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Shockwave Suppression by Vehicle-to-Vehicle Communication

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Abstract
The rapid development of wireless communication and information technologies has increased research interests in inter-vehicle communication systems and their effect on traffic flow. One of the most complex traffic phenomena on freeways are shockwaves. Shockwaves are recognized as the sudden, substantial change in the state of the traffic flow, which acts as an active or moving bottleneck. They have significant impact on freeway capacity and safety.
For this study, a microscopic traffic simulation was used to determine the extent to which inter-vehicle communication and change in the driving strategy after the recognition of a shockwave can influence the propagation and dissolving of shockwaves on freeways. We also briefly introduce the shockwave theory and our communication algorithm. Then we present the simulation result with different penetration rates of communicative vehicles, which are randomly dispersed in traffic flow, through performance measures for traffic flow with shockwaves.

Keywords: Shockwaves, vehicle-to-vehicle communication, freeways, simulation, traffic jam ahead warning

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1 Introduction

Shockwaves are an important flow processes in traffic flow theory. The speed of the jam front defines the congestion patterns and impacts. They have a great influence on highway capacity, number of rear-end collisions on the freeway, fuel consumption and emissions. Several types of shockwaves can be found depending on the traffic conditions that lead to their formation. These include frontal stationary, forward forming and forward recovery, rear stationary, backward forming, backward recovery shockwaves. So far lots of studies focused on different aspects of shockwaves such as: characteristic of shockwaves based on L-W Fluid Theory (Kuhne, et al., 2000), highway bottleneck queue length and delay time (Smith, et al., 2003), (Munoz, et al., 2002), traffic flow stability (Zhang, 1999), traffic accidents due to shockwaves (Lee, et al., 2010), (Yu, 2012), shockwave prevention control methods (Hegyi, et al., 2005), (Breton, et al., 2002), evaluation of the inter-vehicle communication

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(Suijs, et al., 2014) and highly automated vehicles (Motamedidehkordi, et al., 2015) in dynamic traffic flow and propagation of perturbations along the freeway.

By analyzing real vehicle trajectories, Huang and Wu (2013) figured out that the driver reaction time in shockwave situations seemed larger than in normal situations, showing that in shockwave situations drivers' behavior changes. It seemed that during the propagation, downstream drivers recognize that the upstream vehicles are in shockwave situations and adjust their driving behavior accordingly (Huang, et al., 2013). This means that the drivers change their driving towards a shockwave propagation in order to reduce the negative effects. Therefore, the drivers can recognize the shockwave situation by observing their lead vehicles, which leads already to a damping of the shockwave.

The recent development of the internet of things, provides new ideas and ways for shockwave damping solutions. An early recognition of the shockwave is the key to be able to damp the shockwave. Therefore, by telling the drivers in advance, that they might face a shockwave situation, they might adapt their driving behavior prior to reaching the congestion situation.

The paper introduces the analysis of shockwaves on freeways, the development and application of the methods to estimate the shockwave propagation speed and discusses the changes in shockwave characteristics by deploying the vehicle-to-vehicle (V2V) communication technology, specifically the traffic jam ahead warning application.

Section 2 of this paper discusses shockwave theories and state of art V2V communication technologies. Section 3 describes the empirical data used for this study. The simulation framework and the simulation scenarios are presented in section 4. Section 5 presents the result of the simulation and the change in shockwave characteristics for different penetration rates. The last section summarizes the study findings and outlines further research directions.

1.1 Shockwave theory

Shockwaves can be defined as a boundary condition between two traffic states that are characterized by different densities, speeds and/or flow rates on the road. In order to study the congestion patterns and impacts and designing traffic management studies, one needs to understand the formation, the dissolving and the characteristics of shockwaves. The findings on shockwave characteristics and propagation speed can be used to identify the spatial and temporal impacts of a congestion, and to develop and calibrate traffic flow models (Xiao-Yun, et al., 2007).

