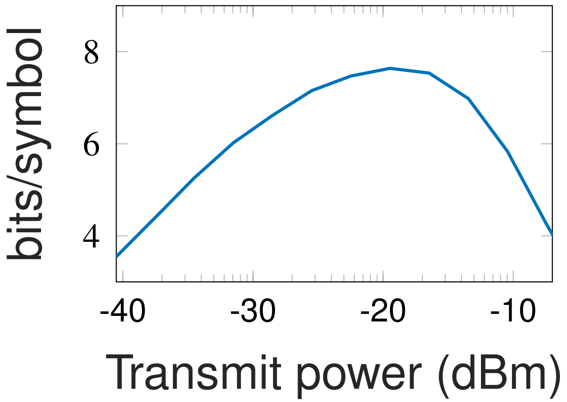
Achievable Rates of Nonlinear Fourier Transform-based Optical Communication Systems

Javier García
Technical University of Munich
Department of Electrical and Computer Engineering
Institute for Communications Engineering
November 2017

Institute for Communications Engineering
Department of Electrical and Computer Engineering
Technical University of Munich

Motivation

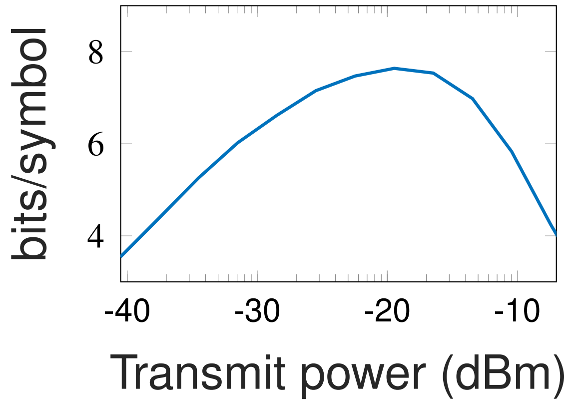
- The nonlinearity of the fiber optic channel imposes a capacity peak on linear transmission systems



5 WDM channels @ 20 GHz
Guardband: 5 GHz
Distance: 2000 km
RRC pulses, multi-ring modulation, 64 rings, 128 phases

Motivation

- The nonlinearity of the fiber optic channel imposes a capacity peak on linear transmission systems

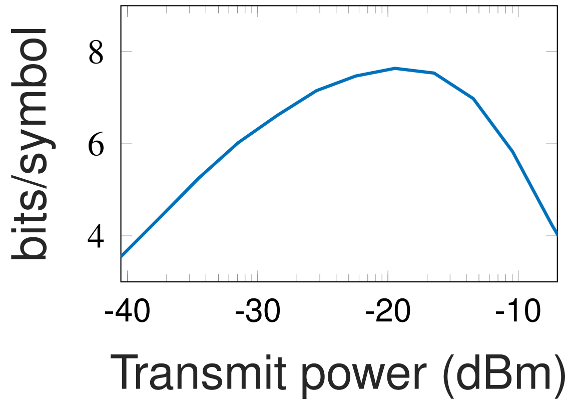


5 WDM channels @ $20\,\mathrm{GHz}$
Guardband: $5\,\mathrm{GHz}$
Distance: $2000\,\mathrm{km}$
RRC pulses, multi-ring modulation, 64 rings, 128 phases

- The Nonlinear Fourier Transform (NFT) provides a domain in which the noise-free channel is multiplicative

Motivation

- The nonlinearity of the fiber optic channel imposes a capacity peak on linear transmission systems



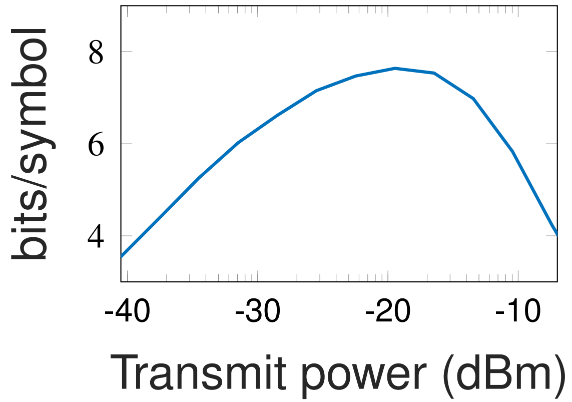
5 WDM channels @ $20\ \mathrm{GHz}$
Guardband: $5\ \mathrm{GHz}$
Distance: $2000\ \mathrm{km}$
RRC pulses, multi-ring modulation, 64 rings, 128 phases

- The Nonlinear Fourier Transform (NFT) provides a domain in which the noise-free channel is multiplicative
- Instability of NFT algorithms and low spectral efficiency still make current NFT-based systems uncompetitive

Institute for Communications Engineering
Department of Electrical and Computer Engineering
Technical University of Munich

Motivation

- The nonlinearity of the fiber optic channel imposes a capacity peak on linear transmission systems



5 WDM channels @ $20\,\mathrm{GHz}$
Guardband: $5\,\mathrm{GHz}$
Distance: $2000\,\mathrm{km}$
RRC pulses, multi-ring modulation, 64 rings, 128 phases

- The Nonlinear Fourier Transform (NFT) provides a domain in which the noise-free channel is multiplicative
- Instability of NFT algorithms and low spectral efficiency still make current NFT-based systems uncompetitive
- This talk: some mathematical and numerical insight to aid in the design of more efficient NFT-based systems

The Nonlinear Schrödinger Equation (NLSE)

$$\frac{\partial Q(Z,T)}{\partial Z} = -j\frac{\beta_2}{2}\frac{\partial^2 Q(Z,T)}{\partial T^2} + j\gamma|Q(Z,T)|^2 Q(Z,T) + N(Z,T)$$

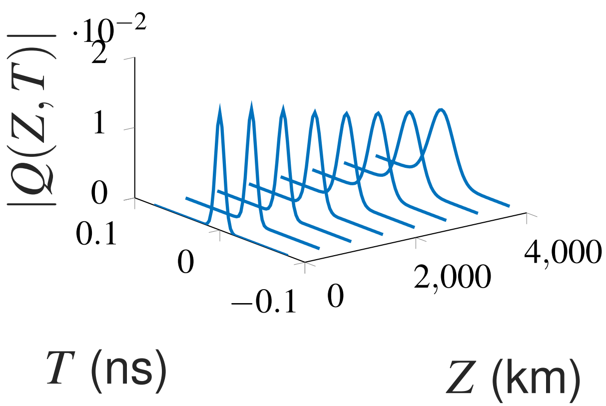
Institute for Communications Engineering
Department of Electrical and Computer Engineering
Technical University of Munich

The Nonlinear Schrödinger Equation (NLSE)

$$\frac{\partial Q(Z,T)}{\partial Z} = -j\frac{\beta_2}{2}\frac{\partial^2 Q(Z,T)}{\partial T^2} + j\gamma|Q(Z,T)|^2 Q(Z,T) + N(Z,T)$$

Dispersion

- Linear term
- Causes temporal broadening



The Nonlinear Schrödinger Equation (NLSE)

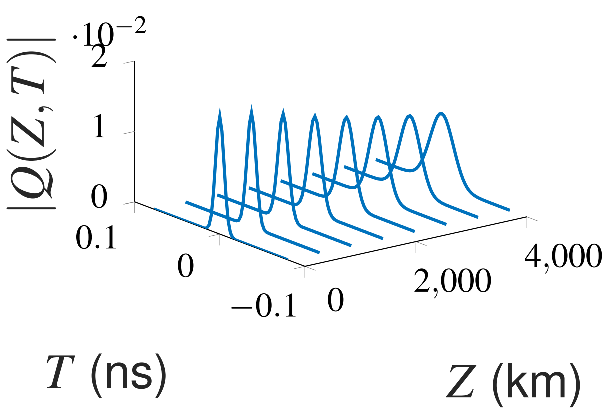
$$\frac{\partial Q(Z,T)}{\partial Z} = -j\frac{\beta_2}{2}\frac{\partial^2 Q(Z,T)}{\partial T^2} + j\gamma|Q(Z,T)|^2 Q(Z,T) + N(Z,T)$$

