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Automatic Adaptive Sampling in Parametric Model Order Reduction by Matrix Interpolation

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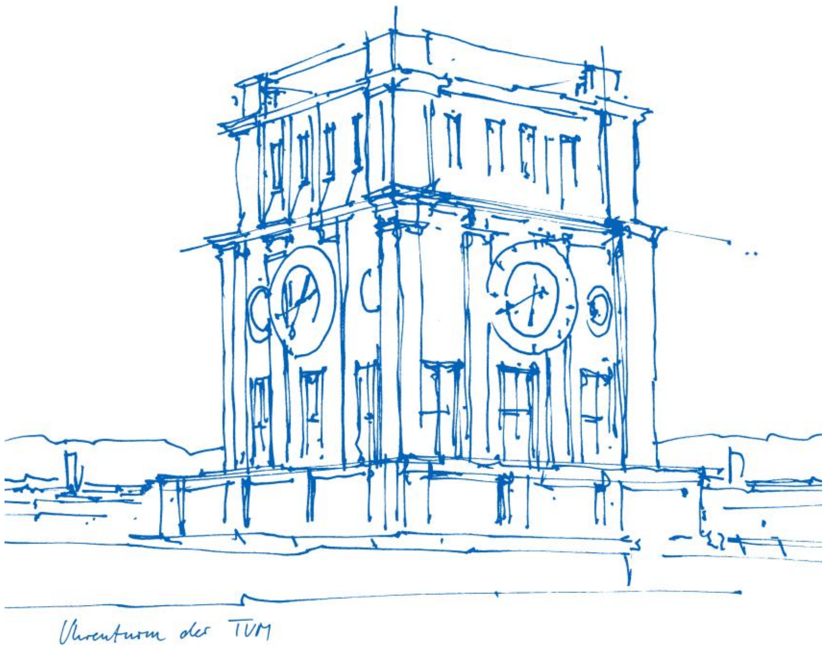
July 4th, AIM 2017



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Motivation

Large-scale full order model (FOM)

$$\mathbf{E}\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{B}\mathbf{u}$$

$$\mathbf{y} = \mathbf{C}\mathbf{x}$$

$$\det(\mathbf{E}) \neq 0$$

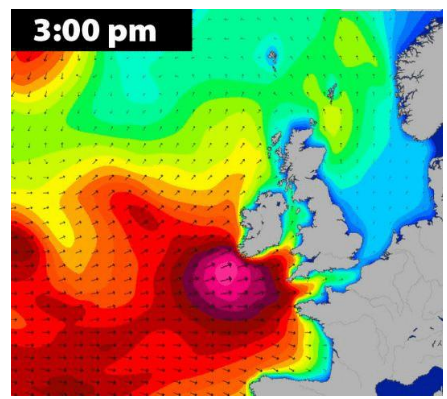
MOR

Reduced order model (ROM)

$$\mathbf{E}_r\dot{\mathbf{x}}_r = \mathbf{A}_r\mathbf{x}_r + \mathbf{B}_r\mathbf{u}$$

$$\mathbf{y}_r = \mathbf{C}_r\mathbf{x}_r$$

$$\mathbf{x}_r \in \mathbb{R}^r, \; r \ll n$$



n: 10^3 — 10^4 — 10^5 — 10^6

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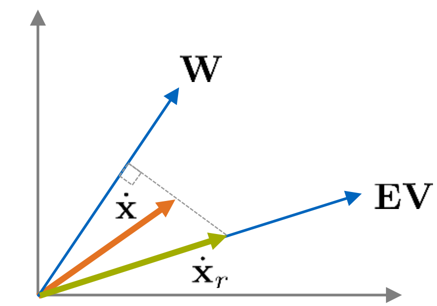
Projective Model Order Reduction

Approximation of the state vector:

$$\mathbf{x} = \mathbf{V}\mathbf{x}_r + \mathbf{e}, \qquad \mathbf{V} \in \mathbb{R}^{n\times r}$$



Petrov-Galerkin projection:

$$\overbrace{\mathbf{W}^T\mathbf{E}\mathbf{V}}^{\mathbf{E}_r}\dot{\mathbf{x}}_r = \overbrace{\mathbf{W}^T\mathbf{A}\mathbf{V}}^{\mathbf{A}_r}\mathbf{x}_r + \overbrace{\mathbf{W}^T\mathbf{B}}^{\mathbf{B}_r}\mathbf{u}$$

$$\mathbf{y} \approx \mathbf{y}_r = \underbrace{\mathbf{C}\mathbf{V}}_{\mathbf{C}_r}\mathbf{x}_r$$

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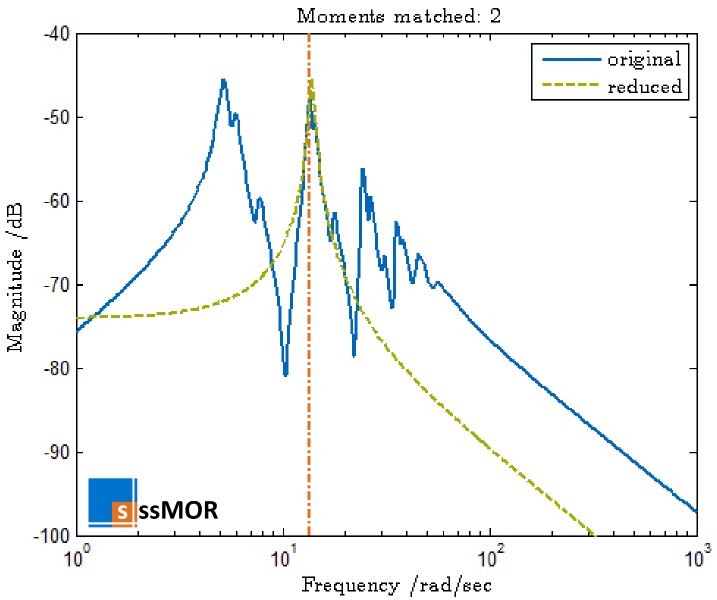
Rational Interpolation by Krylov subspace methods



Moments of a transfer function

$$\mathbf{G}(s) = \mathbf{C}(s\mathbf{E}-\mathbf{A})^{-1}\mathbf{B}$$
$$= \mathbf{G}(\Delta s + s_0) = -\sum_{i=0}^{\infty} \mathbf{M}_i(s_0)(s-s_0)^i$$

$\mathbf{M}_i(s_0)$: i-th moment around s_0

Krylov subspace:

$$\mathcal{K}_r(\mathbf{M},\mathbf{v}) = \left[\mathbf{v}, \mathbf{M}\,\mathbf{v}, \mathbf{M}^2\,\mathbf{v}, \cdots, \mathbf{M}^{r-1}\mathbf{v}\right]$$

Moment Matching by Rational Krylov (RK) subspaces

Bases for input and output Krylov-subspaces:

$$\mathrm{Im}(\mathbf{V}) = \left[\mathbf{A}_{s_0}^{-1}\mathbf{B},\quad \mathbf{A}_{s_0}^{-1}\mathbf{E}\mathbf{A}_{s_0}^{-1}\mathbf{B},\quad \ldots,\quad (\mathbf{A}_{s_0}^{-1}\mathbf{E})^{r-1}\mathbf{A}_{s_0}^{-1}\mathbf{B}\right]$$
$$\mathrm{Im}(\mathbf{W}) = \left[\mathbf{A}_{s_0}^{-T}\mathbf{C}^T,\quad \mathbf{A}_{s_0}^{-T}\mathbf{E}^T\mathbf{A}_{s_0}^{-T}\mathbf{C}^T,\quad \ldots,\quad (\mathbf{A}_{s_0}^{-T}\mathbf{E}^T)^{r-1}\mathbf{A}_{s_0}^{-T}\mathbf{C}^T\right]$$

$$\Rightarrow \quad \boxed{\mathbf{M}_i(s_0) = \mathbf{M}_{r,i}(s_0)}$$
$$\text{for } i = 0, \ldots, 2r$$

Moments from full and reduced order model around certain shifts **match!**

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Parametric Model Order Reduction (pMOR)

Large-scale parametric model

$$\mathbf{E}(\mathbf{p})\,\dot{\mathbf{x}} = \mathbf{A}(\mathbf{p})\,\mathbf{x} + \mathbf{B}(\mathbf{p})\,\mathbf{u}$$
$$\mathbf{y} = \mathbf{C}(\mathbf{p})\,\mathbf{x}$$

$$\boxed{\mathbf{p} \in \mathcal{D} \subset \mathbb{R}^d}$$

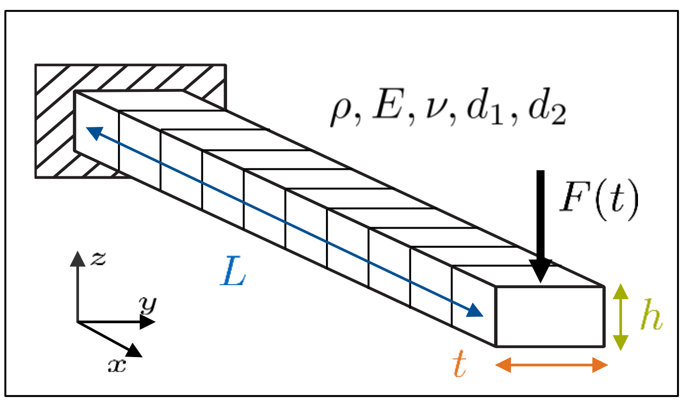
pMOR

Reduced order parametric model

$$\mathbf{E}_r(\mathbf{p})\,\dot{\mathbf{x}}_r = \mathbf{A}_r(\mathbf{p})\,\mathbf{x}_r + \mathbf{B}_r(\mathbf{p})\,\mathbf{u}$$
$$\mathbf{y}_r = \mathbf{C}_r(\mathbf{p})\,\mathbf{x}_r$$

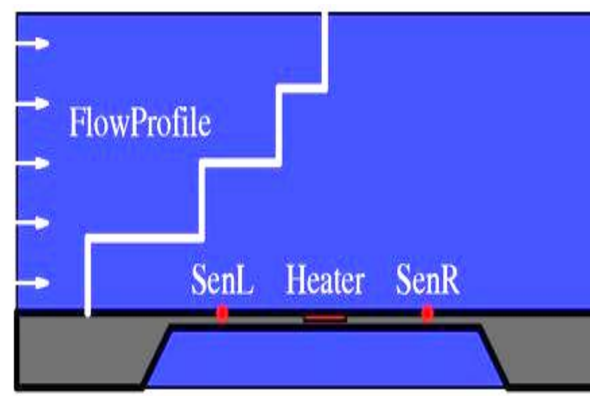
$$\mathbf{x}_r \in \mathbb{R}^r,\ r \ll n$$

