A new Solar Radiation Pressure Model for the GPS Satellites *

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Abstract

The largest error source in GPS orbit modeling is due to the effect of the solar radiation pressure. Over the last few years many improvements were made in modeling the orbits of GPS satellites within the IGS. However, most improvements were achieved by increasing the number of estimated orbit and/or solar radiation pressure parameters. This increase in the number of estimated satellite parameters weakens the solutions of all estimated parameters. Due to correlations the additional parameters may cause biases in other estimated quantities like, e.g., the length of day.

In this paper a recently developed solar radiation pressure model for the GPS satellites is presented. This model is based on experiences and results acquired at the Center of Orbit Determination in Europe (CODE) Analysis Center during its IGS activities since June 1992. The new model outperforms the existing ROCK models by almost an order of magnitude. It also allows a reduction of the number of orbit parameters that have to be estimated!

Introduction

The largest non-gravitational effect on the GPS satellite orbits is the solar radiation pressure (RPR). To underline this, Table 1 shows the effect of different perturbations on the GPS orbits. The results in Table 1 are based on integrating a given set of osculating Keplerian elements over a time period of one day (24 hours) with the respective perturbations turned on or off. Table 1 gives the RMS of the differences between the resulting two orbits, one with the perturbation turned on and one with the perturbation turned off. The RMS was computed using the full 24-hour arc length and all satellites (using the full satellite constellation of January 1, 1998). It can be seen that the size of the perturbation caused by the solar radiation pressure is only exceeded by the effects of the Earth oblateness, the gravitational

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* **reference:** *IGS 1998 Analysis Center Workshop*, ESOC, Darmstadt, Germany, February 9--11 1998

Table 1. Effect of different perturbations on the GPS satellites over 24 hours

Perturbation	Magnitude (m)			
	Radial	Along	Cross	Total
Earth oblateness	1335	12902	6101	14334
Moon (gravitation)	191	1317	361	1379
Sun (gravitation)	83	649	145	670
C(2,2), S(2,2)	32	175	9	178
Solar Radiation Pressure	29	87	3	92
C(n,m), S(n,m) (n,m=3..8)	6	46	4	46

attraction by Sun and Moon, and the lower harmonics (C(2,2) and S(2,2)) of the Earth's gravity field. The results clearly show that an accurate solar radiation pressure model for the GPS satellites is as important as an accurate gravity model of the Earth.

The basis of the RPR models currently used was furnished by Rockwell International, the spacecraft contractor for the Block I, II and IIa satellites [*Fliegel* et al., 1992]. The computer programs that embody this model became known as ROCK4 for Block I [*Fliegel* et al., 1985], and ROCK42 for Block II and IIa satellites [*Fliegel and Gallini,* 1989]. They are also known as ‘‘the Porter models’’. The ROCK models are expressed in the satellite-fixed coordinate system. This system has its origin in the center of mass of the satellite. Its Z-axis points in the direction of the center of mass of the Earth, and therefore along the satellite antennas. The X-axis is positive toward the half plane that contains the Sun and the Y-axis completes a right-handed system and points along one of the solar panel beams.

For high precision geodetic work it is necessary to estimate a scale term and a force in the Y-direction, the Y-bias, in addition to using the ROCK model. The ROCK model, therefore, only serves as a priori information. Both, the scale term and the Y-bias, are parameters which are supposed to vary slowly in time. Although the cause for the Y-bias is unknown, its effect on the orbit is significant. The claimed accuracy of the ROCK models is about 3%. Taking the nominal value of $1 \cdot 10^{-7} m/s^2$ for the solar radiation pressure acceleration of a GPS satellite and the claimed accuracy of 3% for the T20 model, the expected error in acceleration is approximately $3 \cdot 10^{-9} m/s^2$. Furthermore the size of the Y-bias, which is not included in the ROCK models, is about $1 \cdot 10^{-9} m/s^2$. The effect of both error sources is about 3 meters (RMS) over 24-hours! Of course, we have to keep in mind that the ROCK models were developed for orbit estimation using pseudo-range data. With pseudo-ranges, position estimates with an accuracy of about 1 meter may be obtained. For this type of accuracy the ROCK models are adequate to serve as (a priori) model, provided, the scale term and the Y-bias are estimated.

The ROCK models are inadequate for IGS-type accuracies, e.g., centimeter-type orbit accuracies, even if the scale and Y-bias are estimated. This is also obvious from the additional orbit and solar radiation pressure parameters which most of the IGS ACs are estimating, be they deterministic and/or stochastic in nature. However, additional orbit parameters may weaken the GPS solutions significantly, especially the the LOD (length of day) estimates. Therefore it would be advantageous if an improved RPR model could be developed. The experiences gained from our IGS analysis efforts in recent years have indicated that it will indeed be possible to derive an improved RPR model [*Springer* et al., 1998].