In real traffic conditions, the behavior of traffic flow is similar to the behavior of fluid waves. We assume that there are two different areas (A and B) with different traffic densities. The border between these two areas is called shockwave front S and the speed of S is v_s. The respective vehicle speeds in area A and B are v_u and v_d. We can calculate the number of vehicles (N) passing by the interface S within the time t as follows:

$$N = (v_u - v_s)k_u t = (v_d - v_s)k_d t \tag{1}$$

Based on the theory of Lighthill-Whitham-Richard (LWR) we can get the speed of the traffic shockwave:

$$v_S = \frac{q_d - q_u}{k_d - k_u} \tag{2}$$

A shockwave describes the conversion of two different traffic conditions and the speed of the shockwave describes the direction and the process of converting. In other words, the speed of the shockwave equals the jump in the flow over the wave divided by the jump in the density. When $v_s > 0$ this means that the shockwave is moving in the direction of traffic, whereas when $v_s < 0$ the shockwave is moving in the opposite direction of traffic and when $v = 0$, the wave does not move and is stable in its location.

1.2 Vehicle-to-vehicle communication

The rapid development of wireless communication and information technologies has increased research interest in inter-vehicle communication systems, which might improve the comfort, safety and operational efficiency of transportation systems (Lu, et al., 2009). It is foreseen, that by 2020, there will be 250 million connected vehicles on the road (van der Meulen, et al., 2015). Vehicle-to-X (V2X) communication allows information exchange both between vehicles and between a road user and the transportation infrastructure or the traffic control center. Following and oncoming road users will be notified of potentially dangerous situations to enable them to react on time and appropriately. V2V communication technology aims to ensure that vehicles are warned or that they warn the other traffic in potential dangerous situation.

The communication system within this project only sends the Traffic Jam Ahead Warning (TJAW) to the driver and does not take control of the vehicle. This TJAW application alerts vehicle drivers of particular traffic situations. The driver is informed that it is necessary to slow down regardless of the nature of the traffic problem. The alert message informing of a slow down or traffic jam must be transmitted to other vehicles efficiently and quickly, therefore the transmission delay is assumed to be neglectable. After receiving the alert, the drivers drive more carefully, would slow down and follow the proposed speed which increases the safety.

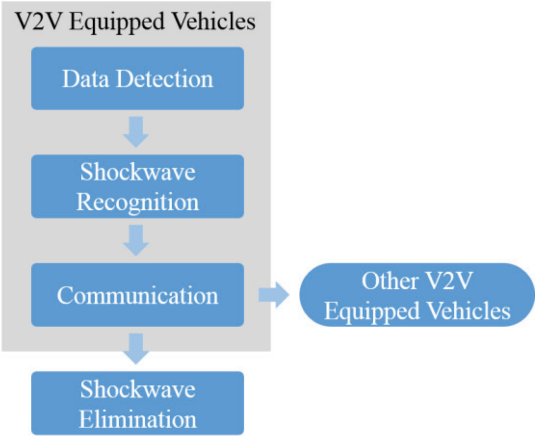

Figure 1: Shockwave elimination method

2 Sim^{TD} data analysis

The main source of data for this research is from the German sim^{TD} project. Sim^{TD} was a research project realized in a Field Operational Trial (FOT) infrastructure around the Hessian city of Frankfurt am Main. The project paved the way for the political, economic and technological framework to successfully set up V2V and vehicle-to-infrastructure (V2I) networking. The FOT testbed consisted of routes on several freeways, rural highways and also a network of urban streets where a fleet of up to 120 sim^{TD}-equipped vehicles was driven in pre-planned and controlled tests with naturalistic driving behavior and in a naturalistic environment together with the normal traffic. In freeway scenarios for instance traffic jam ahead warning, traffic sign assistance, traffic information and route deviation management were tested (simTD, 2015). The goal was to evaluate the different applications in real traffic situations with regard to their technical functionality and to assess the impact on traffic efficiency and traffic safety (Schimandl, et al., 2013). Therefore, the analyses with such a fleet of equipped vehicles allowed for the determination of the effects of the sim^{TD} system on driver-vehicle-units with versus such without a cooperative system. However, the estimation of the effect of cooperative systems on the overall traffic is quite limited, even with 120 equipped vehicles. Hence, this work aims at using the available data from the FOT, including vehicle sensor and stationary detector data, to determine the impact of V2V communication on the overall traffic, like described in the following.