- Linear dynamic systems with design parameters (e.g. material / geometry parameters,…)
- **Goal:** numerically efficient reduction with **preservation of the parameter dependency**



Timoshenko beam

Solar panels



Flow sensing anemometer

pMOR by Matrix Interpolation – Main Idea

$\mathbf{p}_1$ $\quad$ $\mathbf{p}^{\text{int}}$? $\quad$ $\mathbf{p}_2$ $\quad$ $\mathbf{p}$

Model at $\mathbf{p}_1$	Model at $\mathbf{p}^{\text{int}}$	Model at $\mathbf{p}_2$
$\mathbf{V}_1, \mathbf{W}_1$	$\mathbf{V}_{\mathbf{p}^{\text{int}}}, \mathbf{W}_{\mathbf{p}^{\text{int}}}$	$\mathbf{V}_2, \mathbf{W}_2$
Reduced model at $\mathbf{p}_1$	Reduced model at $\mathbf{p}^{\text{int}}$	Reduced model at $\mathbf{p}_2$

Interpolation of
reduced matrices

Reduced model at $\mathbf{p}^{\text{int}}$ from reduced models at sample points $\mathbf{p}_1$ & $\mathbf{p}_2$

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pMOR by Matrix Interpolation – Procedure

[Panzer et al. '10]

$\mathbf{p}_1$ $\quad$ $\mathbf{p}^{\mathrm{int}}$? $\quad$ $\mathbf{p}_2$ $\quad$ $\mathbf{p}$

$$\mathbf{E}_1\dot{\mathbf{x}} = \mathbf{A}_1\mathbf{x} + \mathbf{B}_1\mathbf{u}$$
$$\mathbf{y} = \mathbf{C}_1\mathbf{x}$$

$\mathbf{V}_1, \mathbf{W}_1$

$$\mathbf{E}_{r,1}\dot{\mathbf{x}}_{r,1} = \mathbf{A}_{r,1}\mathbf{x}_{r,1} + \mathbf{B}_{r,1}\mathbf{u}$$
$$\mathbf{y}_{r,1} = \mathbf{C}_{r,1}\mathbf{x}_{r,1}$$

$$\mathbf{E}_2\dot{\mathbf{x}} = \mathbf{A}_2\mathbf{x} + \mathbf{B}_2\mathbf{u}$$
$$\mathbf{y} = \mathbf{C}_2\mathbf{x}$$

$\mathbf{V}_2, \mathbf{W}_2$

$$\mathbf{E}_{r,2}\dot{\mathbf{x}}_{r,2} = \mathbf{A}_{r,2}\mathbf{x}_{r,2} + \mathbf{B}_{r,2}\mathbf{u}$$
$$\mathbf{y}_{r,2} = \mathbf{C}_{r,2}\mathbf{x}_{r,2}$$

direct interpolation
not meaningful

1.) Individual reduction

$$\mathbf{E}_{r,i}\dot{\mathbf{x}}_{r,i}(t) = \mathbf{A}_{r,i}\mathbf{x}_{r,i}(t) + \mathbf{B}_{r,i}\mathbf{u}(t) \qquad \mathbf{E}_{r,i} = \mathbf{W}_i^T\mathbf{E}_i\mathbf{V}_i, \quad \mathbf{A}_{r,i} = \mathbf{W}_i^T\mathbf{A}_i\mathbf{V}_i$$
$$\mathbf{y}_{r,i}(t) = \mathbf{C}_{r,i}\mathbf{x}_{r,i}(t) \qquad \mathbf{B}_{r,i} = \mathbf{W}_i^T\mathbf{B}_i, \quad \mathbf{C}_{r,i} = \mathbf{C}_i\mathbf{V}_i$$

$$\mathbf{p}_i, \; i = 1, \ldots, K$$
$$\mathbf{V}_i := \mathbf{V}(\mathbf{p}_i)$$
$$\mathbf{W}_i := \mathbf{W}(\mathbf{p}_i)$$

pMOR by Matrix Interpolation – Procedure

[Panzer et al. '10]

$\mathbf{p}_i, \; i = 1, \dots, K$

$\mathbf{V}_i := \mathbf{V}(\mathbf{p}_i)$

$\mathbf{W}_i := \mathbf{W}(\mathbf{p}_i)$

1.) Individual reduction

$$\mathbf{E}_{r,i}\dot{\mathbf{x}}_{r,i}(t) = \mathbf{A}_{r,i}\mathbf{x}_{r,i}(t) + \mathbf{B}_{r,i}\mathbf{u}(t) \qquad \mathbf{E}_{r,i} = \mathbf{W}_i^T\mathbf{E}_i\mathbf{V}_i, \quad \mathbf{A}_{r,i} = \mathbf{W}_i^T\mathbf{A}_i\mathbf{V}_i$$

$$\mathbf{y}_{r,i}(t) = \mathbf{C}_{r,i}\mathbf{x}_{r,i}(t) \qquad \mathbf{B}_{r,i} = \mathbf{W}_i^T\mathbf{B}_i, \quad \mathbf{C}_{r,i} = \mathbf{C}_i\mathbf{V}_i$$

2.) Transformation to generalized coordinates

$$\overbrace{\mathbf{M}_i^T\mathbf{E}_{r,i}\mathbf{T}_i}^{\mathbf{E}_{r,i}^*}\dot{\mathbf{x}}_{r,i}^*(t) = \overbrace{\mathbf{M}_i^T\mathbf{A}_{r,i}\mathbf{T}_i}^{\mathbf{A}_{r,i}^*}\mathbf{x}_{r,i}^*(t) + \overbrace{\mathbf{M}_i^T\mathbf{B}_{r,i}}^{\mathbf{B}_{r,i}^*}\mathbf{u}(t)$$

$$\mathbf{y}_{r,i}(t) = \underbrace{\mathbf{C}_{r,i}\mathbf{T}_i}_{\mathbf{C}_{r,i}^*}\mathbf{x}_{r,i}^*(t)$$

$$\mathbf{T}_i = (\mathbf{R}^T\mathbf{V}_i)^{-1}$$

$$\mathbf{M}_i = (\mathbf{R}^T\mathbf{W}_i)^{-1}$$

$\mathbf{x}_{r,i} = \mathbf{T}_i\mathbf{x}_{r,i}^*$

$\mathbf{V}_{\text{all}} = [\mathbf{V}_1, \dots, \mathbf{V}_K]$

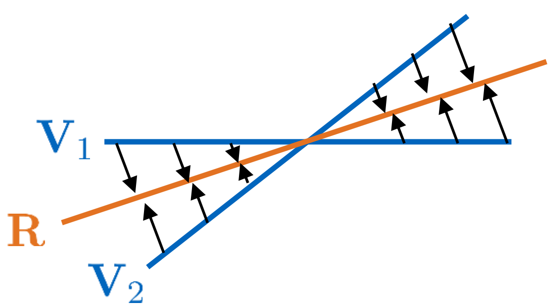
$\mathbf{V}_{\text{all}} \overset{\text{SVD}}{=} \mathbf{U\Sigma N}^T$

$\mathbf{R} = \mathbf{U}(:, 1:r)$

How to choose $\mathbf{T}_i$ and $\mathbf{M}_i$?

Goal: Adjustment of the local bases $\mathbf{V}_i$ to $\mathbf{V}_i^* = \mathbf{V}_i\mathbf{T}_i$, in order to make the gen. coordinates $\mathbf{x}_{r,i}^*$ compatible w.r.t. a reference subspace $\mathbf{R}$.

Dual procedure for the local bases $\mathbf{W}_i$



High correlation $\mathbf{V}_i^* \longleftrightarrow \mathbf{R}$:

$\mathbf{T}_i^T\mathbf{V}_i^T\mathbf{R} \overset{!}{=} \mathbf{I}$

pMOR by Matrix Interpolation – Procedure

[Panzer et al. '10]

1.) Individual reduction

$$\mathbf{E}_{r,i}\dot{\mathbf{x}}_{r,i}(t) = \mathbf{A}_{r,i}\mathbf{x}_{r,i}(t) + \mathbf{B}_{r,i}\mathbf{u}(t) \qquad \mathbf{E}_{r,i} = \mathbf{W}_i^T\mathbf{E}_i\mathbf{V}_i, \quad \mathbf{A}_{r,i} = \mathbf{W}_i^T\mathbf{A}_i\mathbf{V}_i$$

$$\mathbf{y}_{r,i}(t) = \mathbf{C}_{r,i}\mathbf{x}_{r,i}(t) \qquad \mathbf{B}_{r,i} = \mathbf{W}_i^T\mathbf{B}_i, \quad \mathbf{C}_{r,i} = \mathbf{C}_i\mathbf{V}_i$$

$$\mathbf{p}_i,\ i = 1,\ldots,K$$
$$\mathbf{V}_i := \mathbf{V}(\mathbf{p}_i)$$
$$\mathbf{W}_i := \mathbf{W}(\mathbf{p}_i)$$

2.) Transformation to generalized coordinates

$$\overbrace{\mathbf{M}_i^T\mathbf{E}_{r,i}\mathbf{T}_i}^{\mathbf{E}^*_{r,i}}\dot{\mathbf{x}}^*_{r,i}(t) = \overbrace{\mathbf{M}_i^T\mathbf{A}_{r,i}\mathbf{T}_i}^{\mathbf{A}^*_{r,i}}\mathbf{x}^*_{r,i}(t) + \overbrace{\mathbf{M}_i^T\mathbf{B}_{r,i}}^{\mathbf{B}^*_{r,i}}\mathbf{u}(t)$$

$$\mathbf{y}_{r,i}(t) = \underbrace{\mathbf{C}_{r,i}\mathbf{T}_i}_{\mathbf{C}^*_{r,i}}\mathbf{x}^*_{r,i}(t)$$

$$\mathbf{x}_{r,i} = \mathbf{T}_i\mathbf{x}^*_{r,i}$$

$$\mathbf{T}_i = (\mathbf{R}^T\mathbf{V}_i)^{-1}$$
$$\mathbf{M}_i = (\mathbf{R}^T\mathbf{W}_i)^{-1}$$

$$\mathbf{V}_{\text{all}} = [\mathbf{V}_1, \ldots, \mathbf{V}_K]$$
$$\mathbf{V}_{\text{all}} \overset{\text{SVD}}{=} \mathbf{U}\boldsymbol{\Sigma}\mathbf{N}^T$$
$$\mathbf{R} = \mathbf{U}(:, 1:r)$$

3.) Interpolation

$$\mathbf{E}^*_r(\mathbf{p}^{\text{int}}) = \sum_{i=1}^K \omega_i(\mathbf{p})\mathbf{E}^*_{r,i}, \quad \mathbf{A}^*_r(\mathbf{p}^{\text{int}}) = \sum_{i=1}^K \omega_i(\mathbf{p}^{\text{int}})\mathbf{A}^*_{r,i}$$

$$\mathbf{B}^*_r(\mathbf{p}^{\text{int}}) = \sum_{i=1}^K \omega_i(\mathbf{p})\mathbf{B}^*_{r,i}, \quad \mathbf{C}^*_r(\mathbf{p}^{\text{int}}) = \sum_{i=1}^K \omega_i(\mathbf{p}^{\text{int}})\mathbf{C}^*_{r,i}$$

$$\sum_{i=1}^K \omega_i(\mathbf{p}^{\text{int}}) = 1$$

But: how to choose the sample points??