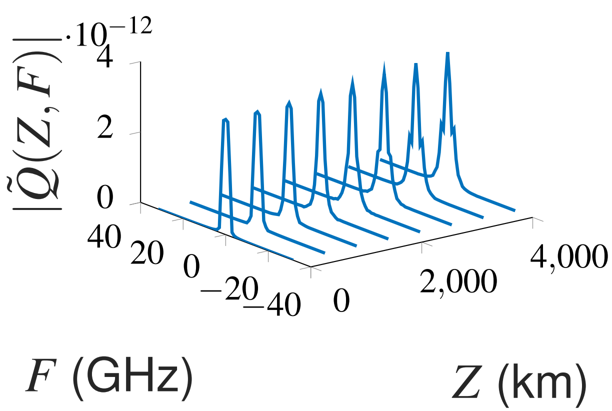
Dispersion

- Linear term
- Causes temporal broadening

Nonlinearity

- Causes frequency mixing (spectral broadening, SPM, XPM, FWM)





Institute for Communications Engineering
Department of Electrical and Computer Engineering
Technical University of Munich

The Nonlinear Schrödinger Equation (NLSE)

$$\frac{\partial Q(Z,T)}{\partial Z} = -j\frac{\beta_2}{2}\frac{\partial^2 Q(Z,T)}{\partial T^2} + j\gamma |Q(Z,T)|^2 Q(Z,T) + N(Z,T)$$

Dispersion

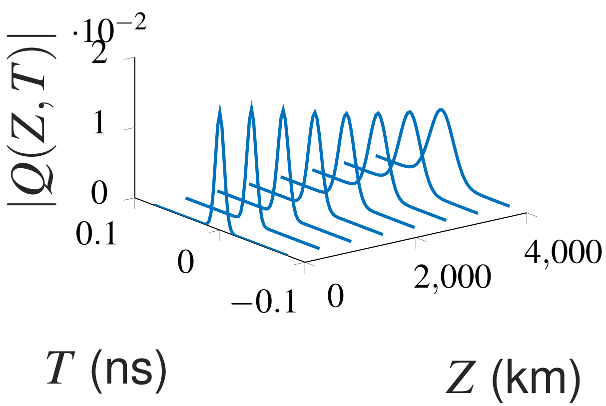
- Linear term
- Causes temporal broadening

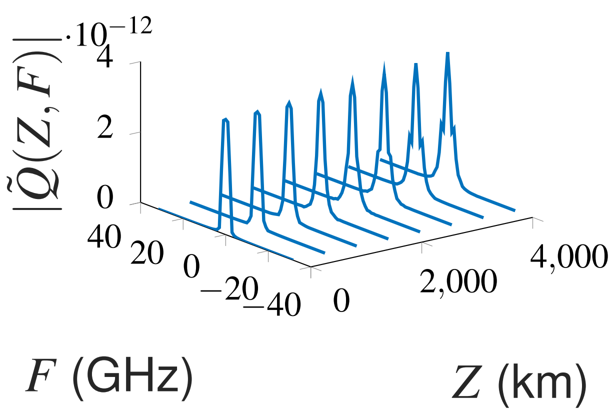
Nonlinearity

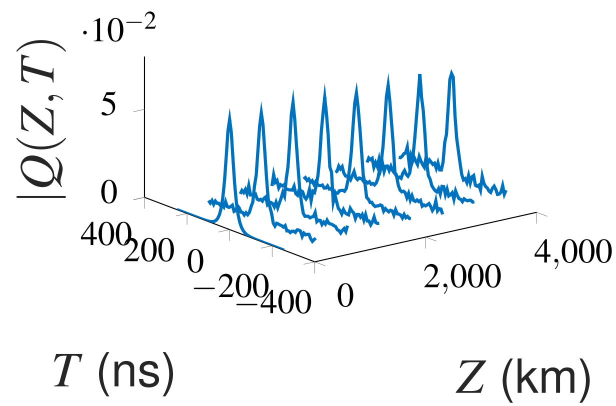
- Causes frequency mixing (spectral broadening, SPM, XPM, FWM)

Noise

- Distributed along the fiber
- Mixes nonlinearly with signal!







Institute for Communications Engineering
Department of Electrical and Computer Engineering
Technical University of Munich

Normalization of the NLSE (focusing case, $\beta_2 < 0$)

$$T = T_0 \cdot t$$

$$Z = 2\frac{{T_0}^2}{|\beta_2|} \cdot z$$

$$Q(Z,T) = \frac{1}{T_0}\sqrt{\frac{|\beta_2|}{\gamma}} \cdot q(z,t)$$

$$\mathbb{E}\left[N(Z,T)N^*(Z',T')\right] = \frac{\beta_2^2}{2\gamma {T_0}^4} \cdot \mathbb{E}\left[n(z,t)n^*(z',t')\right]$$

Institute for Communications Engineering
Department of Electrical and Computer Engineering
Technical University of Munich

Normalization of the NLSE (focusing case, $\beta_2 < 0$)

$$T = T_0 \cdot t$$

$$Z = 2\frac{{T_0}^2}{|\beta_2|} \cdot z$$

$$Q(Z,T) = \frac{1}{T_0}\sqrt{\frac{|\beta_2|}{\gamma}} \cdot q(z,t)$$

$$\mathbb{E}\left[N(Z,T)N^*(Z',T')\right] = \frac{\beta_2^2}{2\gamma {T_0}^4} \cdot \mathbb{E}\left[n(z,t)n^*(z',t')\right]$$

$$\frac{\partial}{\partial z}q(z,t) = j\frac{\partial^2}{\partial t^2}q(z,t) + 2j|q(z,t)|^2 q(z,t) + n(z,t)$$

Institute for Communications Engineering
Department of Electrical and Computer Engineering
Technical University of Munich

Normalization of the NLSE (focusing case, $\beta_2 < 0$)

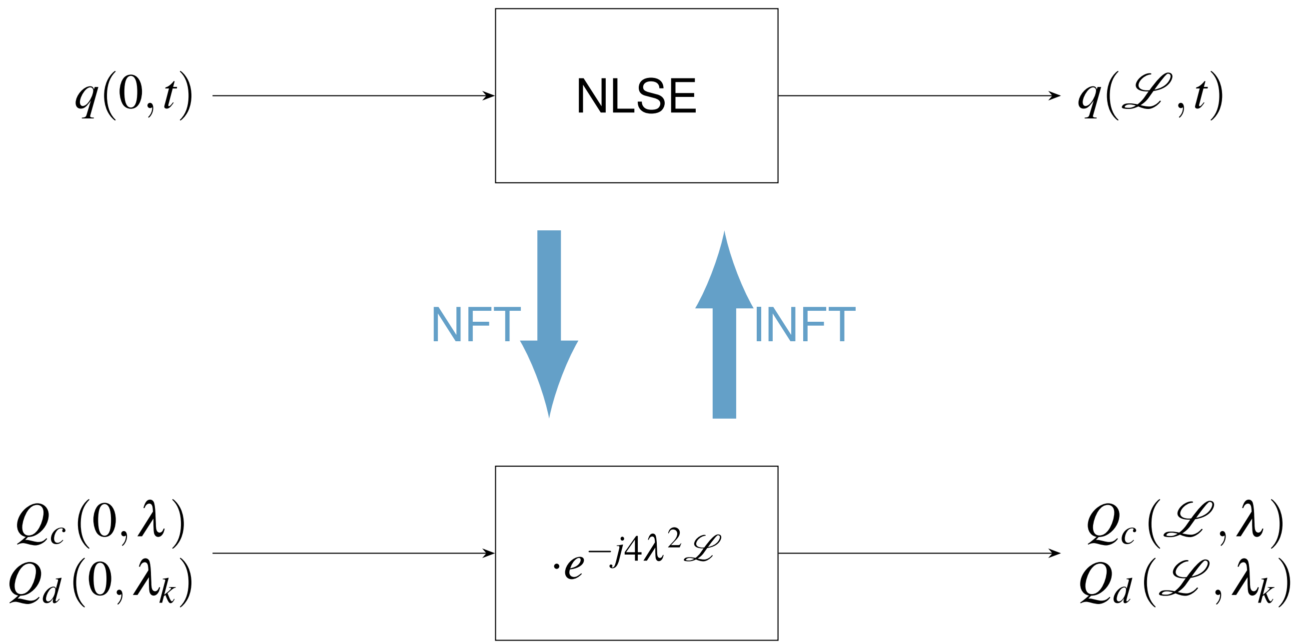
$$T = T_0 \cdot t$$
$$Z = 2\frac{{T_0}^2}{|\beta_2|} \cdot z$$
$$Q(Z,T) = \frac{1}{T_0}\sqrt{\frac{|\beta_2|}{\gamma}} \cdot q(z,t)$$
$$\mathbb{E}\left[N(Z,T)N^*(Z',T')\right] = \frac{\beta_2^2}{2\gamma {T_0}^4} \cdot \mathbb{E}\left[n(z,t)n^*(z',t')\right]$$