Below we first present a summary of the most recent orbit results. Because these results are based on the extended CODE orbit model (ECOM) [*Beutler* et al., 1994], a short description of the ECOM is given as well. Secondly, results from orbit estimation using the GPS (IGS) precise orbits as pseudo-observations (orbit fit) are presented. Based on these results an ‘‘optimal’’ orbit parameterization is proposed. Using this parameterization and all CODE final orbits and EOPs, as submitted to the IGS, a long time series for the selected (optimal) set of RPR parameters is generated. The last two years (1996 and 1997) of this time series are then used to generate our new solar radiation pressure model. The article concludes with an evaluation of the quality of our RPR model and a short summary.

The Extended CODE Orbit Model

In *Beutler* et al. [1994] our orbit model, ECOM, is presented and discussed in detail. We only summarize its basic characteristics. The considerations behind the ECOM are similar to those underlying the Colombo model [*Colombo*, 1989]. The principal difference resides in the fact that the ECOM considers the Sun as the major ‘‘error source’’ for the orbits, whereas the gravity field of the Earth plays this role in the Colombo model. The Colombo model uses the radial, along-, and cross-track directions as the three orthogonal directions whereas, the D-, Y-, and B-directions are used by the ECOM. Notice that in earlier publication the B-axis was referred to as X-axis. To avoid confusion with the X-axis of the ROCK models, the B-axis is introduced here to designate the third axis of the ECOM. *Beutler* et al. [1994] demonstrated that the performance of the ECOM is superior to that of the Colombo model, which is a clear indication that solar radiation pressure is indeed the major error source in the GPS satellite orbit model. In the ECOM the acceleration $\vec{a}_{rpr}$ due to the solar radiation pressure is written as:

$$\vec{a}_{rpr} = \vec{a}_{ROCK} + \vec{a}_D + \vec{a}_Y + \vec{a}_B \tag{1}$$

where $\vec{a}_{ROCK}$ is the acceleration due to the ROCK model, and

$$\begin{aligned}
\vec{a}_D &= [a_{D0} + a_{DC} \cdot \cos u + a_{DS} \cdot \sin u] \cdot \vec{e}_D = D(u) \cdot \vec{e}_D \\
\vec{a}_Y &= [a_{Y0} + a_{YC} \cdot \cos u + a_{YS} \cdot \sin u] \cdot \vec{e}_Y = Y(u) \cdot \vec{e}_Y \\
\vec{a}_B &= [a_{B0} + a_{BC} \cdot \cos u + a_{BS} \cdot \sin u] \cdot \vec{e}_B = B(u) \cdot \vec{e}_B
\end{aligned} \tag{2}$$

where a_{D0}, a_{DC}, a_{DS}, a_{Y0}, a_{YC}, a_{YS}, a_{B0}, a_{BC}, and a_{BS} are the nine parameters of the ECOM, and

$\vec{e}_D$ is the unit vector Sun-satellite,
$\vec{e}_Y$ is the unit vector along the spacecraft's solar-panel axis,
$\vec{e}_B$ $= \vec{e}_Y \times \vec{e}_D, and$
u is the argument of latitude

The ECOM is a generalization of the standard orbit model which uses only two parameters to account for the solar radiation pressure, namely a_{D0} and a_{Y0}. Note that the Y-direction of the ECOM corresponds to the Y-direction of the body-fixed coordinate system. Although not really a solar radiation pressure model in the sense of the ROCK models, the ECOM does consider solar radiation pressure to be the major perturbing force acting on the GPS satellites. Therefore the ECOM provides an excellent tool to study the effects of the solar radiation pressure on the GPS satellites. It allows to detect in which direction the most significant unmodeled RPR forces act on the GPS satellites.

There are two methods to study the effects different parameters of the ECOM have on the orbit estimates. **The first**, and most reliable **method** is to use the ECOM in real orbit estimation procedures using GPS observations, very much like the routine orbit estimation performed at CODE as part of its IGS activities. **The second method** is to use the orbits as provided by CODE as ‘‘pseudo-observations’’ estimating an arc extending over several days which gives the best fit, in a least squares sense, to the observations. This second method is less correct but computationally much more efficient than the first. The generation of a 3-day arc using the ‘‘orbit fit’’ method typically takes 1 minute whereas the ‘‘orbit estimation’’ method will take several hours. Results from both methods, orbit estimation and orbit fit, will be discussed in the following sections.

Orbit estimation using GPS observations

In 1996 the ECOM was fully implemented into the Bernese GPS Software. It was expected that not all nine parameters of the ECOM can (and should) be estimated when estimating 3-day arcs from real GPS data. Initial tests [*Springer* et al., 1996] indicated that it is best not to solve for ‘‘B-terms’’, but to estimate the constant and periodic terms in the D- and Y-directions plus small velocity changes (pseudo-stochastic pulses) in the radial and along-track directions. A careful analysis of this parameterization showed that it leads to a significant degradation of the quality of the LOD estimates. It was therefore decided to systematically test the different parameters of the ECOM in order to find the ‘‘optimal’’ parameterization. In *Springer* et al. [1998] a detailed description of the results from two extensive tests using the ECOM is given. The difference between the two extensive test series is, that in the first test series pseudo-stochastic pulses [*Beutler* et al., 1996] were always estimated (stochastic test series) whereas in the second test series they were never estimated (deterministic test series).