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Adaptive Sampling

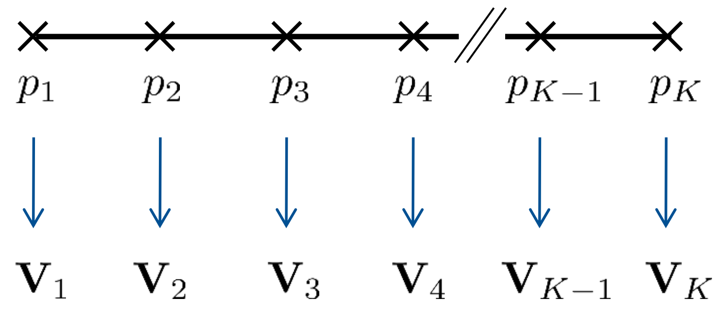
Requirements:

- Parametric space should be adequately sampled
- Avoid undersampling and oversampling
- More parameter samples should be placed in highly sensitive zones

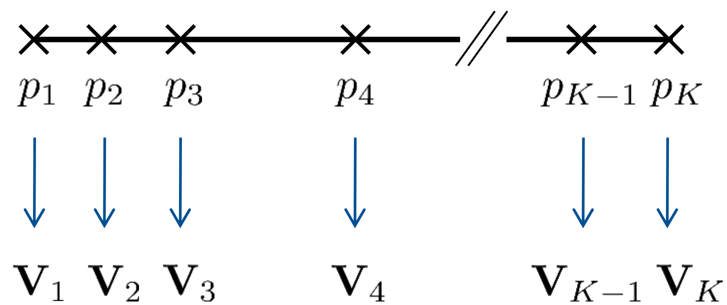
Quantification of parametric sensitivity:

- System-theoretic measure that quantifies the parametric sensitivity is needed in order to guide the adaptive refinement
- Adaptive sampling using angle between subspaces

Uniform Sampling:

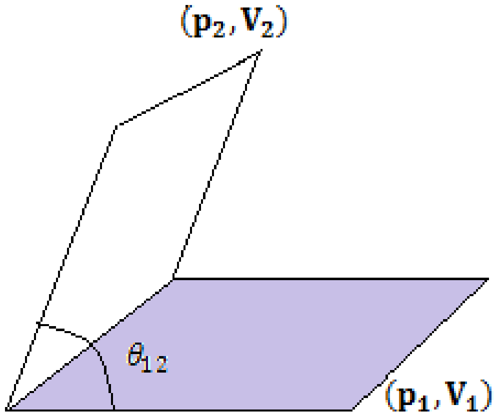


Adaptive Sampling:



Adaptive Sampling via subspace angles

Concept of subspace angles:



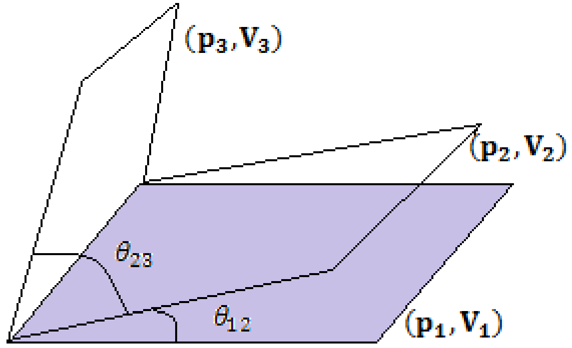
- $\mathbf{V}_1$ and $\mathbf{V}_2$ are orthonormal bases for the subspaces $\mathcal{V}_1$ and $\mathcal{V}_2$
- The largest angle between the subspaces can be determined by

$$\boxed{\theta_{12} = \arcsin\left(\sqrt{1-\sigma_r^2}\right) = \arccos(\sigma_r)}$$

σ_r : smallest singular value of $\mathbf{V}_1^T \mathbf{V}_2$

Usage for adaptive grid refinement:

- The larger the subspace angle, the more different are the projection matrices, and thus:
  - the higher the parametric sensitivity
  - and the more sample points can be introduced in the respective sub-span



Automatic Adaptive Sampling: Pseudo-Code

1) Input $\theta_{\max}$
2) Divide the entire parameter range into a uniform grid, calling it $p_1, p_2, \cdots, p_K$
3) **While** all $l_{i,i+1} > 1$ **do**
   a) Calculate the projection matrices $\mathbf{V}_1, \mathbf{V}_2, \cdots, \mathbf{V}_K$ corresponding to each of these values $p_1, p_2, \cdots, p_K$
   b) Compute subspace angles $\theta_{12}, \theta_{23}, \cdots, \theta_{K-1,K}$ between these $\mathbf{V}_i$´s, each taken pairwise
   c) Calculate

$$l_{12} = \left\lceil \frac{\theta_{12}}{\theta_{\max}} \right\rceil, l_{23} = \left\lceil \frac{\theta_{23}}{\theta_{\max}} \right\rceil, \cdots, l_{K-1,K} = \left\lceil \frac{\theta_{K-1,K}}{\theta_{\max}} \right\rceil$$

   d) Divide the interval between p_1 and p_2 into l_{12} further intervals. Likewise, do the same for all the other intervals.
   e) Obtain new grid points $p_1, p_2, \cdots, p_N$, whereas $N > K$

End While

Local reduction at sample points possible using any preferred MOR technique

```
theta(i) = subspace(Vp{i},Vp{i+1})
```

Quantitative indicator of how many pieces each parameter interval is to be further broken

Stopping criterion:
1. All ratios are equal to 1
2. Specified maximum number of samples points reached

Next iteration: local reduction, etc. <u>only</u> at points that got added in the last while-loop iteration (efficient!)

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Numerical example: Timoshenko Beam

- Finite element 3D model of a Timoshenko beam
- Parameter is the length of the beam: $p \equiv L$
- One-sided Krylov reduction with shifts at $s_0 = 0$
- $\theta_{\max} = 10^{\circ}$ chosen

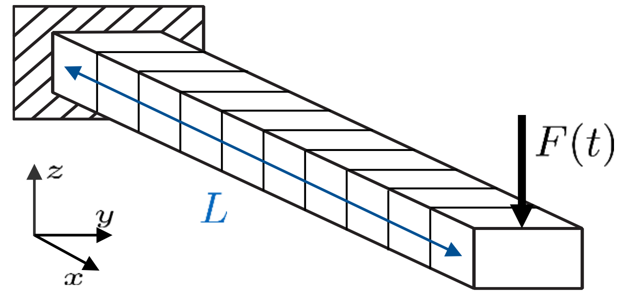

Table 1: Sample points p_i, subspace angles $\theta_{i,i+1}$ and ratios $l_{i,i+1}$

iter 1	p_i[m]	0.5								1.5				2.5		3.5		4.5		5.5
	$\theta_{i,i+1}[^{\circ}]$		25.79								14.20				8.61		7.06		5.73	
	$l_{i,i+1}$		3								2				1		1		1	
iter 2	p_i[m]	0.5				0.833		1.167		1.5		2		2.5		3.5		4.5		5.5
	$\theta_{i,i+1}[^{\circ}]$		10.15				8.59		7.05		8.18		6.03		8.61		7.06		5.73	
	$l_{i,i+1}$		2				1		1		1		1		1		1		1	
iter 3	p_i[m]	0.5		0.667		0.833		1.167		1.5		2		2.5		3.5		4.5		5.5
	$\theta_{i,i+1}[^{\circ}]$		5.26		4.89		8.59		7.05		8.18		6.03		8.61		7.06		5.73	
	$l_{i,i+1}$		1		1		1		1		1		1		1		1		1	

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Numerical example: Timoshenko Beam

Initial uniform grid with $K = 6$

Adaptive Sampling Scheme

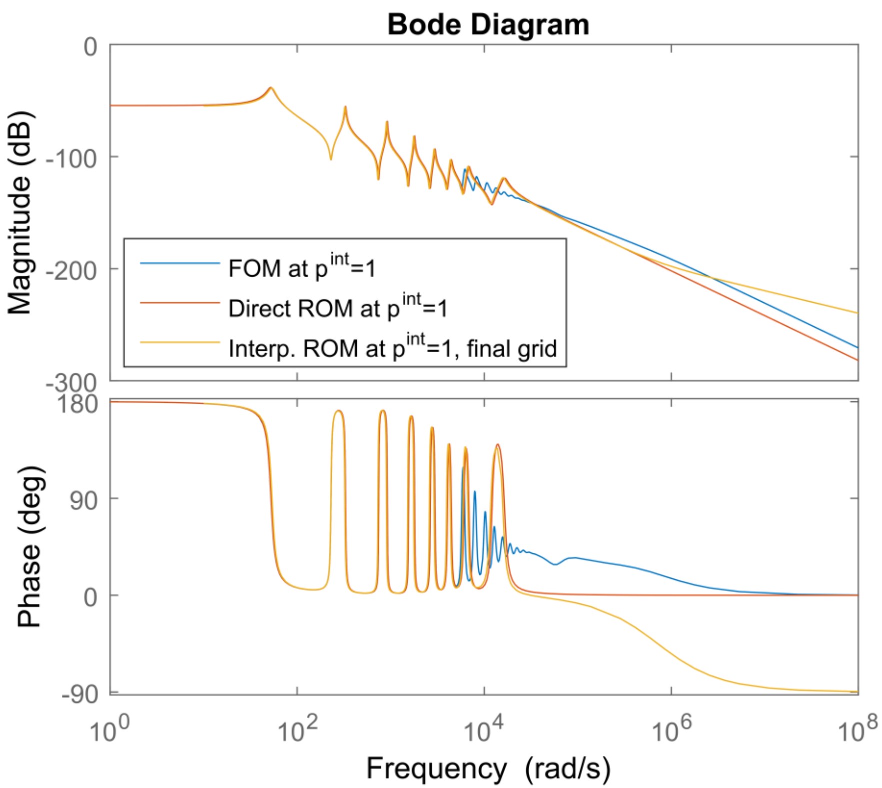
Table 1: Sample points p_i, subspace angles $\theta_{i,i+1}$ and ratios $l_{i,i+1}$

iter 1	p_i[m]	0.5								1.5				2.5		3.5		4.5		5.5
	$\theta_{i,i+1}[°]$								25.79			14.20			8.61		7.06		5.73	
	$l_{i,i+1}$								3			2			1		1		1	
iter 2	p_i[m]	0.5				0.833		1.167		1.5		2		2.5		3.5		4.5		5.5
	$\theta_{i,i+1}[°]$			10.15			8.59		7.05		8.18		6.03		8.61		7.06		5.73	
	$l_{i,i+1}$			2			1		1		1		1		1		1		1	
iter 3	p_i[m]	0.5		0.667		0.833		1.167		1.5		2		2.5		3.5		4.5		5.5
	$\theta_{i,i+1}[°]$		5.26		4.89		8.59		7.05		8.18		6.03		8.61		7.06		5.73	
	$l_{i,i+1}$		1		1		1		1		1		1		1		1		1	