$$\frac{\partial}{\partial z} q(z,t) = j\frac{\partial^2}{\partial t^2} q(z,t) + 2j|q(z,t)|^2 q(z,t) + n(z,t)$$

T_0 is a **free parameter**. Can be used to jointly set *power*, *duration* and *bandwidth* in pure soliton systems.

The Nonlinear Fourier Transform (NFT)

- Motivation: find a domain in which the noise-free NLSE channel is multiplicative (similar to FT in LTIs):



The Nonlinear Fourier Transform (NFT)

- Lax pair: two operators L and M

$$L = j \begin{pmatrix} \frac{\partial}{\partial t} & q(z,t) \\ -q^*(z,t) & \frac{\partial}{\partial t} \end{pmatrix}, \quad M = \begin{pmatrix} 2j\lambda^2 - j|q(z,t)|^2 & -2\lambda q(z,t) - jq_t(z,t) \\ 2\lambda q^*(z,t) - jq_t^*(z,t) & -2j\lambda^2 + j|q(z,t)|^2 \end{pmatrix}$$

such that the condition:

$$L_z = ML - LM$$

implies the NLSE.

The Nonlinear Fourier Transform (NFT)

- Lax pair: two operators L and M

$$L = j\begin{pmatrix} \frac{\partial}{\partial t} & q(z,t) \\ -q^*(z,t) & \frac{\partial}{\partial t} \end{pmatrix}, \quad M = \begin{pmatrix} 2j\lambda^2 - j|q(z,t)|^2 & -2\lambda q(z,t) - jq_t(z,t) \\ 2\lambda q^*(z,t) - jq_t^*(z,t) & -2j\lambda^2 + j|q(z,t)|^2 \end{pmatrix}$$

  such that the condition:

$$L_z = ML - LM$$

  implies the NLSE.
- Main idea: the eigenvalues λ of L are invariant under propagation along z

The Nonlinear Fourier Transform (NFT)

- Step 1: solve the (linear, differential) eigenvalue equation:

$$Lv(t,\lambda) = \lambda v(t,\lambda); \quad v(t,\lambda) \underset{t\to-\infty}{\to} \begin{pmatrix} 1 \\ 0 \end{pmatrix} e^{-j\lambda t}$$

Institute for Communications Engineering
Department of Electrical and Computer Engineering
Technical University of Munich

The Nonlinear Fourier Transform (NFT)

- Step 1: solve the (linear, differential) eigenvalue equation:

$$Lv(t,\lambda) = \lambda v(t,\lambda); \quad v(t,\lambda) \underset{t\to-\infty}{\rightarrow} \begin{pmatrix} 1 \\ 0 \end{pmatrix} e^{-j\lambda t}$$

- Step 2: obtain the spectral amplitudes:

$$a(\lambda) = \lim_{t\to\infty} v_1 e^{j\lambda t}$$

$$b(\lambda) = \lim_{t\to\infty} v_2 e^{-j\lambda t}$$

The Nonlinear Fourier Transform (NFT)

- Step 1: solve the (linear, differential) eigenvalue equation:

$$Lv(t,\lambda) = \lambda v(t,\lambda); \quad v(t,\lambda) \underset{t\to-\infty}{\to} \begin{pmatrix} 1 \\ 0 \end{pmatrix} e^{-j\lambda t}$$

- Step 2: obtain the spectral amplitudes:

$$a(\lambda) = \lim_{t\to\infty} v_1 e^{j\lambda t}$$

$$b(\lambda) = \lim_{t\to\infty} v_2 e^{-j\lambda t}$$

- Step 3: obtain the NFT as:
  - **Continuous spectrum:** $Q_c(\lambda) = \frac{b(\lambda)}{a(\lambda)}, \lambda \in \mathbb{R}$
  - **Discrete spectrum:** $Q_d(\lambda_k) = \frac{b(\lambda_k)}{a_\lambda(\lambda_k)}, \quad \lambda_k \in \mathbb{C}^+, a(\lambda_k) = 0$

The Nonlinear Fourier Transform (NFT)

- Step 1: solve the (linear, differential) eigenvalue equation:

$$Lv(t,\lambda) = \lambda v(t,\lambda); \quad v(t,\lambda) \underset{t\to-\infty}{\to} \begin{pmatrix} 1 \\ 0 \end{pmatrix} e^{-j\lambda t}$$

- Step 2: obtain the spectral amplitudes:

$$a(\lambda) = \lim_{t\to\infty} v_1 e^{j\lambda t}$$

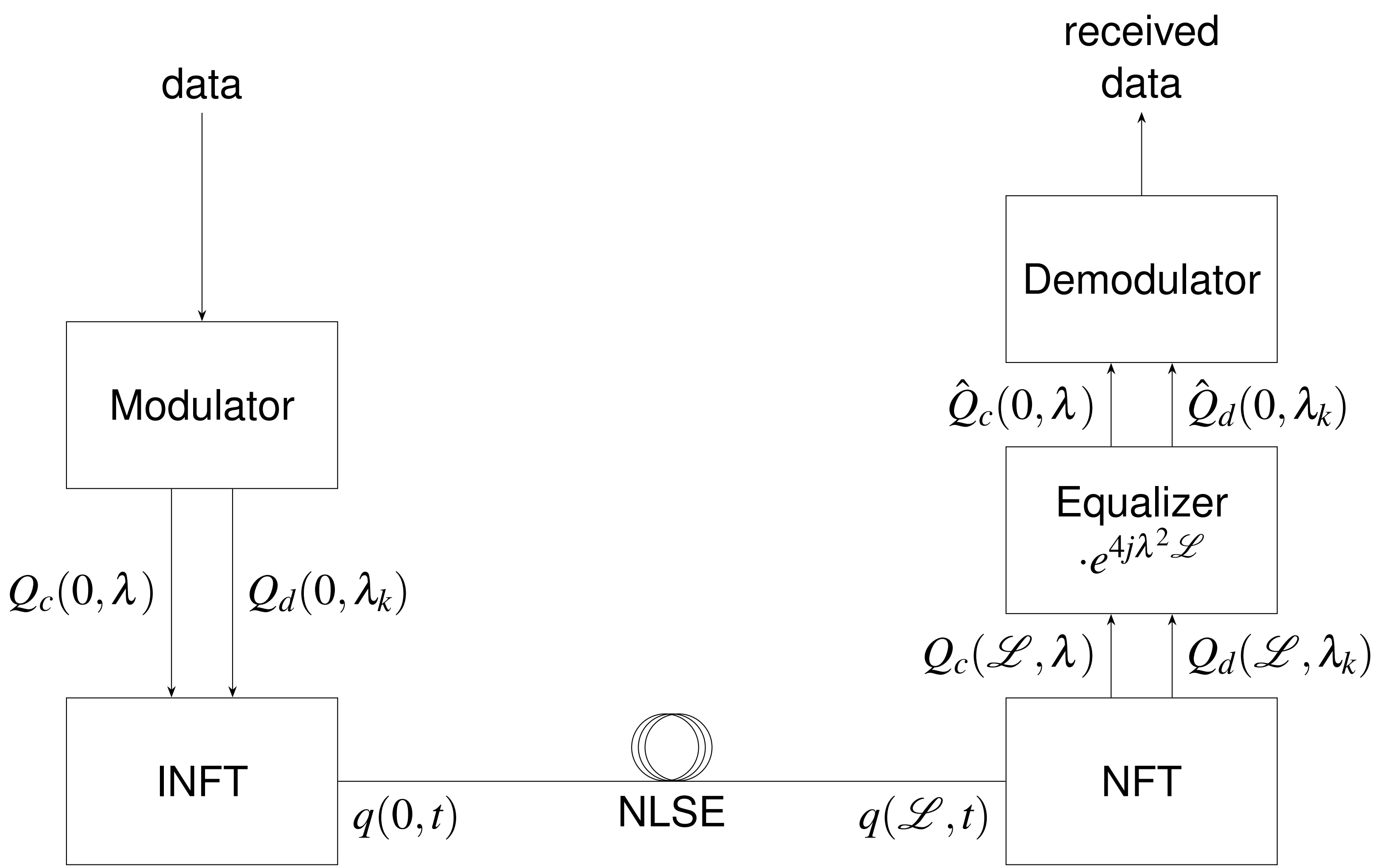
$$b(\lambda) = \lim_{t\to\infty} v_2 e^{-j\lambda t}$$

