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Spectral stochastic finite element method in vibroacoustic analysis of fiber-reinforced composites

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Abstract

The attractive high strength and low weight of fiber-reinforced composites (FRC) make them ideal in many applications as defense and automotive industry. Such materials, however, possess some level of uncertainties which have to be taking in account for a reliable performance and quality control. In this paper, non-sampling based spectral stochastic finite element method is applied to vibroacoustic analysis of FRC plates with uncertain elastic and damping parameters. The impact of parameter uncertainties on the acoustical transmission loss is investigated. The results denote various impacts of uncertainties in different frequency ranges. Further, the calculated random transmission loss with few realizations on collocation points show a reasonable accuracy compared to results obtained from large number of realizations by sampling-based stochastic Monte Carlo simulations, and consequently, very efficient in terms of computational time.

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Keywords: Fiber-reinforced composites; stochastic FEM; uncertain parameter; polynomial chaos; acoustic transmission loss

1. Introduction

The high strength to weight of fiber-reinforced composites (FRC) makes them ideal for weight reduction and consequently energy saving in many industries. This is, in the most cases, in contrast to the optimal vibroacoustical behavior of the structures made from such materials. In many applications, the exact prediction of the structural reliability and performance is a very critical issue, such as airspace industry. Achieving reliable results depends on how one includes the associated uncertainty in the prediction model. Such an uncertainty is intrinsic to all endeavors and despite the best knowledge and all efforts, it is impossible to eliminate them. In past decades, traditionally, such uncertainties have been compensated through the use of empirical factors of safety. This, however, may lead to unrealistic results or overdesign conditions. On the other side, developed models based on probabilistic analysis allow

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the designer to include uncertainties based on explicit consideration of the likely source of parameter uncertainties to achieve a desirable level of performance and reliability of the structures.

Ignoring the epistemic type uncertainties [1], the aleatory uncertainties result from the varying properties of the individual layers and the way they are connected together [2–5]. The impact of this irreducible uncertainty type influences the macroscale properties of structures, e.g. material and damping parameters. The uncertain parameters may represent as random variable having an expected mean value and a variation range denoted by the standard deviation. This is, however, true if one makes sure that parameters can be estimated as Gaussian random variables. For non-Gaussian uncertain parameters, the entire range of uncertainty cannot be captured only by means of the mean value and the standard deviation. For that reason, in this paper the spectral based method [6] for uncertainty quantification is employed in which the generalized polynomial chaos (gPC) expansion [6–11] plays the major role. The application of the method on dynamic analysis of FRC structures have been investigated in [12–14].

This work uses the gPC expansion for representation of the damping and elastic parameters of the FRC structures. The random parameters then are used as inputs to the stochastic FEM (SFEM) [15] of the FRC vibroacoustic analysis to estimate the random transmission loss (rTL). The rTL is considered as random process and approximated by means of a gPC expansion with unknown frequency-dependent deterministic functions. A nominal FEM model of the problem operates as black-box to realize samples of the responses, here the nodal pressures, on a set of collocation points generated in the random space. The realizations then are served to estimate the unknown functions. This provides the major advantage to use limited number of realizations to calculate the rTL which normally requires large number of realizations when using the Monte Carlo (MC) method. Furthermore, any commercial or developed in-house FEM code can be used to accomplish desirable structural responses considering parameter uncertainty.

This paper is organized as follows: next section presents the spectral-based SFEM formulation of vibroacoustic problems. Random transmission loss is discussed in section 3 and numerical results are given in the section 4. The final section discusses the conclusions.