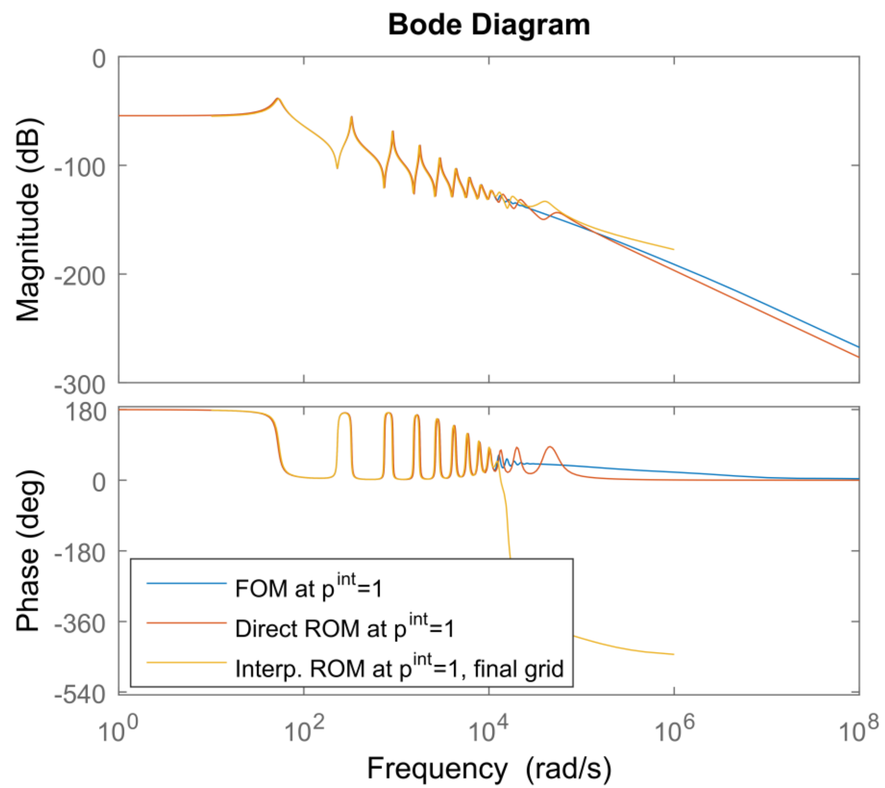
Interpolation point $p^{\text{int}} = 1.0$
between ROM 3 & ROM 4