- Step 3: obtain the NFT as:
  - **Continuous spectrum:** $Q_c(\lambda) = \frac{b(\lambda)}{a(\lambda)}, \lambda \in \mathbb{R}$
  - **Discrete spectrum:** $Q_d(\lambda_k) = \frac{b(\lambda_k)}{a_\lambda(\lambda_k)}, \lambda_k \in \mathbb{C}^+, a(\lambda_k) = 0$
- Equivalent of Parseval's identity:

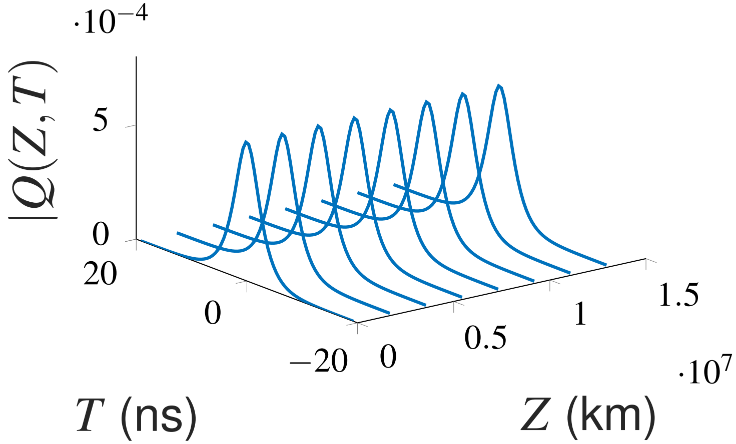
$$\int_{-\infty}^{\infty} |q(t)|^2 \, \mathrm{d}t = \frac{1}{\pi} \int_{-\infty}^{\infty} \log\left(1 + |Q_c(\lambda)|^2\right) \mathrm{d}\lambda + 4 \sum_{k=1}^{K} \Im\{\lambda_k\}$$

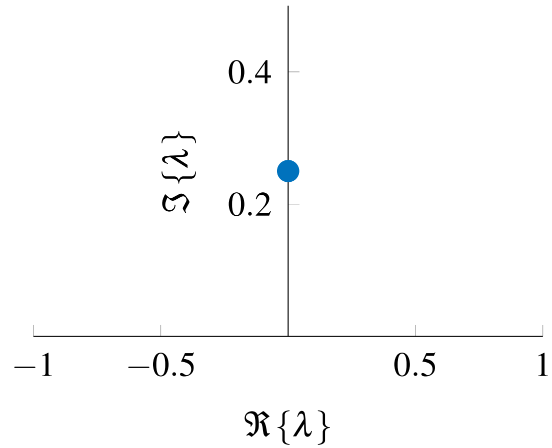
Institute for Communications Engineering
Department of Electrical and Computer Engineering
Technical University of Munich

Information transmission using the NFT



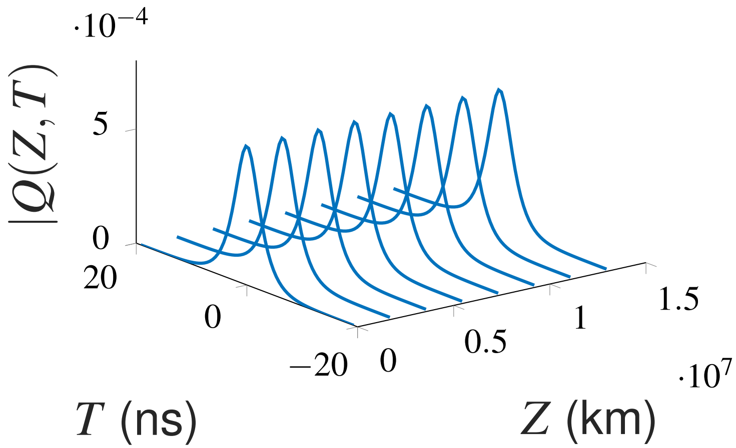
Modulation of the discrete spectrum: solitons

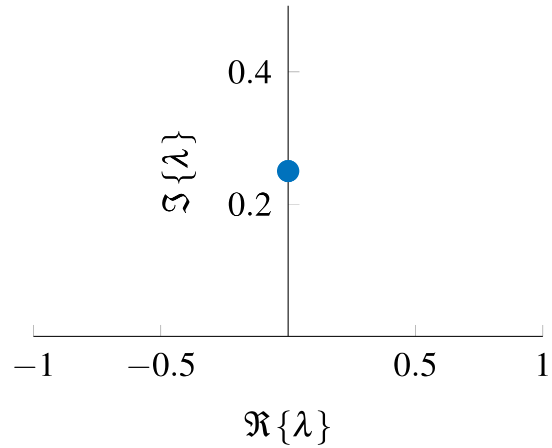


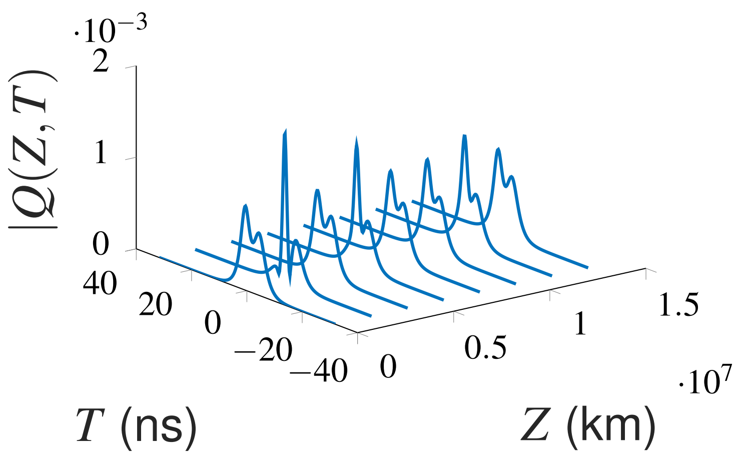


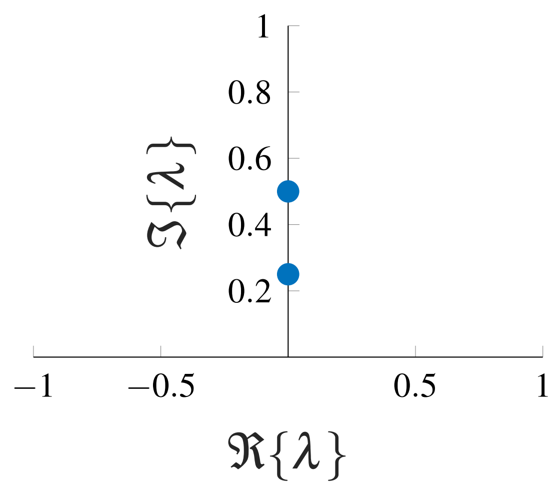
Institute for Communications Engineering
Department of Electrical and Computer Engineering
Technical University of Munich