2. Spectral based stochastic FEM of vibroacoustic problems

Vibroacoustic analysis and design of structures and components made of FRC materials requires the study of sound-structure interaction. Accordingly, many computational techniques mostly based on FEM and BEM and numerous commercial and in-house computer programs have been developed in past decades. The effect of uncertainties, however, has been ignored. The stochastic FEM (SFEM) formulations for the coupled structure-acoustic involves the FEM model derived from the weak formulation of the equation of motion for structure part and from the wave equation for the fluid part. The fluid is assumed to be inviscid, irrotational and only under small translations and the added fluid mass per volume is ignored. The coupled FEM equations require a carefully construction of the model to accuracy representation of the fluid and solid parts and as well as the interface. For that, the elements around the structure part have to be able to capture the pressure and displacement fields at the same time. As a classical vibroacoustic problem, it is assumed that a FRC plate with uncertain parameters is used to separate two acoustic rooms. A noise source is dominated in the first room. The SFEM formulations include the fluid-solid interaction (FSI) to calculate the random nodal pressures in the second room from which the rTL is estimated. It is assumed that the randomness in elastic and damping parameters can be represented by the vector of random variables $\boldsymbol{\xi} = \{\xi_1, \xi_2, \ldots, \xi_m\}$. This randomness affects the structural stiffness and damping matrices as well as the system responses. That is by assembled SFEM model, the discretized FSI problem is written as

$$\begin{bmatrix} \boldsymbol{M}_s & 0 \\ \rho_f \boldsymbol{R}^T & \boldsymbol{M}_f \end{bmatrix} \begin{Bmatrix} \ddot{\boldsymbol{u}}_e(\cdot,\boldsymbol{\xi}) \\ \ddot{\boldsymbol{p}}_e(\cdot,\boldsymbol{\xi}) \end{Bmatrix} + \begin{bmatrix} \boldsymbol{C}_s(\boldsymbol{\xi}) & 0 \\ 0 & \boldsymbol{C}_f \end{bmatrix} \begin{Bmatrix} \dot{\boldsymbol{u}}_e(\cdot,\boldsymbol{\xi}) \\ \dot{\boldsymbol{p}}_e(\cdot,\boldsymbol{\xi}) \end{Bmatrix} + \begin{bmatrix} \boldsymbol{K}_s(\boldsymbol{\xi}) & -\boldsymbol{R} \\ 0 & \boldsymbol{K}_f \end{bmatrix} \begin{Bmatrix} \boldsymbol{u}_e(\cdot,\boldsymbol{\xi}) \\ \boldsymbol{p}_e(\cdot,\boldsymbol{\xi}) \end{Bmatrix} = \begin{Bmatrix} \boldsymbol{f}_s \\ \boldsymbol{f}_f \end{Bmatrix} \tag{1}$$

in which $\boldsymbol{p}_e$ and $\boldsymbol{u}_e$ are nodal pressure and displacement vectors, $\boldsymbol{M}_s, \boldsymbol{K}_s, \boldsymbol{C}_s$ and $\boldsymbol{M}_f, \boldsymbol{K}_f, \boldsymbol{C}_f$ denote structural and fluid mass, stiffness and damping matrices, respectively, $\boldsymbol{R}$ is the coupling matrix and ρ_f denotes the fluid mass density. The structural and fluid force vectors, $\boldsymbol{f}_s$ and $\boldsymbol{f}_s$, are assumed to be deterministic and $(\cdot)$ refers to any deterministic variable such as time or frequency. For sake of simplicity, Eq. (1) is written in the compact form as

$$\mathcal{M}\ddot{\mathcal{U}}(\cdot,\boldsymbol{\xi}) + \mathcal{C}(\boldsymbol{\xi})\dot{\mathcal{U}}(\cdot,\boldsymbol{\xi}) + \mathcal{K}(\boldsymbol{\xi})\mathcal{U}(\cdot,\boldsymbol{\xi}) = \mathcal{F} \tag{2}$$

where $\mathcal{U} = \{\boldsymbol{u}_e\ ,\ \boldsymbol{p}\}^T$ and $\mathcal{M}_{2\times2}$, $\mathcal{C}_{2\times2}$ and $\mathcal{K}_{2\times2}$ are generalized mass, damping and stiffness matrices, respectively, and $\mathcal{F} = \{\boldsymbol{f}_s\ ,\ \boldsymbol{f}_f\}^T$. Equation (2) is the general SFEM model for the random vibroacoustic analysis of structure-acoustic problem. The numerical solution of this equation requires random space discretization analogous to physical spatial and time discretization. The associated global random stochastic stiffness and damping matrices are represented by means of the truncated gPC as [5,11]