Final refined grid with $N = 10$

Numerical results – Direct vs. Interpolated ROM



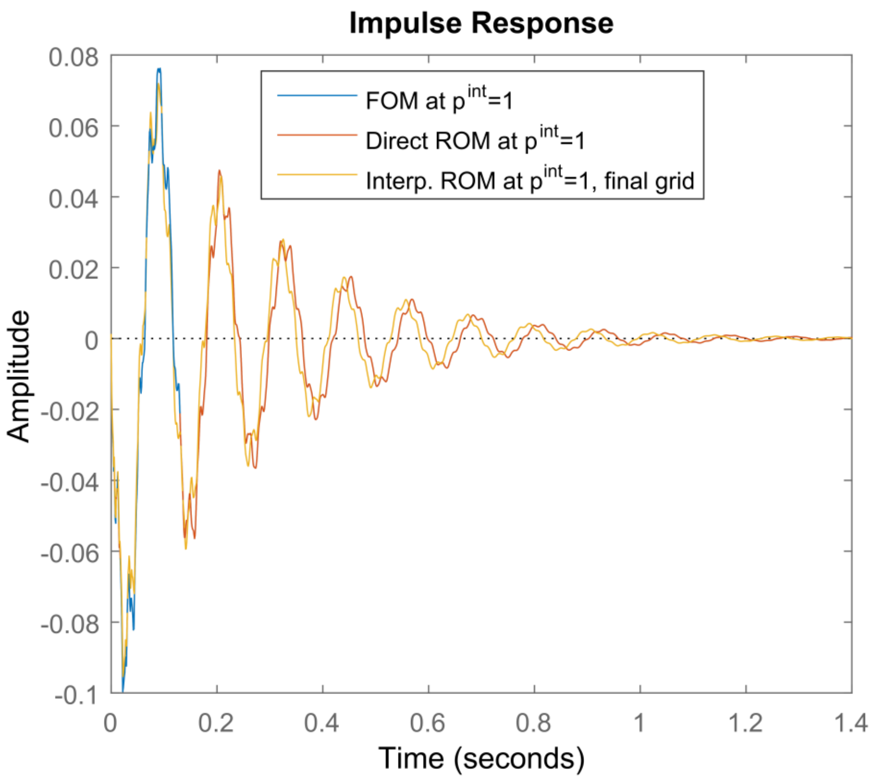
FOM size n = 240, ROMs size r = 17

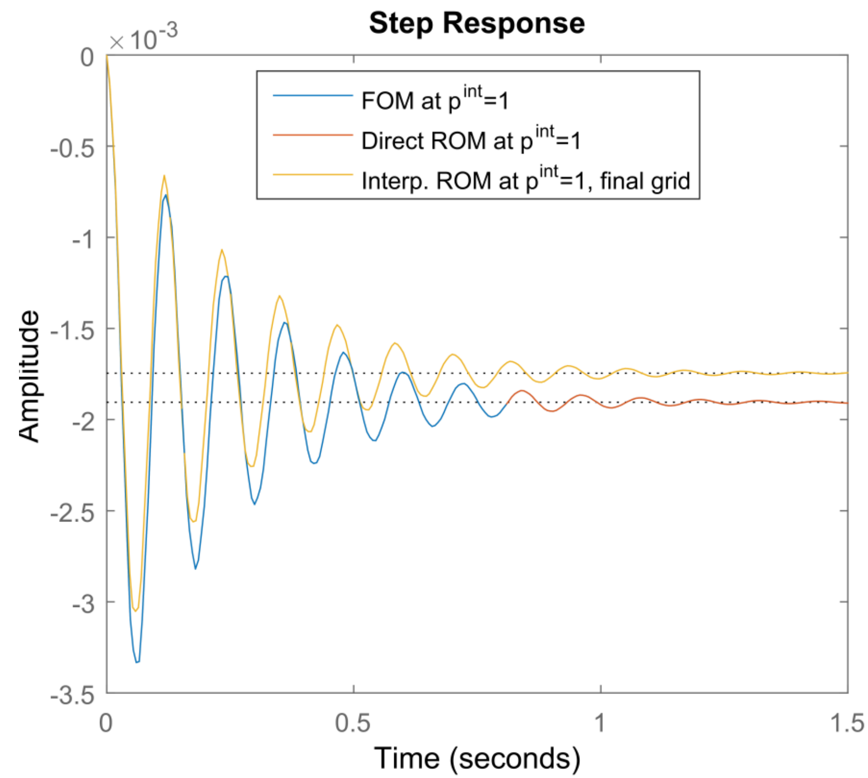


FOM size n = 2400, ROMs size r = 25

Two errors: model reduction error + interpolation error

Numerical results – Direct vs. Interpolated ROM



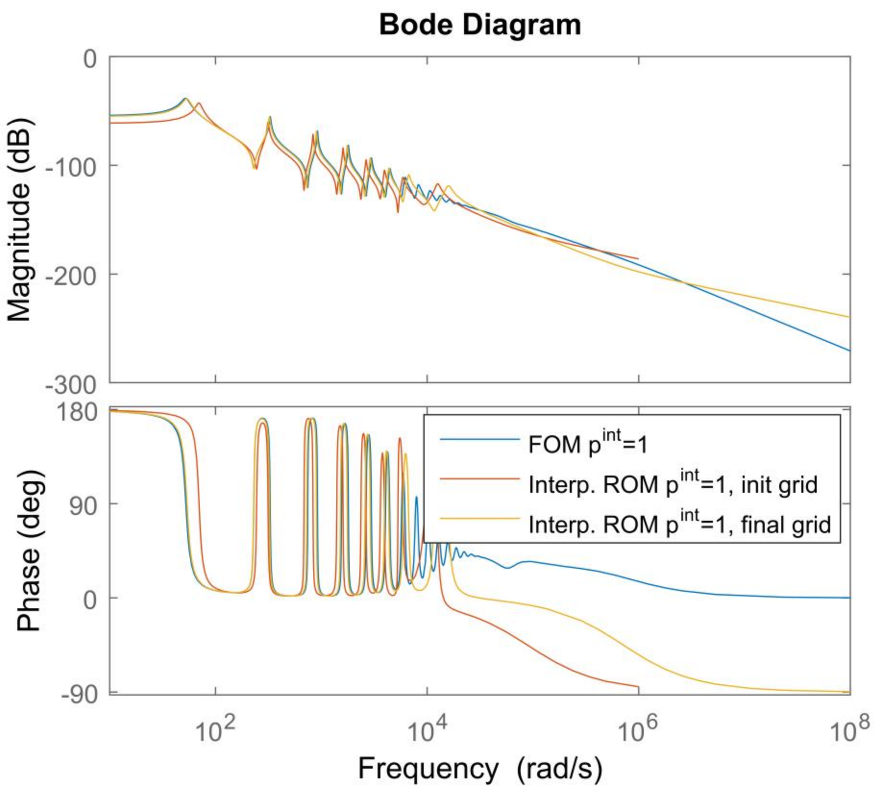


FOM size n = 2400, ROMs size r = 25

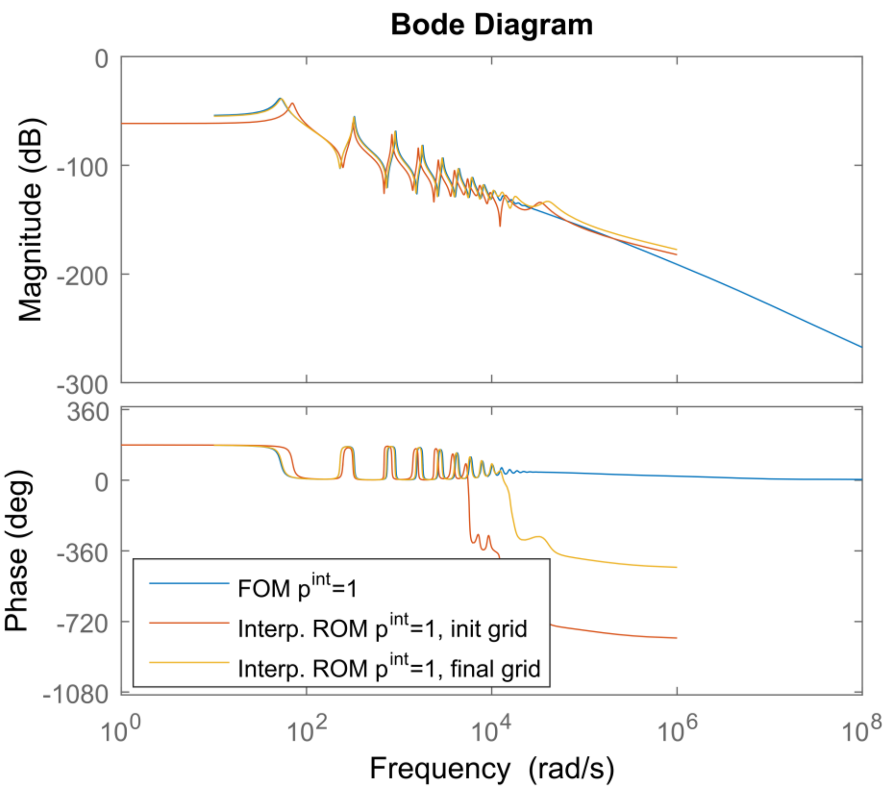
FOM size n = 2400, ROMs size r = 25

With MatrInterp: no need to reduce the model for every new parameter value

Numerical results – Initial vs. Final Grid



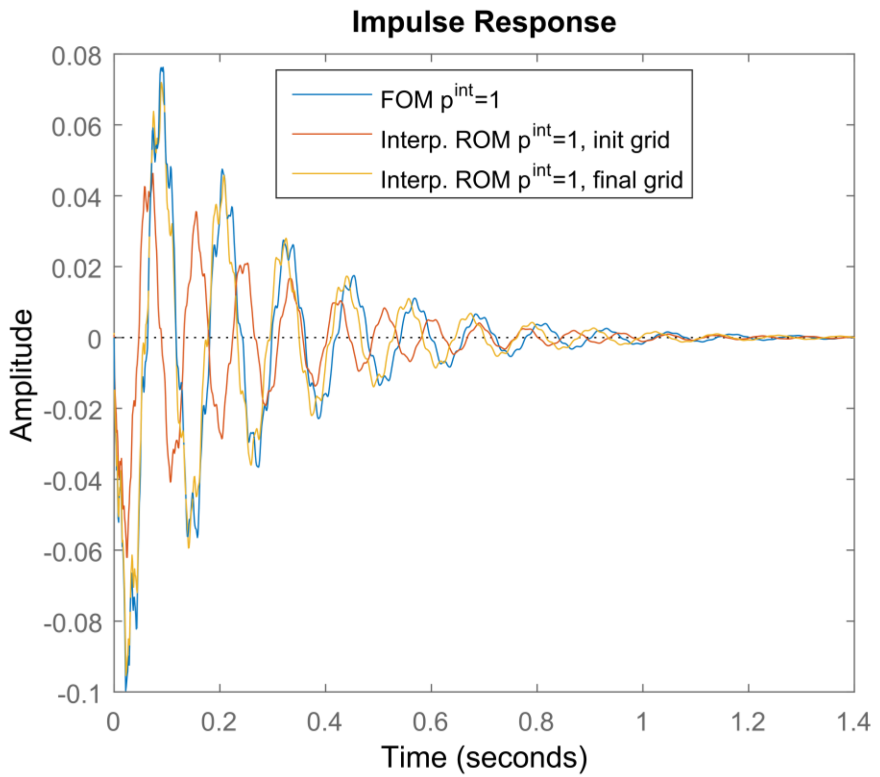
FOM size n = 240, ROMs size r = 17

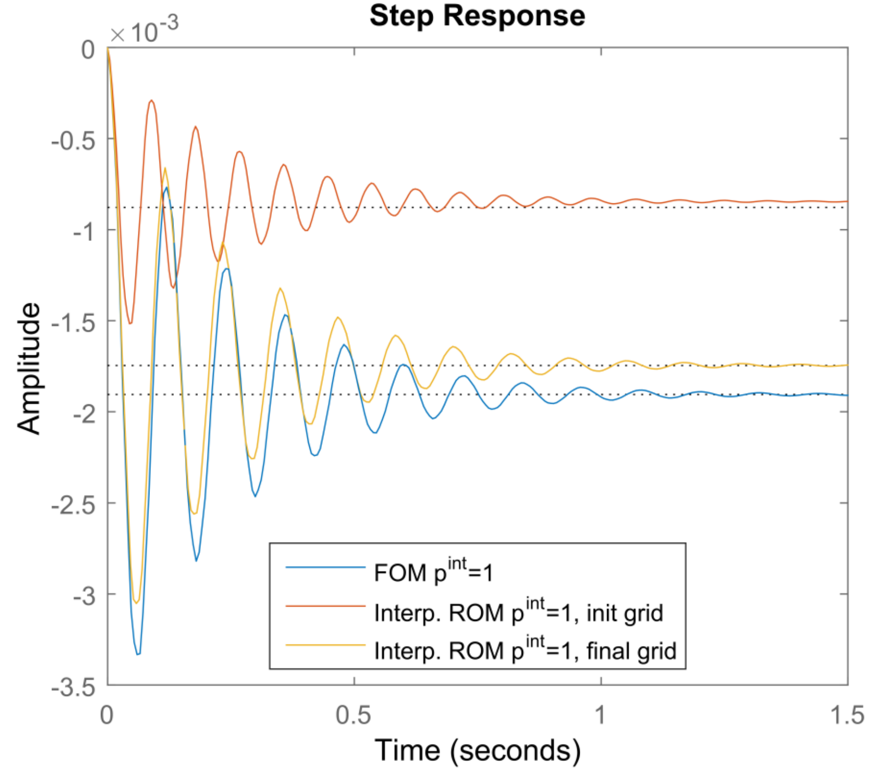


FOM size n = 2400, ROMs size r = 25

ROMs calculated with the final grid yield better approximations

Numerical results – Initial vs. Final Grid





FOM size n = 2400, ROMs size r = 25

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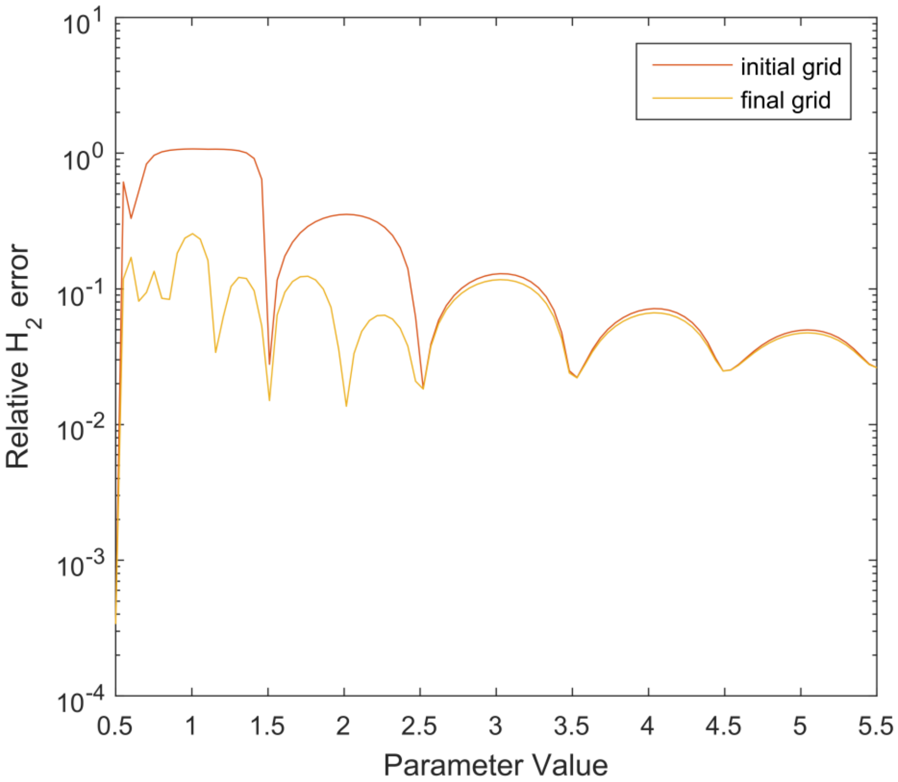

Relative H2 error between FOMs and interpolated ROMs for different parameter values and grids: FOM size n = 240, ROM size r = 17

- **Quantitative evaluation of the approximation**
- **Relative H2 error for** `nP=100` **different query points** p^{int}
- **Errors particularly small in the proximity of the sample points**
- **Final grid yields smaller errors for smaller beam lengths due to the adaptive refinement in this region**

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psssMOR Toolbox – Analysis and Reduction of Parametric Models in MATLAB

- ✓ Definition of **parametric sparse state-space models**

```
psys = loadFemBeam3D(Opts)
psys = loadAnemometer3parameter
```

- ✓ Manipulation of **psss-class objects**

```
psys = fixParameter(psys,2,1.7)
psys = unfixParameter(psys,3)
```

- ✓ **Compatible** with the sss & sssMOR toolboxes

```
param = [p1, p2, p3, p4]
sys = psys(param)

bode(psys,param); step(sys);
```

- ✓ Different **parametric reduction methods** available (offline- & online-phase)

```
psysr = matrInterpOffline
(psys,param,r,Opts);

psysr = globalPmorOffline
(psys,param,r,Opts)

sysr = psysr(pInterp)
```

- ✓ `localReduction` & `adaptiveSampling` as **core functions**

www.rt.mw.tum.de/?morlab

Summary & Outlook

Takehome Messages:

- A simple automatic sampling strategy is presented for **adaptively choosing sample points** in parametric model order reduction
- Scheme uses concept of **subspace angles** to measure need of further sampling points
- Adaptive approach is **fully automated** and **embedded in the matrix interpolation framework**
- Algorithm is applied to a **Timoshenko beam model**, achieving satisfactory results.