Modulation of the discrete spectrum: solitons



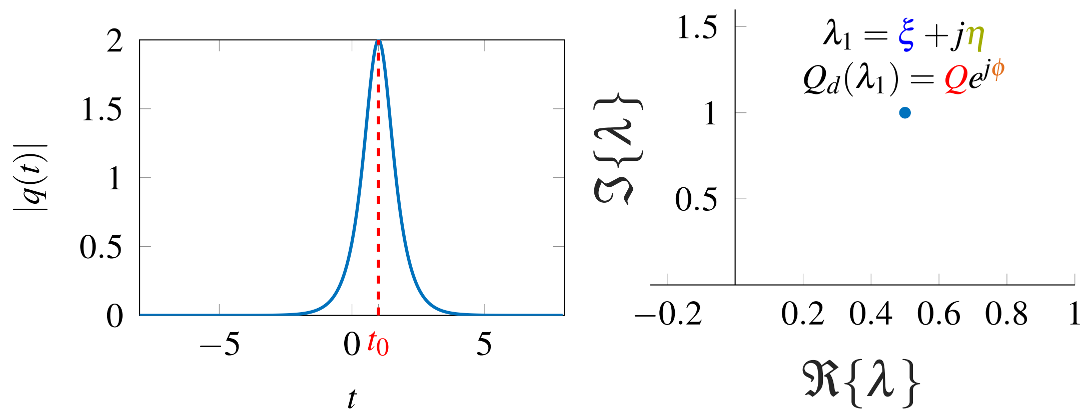






Institute for Communications Engineering
Department of Electrical and Computer Engineering
Technical University of Munich

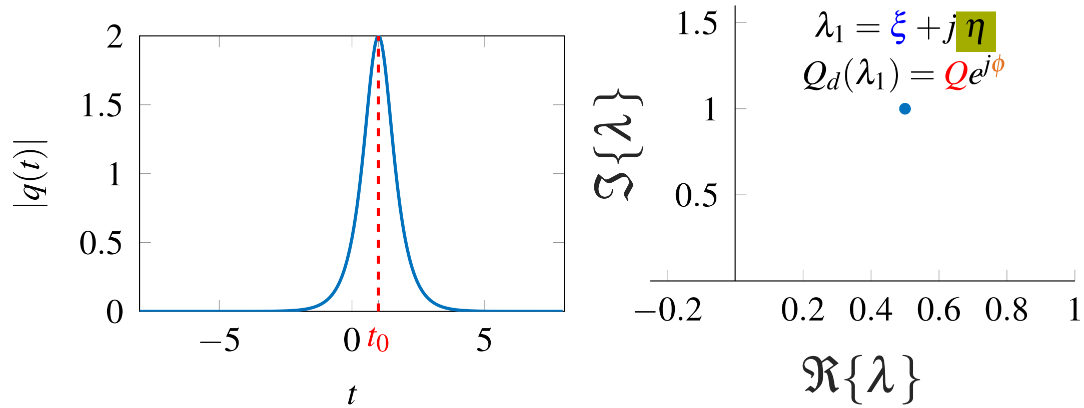
Parameters of a 1-soliton



$$q(z,t) = -je^{-j\phi}e^{-4j(\xi^2-\eta^2)z}e^{-2j\xi t}2\eta\,\mathrm{sech}\left(2\eta t + 8\xi\eta z - \ln\frac{Q}{2\eta}\right)$$

Institute for Communications Engineering
Department of Electrical and Computer Engineering
Technical University of Munich

Parameters of a 1-soliton

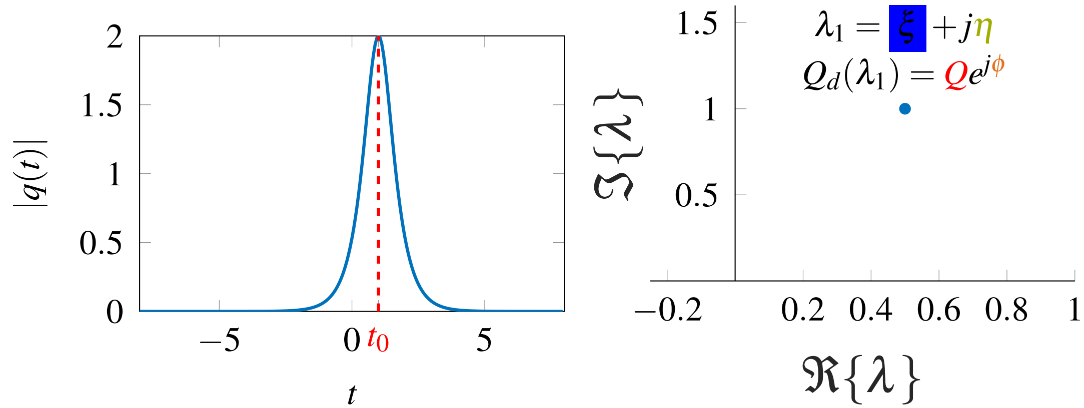


$$q(z,t) = -je^{-j\phi}e^{-4j(\xi^2-\eta^2)z}e^{-2j\xi t}2\eta\,\mathrm{sech}\left(2\eta t + 8\xi\eta z - \ln\frac{Q}{2\eta}\right)$$

- Energy: $E = 4\eta$
- Duration: $T = 2.6467/\eta$
- Bandwidth: $B = 1.0726\eta$

Institute for Communications Engineering
Department of Electrical and Computer Engineering
Technical University of Munich

Parameters of a 1-soliton



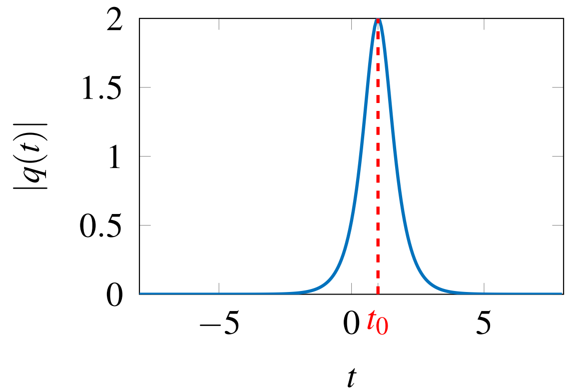
$$q(z,t) = -je^{-j\phi}e^{-4j(\xi^2-\eta^2)z}e^{-2j\xi t}2\eta \operatorname{sech}\left(2\eta t + 8\xi\eta z - \ln\frac{Q}{2\eta}\right)$$

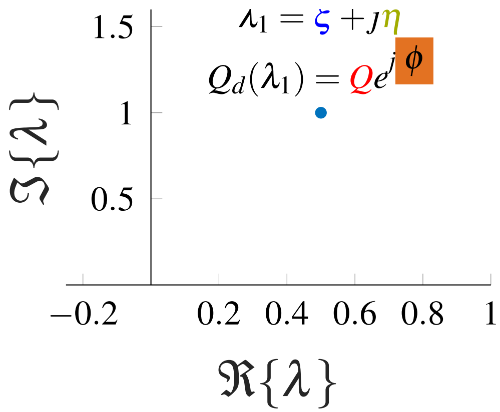
- Energy: $E = 4\eta$
- Duration: $T = 2.6467/\eta$
- Bandwidth: $B = 1.0726\eta$

- Center frequency: $f_0 = -\xi/\pi$
- Group velocity: $v_g = 4\xi$

Institute for Communications Engineering
Department of Electrical and Computer Engineering
Technical University of Munich