2.1 Microscopic data

The available microscopic data from the project includes detailed sub-second information about vehicle trajectories over time. It also includes sub-second vehicle position, speed, acceleration, headway and spacing information form the 120 equipped vehicles which took part in the field trials. Vehicle trajectories can provide valuable information for the calibration of microsimulation models. In the sim^{TD} project the number of provided vehicle trajectories were limited. For this reason, the calibration based on the vehicle trajectories, is not considered in this study and it was only analyzed to gather information that can be used in calibrating the microscopic driving behavior.

2.2 Macroscopic data

Macroscopic data which was delivered in the project includes aggregated vehicle information. This data is typically obtained from inductive loops. The aggregation interval varies from 20 seconds to 15 minutes. Available macroscopic data are based on loop detectors and include one minute aggregated volumes, speeds and occupancy counts.

Using the macroscopic data from the A5 freeway in Germany between Friedberg and Bad Homburger Kreuz, speed and flow contour plots were drawn. These contour graphs have a wealth of information. Information that can be obtained from contours include the extent (space and time) of the congestion, the location of the bottleneck, speed, flow or density information as well as queue lengths. In figure 2 below, time-space diagrams for speeds as well as flows are shown. In the left figure the average speed (per minute per road section) on the network is visualized through time. A clear indication for shockwaves can be seen on the network. Through time it can be seen, that the shockwave moves upstream through the network. The same shockwave can also be recognized in the right figure in which the flow is visualized through time and space. When the shockwave passes an inductive loop, the measured flow is clearly reduced. Not only can the presence of a shockwave be deduced from the analysis of the speed- and flow diagrams, also the movement speed of the shockwave can be deduced.

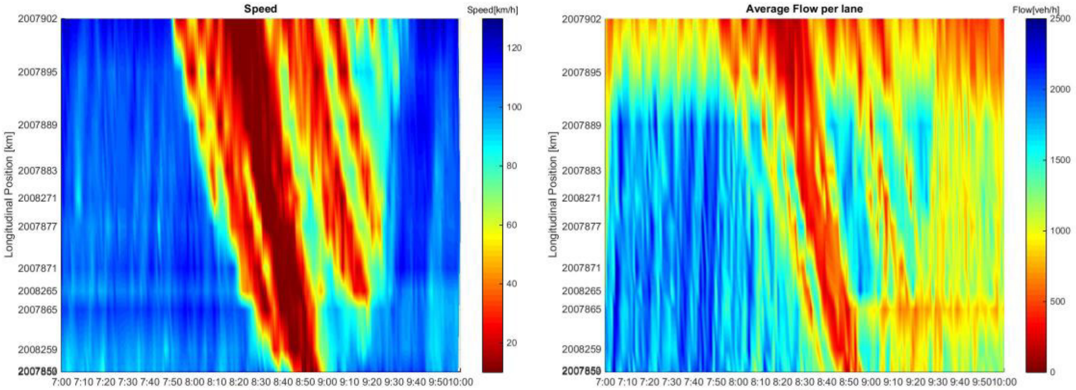

Figure 2: Speed and flow contour plots

3 Simulation frameworks

In this section, a description of the software used and the calibration and validation procedure of the simulation experiment are explained. Later, the scenarios which were considered in this study are discussed.

3.1 Simulation software

PTV Vissim 6 microscopic simulation software was used for this study. Vissim is a discrete, stochastic, time step based simulation software. The traffic flow model used by the traffic simulator relies on one of the two following models: the "Wiedemann 74" car following model or the "Wiedemann 99" car following model. Vissim car following models are classified as psycho-physical car following models, which are based on modeling human perception and reaction thresholds in a car following process (Menneni, et al., 2009). Wiedemann (Wiedemann, 1974) defines four different car following regimes in both the car following models:

- Free Flow: The vehicle is not influenced by other vehicles; the vehicle tries to keep its desired speed, but fluctuates around its desired speed due to imperfect throttle control.
- Approaching: Once the vehicle realizes it is approaching another vehicle, it decelerates to match the lead vehicle's speed as it reaches its desired safety distance.
- Following: In following state, the following vehicle unconsciously follows the lead vehicle as it keeps the speed difference and acceleration low.
- Emergency: If the vehicle's distance falls below a desired following distance, it reacts by applying maximum deceleration in order to avoid collision.