Future Extensions / Ongoing Work:

- Extension of proposed adaptive sampling scheme to 2D and 3D parametric case
- Higher dimensional case (d > 3) with adaptive sparse grids is topic of future research
- psssMOR toolbox is being actively developed and will be available open-source very soon!!

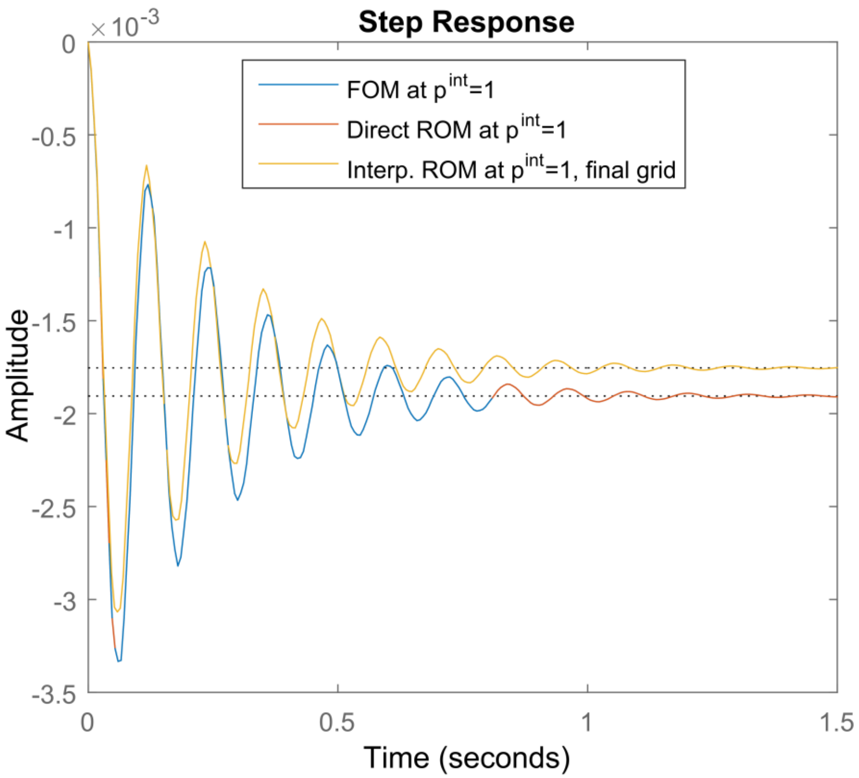
Thank you for your attention!

Backup

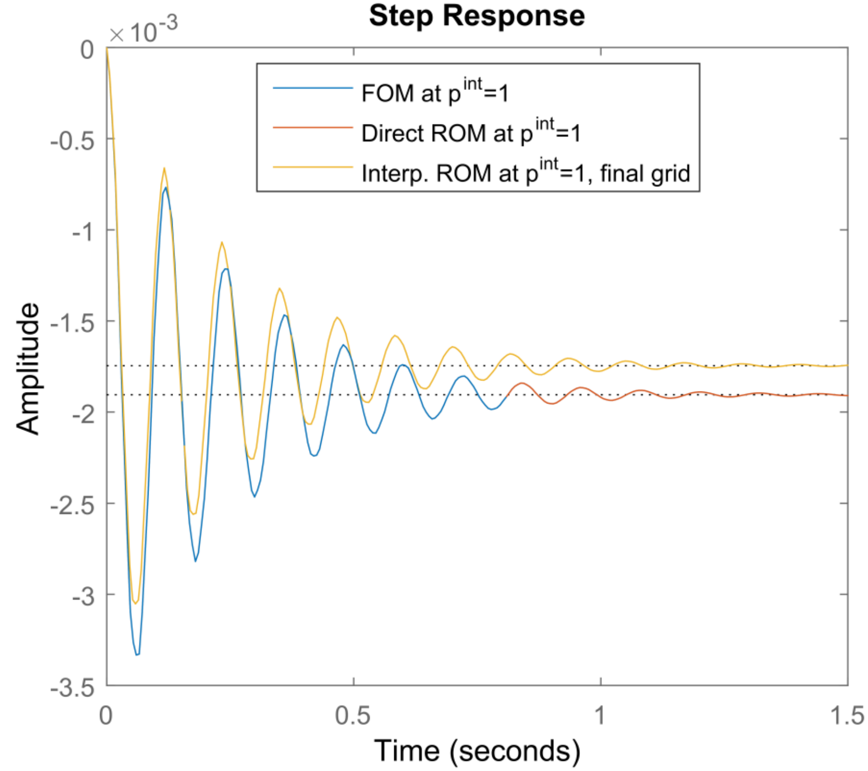
References

[Amsallem '10]	Interpolation on manifolds of CFD-based fluid and finite element-based structural reduced-order models for on-line….
[Bazaz et al. '15]	Adaptive Parameter Space Sampling in Matrix Interpolatory pMOR.
[Benner et al. '15]	A survey of projection-based model reduction methods for parametric dynamical systems.
[Baur et al. '15]	Comparison of methods for parametric model order reduction of instationary problems.
[Geuss et al. '08]	On Parametric Model Order Reduction by Matrix Interpolation.
[Panzer et al. '10]	Parametric Model Order Reduction by Matrix Interpolation.

Numerical results – Direct vs. Interpolated ROM



FOM size n = 240, ROMs size r = 17



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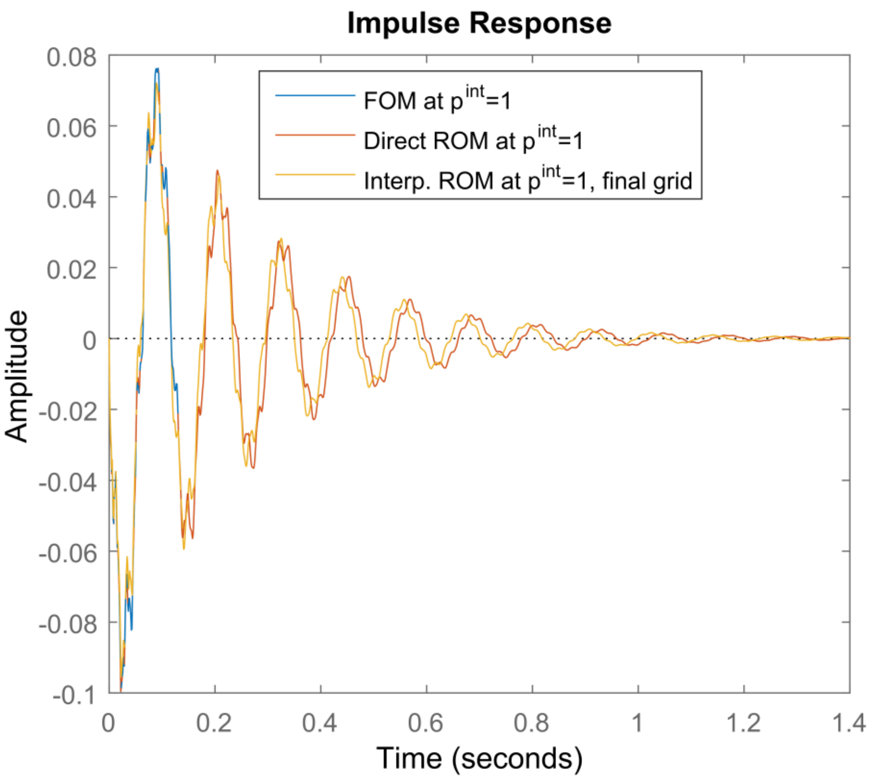
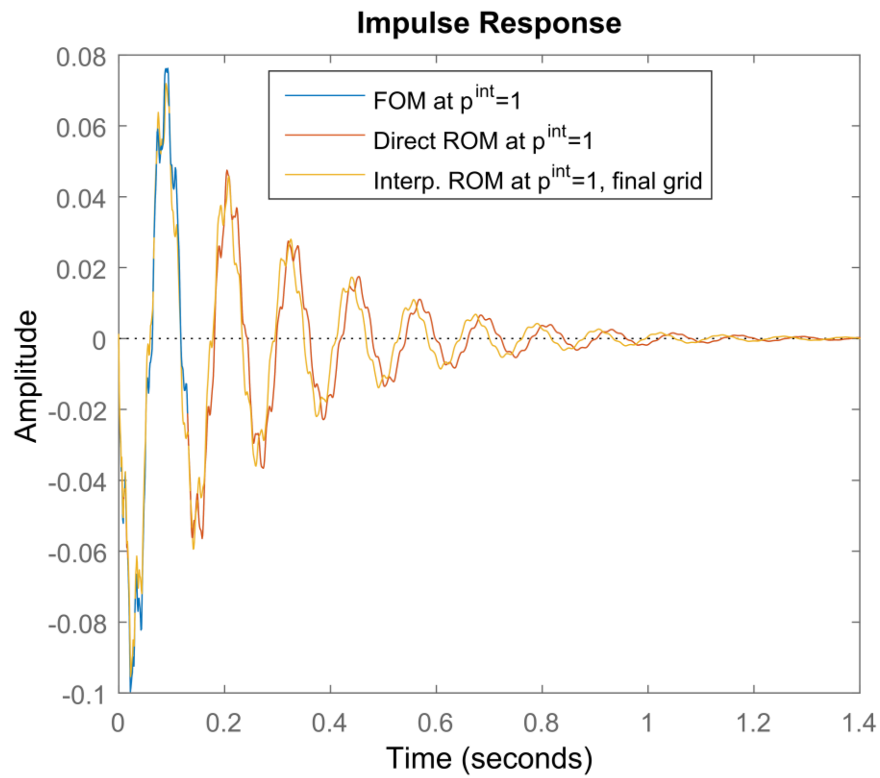
Numerical results – Direct vs. Interpolated ROM