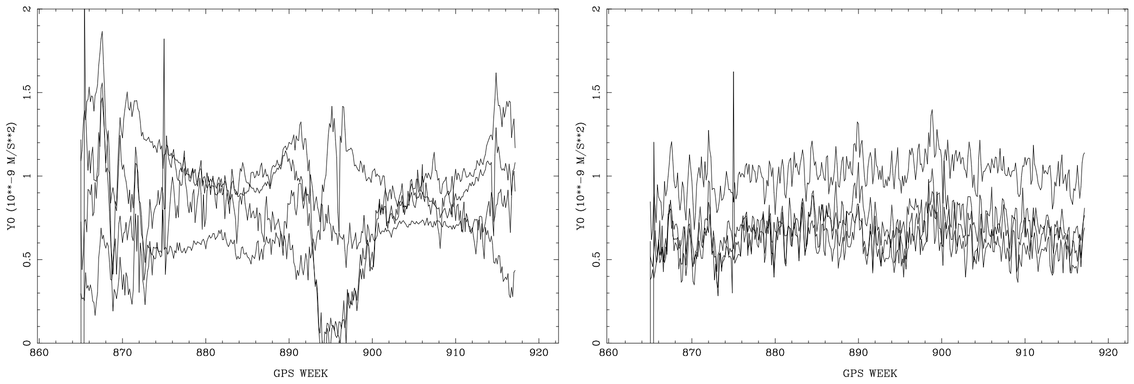
The stochastic test series showed that the estimation of the constant and periodic terms in the B-direction, in addition to the estimation of the constant terms in the D- and Y-direction, and the pseudo-stochastic pulses in the radial and along-track directions, significantly improves the quality of the orbit estimates. An improvement was seen in all estimated parameters: orbits, station coordinates, and EOPs. Only for the LOD estimates a small, but significant, degradation in quality was observed. The improvement of the orbit quality was estimated to be a factor of two to three, compared to the results without estimating the three terms in the B-direction! As a direct consequence of these tests the estimation of the B-terms was implemented for the generation of the CODE contributions to the IGS on September 29, 1996.

The second, deterministic, test series confirmed that the periodic terms in the B-direction most significantly reduce the orbit model deficiencies. Evidence was presented that the periodic signals in the Y-direction reduce the orbit model deficiencies as well. The periodic signals in the B-direction, however, were shown to be more important than those in the Y-direction. The deterministic test series further showed that a purely deterministic orbit parameterization, consisting of the constant terms in the D- and Y-directions plus periodic terms in the D- and B-directions, gives excellent orbit results. Because of a degradation of the LOD estimates this deterministic orbit model is currently not considered for the IGS activities at CODE.

The results based on one full year of routine orbit estimates using the standard (R3) and new (X3) orbit parameterizations (3 B-terms) showed that the behaviour of the estimated stochastic and deterministic orbit parameters significantly improves with the new orbit parameterization. This is true in particular for the Y-bias (see Figure 1), and the radial pseudo-stochastic pulses (see Figure 2). Tests without estimating the radial pseudo-stochastic pulses showed that with the new orbit parameterization these pulses no longer have to be estimated. Considering the fact that pseudo-stochastic pulses are meant to absorb orbit model deficiencies it is clear that the modeling deficits are significantly reduced in the new orbit parameterization. Because the behaviour of all estimated RPR parameters in the X3 solutions is ‘‘predictable’’ it is expected that an improved RPR model may be developed.

Orbit estimation using satellite positions as pseudo-observations

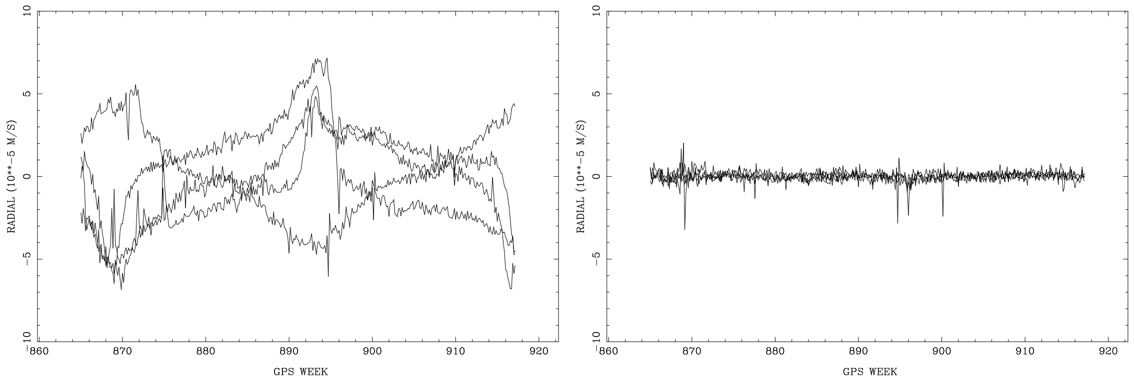
The essential difference between orbit estimation, using real (GPS) data, and orbit fit, using previously determined satellite positions as pseudo-observations, lays in the type of observations used. For the orbit estimation double-difference GPS carrier phase observations are used whereas for the orbit fit the observations are the position vectors of the satellites. The position vectors are very strong observations for orbit estimation whereas the double-difference carrier phases do not contain very much information about the satellite position. Only thanks to the dense global network it is possible to get accurate GPS orbit estimates based on carrier phase data. Due to this significant difference the