$$\boldsymbol{K}_s(\boldsymbol{\xi}) = \sum_{i_k=0}^{N_k} [k]_{i_k} \Psi_{i_k}(\boldsymbol{\xi}) = \boldsymbol{\mathcal{K}}^T \boldsymbol{\Psi}(\boldsymbol{\xi}), \qquad \boldsymbol{C}_s(\boldsymbol{\xi}) = \sum_{i_c=0}^{N_c} [c]_{i_c} \Psi_{i_c}(\boldsymbol{\xi}) = \boldsymbol{C}^T \boldsymbol{\Psi}(\boldsymbol{\xi}) \tag{3}$$

where $[k]_{i_k}$ and $[c]_{i_c}$ are the matrix coefficients and $\boldsymbol{\Psi}$ argues random orthogonal basis. Accordingly, the nodal response vector $\boldsymbol{\mathcal{U}}$ is approximated using the gPC as

$$\boldsymbol{\mathcal{U}}(\cdot, \boldsymbol{\xi}) = \sum_{i_u=0}^{N_u} \{U(\cdot)\}_{i_u} \Psi_{i_u}(\boldsymbol{\xi}) = \boldsymbol{U}^T \boldsymbol{\Psi}(\boldsymbol{\xi}) \tag{4}$$

in which $\{U(\cdot)\}_{i_u}$ are deterministic nodal gPC coefficients. The expansions of random matrices and random responses are substituted in Eq. (2), this yields to

$$\boldsymbol{\mathcal{M}}\ddot{\boldsymbol{U}}^T \boldsymbol{\Psi}(\boldsymbol{\xi}) + \boldsymbol{C}^T \boldsymbol{\Psi}(\boldsymbol{\xi})\dot{\boldsymbol{U}}^T \boldsymbol{\Psi}(\boldsymbol{\xi}) + \boldsymbol{\mathcal{K}}^T \boldsymbol{\Psi}(\boldsymbol{\xi})\boldsymbol{U}\boldsymbol{\Psi}(\boldsymbol{\xi}) = \boldsymbol{\mathcal{F}} \tag{5}$$

This denotes the spectral SFEM model of the coupled structure-acoustic problem due to the structural material and damping uncertainties. Analogy to error minimization in deterministic FEM modeling, the error associated to the random space discretization by means of the gPC expansions has to be minimized to estimate the unknown coefficients of the responses. The stochastic error is defined as

$$\boldsymbol{\epsilon}(\cdot, \boldsymbol{\xi}) = \boldsymbol{\mathcal{M}}\ddot{\boldsymbol{U}}^T \boldsymbol{\Psi}(\boldsymbol{\xi}) + \boldsymbol{C}^T \boldsymbol{\Psi}(\boldsymbol{\xi})\dot{\boldsymbol{U}}^T \boldsymbol{\Psi}(\boldsymbol{\xi}) + \boldsymbol{\mathcal{K}}^T \boldsymbol{\Psi}(\boldsymbol{\xi})\boldsymbol{U}^T \boldsymbol{\Psi}(\boldsymbol{\xi}) - \boldsymbol{\mathcal{F}} \tag{6}$$

The unknown deterministic coefficients of responses have to calculated so that the error to be minimum. Various methods are used for spectral error minimization, among them, two methods are mostly used in the literature [15]; intrusive and non–intrusive methods. In the first method, known also as stochastic Galerkin projection, the minimization test functions are chosen to be the known random orthogonal basis used to construct the gPC expansions. This requires access to the data structures of the FEM model or analytical governing equations to derive a system of deterministic equations for the stochastic modes of the solution. In the second method, sample collocation points in the random space are generated and the error $\boldsymbol{\epsilon}$ is vanished at these points. Here, the FEM model or the governing equations are employed as third party solver or black-box. This provides us with the major advantage to use deterministic numerical solver for solution of the SFEM model. A collocation based procedure is adopted in this work to realize the system responses. The realizations then are used to estimate the unknown deterministic coefficients/functions of the responses.