FOM size n = 240, ROMs size r = 17



FOM size n = 2400, ROMs size r = 25

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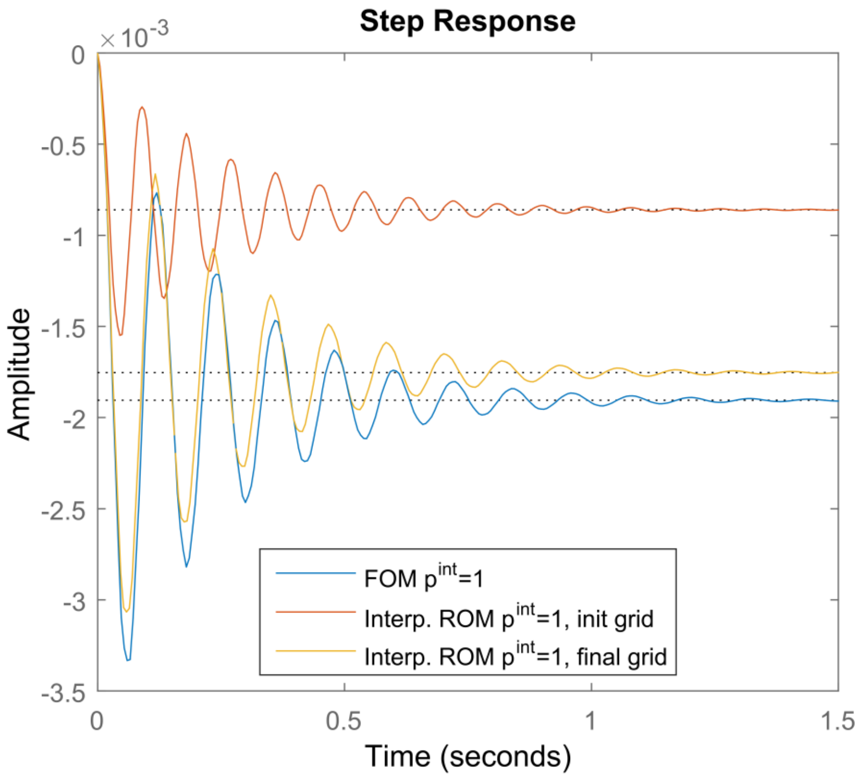
Numerical results – Initial vs. Final Grid



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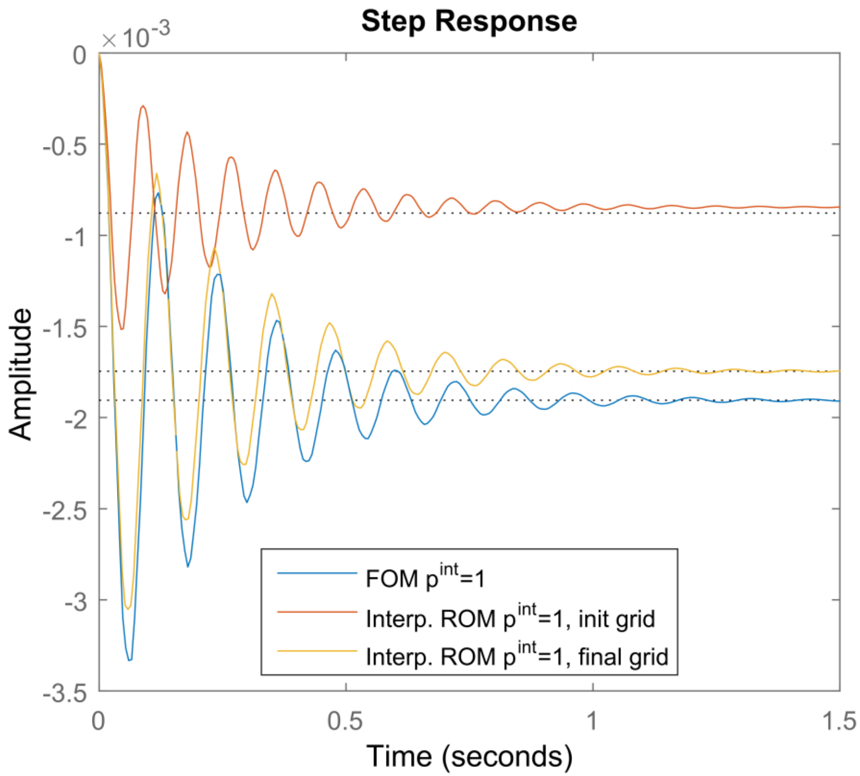

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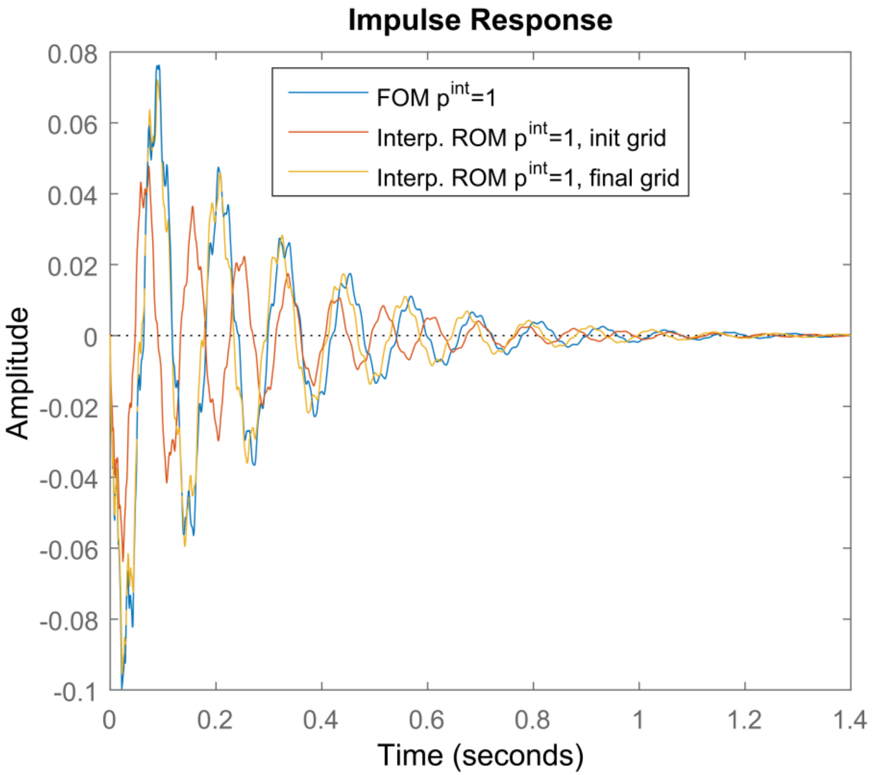
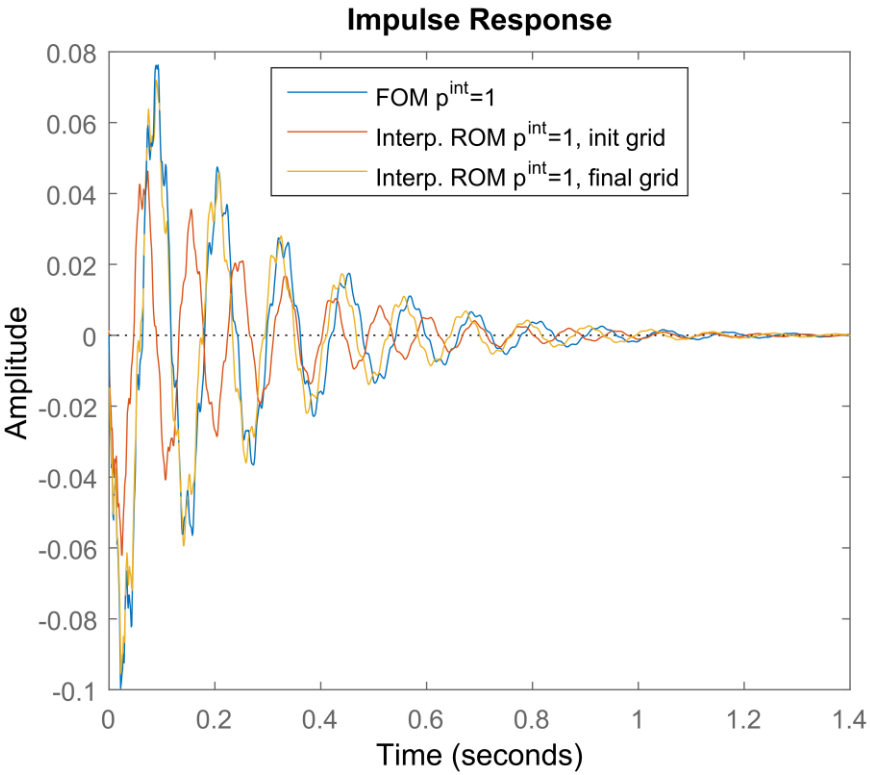
Numerical results – Initial vs. Final Grid

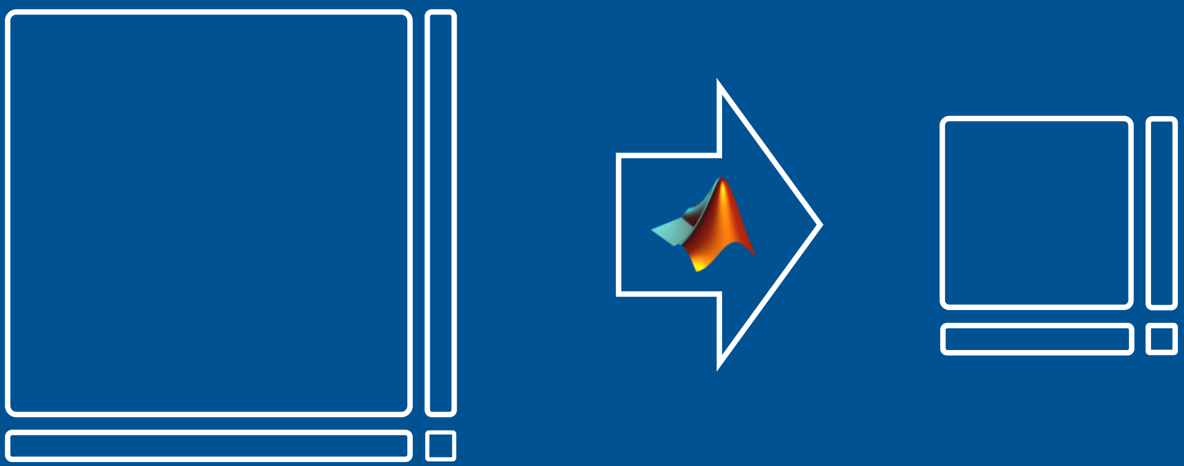


FOM size n = 240, ROMs size r = 17



FOM size n = 2400, ROMs size r = 25

sss & sssMOR – MATLAB Toolboxes



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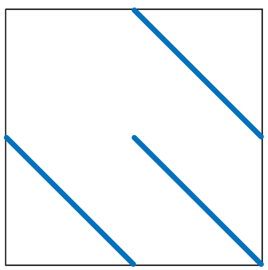
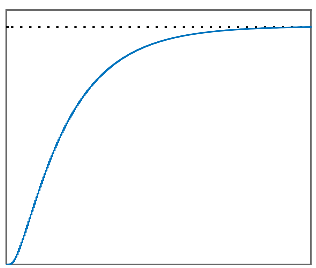
Toolboxes for sparse, large-scale models in

sss

sssMOR

```
sys = sss(A,B,C,D,E);
```

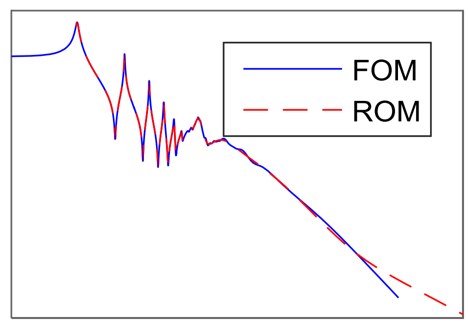


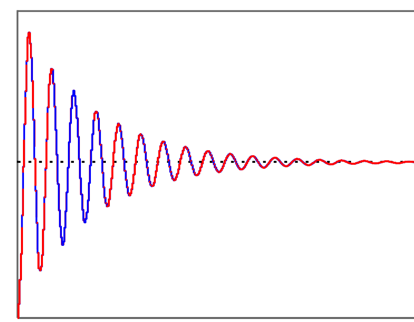


```
bode(sys), sigma(sys)
step(sys), impulse(sys)
norm(sys,2), norm(sys,inf)

c2d, lsim, eigs, connect,…
```

```
sysr = tbr(sys,r)
sysr = rk(sys,s0)
sysr = irka(sys,s0)
sysr = cure(sys)
sysr = cirka(sys,s0)
       ⋮
```