(a) Standard Orbit Parameterization

(b) New Orbit Parameterization

Figure 1. Estimated Y-bias (Y_0) using the two different CODE orbit parameterizations. Only PRNs 3, 6, 7, and 31 in orbital plane C are shown.



(a) Standard Orbit Parameterization

(b) New Orbit Parameterization

Figure 2. Estimated radial pseudo-stochastic pulses using the two different CODE orbit parameterizations. Only PRNs 3, 6, 7, and 31 in orbital plane C are shown.

results from orbit fit tests always have to be verified using real GPS data analysis.

The goal of the orbit fit tests was to find the optimal orbit parameterization. For this purpose a ‘‘standard test’’ was developed in order to be able to compare the results. The selected standard test consists out of a 7-day orbit fit using the CODE final products, e.g., precise orbits plus their respective Earth Orientation Parameters (EOP). The resulting 7-day arc is extrapolated, using orbit integration, for 48 hours. The last 24 hours of this orbit prediction are compared to the CODE final orbit for the same day. The period from March 13 to March 21 in 1997 was selected as test interval. The following quantities are considered as quality indicators for the orbit parameterization:

- the RMS of the residuals of the 7-day fit,
- the RMS of the residuals of the orbit prediction comparison, and
- the median of the residuals of the orbit prediction comparison.

First, we studied whether our standard orbit fit test gives similar results as the deterministic orbit estimation test discussed earlier. It was verified that the results were indeed very similar. Only one small anomaly was detected in the estimation of the periodic terms in the Y- and B-direction. In the orbit fit the effect of the periodic terms in these two directions are almost identical whereas in the orbit estimation test a significant difference was observed favoring the periodic terms in the B-direction.

As mentioned above the orbit fit method is based on a strong observation type making it possible to estimate a large number of orbit parameters. Therefore, our software was enhanced to estimated periodic terms up to six times per orbital revolution. Furthermore, modifications were made to allow for periodic terms in two other coordinate systems: the satellite-fixed reference frame (Z, Y, X) and the ‘‘classical’’ orbit system radial, along-, and cross-track (R, S, W). In addition, the argument for the periodic terms was slightly changed to account for the position of the Sun with respect to the ascending node. This change is a consequence of the assumption that the solar radiation pressure is the major ‘‘error source’’ in the GPS orbit modeling. It is therefore logical to relate the time argument of the periodic signals to the position of the Sun in the orbital plane. Thus, the argument of latitude is corrected for the argument of latitude of the Sun in the orbital plane (u_0), [*Rothacher* et al., 1995]. After extensive tests, using many different combinations of the available parameters, a small set of optimal orbit parameterizations was found. Table 2 lists these ‘‘best’’ parameterizations.

All candidate parameterizations were subsequently used in real orbit estimation using one full week of GPS data. This test confirmed that all 5 parameterizations perform very well apart from some correlation with the LOD. Because of the slightly better performance and its resemblance with the ROCK model, ‘‘model 5’’ (Table 2) was selected as the ‘‘optimal’’ orbit parameterization. It consists of three constant terms in the D-, Y-, and B-directions

Table 2. Selected ''optimal'' orbit parameterizations.

Model	Constant Terms	Periodic Terms
1	D, Y, and B	B sin$(u - u_0)$ and D sin$(u - u_0)$
2	D, Y, and B	B sin$(u - u_0)$ and B sin$(2u - u_0)$
3	D, Y, and B	Z sin$(u - u_0)$ and X sin$(u - u_0)$
4	D, Y, and B	Z sin$(u - u_0)$ and X sin$(3u - u_0)$
5	D, Y, and B	Z sin$(u - u_0)$, X sin$(u - u_0)$, and X sin$(3u - u_0)$

