Comparison of Different Gravity Field Solution Methods Applied to CHAMP Gravity Field Modelling

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Abstract:

The new gravity field satellite missions CHAMP, GRACE and GOCE are expected to deliver a new generation of gravity field models with unprecedented accuracy and resolution. However the large number of measurements as well as the large number of unknowns are posing new challenges in terms of computing strategies. Several competing methods for gravity field analysis, each having advantages and drawbacks, have been developed during the past years. A selection of them, namely the Direct Approach, the Pre-Conditioned Conjugate-Gradient Multiple-Adjustment (PCGMA) (cf. Schuh, 1996) and the Lumped-Coefficient Approach (cf. Sneeuw, 2000) are compared and evaluated here with real data delivered by CHAMP. The different gravity field solution methods and the resulting potential coefficients are tested and compared in terms of computation time, accuracy and spatial resolution.

1 Introduction

While in the case of the Direct Approach the full system of normal equations is generated, associated with heavy demands in terms of computer time and memory, the other methods introduce simplifying assumptions, which speed up the computation, but require an iterative strategy. It seems therefore worthwhile to compare these methods in terms of precision, formal quality measures and computational load.

The study is based on time series of disturbance potential values given along six months of orbit trajectories of CHAMP with a sampling period of 30s. They are derived using the energy integral method (cf. Gerlach, 2003). To be independent from any prior gravity models, purely kinematic CHAMP-orbits were computed from GPS high-low satellite-to-satellite tracking (cf. Švehla and Rothacher, 2002). Kinematic velocities are derived from kinematic positions by numerical differentiation using smoothing splines. In addition, the effect of the measured non-gravitational accelerations and of all tidal effects is taken into account.

2 Gravity Solution Methods

The Direct approach uses a classical least squares adjustment to obtain the potential coefficients. Formally it can be written as

$$\begin{bmatrix} \overline{C}_{lm} \\ \overline{S}_{lm} \end{bmatrix} = \left(\mathbf{A}^{\mathrm{T}} \mathbf{P}_y \mathbf{A} + \alpha \mathbf{R} \right)^{-1} \mathbf{A}^{\mathrm{T}} \mathbf{P}_y \mathbf{y} \,, \tag{1}$$

where (Cl_m, S_{lm}) is the vector of estimated potential coefficients, **A** the matrix of partial derivatives, **R** a regularisation matrix weighted by a factor α (which was set to 0 for this

study), and **y** the vector containing the time series of potential values along the orbit. For simplification the weight-matrix $\mathbf{P}_y$ is replaced by a unit matrix.

The advantages of the Direct Method are, that (theoretically) the most accurate solution is obtained, as no assumptions are made, and that it is the only method, where a full variance-covariance matrix is obtained. The disadvantages are the high memory requirement, as the normal equation matrix contains about L^4_{max} elements, and the required computation time.

The PCGMA approach (pre-conditioned conjugate-gradient multiple adjustment) (cf. Schuh, 1996) is an iterative numerical method to solve large systems of normal equations with much less computational effort. Instead of solving the full normal equation system, the adjustment is carried out with a simplified pre-conditioner matrix. The pre-conditioner has to be a good approximation of the full matrix, and has to be sparse (the majority of the elements are zero), so that the system can be solved quicker. As the original normal equation matrix is dominantly block-diagonal, its block-diagonal elements are used as pre-conditioner. The errors made by this simplification have to be compensated by iteration.

The Semi-Analytical approach is divided into two steps. The first step is to compute lumped-coefficients by FFT-techniques, and the second is the adjustment of potential coefficients from the lumped-coefficients as pseudo-observables (cf. Sneeuw 2000). The lumped-coefficients A_{lmk} and B_{lmk} are linear combinations of the potential coefficients. The formula for the gravitational potential in orbit co-ordinates (u, Λ), can be rewritten as a two-dimensional Fourier series. The argument of latitude u is the sum of the argument of perigee ω and the true anomaly f, and Λ is the angle between the Greenwich meridian and the ascending node.

$$V(u,\Lambda) = \sum_{m=0}^{L} \sum_{k=-L}^{L} A_{mk} \cos\psi_{mk} + B_{mk} \sin\psi_{mk} \; . \tag{2}$$