Parameters of a 1-soliton





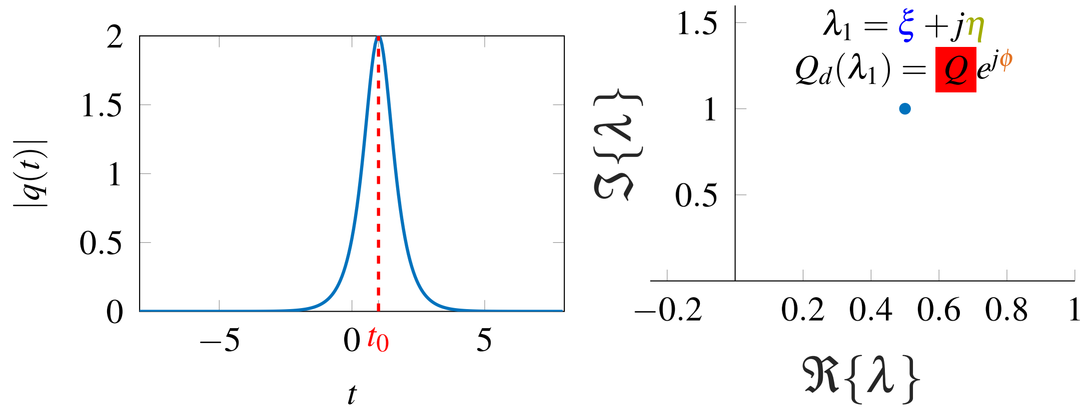
$$q(z,t) = -je^{-j\phi} e^{-4j(\xi^2-\eta^2)z} e^{-2j\xi t} 2\eta \operatorname{sech}\left(2\eta t + 8\xi\eta z - \ln\frac{Q}{2\eta}\right)$$

- Energy: $E = 4\eta$
- Duration: $T = 2.6467/\eta$
- Bandwidth: $B = 1.0726\eta$

- Center frequency: $f_0 = -\xi/\pi$
- Group velocity: $v_g = 4\xi$
- Constant phase shift: $-\frac{\pi}{2} - \phi$

Institute for Communications Engineering
Department of Electrical and Computer Engineering
Technical University of Munich

Parameters of a 1-soliton



$$q(z,t) = -je^{-j\phi}e^{-4j(\xi^2-\eta^2)z}e^{-2j\xi t}2\eta\,\mathrm{sech}\left(2\eta t + 8\xi\eta z - \ln\frac{Q}{2\eta}\right)$$

- Energy: $E = 4\eta$
- Duration: $T = 2.6467/\eta$
- Bandwidth: $B = 1.0726\eta$

- Center frequency: $f_0 = -\xi/\pi$
- Group velocity: $v_g = 4\xi$
- Constant phase shift: $-\frac{\pi}{2} - \phi$
- Pulse delay: $t_0 = \frac{1}{2\eta}\ln\frac{Q}{2\eta}$

Perturbation analysis of a 1-soliton

$$\frac{\partial}{\partial z}q(z,t) = j\frac{\partial^2}{\partial t^2}q(z,t) + 2j\left|q(z,t)\right|^2 q(z,t) + \varepsilon n(z,t)$$

where $\varepsilon \ll 1$.

J. Yang, "Soliton Perturbation Theories and Applications," *Nonlinear Waves in Integrable and Nonintegrable Systems*, ch. 4, pp. 119–162, 2010

Perturbation analysis of a 1-soliton

$$\frac{\partial}{\partial z}q(z,t) = j\frac{\partial^2}{\partial t^2}q(z,t) + 2j\left|q(z,t)\right|^2 q(z,t) + \varepsilon n(z,t)$$

where $\varepsilon \ll 1$.

Multi-scale perturbation analysis:

$$q(z,t) = q_0(z,t) + \varepsilon q_1(z,t) + \varepsilon^2 q_2(z,t) + \cdots$$

J. Yang, “Soliton Perturbation Theories and Applications,” *Nonlinear Waves in Integrable and Nonintegrable Systems*, ch. 4, pp. 119–162, 2010

Perturbation analysis of a 1-soliton

$$\frac{\partial}{\partial z} q(z,t) = j \frac{\partial^2}{\partial t^2} q(z,t) + 2j \left| q(z,t) \right|^2 q(z,t) + \varepsilon n(z,t)$$

where $\varepsilon \ll 1$.

Multi-scale perturbation analysis:

$$q(z,t) = q_0(z,t) + \varepsilon q_1(z,t) + \varepsilon^2 q_2(z,t) + \cdots$$

- Solution of $\mathcal{O}(1)$ equation:

$$q_0(z,t) = -je^{-j\phi} e^{-4j\left(\xi^2 - \eta^2\right)z} e^{-2j\xi t} 2\eta \operatorname{sech}\left(2\eta\left(t - t_0\right) + 8\xi\eta z\right)$$

where the four parameters depend on the **slow distance** $Z = \varepsilon z$:

$$\eta = \eta(Z) \quad \xi = \xi(Z) \quad \phi = \phi(Z) \quad t_0 = t_0(Z)$$

J. Yang, "Soliton Perturbation Theories and Applications," *Nonlinear Waves in Integrable and Nonintegrable Systems*, ch. 4, pp. 119–162, 2010

Perturbation analysis of a 1-soliton

Substituting $q_0(z,t)$ into the $\mathscr{O}(\varepsilon)$ equation yields:

$$\frac{\mathrm{d}\eta}{\mathrm{d}Z} \sim \mathscr{N}_{\mathbb{R}}(0,\eta/2) \qquad \frac{\mathrm{d}\xi}{\mathrm{d}Z} \sim \mathscr{N}_{\mathbb{R}}(0,\eta/6)$$

$$\frac{\mathrm{d}t_0}{\mathrm{d}Z} \sim \mathscr{N}_{\mathbb{R}}\left(0,\frac{\pi^2}{96\eta^3}\right) \qquad \frac{\mathrm{d}\phi}{\mathrm{d}Z} \sim \mathscr{N}_{\mathbb{R}}\left(0,\frac{1}{72\eta}\left(12+\pi^2\right)+\frac{\pi^2\xi^2}{24\eta^3}\right)$$

Institute for Communications Engineering
Department of Electrical and Computer Engineering
Technical University of Munich

Perturbation analysis of a 1-soliton

Substituting $q_0(z,t)$ into the $\mathscr{O}(\varepsilon)$ equation yields:

$$\frac{\mathrm{d}\eta}{\mathrm{d}Z} \sim \mathscr{N}_{\mathbb{R}}(0, \eta/2) \qquad \frac{\mathrm{d}\xi}{\mathrm{d}Z} \sim \mathscr{N}_{\mathbb{R}}(0, \eta/6)$$

$$\frac{\mathrm{d}t_0}{\mathrm{d}Z} \sim \mathscr{N}_{\mathbb{R}}\left(0, \frac{\pi^2}{96\eta^3}\right) \qquad \frac{\mathrm{d}\phi}{\mathrm{d}Z} \sim \mathscr{N}_{\mathbb{R}}\left(0, \frac{1}{72\eta}(12+\pi^2) + \frac{\pi^2\xi^2}{24\eta^3}\right)$$

Assuming η and ξ do not change much along propagation:

$$\eta(\mathscr{L}) \sim \mathscr{N}_{\mathbb{R}}\left(\eta(0), \frac{\eta(0)}{2} N_{\text{ASE}}\mathscr{L}\right)$$