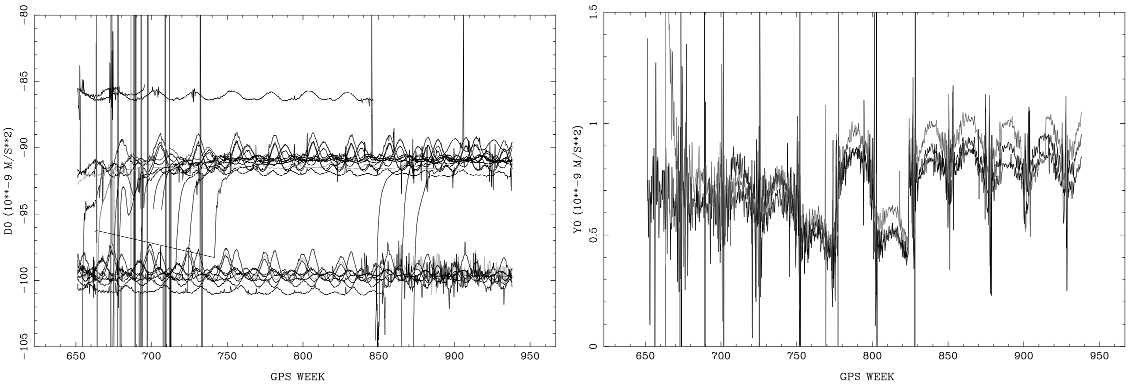
and three periodic sine terms: once-per-revolution terms in the Z-, and X-direction and one three-times per revolution term in the X-direction.

The New Solar Radiation Pressure Model

Using the ''optimal'' orbit parameterization (model 5 in Table 2) all final CODE orbits with their respective EOPs, as submitted to the IGS since June 1992, were used in an orbit fit. An arc length of 5 days was chosen and **no** a priori solar radiation pressure model was used. This resulted in a long time series, covering 5.5 years, of estimates for the selected (optimal) set of RPR parameters. It was hoped that, after careful analysis, this time series may be used to derive a new solar radiation pressure model. Figure 3 shows the estimated values for the direct solar radiation pressure (D_0) and for the Y-bias (Y_0) accelerations as function of time over the full 5.5 years. Jumps are visible in the Y-bias time series. These jumps are related to the ''bias'' changes in the attitude system of the GPS satellites. These biases have been kept constant since approximately November 1995. Futhermore, the eclipse phases can clearly be seen in the Y-bias estimates: the estimates are somewhat anomalous during these phases.

A careful analysis of the estimated parameters as a function of time showed that the behaviour of satellites within one orbital plane is very similar. Clear annual and semiannual signals are present. Assuming that the Sun causes the observed signals it is logical to study the behaviour of the RPR parameters as function of the angle of the Sun above the orbital plane (angle β_0). Note that, if the absolute value of this angle is $\leq 14^o$, the satellite is in eclipse. In Figure 4 the same two time series for the direct solar radiation pressure and Y-bias accelerations are shown but now as function of the angle of the Sun above the orbital plane. For the Y-bias a shorter time interval was selected to exclude the observed jumps, but more satellites are shown.

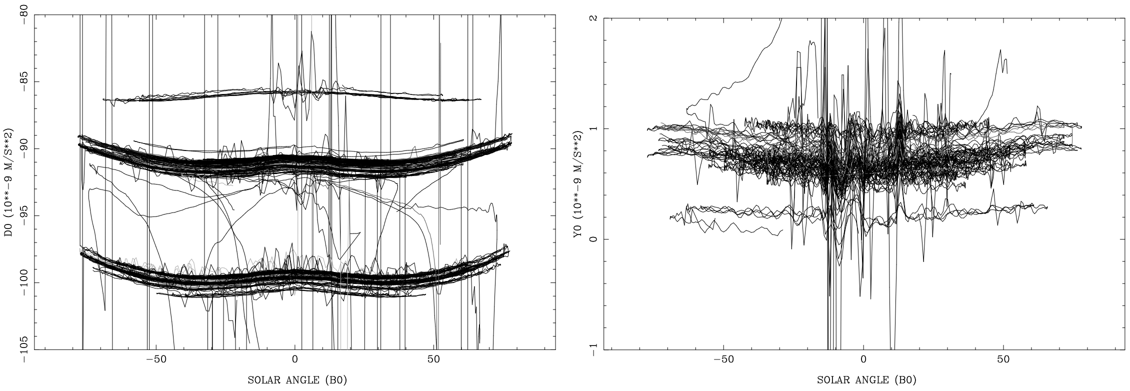
Clearly, the behaviour of the estimates of the direct solar radiation pressure acceleration and the Y-bias acceleration is very similar for all (Block II and IIa) satellites which indicates that a model can be easily derived for these parameters. The same is true for the constant term in the B-direction and for the once per revolution periodic term in the Z-direction. Both periodic signals in the X-direction do not show a very clear signal, nevertheless a model was estimated for these parameters as well. The Block I satellites, the uppermost



(a) Direct solar radiation pressure acceleration for the Block I, II, and IIa satellites

(b) Y-bias acceleration for the satellites in orbital plane A (PRNs 9, 25, 27)

Figure 3. Estimated direct solar radiation pressure acceleration (D_0) and Y-bias (Y_0) acceleration as function of time over the interval from June 1992 to 1997



(a) Direct solar radiation pressure acceleration for the Block I, II, and IIa satellites

(b) Y-bias acceleration for the Block II and IIa satellites

Figure 4. Estimated direct solar radiation pressure acceleration (D_0) and Y-bias (Y_0) acceleration as function of the angle of the Sun above to the orbital plane. (for D_0 the complete interval from June 1992 to 1997 is shown whereas for Y_0 only the last two years (1996, 1997) are included)

lines in both plots showing the D_0 estimates, behave in a slightly different way. Although their behaviour is also predictable, no attempts were made to create a model for the Block I satellites. Due to the ‘‘jumps’’ in the estimates of the Y-bias, the model for this parameter had to be based on the estimates since 1996 only. It turned out that the performance of the complete model was better if all model parameters were uniquely based on the most resent results (since 1996). Apparently the (significant) modeling improvements made over the last few years are important when deriving a solar radiation pressure model.