The adjustment can be written as

$$\begin{bmatrix} \overline{C}_{lm} \\ \overline{S}_{lm} \end{bmatrix} = \left(\mathbf{H}^{\mathrm{T}}_{lmk} \mathbf{P}_y \mathbf{H}_{lmk} + \alpha \mathbf{R} \right)^{-1} \mathbf{H}^{\mathrm{T}}_{lmk} \mathbf{P}_y \begin{bmatrix} A_{mk} \\ B_{mk} \end{bmatrix}, \tag{3}$$

where

$$H_{lmk} = \frac{GM}{R} \left(\frac{R}{r} \right)^{l+1} \lambda_{lk} \overline{F}_{lmk}(i) \, . \tag{4}$$

In this adjustment only coefficients with the same order m are dependent from each other, and it can be carried out block-wise, which speeds up the whole process and requires much less memory.

The lumped coefficients can be obtained by two different methods – the 1D-FFT method and the 2D-FFT (or torus-) method. For both methods the assumption is made, that the orbit height r and the inclination I are constant. The orbit can then be transformed from Keplerian elements to (u, Λ)-co-ordinates, which form a torus. The potential can be mapped to a regular grid on the torus by interpolating block mean values from the original values. After a 2D-FFT

transformation and reordering of the coefficients, the lumped coefficients A_{lmk} and B_{lmk} are obtained. The errors comitted by the interpolation and the assumption, that r and I are constant are compensated by iteration.

For the 1D-FFT method a repeat-orbit is required in addition. If the orbit closes after α revolutions in β days (α and β having no common divisor), a spectral projection from the one-dimensional domain of the measurement time series to the two-dimensional domain of the lumped coefficients can be made: $(m,k) \mapsto n$ with $n = k\beta - m\alpha$. The lumped coefficients are simply obtained by a 1D-FFT of the potential time series, and a reordering of the coefficients. Although it has been shown in (Pail and Wermuth 2003), that small deviations from the repeat-condition pose no problem, a sufficiently constant repeat-cycle could not be found for the CHAMP orbit, as due to the decreasing orbit height, the satellite is passing different repeat-cycles. This makes the 1D-FFT method unsuitable for the CHAMP-mission, but should still be considered an option for the GOCE-mission, as the GOCE satellite will maintain its orbit height.

3 Comparison of Results

Our data contained about 450.000 disturbing potential values along the orbit (6 months) derived from GPS-measurements via the energy integral, and the solution was computed up to degree L_{max} = 100 on one Pentium IV 3GHz processor with 1 GByte of memory. At the moment the memory limits the Direct Method to L_{max} = 100, while it is not a critical issue for the other methods. The computation time was 48 hours for the Direct Method. For both, the PCGMA and the Torus method, convergence was achieved after 10 iterations, and the computation time was 1 hour for the PCGMA approach and 10 minutes for the Torus Method, which shows a big advantage for the iterative methods.