Both models, Wiedemann 74 and Wiedemann 99, are based on human perception thresholds but the calculation of these perception thresholds are different in both models. Figure 3 describes a typical goal-seeking behavior of a following car in a car following process.

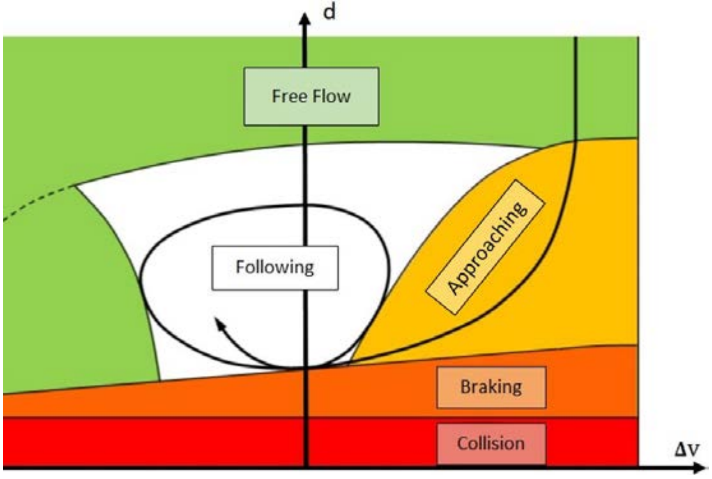

Figure 3: Car following model and driving states **(PTV AG, 2015)**

3.1.1. Wiedemann 74 car following model

In the Wiedemann 74 model, the minimum safety distance is calculated based on formula 3 (Wiedemann, 1974). The minimum desired distance of a vehicle in the following process is proportional to the square root of the slower vehicle's speed. The slower one can be either the lead vehicle or the following vehicle.

$$ABX = AX + (bx_{add} + bx_{mult} \cdot z) \cdot \sqrt{v} \qquad (3)$$

v: Speed of the slower vehicle [m/s].

z: Is a value in the range [0,1] which is normally distributed around 0.5 with a standard deviation of 0.15.

AX: Average standstill distance, which defines the average desired distance between two vehicles.

bx_{add}: Additive part of the following distance, which allows adjusting the time requirement values.

bx_{mult}: Multiplicative part of the following distance, which allows adjusting the time requirement values.

3.1.2. Wiedemann 99 car following model

The Wiedemann 99 car following model is very similar to the Wiedemann 74 model. The core execution or logic in Wiedemann 99 remained the same. However, some of the thresholds are calculated differently from the Wiedemann 74 model. The Wiedemann calibration parameters are described as follows:

- CC0: Defines the desired rear bumper-to-front bumper distance between stopped cars. This parameter has no variation.

  $$AX = CC0 + L_{n-1} \tag{4}$$

- CC1: Defines the time (in seconds) the following driver wishes to keep.

  $$ABX = AX + CC1 \cdot v_{slower} \tag{5}$$

- CC2: Restricts the longitudinal oscillation during following condition. In other words, it defines how much more distance than the desired safety distance (ABX) before the driver intentionally moves to the lead vehicle.

  $$SDX = ABX + CC2 \tag{6}$$

- CC3: Defines the start (in seconds) of the deceleration process.

  $$SDV: \Delta X = CC3. \Delta v + CC3 \cdot (-CC4) \tag{7}$$

- CC4 and CC5: Define the speed difference (in m/s) during the following process. CC4 controls the speed differences during the closing process while CC5 controls the speed differences during an opening process.

  $$CLDV = -CC4 \tag{8}$$

  $$OPDV = -CC5 \tag{9}$$

- CC6: Defines the influence of distance on speed oscillation during following condition. Increasing CC6 increases the oscillation of speed with increasing distance.

- CC7: Defines the actual acceleration during oscillation in a following process.

- CC8: Defines the desired acceleration when starting from a standstill.

- CC9: Defines the desired acceleration when at 80 km/h.