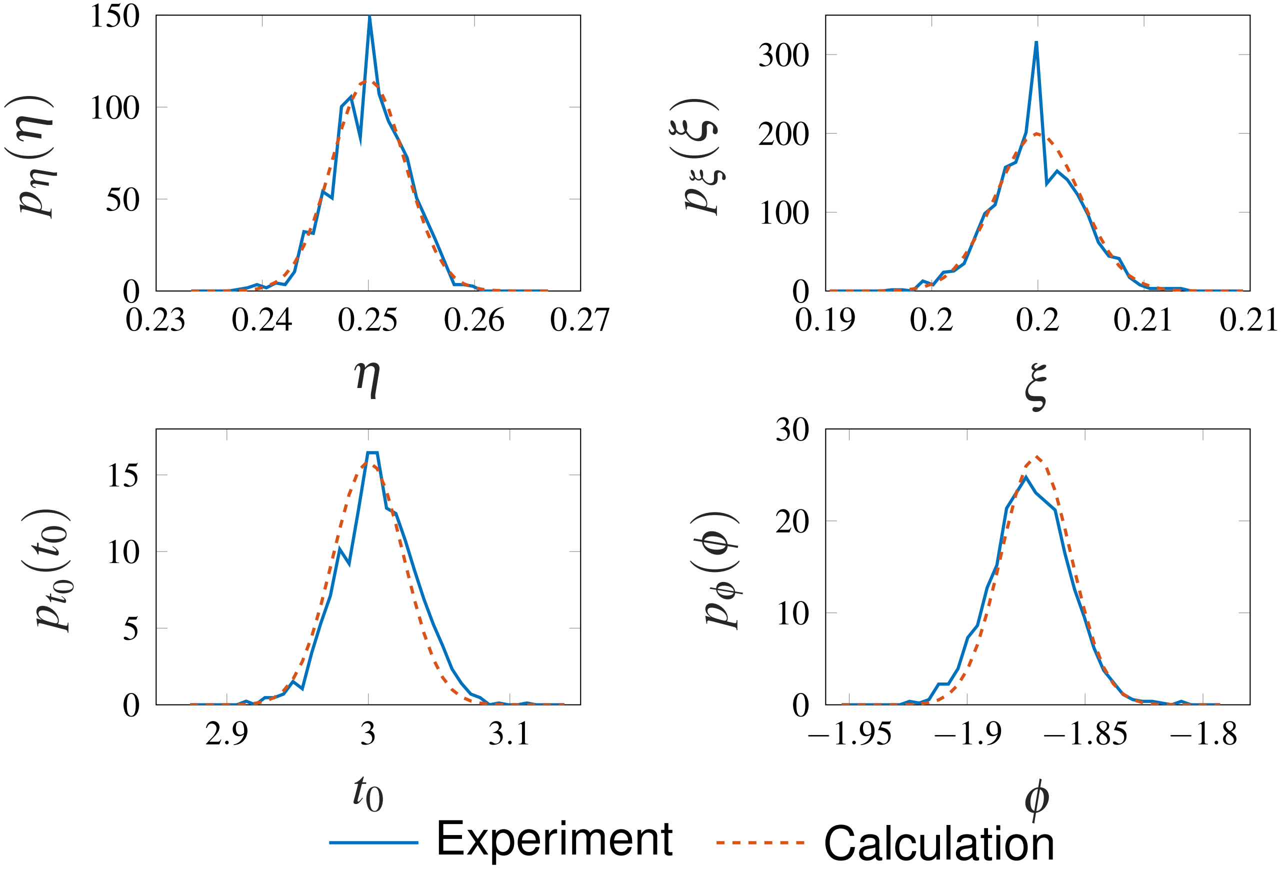
$$\xi(\mathscr{L}) \sim \mathscr{N}_{\mathbb{R}}\left(\xi(0), \frac{\eta(0)}{6} N_{\text{ASE}}\mathscr{L}\right)$$

$$t_0(\mathscr{L}) \sim \mathscr{N}_{\mathbb{R}}\left(t_0(0), \frac{\pi^2}{96\eta(0)^3} N_{\text{ASE}}\mathscr{L}\right)$$

$$\phi(\mathscr{L}) \sim \mathscr{N}_{\mathbb{R}}\left(\phi(0), \left[\frac{1}{72\eta(0)}(12+\pi^2) + \frac{\pi^2\xi(0)^2}{24\eta(0)^3}\right] N_{\text{ASE}}\mathscr{L}\right)$$

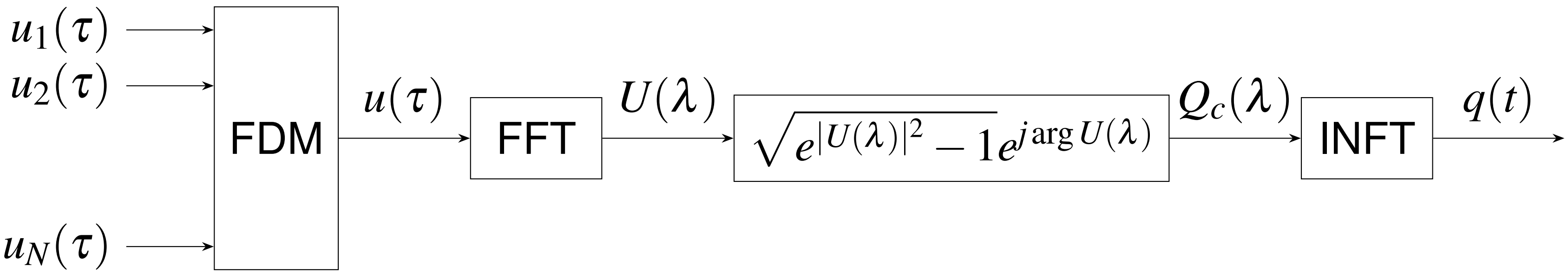
Institute for Communications Engineering
Department of Electrical and Computer Engineering
Technical University of Munich

Perturbation analysis of a 1-soliton ($z = 0.9578$)



Institute for Communications Engineering
Department of Electrical and Computer Engineering
Technical University of Munich

Modulation of the continuous spectrum



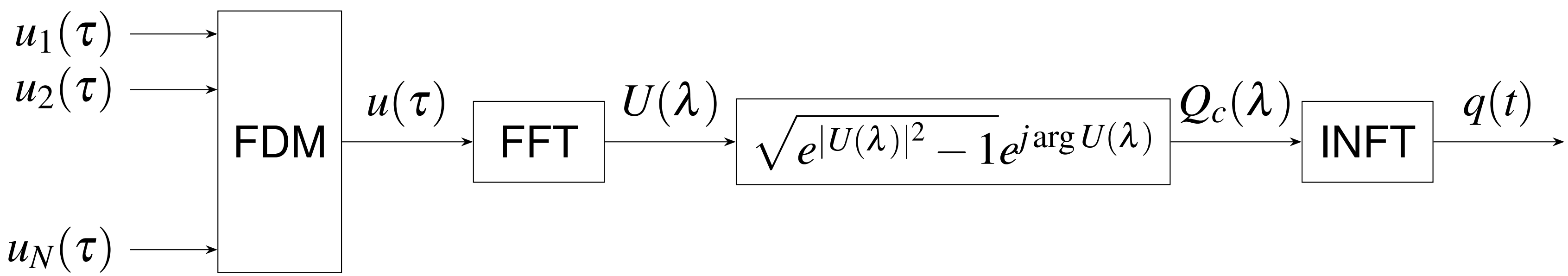
- From Parseval, the signal

$$U(\lambda) = \log\left(1 + |Q_c(\lambda)|^2\right) e^{j\arg Q_c(\lambda)}$$

has energy $E/2$, where E is the energy of $q(z,t)$

Institute for Communications Engineering
Department of Electrical and Computer Engineering
Technical University of Munich

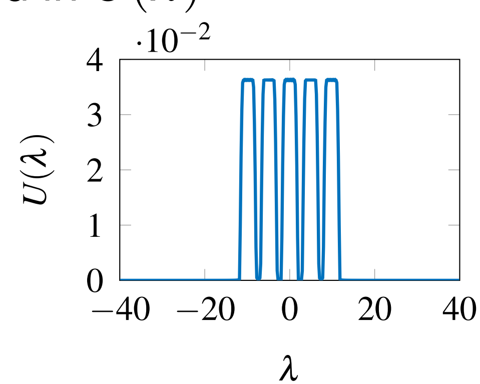
Modulation of the continuous spectrum



- From Parseval, the signal

$$U(\lambda) = \log\left(1+|Q_c(\lambda)|^2\right) e^{j\arg Q_c(\lambda)}$$

  has energy $E/2$, where E is the energy of $q(z,t)$
- User channels are multiplexed in $U(\lambda)$



