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Computational Cutting Pattern Generation using Isogeometric B-Rep Analysis

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Abstract

The cutting pattern plays a major role for the design of structural membranes, since it influences both their aesthetical appearance and structural behavior.
A novel approach towards cutting pattern generation is the so-called Variation of Reference Strategy (VaReS) [1], which minimizes the total potential energy arising from the motion of a planar cutting pattern to its corresponding three-dimensional shape.

With non-uniform rational B-Splines (NURBS) being the standard tool for geometry description in CAD, it is only consequent to use these for analysis as well. Isogeometric B-Rep Analysis (IBRA) [2] follows up on this idea and enriches the original Isogeometric Analysis (IGA), which was introduced by Hughes et al. [3], by the possibility of analysing trimmed NURBS geometries.

This paper presents cutting pattern generation with the Variation of Reference Strategy in the context of IGA/IBRA. With this approach, the whole design of a membrane structure can be represented by NURBS geometries – including blueprint plans. To use the benefits of IBRA for cutting pattern generation, a NURBS-based membrane-element was developed for the VaReS routine.
A developable surface serves as a benchmark example, since its analytical cutting pattern is known. Examples of double-curved geometries show the applicability and benefits of the proposed procedure for real structures.

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1. Introduction and Motivation

Structural membranes as lightweight structures are characterized by a considerable gain in strength through a double curved shape, since they carry loads through tension exclusively. This leads to the necessity of generating cutting patterns for the construction, which is often performed by simple geometric projections and seam allowance by empirical data.

The design cycle of membrane structures consists of three major disciplines, namely form-finding, structural analysis and cutting pattern generation. Due to its influence on both the aesthetical appearance and the structural behavior of the final membrane structure, the cutting pattern plays a major role for the design. The double curvature in most of these structures' shapes makes it impossible to find the stress-free planar correspondence, i.e. cutting pattern, to the spatial geometry by means of mere geometric operations. So in a mathematical sense, no unique cutting pattern exists for synclastic and/or anticlastic structures, therefore it needs to be found by an optimization.

The *Variation of Reference Strategy* [1] (VaReS) solves this optimization problem by minimizing the potential energy arising from the movement of the planar geometry (the cutting pattern) into the spatial configuration. The nodal positions of the planar configuration serve as design variables.

Since the development of *Isogeometric Analysis* (IGA) [3], which enables the designer to perform analysis on CAD geometries without creating a Finite Element mesh, it is most desirable to perform the whole design cycle of structural membranes on the CAD-geometry, including cutting pattern generation.

Apart from the advantage of not having to create meshes, which can be highly time-consuming, there is no loss of information on the geometry, since NURBS are able to exactly represent even highly complex geometries. With *Isogeometric B-Rep Analysis* (IBRA) [2], the cutting pattern generation via VaReS can determine optimal cutting patterns for NURBS-based geometries. Additional features such as trimming operations and coupling of B-Rep edges provide further enhancements.

A brief overview of the Variation of Reference Strategy is given in Section 2. Section 3 points out the major traits of IGA and IBRA with respect to cutting pattern generation. The interested reader is referred to the relevant literature for detailed explanations of both VaReS and the concepts of IGA and IBRA, as these would go beyond the scope of this paper. In Section 4 and 5, two examples show the applicability and benefits of using the developed elements, before a conclusion and an outlook are provided in Section 6.

2. Variation of Reference Strategy

The Variation of Reference Strategy was developed at the authors' chair (see Widhammer [1] and Dieringer [6]), and follows the principle of minimizing the potential energy which results out of the movement of a geometry from a planar configuration to a spatial one. The spatial configuration is identified as the target of the planar one's movement and is thus held fixed. In a continuum mechanical sense, the planar configuration is the reference configuration Ω_0 and the spatial one serves as the current configuration Ω. In contrast to most analyses, the reference configuration holds the design variables in VaReS.