Based on the careful analysis of the orbit fit results the following terms were included in the radiation pressure model:

$$\boxed{\begin{array}{lcl} a_D & = & D_0 + D_{C2}\cos(2\beta_0) + D_{C4}\cos(4\beta_0) \\ \\ a_Y & = & Y_0 + Y_C\cos(2\beta_0) \\ \\ a_B & = & B_0 + B_C\cos(2\beta_0) \\ \\ a_{Zp} & = & \{Z_0 + Z_{C2}\cos(2\beta_0) + Z_{S2}\sin(2\beta_0) \\ & & +Z_{C4}\cos(4\beta_0) + Z_{S4}\sin(4\beta_0)\}\sin(u-u_0) \\ \\ a_{Xp} & = & \{X1_0 + X1_C\cos(2\beta_0) + X1_S\sin(2\beta_0)\}\sin(u-u_0) \\ & & +\{X3_0 + X3_C\cos(2\beta_0) + X3_S\sin(2\beta_0)\}\sin(3u-u_0) \end{array}} \qquad (3)$$

Note that all three constants ($D_0,\ Y_0,\ B_0$) were chosen to be satellite-specific and that the Z_0-term was chosen to be Block type dependent. The values for all of the above parameters are given in the Appendix. Please note that the model is only valid for Block II and IIa satellites. Furthermore, the values given for PRN8 should be used with care because this satellite was launched late in 1997 and by the end of the year was still in its ‘‘outgassing’’ phase. Of course, satellite PRN23 should be used with care as well due to the problems with the orientation of its solar panels. The results indicate, however, that it should be possible to derive a tailored RPR model for PRN23.

Evaluation of the New Solar Radiation Pressure Model

Four different investigations were performed to evaluate the new solar radiation pressure model.

- The effect of the parameters of our RPR model on the satellite positions was determined to get an idea of the significance of the individual terms of the model.
- An error budget of the model was derived, based on the residuals of the RPR series, to get an idea of the remaining model errors.
- The model was compared, using our standard test, to other RPR models to check its performance.
- Finally, the model was tested in a real parameter estimation, using one full week of GPS observations.

The effects of the different parameters of the new RPR model on the orbit were estimated by integrating a given set of osculating Keplerian elements over a time period of one day (24 hours), once with the parameter turned on and once with the parameter turned off. The RMS of the difference between the two resulting orbits, over the full 24 hour period, was then computed to get an idea of the size of the effect. The results are given in Table 3.

Table 3. Effect of the individual parameters of the new RPR model on the GPS satellite orbits over 24 hours.

Parameter	Effect			
	Radial	Along	Cross	Total
D_0 (m)	29	87	3	92
Y_0 (cm)	49	350	8	354
B_0 (cm)	2	29	3	29
$Z\sin(u-u_0)$ (cm)	15	32	0	36
$X\sin(u-u_0)$ and $X\sin(3u-u_0)$ (cm)	2	11	0	11

As expected the D_0 (direct solar radiation term) and Y_0 (Y-bias) give the largest contributions. However, the contributions of the B_0-term and the periodic term in the Z-direction (radial direction!) are not negligible! Note that the periodic Z-term has a signature very similar to the periodic terms in the B-direction, which CODE uses when computing its IGS orbit products. The periodic terms in the X-direction have an effect of only 11 cm. The typical RMS of the 5-day fits, used for the model development, is at the level of 5 cm. This means that the 11 cm effect is close to the noise level of the solutions. However, the IGS orbit combinations show an orbit consistency of about 4 cm between the orbits of different ACs. Thus, an 11 cm effect may be significant.

The RPR model is based on the time series of parameter estimates computed by fitting 5-day arcs through the final products from CODE. The RMS of the residuals of the parameter estimates, after subtracting the estimated RPR model, is used to estimate the remaining errors in the model. For this purpose the RMS value was introduced as a ‘‘bias’’ in the corresponding RPR parameter and a 24-hour orbit integration was performed with this bias included. The difference between the biased orbit and the original orbit are a measure of the remaining orbit model errors. The results are given in Table 4. The total error budget was estimated by introducing the RMS value for all parameters as bias.