3.2 Calibration and validation

Calibration is one of the most important steps in traffic simulation model development which aims at reaching calibrated models that reasonably replicate local traffic conditions. Different methodologies were developed during the research of traffic simulations like (Cheu, et al., 1998), (Rakhau, et al., 2002), (Hourdakis, et al., 2003) (Brockfeld, et al., 2004), (Gomes, et al., 2004) and (Park, et al., 2005). Many researchers calibrated their models based on single or average values of traffic variables (Aycinf, et al., 1998), (Payne, et al., 1979). In this study, the single parameter method was deployed which is based on measuring the differences between field and simulated parameter values in which the speed recorded by detectors was used as a calibration parameter. In order to determine the statistical goodness-of-fit measure of the speed, based on the research objectives, the Root Mean Squared Error (RMSE) was calculated and the parameter set with the lowest RMSE was chosen. A temporary reduced speed area wad used to artificially make a bottleneck in the network. In order to replicate the driving behavior of cars on the freeway, Wiedemann 74 was used and the parameters AX, bx_{add} and bx_{mult} were calibrated after running the simulation for many parameter sets. The parameter set with $AX = 1$, $bx_{add} = 3$ and $bx_{mult} = 5$ achieved the minimum RMSE; therefore this parameter set was chosen as the best parameter set to replicate the manual driving behavior based on the empirical data.

3.3 Scenarios

The chosen scenario simulated the vehicles which are equipped with V2V communication systems. The driving behavior of these vehicles was like the manually driven vehicles (Wiedemann 74) unless they received a Traffic Jam Ahead Warning (TJAW). In this case the driving behavior changed Wiedemann 99 and the desired speed was set to 70 km/h. The CC3 value, which controls the start of the deceleration process and the vehicle recognition of the slow moving vehicle, was set to -11 instead of -8, which is the default value in Vissim. The more this value is, the sooner the reaction to the vehicle in front is. The equipped vehicles receive the warning in case there is another equipped vehicle 500 meters downstream, whose speed dropped abruptly to values below 30 km/h. The penetration rates of 5 %, 10 %, 20 % and 50 % have been simulated. This means, that in case of a penetration rate of 10%, one out of ten vehicles will apply to the provided speed advice. Figure 4 shows the result for different penetration rates.

4 Results

The simulation results for each penetration rate as well as the performance of each scenario with regard to average network speed, shockwave propagation and dissolving rate are discussed in this section.

4.1 Simulation results

We used Matlab to generate the contour plots of speed and flow for each scenario. The left-hand plots in figure 4 show the speeds for different penetration rates and the right-hand plots show the traffic flow contour plots. The plots confirm that the V2V communication damped the shockwave and with an increase in the penetration rate the length of the congestion and the congested area have decreased. A backward forming congestion can be seen in all the graphs since the capacity of the bottleneck dropped below the demand and the queue was formed upstream of the bottleneck. After removing the restriction causing the capacity reduction – which was a temporary reduced speed area – at the bottleneck, the congestion started to resolve and a backward recovery congestion was formed.