In order to find the minimum of the potential energy, the nodal positions of the reference configuration $\mathbf{X} \in \Omega_0$ are allowed to change their position. The unconstrained optimization problem thus reads as follows,

$$\min_{\mathbf{X}\in\Omega_0} \rightarrow \Pi(\mathbf{X}) = \Pi_\chi(\mathbf{X}) - \Pi_{\text{pre}} = \int_{\Omega_0} \Psi_\chi(\mathbf{C}(\mathbf{X}))\mathrm{d}\Omega_0 - \int_{\Omega_0} \Psi_{\text{pre}}(\mathbf{C}(\mathbf{X}))\mathrm{d}\Omega_0 \tag{1}$$

with the components of potential energy arising from the motion $\Pi_\chi(\mathbf{X})$;and the prestress state Π_{pre} .

The potential energy is defined by means of a hyperelastic material model, which links strains and stresses by a scalar strain-energy function $\Psi(I_1, I_2, I_3) = \Psi(\mathbf{C})$, with the structural invariants $I_1(\mathbf{C}), I_2(\mathbf{C})$ and $I_3(\mathbf{C})$ at a deformation state described by the right Cauchy-Green tensor $\mathbf{C}$. See Holzapfel [4] or Belytschko [5] for further information on hyperelastic material models and the theory of invariants.

The 2[nd] Piola-Kirchhoff stress tensor can be expressed by the strain-energy function in the following way:

$$\mathbf{S} = 2\frac{\partial \Psi(\mathbf{C})}{\partial \mathbf{C}} \tag{2}$$

For the examples shown in this paper, Neo-Hooke material with isotropic material behavior was considered. According to Holzapfel [4], this material can be described by the following entities.

$$\Psi = c_1\left(\lambda_1^2 + \lambda_2^2 + \lambda_3^2 - 3\right) = c_1(\mathrm{I}_1 - 3) \tag{3}$$

The constant c_1 is defined as $c_1 = \mu / 2$, with μ being the shear modulus. The variables λ_a with $a = 1,2,3$ are the principal stretches.

There are numerous possible ways of solving the unconstrained optimization problem stated in Equation (1). However, two major approaches have been used for VaReS thus far, namely:

- A first-order approach with the Steepest Descent method
- A Second-order approach with the Newton-Raphson method.

In the following, both approaches will be introduced briefly, while the examples shown in this paper were conducted using the second-order one.

2.1. First-order approach: Steepest Descent

The method of Steepest Descent directly solves the optimization problem by updating the design variables in the following way:

$$\mathbf{X}^{i+1} = \mathbf{X}^i - \alpha^i \cdot \nabla_{\mathbf{X}} \Pi_{\text{total}}(\mathbf{X}^i) \tag{4}$$

Assuming hyperelastic material behavior, the gradient of the potential energy can be expressed as follows, using Green-Lagrange strains and 2[nd] Piola-Kirchhoff stresses and considering the chain rule.

$$\nabla_{\mathbf{X}} \Psi(\mathbf{E}(\mathbf{X})) = \mathbf{S}(\mathbf{E}(\mathbf{X})) : \nabla_{\mathbf{X}} \mathbf{E}(\mathbf{X}) \quad \text{and} \quad \nabla_{\mathbf{X}} \Psi(\mathbf{E}_{\text{pre}}) = \mathbf{S}(\mathbf{E}_{\text{pre}}) : \nabla_{\mathbf{X}} \mathbf{E}(\mathbf{X}) \tag{5}$$

$$\nabla_{\mathbf{X}} \Pi_{\text{total}}(\mathbf{X}^i) = \int_{\Omega_0} \left[\mathbf{S}\left(\mathbf{E}\left(\mathbf{X}^i\right)\right) - \mathbf{S}\left(\mathbf{E}_{\text{pre}}\right)\right] : \nabla_{\mathbf{X}} \mathbf{E}\left(\mathbf{X}^i\right) d\Omega_0^i + \int_{\Omega_0} \left[\Psi\left(\mathbf{E}\left(\mathbf{X}^i\right)\right) - \Psi\left(\mathbf{E}_{\text{pre}}\right)\right] \nabla_{\mathbf{X}} d\Omega_0^i \tag{6}$$

$$\nabla_{\mathbf{X}} \mathbf{E}((\mathbf{X})) = D_{\delta \mathbf{X}} \mathbf{E}(\mathbf{X}) = \frac{d}{d\varepsilon}\left[\mathbf{E}(\mathbf{X} + \varepsilon \delta \mathbf{X})\right] \Big|_{\varepsilon=0} \tag{7}$$