Powered by: **M-M.E.S.S. toolbox** [Saak, Köhler, Benner] for Lyapunov equations
Available at www.rt.mw.tum.de/?sssMOR
[Castagnotto/Cruz Varona/Jeschek/Lohmann ’17]: **„sss & sssMOR: Analysis and Reduction of Large-Scale Dynamic Systems in MATLAB“**, at-Automatisierungstechnik]

GitLab

Main characteristics

sss

- **State-space models** of very high order on a standard computer $\mathcal{O}\left(\mathbf{10^8}\right)$
- Many Control System Toolbox functions, revisited to **exploit sparsity**
- Allows system analysis in **frequency** (`bode, sigma`, …) and **time domain** (`step,norm,lsim`,…), as well as **manipulations** (`connect,truncate,`…)
- Is **compatible** with the built-in syntax
- **New functionality:** `eigs, residue, pzmap`,…

sssMOR

- **Classical** (`modalMor, tbr, rk`,…) and **state-of-the-art** (`isrk, irka, cirka, cure`,…) reduction methods
- Both **highly-automatized**

```
sysr = irka(sys,n)
```

  and **highly-customizable**

```
Opts.maxiter = 100
Opts.tol = 1e-6
Opts.stopcrit = 'combAll'
Opts.verbose = true
sysr = irka(sys,n,Opts)
```

- `solveLse` and `lyapchol` as **core functions**

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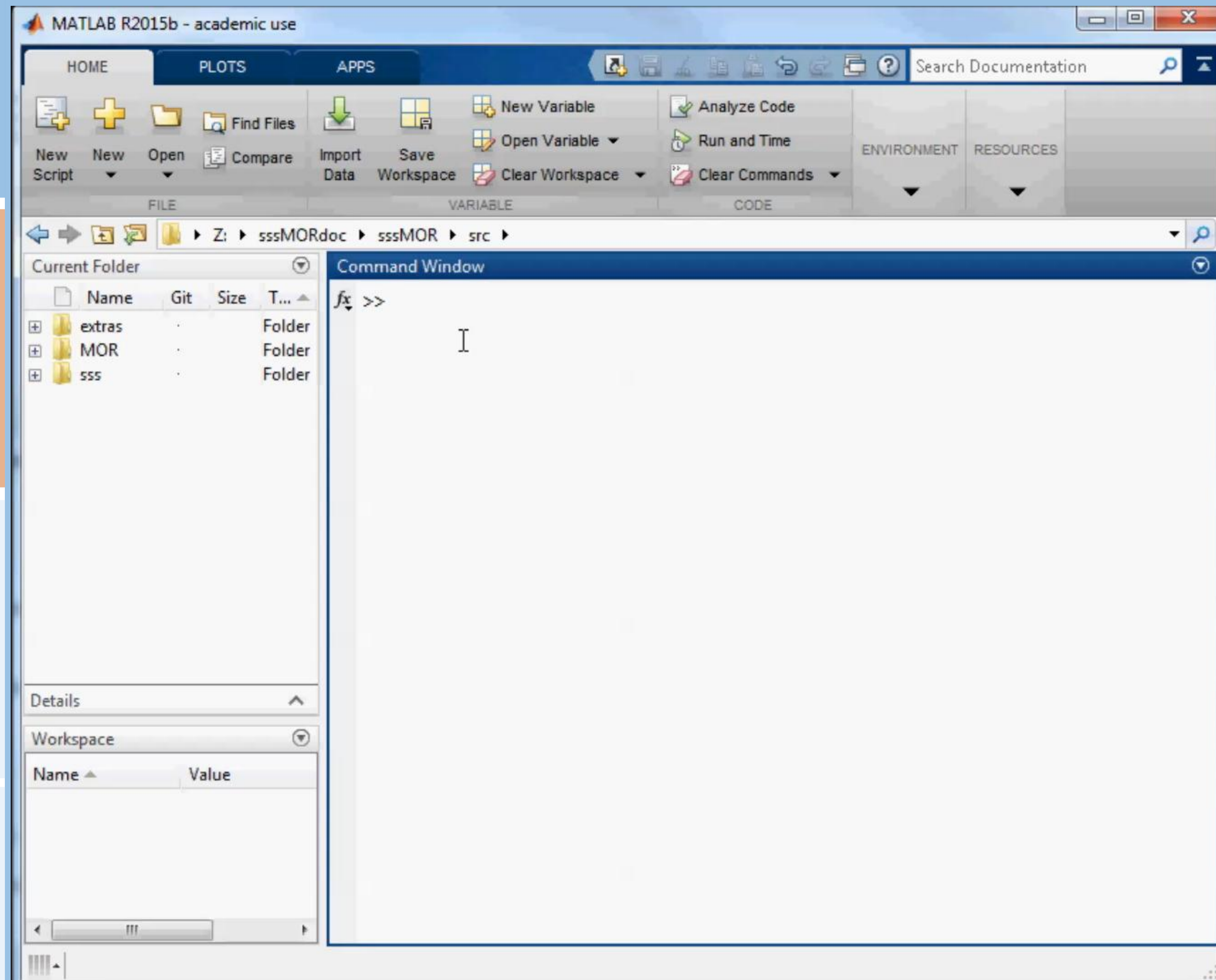
sss sssMOR

Comprehensive **documentation** with examples and references

sssMOR App
graphical user interface

completely **free**
and **open source**
(contributions welcome)



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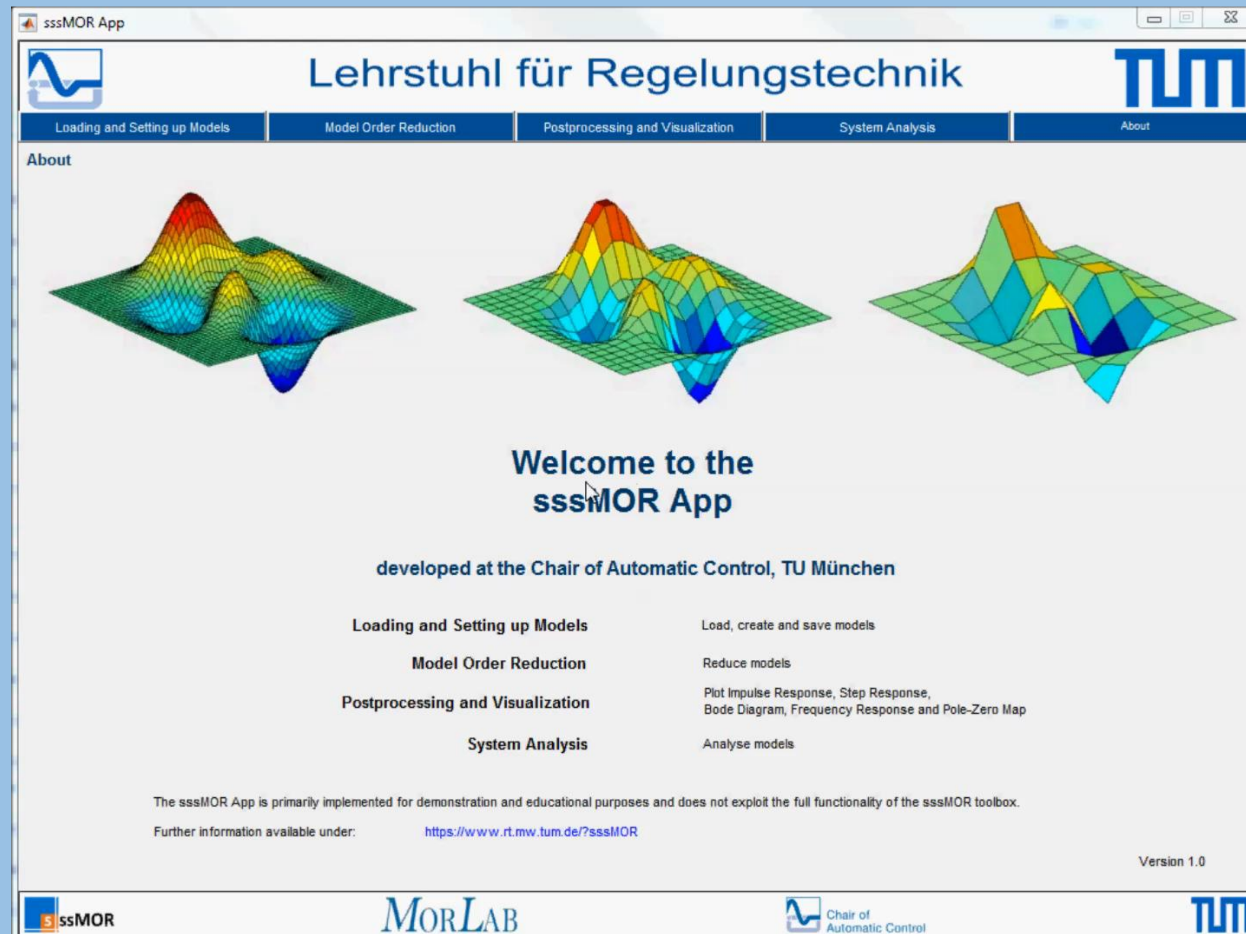

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