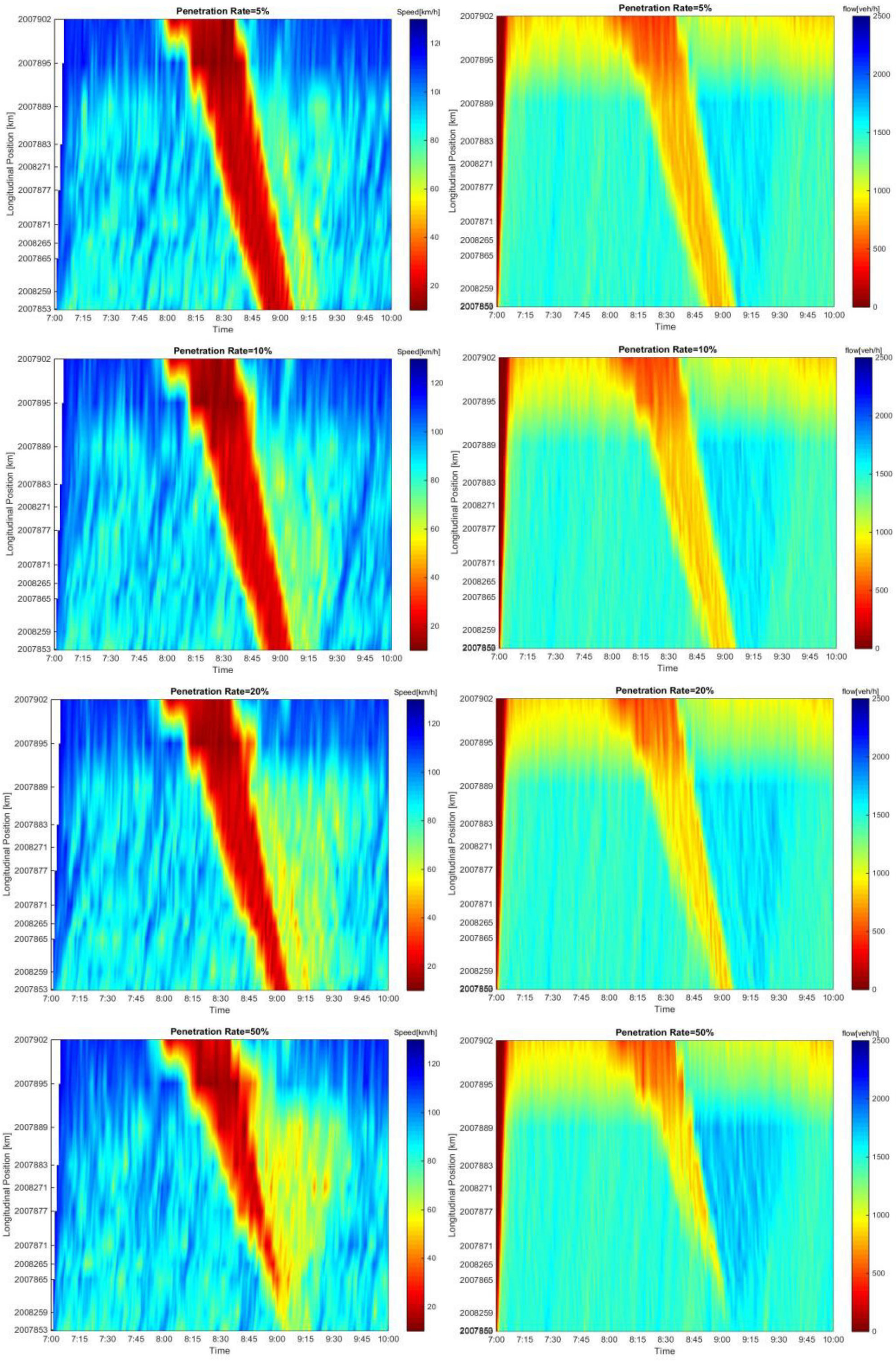

Figure 4: Speed and flow contour plots for different penetration rates

4.2 Performance

In order to analyze the effects of V2V communication and to compare the different scenarios, three performance indicators were considered:

- the average network speed,
- the propagation speed and
- the resolving speed of the congestion along the freeway.

The average network speed performance was calculated based on the formula:

$$V = \frac{\sum_{i=1}^{180} v_i \cdot q_i}{\sum_{i=1}^{180} q_i} \tag{10}$$

v_i: Speed of vehicles over all lanes in i^{th} minute [km/h]

q_i: The number of vehicles which passed that section in i^{th} minute [veh]

According to the shockwave theory, the propagation of the front between two traffic states has the same slope as the line corresponding the two states in the density-flow diagram. Therefore, the abrupt change of speed in the space-time diagram was considered. The border of the jam at each detector was chosen as traffic state change from free flow to speeds below 40 km/h. A simple linear regression was used in order to get the shockwave propagation speed (the backward forming shockwave) and the resolving speed (the backward recovery shockwave) of each scenario. Supposing that we have n data points $\{(xi, yi), i = 1, \ldots, n\}$, best-fit straight line through the data is calculated based on the following minimization problem using least squares method:

$$Find \min Q(a,b), \qquad for\ Q(a,b) = \sum_{i=1}^{n} r_i^2 = \sum_{i=1}^{n} (y_i - a - bx_i)^2 \tag{11}$$

