLR, and GPS nutation series promises to give the most accurate nutation results.

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