3. Random acoustic transmission loss

The FRC structures are used as typical noise control. They offer a combination variety of sound absorption and transmission of sound energy by selecting appropriate fiber orientations and the number of layers. To evaluate the efficiency of such structures for noise control, the acoustical transmission loss is defined. The transmission loss can be estimated directly by measuring the sound pressure levels on each side. The random Transmission Loss (rTL) is calculated as a random quantity from the ratio of average nodal pressures, i.e.

$$\mathrm{rTL}(\omega, \boldsymbol{\xi}) = 20 \log \frac{p_i(\omega)}{p_t(\omega, \boldsymbol{\xi})} \tag{7}$$

In which p_i and p_t stand for incident and transmitted sound pressures, respectively. They are approximated by the average of the nodal pressures in the FEM, that is

$$p_i(\omega) \approx \frac{1}{N_i} \sum_{n=1}^{N_i} p_n(\omega), \qquad p_t(\omega, \boldsymbol{\xi}) \approx \frac{1}{N_t} \sum_{n=1}^{N_t} p_n(\omega, \boldsymbol{\xi}) \tag{8}$$

Where N_i and N_t are number of nodes in incident and transmitted zones, respectively. Note that the sound pressure on the excitation side of the structure is assumed to be deterministic, i.e. not influenced from the structural random parameters. The realization samples of the rTL is estimated at collocation points employing deterministic FEM model and by calculation of the average of nodal pressures using Eqs. (8). The rTL is then approximated using the gPC expansion with the frequency-dependent deterministic coefficients and random orthogonal basis used to represent uncertainty in uncertain system parameters, i.e.

$$\text{rTL}(\omega, \boldsymbol{\xi}) = \sum_{i=0}^{N} b_i(\omega)\Psi_i(\boldsymbol{\xi}) \tag{9}$$

This leads a general approximated from of the rTL considering uncertainty in elastic and damping parameters. The unknown deterministic function b_i are calculated from realizations of the system responses on a set of collocation points. For a 2nd-order gPC expansion with 4-dimensional random vector, i.e. $\boldsymbol{\xi} = \{\xi_1, \xi_2, \xi_3, \xi_4\}$, this leads 15 unknown deterministic coefficients [16]. The numerical algorithm is given in Fig. 1. As demonstrated, the uncertain

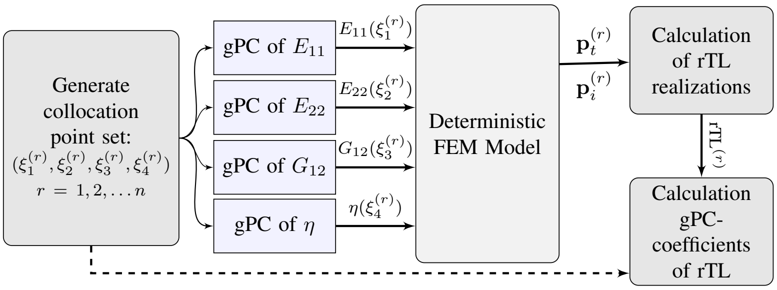

Fig. 1. Numerical algorithm for calculation of coefficients of the gPC expansion of rTL. The nodal pressures $\boldsymbol{p}_t^{(r)}$ and $\boldsymbol{p}_i^{(r)}$ are estimated using Eqs. (8).

parameters, i.e. elastic moduli E_{11}, E_{22}, G_{12} and damping ratio η, are realized for each collocation point set from the associated gPC expansions. The parameter values then are used as deterministic constant inputs to the FEM model to realize the vector of transmitted nodal pressures, $\boldsymbol{p}_t^{(r)}$ and $\boldsymbol{p}_i^{(r)}$, from which the rTL will be calculated. Once a set of rTL realizations are known, the coefficients $b_i(\omega)$ are estimated. For a very good accuracy, one requires the number of collocation points to be larger (about twice) than of the number of unknown coefficients. For that reason, a least-square minimization procedure is employed to calculated the unknown coefficients [16].