$Q(\alpha, \beta)$: Sum of squared residuals of the linear regression model

r_i: Residual of the linear regression model

a: The y-intercept

b: The slope

The result of the performance indicators for different penetration rates of vehicles with communication capability is illustrated in table 1:

Penetration Rate	Indicator 1		Indicator 2		Indicator 3	
	Average Speed [km/h]	Relative Difference	Estimate of Shockwave Propagation Speed [km/h]	Relative Difference	Estimate of Shockwave Resolving Speed [km/h]	Relative Difference
0 %	83.22	-	-11.17	-	-18.08	-
5 %	83.46	0.29 %	-10.77	-3.60 %	-16.60	-8.20 %
10 %	83.46	0.29 %	-10.39	-7.00 %	-17.48	-3.30 %
20 %	81.38	-2.21 %	-9.14	-18.20 %	-16.78	-7.20 %
50 %	81.96	-1.51 %	-7.52	-32.70 %	-16.28	-10.00 %

Table 1: Performance for different penetration rates

Warning the drivers before reaching a traffic jam through V2V communication reduces the speed of the vehicles approaching the traffic jam, but this change does not limit the traffic flow or increase the density noticeably. Since the vehicles get the message 500 meters before, the time to breakdown does not change dramatically by reducing the desired speed of the vehicles and setting them to 70 km/h. However, the congested area becomes smaller over time and space as the penetration rate increases and the safety improves.

5 Conclusions

In this paper we presented the inter-vehicles communication in a shockwave traffic flow scenario with the discussion of performance indicators. With the help of a microscopic simulation experiment, we considered the shockwave propagation speed, the dissolving speed and the average network speed as performance measures for different penetration rates. We found out that, with a low penetration rate, which is expected to be the beginning phase of V2V equipped vehicles on the roads, no significant result should be expected. Besides, with an increasing penetration rate of V2V equipped vehicles, which can receive the traffic jam ahead warning, the congestion propagates on the freeway with slower speed and the congestion area becomes smaller. On the other hand, approaching the congestion tail with lower speed increases the traffic safety.

References

Aycinf, M. F., & Benekohat, B. F. (1998). Linear Acceleration Car-Following Model Development and Validation. *Transportation Research Record: Journal of the Transportation Research Board, No. 1644*, pp. 10-19.

Breton, P., Hegyi, A., De Shutter, B., & Hellendoorn, H. (2002). Shockwave Elimination/Reduction by Optimal Coordination of Variable Speed Limits. *IEEE Transactions on Intelligent transportation Systems*, pp. 225-230.

Brockfeld, E., Kühne, R. D., & Wagner, P. (2004). Calibration and Validation of Microscopic Traffic Flow Models. *Transportation Research Record: Journal of Transportation Research Board, No.1876*, pp. 62-70.

Cheu, R., Jin, X., Ng, K. C., Ng, Y. L., & Sirnivasan, D. (1998). Calibration of FRESIM for Singapore Epressway Using Genetic ALgorithm. *Journal of Transportation Engineering, Vol. 124, No. 6*.

Gomes, G., May, A., & Horowitz, R. (2004). Congested Freeway Microsimulation Model Using VISSIM. *Transportation Research Record: Journal of Transportation Research Board, No. 1876*, pp. 71-81.

Hegyi, A., Schutter, B. D., & Hellendoorn, J. (2005). Optimal Cooperation of Variable Speed Limits to Suppress Shockwaves. *IEEE Transportation Intelligent Transportation Systems 6*, pp. 102-112.

Hourdakis, J., Michalopoulos, P. G., & Kottommannil, J. (2003). Practical Procedure for Calibrating Micoscopic Traffic Simulation Models. *Transportion Research Record: Journal of Transportation Research Board, No. 1852*, pp. 130-139.

Kuhne, R., & Michalopolous, P. (2000). *Continnum Flow Models. Traffic Flow Theory, Chapter 5.* FHWA.

Lee, C., & Volpatti, S. (2010). Effects of Shock Waves on Freeway Crash Likelihood. *The Open Transportation Journal*, pp. 61-70.