Table 4. Estimated model errors based on the parameter residuals

Error Source	Model Fit $(10^{-9} m/s^2)$	Magnitude (cm) Radial	Along	Cross	Total
D_0	0.0724	2	7	0	7
Y_0	0.0416	2	15	1	15
B_0	0.2318	1	15	2	15
$Z\sin(u-u_0)$	0.1187	2	4	0	4
$X\sin(u-u_0)$	0.1454	5	36	1	36
$X\sin(3u-u_0)$	1.5252	8	61	2	62
Total Error budget		11	79	4	79
RMS of 7-day fit (no par. est.)					52

Surprisingly enough the largest error source stems from the two periodic terms in the X-direction. This is remarkable in view of the very small effect these parameters have on the orbit. The estimated errors from the other parameters are all below the 20 cm level. The total error budget is estimated to be about 80 cm. To verify this, our standard test was used without estimating any parameters (except the 6 osculating Keplerian elements). The RMS of this 7-day fit may then be comparable to the estimated model error. The results are comparable but the error budget seems to be somewhat pessimistic. This may be caused by the relatively large error of the X-periodic terms. Also, the arc length of our standard test (7 days) is longer than the arc length used for the RPR parameter estimates (5 days). Therefore the remaining orbit model error is estimated to be of the order of 50 cm only!

Apart from CODE also the JPL analysis center has successfully developed a new RPR model [*Bar-Sever*, 1997]. To test the performance of different RPR models our standard test was used once more. Table 5 gives the results of the standard test using the different RPR models available: ROCK, JPL, and CODE. It shows the RMS of fit using 7 days of precise orbits and the RMS and median of the residuals of the prediction comparison. Again the CODE final products (orbit and EOPs) were used. In all cases only the scale term (or a constant acceleration in the direction sun-satellite (D_0)) and the Y-bias (Y_0) were estimated. Only for the solution labelled ‘‘BEST’’ more RPR parameters (all 9 parameters of the ECOM) were estimated. This solution is given as a reference. Furthermore, the ROCK model was used in two different ways. First, it was used as a priori model and the accelerations D_0 and Y_0 were estimated on top of the model (solution: T20), which represents the way the ROCK model is normally used in the Bernese GPS Software. Secondly, it was used by estimating a scale factor for the complete ROCK model and the acceleration in the Y-direction (solution: T20 scaled), which represents the recommended usage of the ROCK model.

Table 5. Orbit Fit (7 days) and orbit extrapolation (2 days) using different RPR models. Only scale (or D_0) and Y-bias estimated.

RPR-MODEL	RMS of FIT (cm)	Prediction Median (cm)	RMS (cm)
No Model	75	133	159
T20	76	134	161
T20 Scaled	72	119	151
JPL Scaled	10	45	58
CODE	6	17	31
‘‘BEST’’ (9 RPR par.)	5	17	22

Table 5 shows that including the ROCK model as a priori RPR model does hardly gives any improvement, both in the fit and in the prediction, compared to not including an a priori model. Although it was clear for a long time that the ROCK models are not very accurate, this is a surprise! Both the CODE and JPL RPR models perform much better than the ROCK model. The results of the CODE model are close to the ‘‘best possible’’ results. This means that the reduction of the number of estimated RPR parameters (from 9 to 2!), does not significantly degrade the accuracy of the results. This reduction of parameters should make the GPS orbit predictions more reliable! This is very important because it has become clear that the integrity of the predicted orbits is the most crucial factor for real time GPS data analysis [*Martin Mur* et al., 1998].

Finally, the new RPR model was tested in a real GPS data processing experiment using one full week of data (7 days of 3-day solutions). Four different solutions were generated. For the first two solutions our standard ($D_0,\ Y_0$) and new ($D_0,\ Y_0,\ B_0,\ B_p$) orbit solutions were generated using the ROCK model as a priori model. For the second two solutions the same two orbit solutions were generated but now using the new CODE RPR model as a priori model. The results are given in Table 6.

Table 6. Results from real GPS data analysis using both the ROCK and CODE RPR models

	ROCK + $D_0\ Y_0$	CODE + $D_0\ Y_0$	ROCK + $D_0\ Y_0\ B_0\ B_p$	CODE + $D_0\ Y_0\ B_0\ B_p$
Orbit Overlap (mm)	106	34	31	32
Orbit Comparison (mm)	66	54	50	51

A significant improvement can be seen for the standard solution ($D_0,\ Y_0$). In fact, the standard solution using the CODE model has become almost as good as the two X3-type ($D_0,\ Y_0,\ B_0,\ B_p$) solutions. This is an important result because it means that three orbit parameters (the three B-terms) become obsolete! The slight difference in quality is most likely caused by the eclipsing satellites, which are not treated in any special way in the CODE model. The similarity in quality of both X3 ($D_0,\ Y_0,\ B_0,\ B_p$) solutions shows once more that the periodic B-terms behave very similar as the periodic Z-terms.