4. Numerical results

As a case study, in this section, the vibroacoustic analysis of FRC plates is investigated to estimate the rTL under uncertainty in elastic and modal damping parameters. Specimen is made of 12 layers with 60% glass fibers oriented in $[0/90]_s$ in epoxy matrix. All layers have nominal identical properties with dimensions and the mean value of elastic parameters given in Table 1. The uncertain parameters, i.e. elastic moduli and damping coefficient, are all assumed to

Table 1. Nominal plate dimensions; a : length, b : width and h : thickness; with the mean value μ and the standard deviation σ of elastic parameters and the damping ratio. Coordinates 1 and 2 are parallel and perpendicular to fibers, respectively.

Dimensions [mm]	$a = 250$	$b = 125$	$h = 2$
Elastic moduli (μ, σ) [Gpa]	$E_{11} = (47.45, 11.86)$	$E_{22} = (9.73, 2.44)$	$G_{12} = (4.01, 0.62)$
Poisson's ratio [–]	$\nu_{12} = 0.24$	$\nu_{21} = 0.24$	–
Damping ratio [%]	$\mu = 2.5$	$\sigma = 0.625$	–

be distributed with lognormal probability density function (PDF) having the mean value μ and the standard deviation σ given in Table 1. The 3rd-order gPC expansions with orthogonal Hermite basis are employed to represent the parameters. The unknown coefficients the are calculated using Galerkin projection, see [16] for more details. The reconstructed PDFs of uncertain parameters compared to the theoretical solutions are given in Fig. 2. As shown, the 3rd-order expansion show a high accuracy for representation of the uncertain parameters. The deterministic FEM model consists of two rooms separated by a sample FRC plated having dimensions and material parameters given

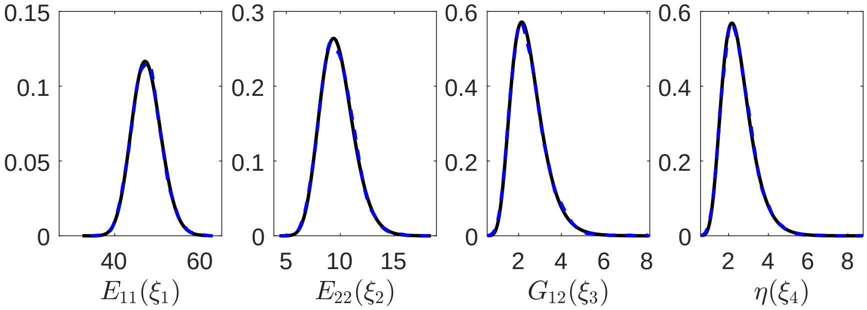

Fig. 2. Reconstructed PDF of elastic parameters (in Mpa) and damping coefficient (in %) from the 3rd-order gPC expansions (dashed lines) compared to theoretical lognormal PDFs (bold lines) .

in Table 1. The rTL is assumed as unknown output and is approximated with 2nd-order gPC expansion as given in Eq. (9). A harmonic sound source is assumed at the bottom left corner of the first room in frequency range of $f = 0$ to 500 Hz. A set of collocation points $(\xi_1^{(r)}, \xi_2^{(r)}, \xi_3^{(r)}, \xi_4^{(r)})$, $r = 1, 2, \ldots, 30$ are generated from the roots of 3rd-order orthogonal Hermite polynomials, i.e. $0, -\sqrt{3}$ and $\sqrt{3}$. Typical collocation points may be $(0, 0, 0, 0)$ and $(0, 0, 0, -\sqrt{3})$ etc. This leads a set of $4^3 = 81$ collocation points from which 30 points are selected to estimated 15 unknown coefficients b_i. The closest point to $(0, 0, 0, 0)$ has priory. The full harmonic analysis is performed to realize the nodal pressures at each selected collocation point and accordingly, the rTL samples. These are used to estimate the coefficients using the least-square minimization procedure. The calculated coefficients are shown in Fig. 3 in which $\omega = 2\pi f$. As demonstrated, $b_0(f)$, is the largest dominated coefficient as shown by bold curve. The effect of