2.2. Second-order approach: Newton-Raphson

Following the variational principle, a virtual material position vector $\delta\mathbf{X}$ is introduced in order to solve the unconstrained optimization problem (Equation 1) with the second-order approach. According to variational calculus, the minimum of a functional is reached at its stationary point, i.e. $\delta\Pi = 0$:

$$\delta\Pi_{\text{total}}(\text{X}) = D_{\delta\text{X}}\Pi_{\text{total}}(\mathbf{X}) = \frac{\text{d}}{\text{d}\varepsilon}\left[\Pi_{\text{total}}(\mathbf{X}+\varepsilon\delta\mathbf{X})\right]\bigg|_{\varepsilon=0} = 0 \tag{8}$$

Again, the chain rule needs to be applied in order to formulate the variation of the total potential energy under consideration of Green-Lagrange strains and 2nd Piola-Kirchhoff strains.

$$\delta\Pi_{\text{total}}(\mathbf{X}) = \int_{\Omega_0}\left[\mathbf{S}(\mathbf{E}(\mathbf{X})) - \mathbf{S}(\mathbf{E}_{\text{pre}})\right] : \delta\mathbf{E}(\mathbf{X})\text{d}\Omega_0 + \int_{\Omega_0}\left[\Psi(\mathbf{E}(\mathbf{X})) - \Psi(\mathbf{E}_{\text{pre}})\right]\delta\text{d}\Omega_0 \overset{!}{=} 0 \tag{9}$$

Equation (9) can be solved by determination of the 1st Variation of the Green-Lagrange strain tensor $\delta\mathbf{E}(\mathbf{X})$,

$$\delta\mathbf{E}(\mathbf{X}) = D_{\delta\text{X}}\mathbf{E}(\mathbf{X}) = \frac{\text{d}}{\text{d}\varepsilon}\left[\mathbf{E}(\mathbf{X}+\varepsilon\delta\mathbf{X}\right]\big|_{\varepsilon=0}, \tag{10}$$

the 1st Variation of the strain-energy functions $\delta\Psi(\mathbf{E}(\mathbf{X}))$ and $\delta\Psi(\mathbf{E}_{\text{pre}})$

$$\delta\Psi(\mathbf{E}(\mathbf{X})) = \mathbf{S}(\mathbf{E}(\mathbf{X})) : \delta\mathbf{E}(\mathbf{X}) \quad \text{and} \quad \delta\Psi(\mathbf{E}_{\text{pre}}) = \mathbf{S}(\mathbf{E}_{\text{pre}}) : \delta\mathbf{E}(\mathbf{X}), \tag{11}$$

and the 1st Variation of the differential material domain

$$\delta\text{d}\Omega_0 = D_{\delta\text{X}}\text{d}\Omega_0(\mathbf{X}) = \frac{\text{d}}{\text{d}\varepsilon}\left[\text{d}\Omega_0(\mathbf{X}+\varepsilon\delta\mathbf{X})\right]\big|_{\varepsilon=0}. \tag{12}$$

3. Isogeometric B-Rep Analysis: discrete formulation

Isogeometric B-Rep Analysis was developed by Breitenberger et al. at the authors' chair (see Breitenberger et al. [2] and Philipp et al. [7]) and enhances *Isogeometric Analysis* (IGA), which was first introduced by Hughes et al. [3], by the inclusion of B-Rep geometries. The usage of *Non-uniform Rational B-Splines* (NURBS) allows for the accurate description of highly complex geometries. IBRA makes use of this accuracy by referring to the full B-Rep description of the CAD-geometry and performing analysis on this geometry with NURBS basis functions.