Summary and Outlook

It has been shown that the new solar radiation pressure model as developed by CODE is superior to the ROCK model by almost one order of magnitude. The remaining model error was estimated to be about 50 cm, whereas for the ROCK model the error is about 300 cm. Although a significant improvement could be achieved with the new RPR model, it should be considered as a ‘‘first attempt’’ only. In the near future more time and effort will (have to) be spent on RPR models. Different models are required for the Block I and IIr satellites and also for PRN23. Furthermore, the behaviour of some of the parameters of the RPR model is significantly different, but not erratic, during the eclipse phases. Therefore, it is very likely that a special eclipse model may be derived. One minor problem was discovered in the model. The so-called X3 orbit solutions, the official CODE solutions since September 1996, show a small scale difference with respect to the standard (R3) solutions and also with respect to the IGS combined orbit. Because the model was based mainly on our X3 orbits (only for the first few months of 1996 the R3 solutions were used) this scale effect has propagated into the RPR model. This means that all orbit estimates generated with this new RPR model will have a small scale difference of approximately 0.2 ppb (5 mm). Keep in mind that the ‘‘true scale’’ is not known.

The implementation of the CODE RPR model may improve the quality of the orbit estimates. In addition, the number or required (orbit) parameters may be reduced. This will strengthen the GPS solutions significantly. Especially the generation of the so-called ‘‘rapid’’ products may profit from this development. The predicted orbits may also improve; maybe not in accuracy, but certainly in integrity, thanks to the reduction of the required number of orbit parameters. Last but not least we hope that the model will enable us to generate GPS orbits based on SLR observations only. So far, the limited number of SLR observations and the large number of required orbit parameters made it almost impossible to generate accurate GPS orbits based on SLR data only. With our new RPR model we are are in a much better position because, it allows a 7-day fit at the 6 cm level solving for only two parameters.

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Appendix

This Appendix gives some statistics of the RPR model estimation and the values of the CODE solar radiation pressure model.

Table 7. Results from the CODE solar radiation pressure model estimation

Parameters	#Est.	RMS ($10^{-9}m/s^2$)
D_0	15961	0.0724
Y_0	15815	0.0416
B_0	15406	0.2318
$Z\sin(u-u_0)$	15348	0.1187
$X\sin(u-u_0)$	15187	0.1454
$X\sin(3u-u_0)$	15760	1.5252

Table 8. The values of the CODE solar radiation pressure model

Parameters	Estimate ($10^{-9}m/s^2$)	Formal Error ($10^{-11}m/s^2$)
D_{C2}	-0.813	0.176
D_{C4}	0.517	0.124
Y_C	-0.067	0.104
B_C	0.385	0.572
Z_0 Block II	1.024	0.299
Z_0 Block IIa	0.979	0.184
Z_{C2}	0.519	0.248
Z_{S2}	0.125	0.149
Z_{C4}	0.047	0.261
Z_{S4}	-0.045	0.164
$X1_0$	-0.015	0.157
$X1_C$	-0.018	0.297
$X1_S$	-0.033	0.168
$X3_0$	0.004	1.655
$X3_C$	-0.046	3.118
$X3_S$	-0.398	1.773

Table 9. The values for the CODE solar radiation pressure model. The values for PRN8 and PRN13 should be used with care. PRN8 was launched by the end of 1997 end was still in its ‘‘outgassing’’ phase. PRN13 was also launched by the end of 1997 and is a completely new type of satellite (Block IIr). This new Block type most like will have a different solar radiation pressure model.

PRN	Block	D_0 $(10^{-9} m/s^2)$	Y_0 $(10^{-9} m/s^2)$	B_0 $(10^{-9} m/s^2)$
2	II	-99.373	0.6362	0.0480
14	II	-99.290	0.9064	-0.2510
15	II	-98.985	0.7048	-0.4749
16	II	-99.108	0.6496	-0.1170
17	II	-99.010	0.6604	-0.0770
18	II	-99.359	0.8683	-0.4783
19	II	-99.850	0.7057	-0.1449
20	II	-100.396	0.6642	-0.4997
21	II	-99.477	0.2592	0.0996
1	IIa	-91.088	0.7458	-0.4868
3	IIa	-90.395	0.5637	-0.3960
4	IIa	-90.502	0.7856	-0.2487
5	IIa	-90.414	0.7612	-0.2309
6	IIa	-90.354	0.7589	-0.3092
7	IIa	-90.238	1.0376	-0.2241
8	IIa	-93.342	1.8394	-0.7143
9	IIa	-90.317	0.7955	-0.3569
10	IIa	-89.546	0.7819	-0.1772
22	IIa	-90.944	0.7319	-0.0179
23	IIa	-78.592	0.7440	-1.0843
24	IIa	-91.436	1.0537	-0.2214
25	IIa	-90.785	0.8556	-0.3851
26	IIa	-90.377	0.9750	-0.4144
27	IIa	-90.291	0.9482	-0.4224
28	IIa	-90.951	0.8210	-0.1303
29	IIa	-91.015	0.9078	-0.5188
30	IIa	-90.455	0.8285	-0.5409
31	IIa	-90.370	0.6269	-0.6173
13	IIr	-99.599	-0.2801	-1.6732