Huang, L., Wu, J. (2013). A Freeway/Expressway Shockwave Elimination Method Based on IoT. In F. Chen, Y. Liu, & G. Hua, *LTLGB 2012, Proceeding of International Conference on Low-carbon Transportation and Logistics, and Green Building.* Heidelberg: Springer.

Lu, W., Bao, Y., Sun, X., & Wang, Z. (2009). *Performance Evaluation of Inter-Vehicle Communication in a Unidirectional Dynamic Traffic Flow with Shockwave.* St. Petersburg: Ultra Modern Telecommunications & Workshops, ICUMT International Conference.

Menneni, S., Sun, C., & Vortisch, P. (2009). An Integrated Microscopic and Macroscopic Calibration for Psycho-Physical Car Following Models. *TRB, Annual Meeting CD-ROM*. Washington, D.C.

Motamedidehkordi, N., Benz, T., & Margreiter, M. (2015). Shockwave Analysis on Motorways and Possibility of Damping by Autonomous Vehicles. In T. Schulze, B. Müller, & G. Meyer, *Advanced Microsystems for Automotive Applications 2015:Smart Systems for Green and Automated Driving* (pp. 37-52). Springer.

Munoz, J. C., & Daganzo, C. F. (2002). The Bottleneck Mechanis of a Freeway Diverge. *Transportation Research Part A*, pp. 483-505.

Park, B., & Qi, H. (2005). Developement of Evaluation of Procedure for the Calibration of Simulation Models. *Transportation Research Record: Journal of the Transportation Research Board, No. 1934*, pp. 208-217.

Payne, H., Thompson, S., Chang, G., & Ge, W. (1979). Calibration of FRESIM to Achieve Desired Capacities. *Transportation Research Record: Journal of the Transportation Research Board, Volume 1591*.

PTV AG. (2015). *Vissim 8 Manual.* Karlsruhe.

Rakhau, H., & Crowther, B. (2002). Comparison of Greenshield, Pipes, and Van Aerde Car-Following and Traffic Stream Models. *Transportation Research Board, No. 1802*, pp. 248-253.

Schimandl, F., Baur, M., Hoffmann, S., Margreiter, M., & Gabloner, S. (2013). *Effects of Cooperative Systems on Traffic Safety and Efficiency.* Munich: mobil.TUM International Scientific Conference on Mobility and Transport.

simTD. (2015). *simTD: Test Field Germany*. Retrieved from http://www.simtd.de/index.dhtml/enEN/Testfeld_Deutschland.html

Smith, B. L., Ling, Q., & Venkatanarayana, R. (2003). Characterization of Freeway Capacity Reduction Resulting from Traffic Accidents. *Journal of Transportation Engineering 123*, pp. 362-368.

Suijs, L. C., Wismans, L. J., Krol, L., & van Berkum, E. C. (2014). Phantom Jam Avoidance Through In-car Speed Advice. *Transportation Research Procedia 8*, pp. 227-236.

van der Meulen, R., & Rivera, J. (2015, January 26). *Gartner Says By 2020, A Quarter Billion Connected Vehicles Will Enable New In-Vehicle Services and Automated Driving Capabilities*. (Gartner) Retrieved September 18, 2015, from Gartner Newsroom: http://www.gartner.com/newsroom/id/2970017

Wiedemann, R. (1974). *Simulation des Strassenverkehrsflusses, Heft 8* (Vol. Band 8). Karlsruhe, Germany.

Xiao-Yun, L., & Skabardonis, A. (2007). *Freeway Traffic Shockwave Analysis: Exploring the NGSIM Trajectory Data.* Washington, D.C.: Transportation Research Board.

Yu, G. L. (2012). Analysis of Traffic Accident Temporal and Spatial Impact Based on Shockwave Theory. In C. Fang, H. Wei, Y. Wang, & J. Zhang, *CICTP 2012: Multimodal Transportation Systems - Convenient, Safe, Cost-Effective, Efficient* (pp. 557-567). American Society of Civil Engineers

Zhang, H. M. (1999). Analyses of the Stability and Wave Properties of a New Continuum Traffic Theory. *Transportation Research Part B, 33*, pp. 399-415.