The shapes of architectural membranes, as well as manufacturing circumstances usually lead to shapes which are constructed by an arrangement of membrane strips. Several techniques of finding these stripes have been developed in the past, with the usage of geodesic lines being a standard approach (see Forster et al. [8] and Dieringer [6]).

By using trimming operations in order to define cutting pattern lines, the variety in strip shapes is enlarged by a considerable amount of possibilities. Trimming operations basically "cut" out geometrical parts and thus create partially visible trimmed surfaces as can be seen in Figure 3.

NURBS-based geometry description allows for the modeling of highly complex shapes with respect to curvature. By enriching the design possibilities through the usage of trimmed multipatches, an even larger variety of shapes can possibly be dealt with for the analysis of architectural membranes.

The basic idea behind the analysis of trimmed multipatches (developed by Breitenberger et al. [2], [7]) is based on the coupling of edges.

With respect to cutting patterns, the potential of working with multipatches lies in the ability of adding constraints to the edges. Thinking of seam lines, multipatches certainly lead to an important enhancement of the field of cutting pattern generation. The cutting pattern analysis with VaReS relates to the real erection process of a membrane structure in an acceptable manner, once coupled edges can be considered by the analysis.

4. Example of a developable surface: cylinder segment

As a first example of patterning with VaReS-based IBRA elements, a cylinder segment was chosen. Being curved in one direction only, the cylinder segment portrays a developable surface. An analytical solution to the cutting pattern thus exists and allows for a benchmark example.

The chosen 180° cylinder segment has a radius of 10 m and a length of 20 m in lateral direction. The discretization is depicted in Figure 1 and the material data is given in Table 1.

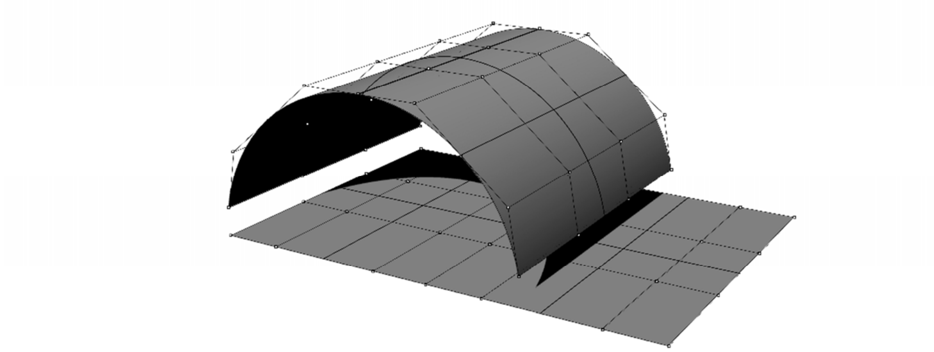

Fig. 1. Cutting pattern of a 180° cylinder segment with its control-points.

Table 1. Material data

Young's modulus E [Pa]	Poisson's ratio ν [-]	time steps (Newton-Raphson)
1.0e+03	0.10	5

The design variables are positioned at the control-points of the planar geometry (reference configuration). Each control-point has two degrees of freedom in the plane.

Since the analytical solution for the curve length and surface area was met by the cutting pattern analysis with the NURBS-based VaReS elements, the functionality of the analysis in the IBRA context could be shown. The exact geometry description by NURBS yields an exact solution for the cutting pattern of the cylinder segment. This could never be reached by classical finite elements due to the necessity of meshing and thus only approximating the geometry by a facette-type surface.

Apart from the advantages with respect to the accuracy in shape-description, the small amount of design variables needed for the analysis with IBRA should be mentioned.

5. Examples of a double-curved surface: Schwarz-like minimal surface

In order to show the applicability of VaReS with NURBS for double-curved surfaces, a four-point sail was chosen (Schwarz-like minimal surface). The possibility of considering trimmed surfaces is shown for this example. Again, the examples shown here are the results of an optimization with the Newton-Raphson method (second-order approach).