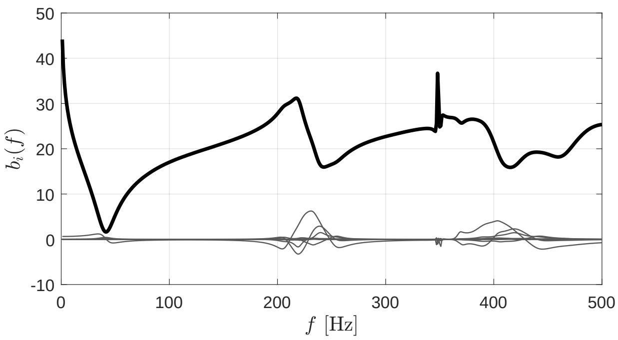

Fig. 3. The gPC coefficients of the rTL (in dB) estimated from 30 collocation points. The bold curve shows $b_0(f)$, as the expected value of th rTL.

uncertainty in some frequency ranges can be ignored where the higher order coefficients converge to zero. To show the accuracy of the method, the mean value and the standard deviation of the rTL obtained from the 2nd-order gPC expansion compared to MC simulations with 1000 samples are shown in Fig. 4. As shown, the maximum variation of the rTL is about ±6 dB around frequency 230 Hz. The high accuracy of the collocation-based simulation compared to MC results is remarkable. Furthermore, while the computational time for the method with 30 collocation points is 51 minutes, this yields 25 hours and 52 minutes for the MC simulations.

5. Conclusions

The spectral stochastic FE method has been employed to investigate the acoustical transmission loss of fiber-reinforced composite plates under random elastic and damping parameters. The gPC method is used to discretize the parameters as well as the random transmission loss (rTL). The deterministic FE model has been operated to realize the rTL on a set of collocation points generated in random space. The realization samples are then used to calculate the unknown frequency-dependent coefficients of the gPC expansion of rTL. The results have demonstrated that while the parameter uncertainties had strong impact on the rTL in specific frequency domains, they have minor influences

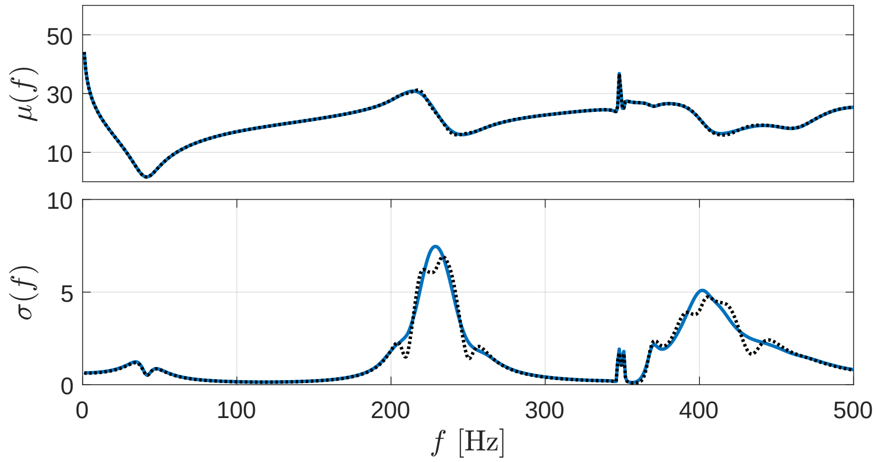

Fig. 4. The mean value, $\mu(f)$, and the standard deviation, $\sigma(f)$, both in dB, obtained from 2nd-order gPC expansion (dotted lines) compared to MC simulation with 1000 realization samples (bold lines)

in other frequency ranges. The results obtained from 30 collocation points also demonstrated a very good agreement with MC simulations with 1000 realizations and, consequently, very efficient in terms of computational time.

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