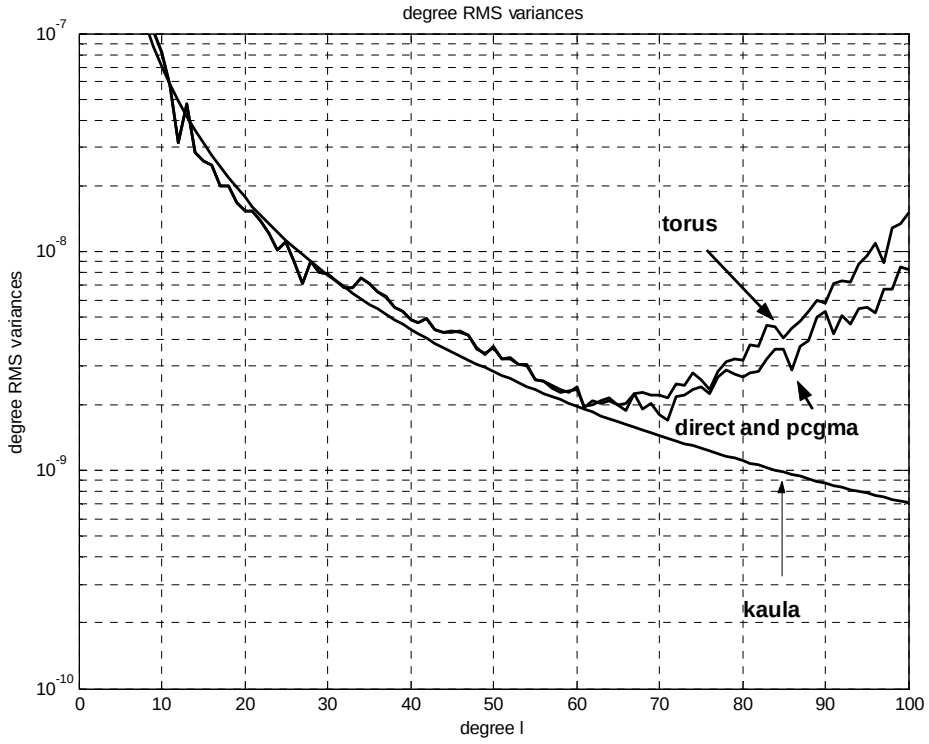

Figure 1: *Degree RMS Variances of the Different Solutions.*

As can be seen in Figure 1, the degree variances for all methods follow Kaula's rule of thumb until degree 60. For higher degrees the coefficients increase due to a lack of signal in the data which results in an ill-determination of the normal equations. The curves for the Direct Method and the PCGMA approach are almost identical, while the curve for the Torus Method is increasing even stronger. Figure 2 shows, that the difference in coefficients between the direct solution and the PCGMA solution varies between 10^{-13} for low and 10^{-11} for high coefficients, while the difference between the direct and torus solutions is two orders higher.

For the following test all solutions were truncated at degree 60, as because of the degradation of the high-frequent signal, higher coefficients showed no significant values. For a set of 5168 points in USA the geoid heights computed from the different models were compared to the geoid heights obtained from GPS/levelling. The RMS differences displayed in Table 1 show again, that the solutions of the Direct Method and the PCGMA approach are very similar, while the result of the Torus Method is slightly worse. It also shows, that all three solutions range between the official EIGEN-solutions (European Improved Gravity model of the Earth by New techniques) computed by GFZ (GeoForschungsZentrum Potsdam).

model	EIGEN-1S	EIGEN-2	direct	pcgma	torus
RMS [m]	0.968	0.893	0.904	0.904	0.926

Table 1: *Degree RMS Differences of Comparison with 5168 GPS/levelling points in USA.*