Figure 2 and 3 show how little elements are necessary for a Variation of Reference Strategy analysis with NURBS-based elements.

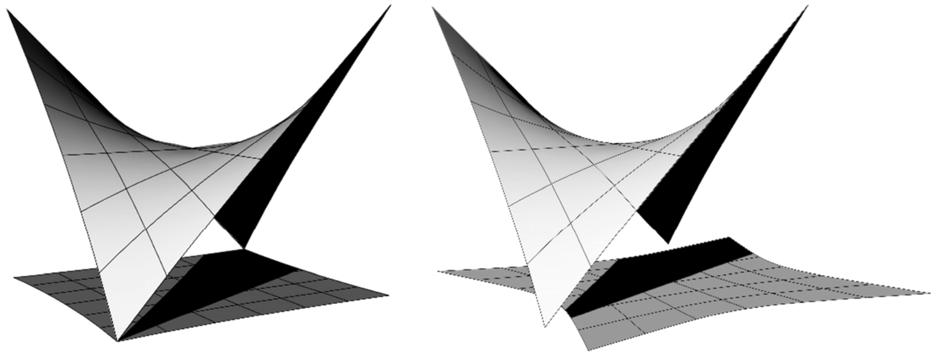

Fig. 2. Cutting pattern of a four-point sail: projection into the plane (left) and converged optimized cutting pattern after 5 time steps (right)

For the analysis of trimmed NURBS-surfaces, only the untrimmed part of the geometry is considered for the analysis. The underlying theory can be found in Breitenberger et al. [2] and Philipp et al. [7]. In Figure 3, the control-point polygons of the trimmed 4-point sail can be seen and illustrate the mentioned method.

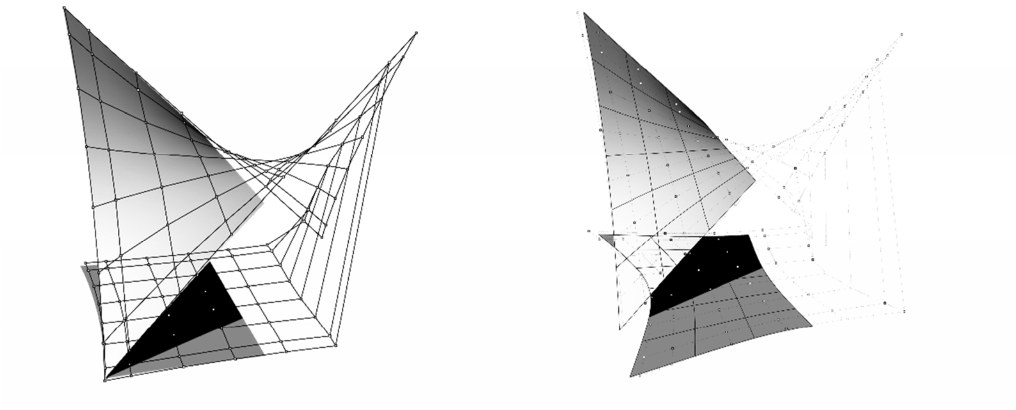

Fig. 3. Cutting pattern of a trimmed four-point sail: projection into the plane (left) and converged optimized cutting pattern after 5 time steps (right)

6. Conclusion and Outlook

A new approach towards cutting pattern generation was shown by integrating Isogeometric B-Rep Analysis into the Variation of Reference Strategy. Through the usage of NURBS as basis functions and the advantages of B-Rep geometry description, VaReS based on NURBS proves to be very promising for the cutting pattern generation of complex geometries. A developable and a double-curved geometry showed both the accuracy and potential of the developed procedure.

The design-cycle of membrane structures can thus be solely performed on the CAD-geometry, as is the idea of Analysis in Computer Aided Design (AiCAD) [7].

In order to consider all influences on architectural membranes and to fully model the construction process, prestress needs to be included into the cutting pattern generation, as was shown in equations (1) to (12).

The extension of VaReS to multipatches would provide the framework to generate cutting patterns for large complex geometries including kinks.

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