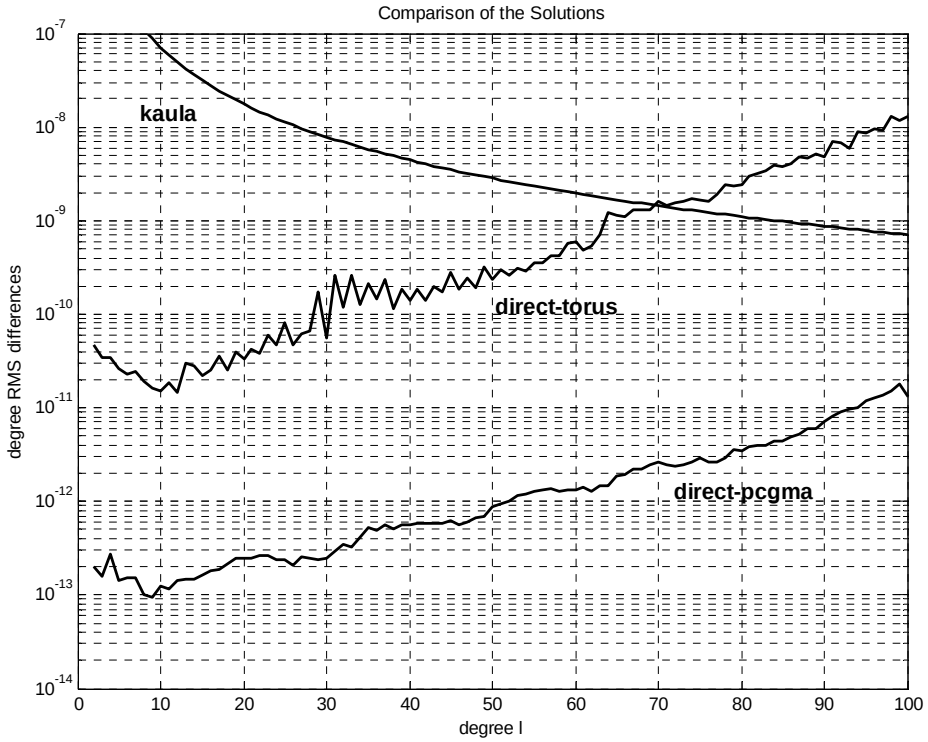

Figure 2: *Degree RMS Differences between the Different Solutions.*

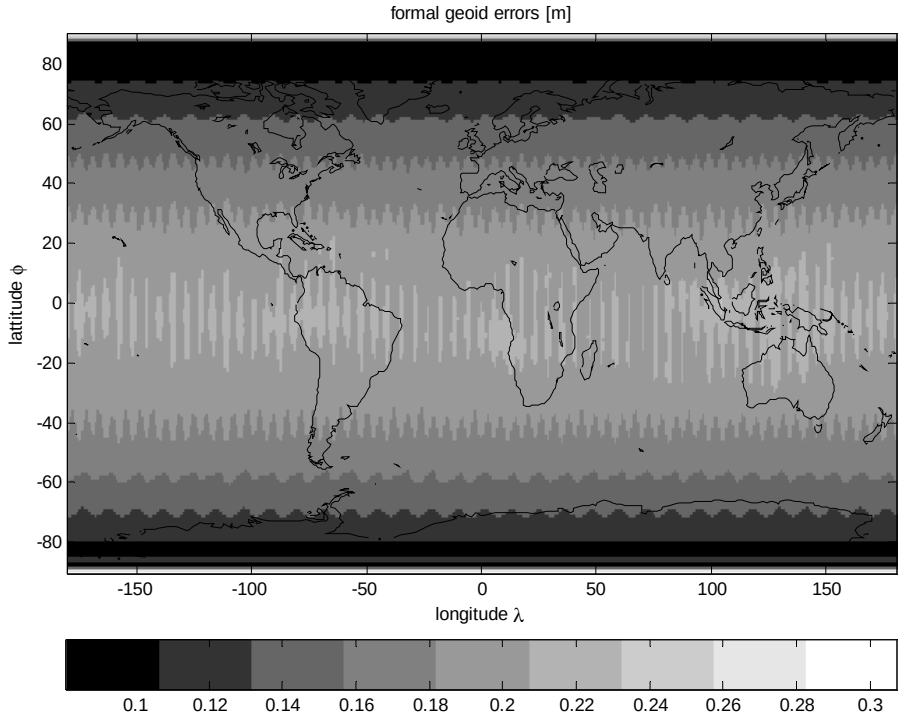

Figure 3: *Error Propagation for geoid height errors from the Variance-Covariance Matrix.*

Figure 3 shows the geoid height error as obtained by error propagation of the full variance-covariance matrix, which is only obtained from the Direct Method. The formal error lies between 10 and 30 cm, with a maximum at the equator. It decreases to the high latitudes due to the higher density of satellite tracks and has a negligible polar gap effect.

4 Conclusions

It could be shown, that the Direct Method and the PCGMA approach produce nearly identical results. While the PCGMA approach requires much less computation time and is not so strongly limited in maximum degree, it has the drawback, that no full variance-covariance information is obtained. The Torus method proved to be by far the fastest method, but it produced slightly inferior results due to interpolation errors and the fact, that the condition of a constant orbit height is not met by the descending CHAMP-orbit. Nevertheless due to the limited spatial resolution of the CHAMP mission and a computation time of "only" 48 hours, the Direct Method is a good option for CHAMP.

For the GOCE mission things look different, as it will produce several millions of observations, and the maximum degree is expected to be 250 or even 300. As the memory and computation time requirements increase with $\sim L^4$, the direct solution requires multi-processor computers, and considerable processing time. There the Semi-Analytical approach is recommended as Quicklook-Tool for a preliminary analysis of the data. It is expected to perform better for GOCE than for CHAMP, and this for two reasons: The constant height is almost met, and improved torus-interpolations are under development. The PCGMA approach also should be considered as an option for the GOCE-processing, as it combines high accuracy with relatively low computer requirements.

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