Institute for Communications Engineering
Department of Electrical and Computer Engineering
Technical University of Munich

Continuous spectrum: simulation parameters

- 5 FDM channels, Root Raised Cosine pulses with roll-off $\beta = 0.25$

Institute for Communications Engineering
Department of Electrical and Computer Engineering
Technical University of Munich

Continuous spectrum: simulation parameters

- 5 FDM channels, Root Raised Cosine pulses with roll-off $\beta = 0.25$
- 1 symbol per block

Continuous spectrum: simulation parameters

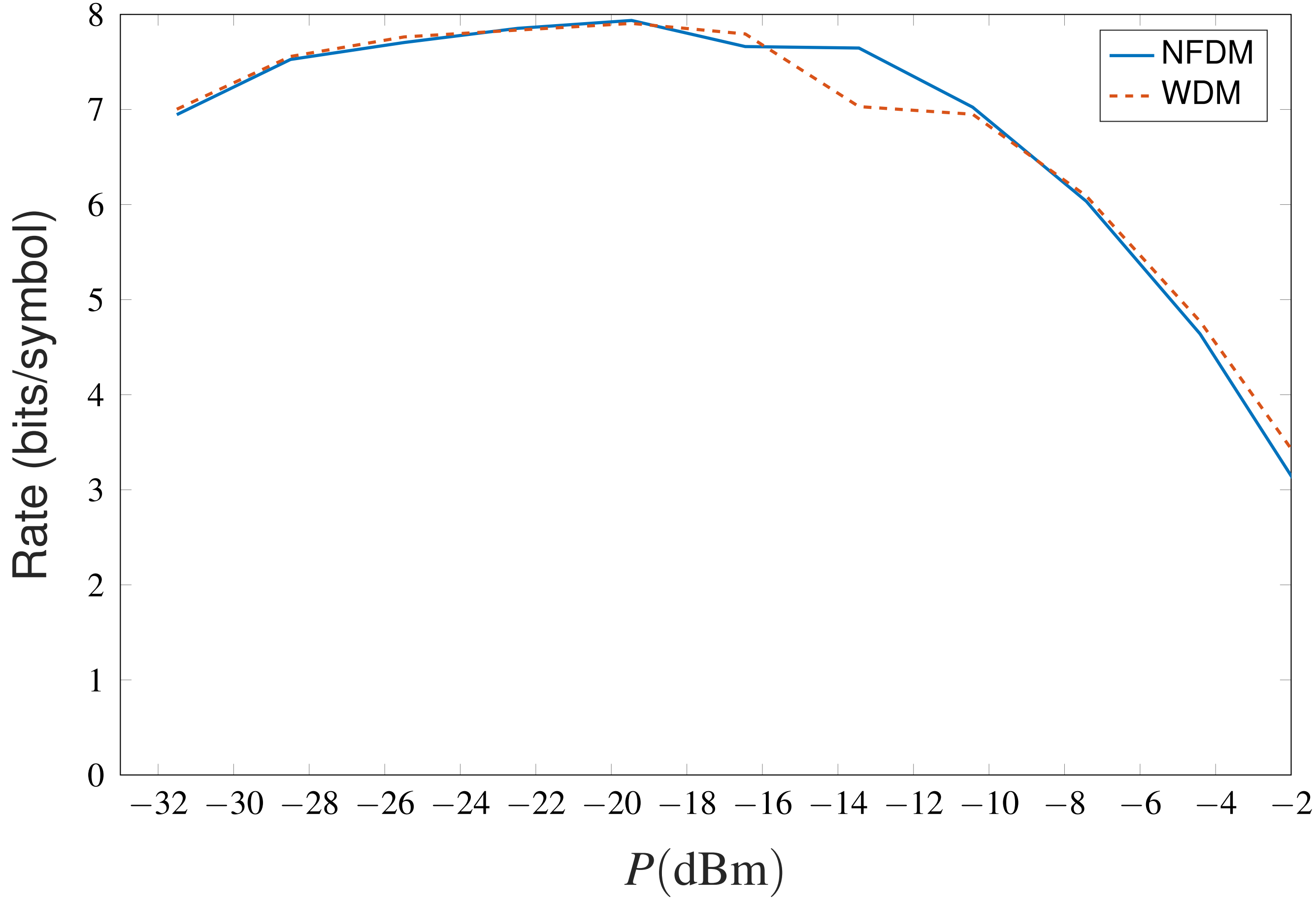
- 5 FDM channels, Root Raised Cosine pulses with roll-off $\beta = 0.25$
- 1 symbol per block
- Multi-ring modulation, 8 rings with 32 phases.

Continuous spectrum: simulation parameters

- 5 FDM channels, Root Raised Cosine pulses with roll-off $\beta = 0.25$
- 1 symbol per block
- Multi-ring modulation, 8 rings with 32 phases.

Parameter	Symbol	Value
Dispersion coefficient	β_2	$-21.667\ \mathrm{ps}^2/\mathrm{km}$
Nonlinearity parameter	γ	$1.2578\ \mathrm{W}^{-1}\mathrm{km}^{-1}$
Fiber length	$\mathscr{L}$	$250\ \mathrm{km}$
Channel bandwidth	B	$10\ \mathrm{GHz}$
Guard band	B_{guard}	$2.5\ \mathrm{GHz}$
Noise spectral density	N_{ASE}	$6.4893 \cdot 10^{-19}\ \mathrm{W} \cdot \mathrm{s}$

Continuous spectrum: simulation results



Institute for Communications Engineering
Department of Electrical and Computer Engineering
Technical University of Munich

Conclusions

- The Nonlinear Fourier Transform (NFT) provides a domain in which the NLSE channel is multiplicative

Conclusions

- The Nonlinear Fourier Transform (NFT) provides a domain in which the NLSE channel is multiplicative
- Higher energy solitons in the presence of distributed noise have
  - less robust eigenvalue position λ_1
  - but more robust spectral amplitude $Q_d(\lambda_1)$

  as compared to lower energy solitons.

Conclusions

- The Nonlinear Fourier Transform (NFT) provides a domain in which the NLSE channel is multiplicative
- Higher energy solitons in the presence of distributed noise have
  - less robust eigenvalue position λ_1
  - but more robust spectral amplitude $Q_d(\lambda_1)$

  as compared to lower energy solitons.
- With several eigenvalues, those with higher energy seem less robust at high power

Conclusions

- The Nonlinear Fourier Transform (NFT) provides a domain in which the NLSE channel is multiplicative
- Higher energy solitons in the presence of distributed noise have
  - less robust eigenvalue position λ_1
  - but more robust spectral amplitude $Q_d(\lambda_1)$

  as compared to lower energy solitons.
- With several eigenvalues, those with higher energy seem less robust at high power
- Discrete spectrum modulation has too low spectral efficiency, but the addition of eigenvalues to a continuous spectrum signal could bring improvements

Institute for Communications Engineering
Department of Electrical and Computer Engineering
Technical University of Munich

Institute for Communications Engineering
Department of Electrical and Computer Engineering
Technical University of Munich

Perturbation analysis of a 1-soliton

$$j\frac{\partial}{\partial z}q(z,t)+\frac{\partial^2}{\partial z^2}q(z,t)+2|q(z,t)|^2q(z,t)=\varepsilon F\left(q(z,t)\right)$$

$$F_0(z,t)=je^{j\phi}e^{4j(\xi^2-\eta^2 z)}e^{2j\xi t}F\left(q_0(z,t)\right)$$

$$\frac{\mathrm{d}\eta}{\mathrm{d}Z}=\frac{1}{2}\int_{-\infty}^{\infty}\Im\left\{F_0(z,t)\right\}\cdot 2\eta\,\mathrm{sech}(2\eta t)\,\mathrm{d}t$$

$$\frac{\mathrm{d}\xi}{\mathrm{d}Z}=-\frac{1}{2}\int_{-\infty}^{\infty}\Re\left\{F_0(z,t)\right\}\cdot 2\eta\,\mathrm{sech}(2\eta t)\tanh(2\eta t)\,\mathrm{d}t$$

$$\frac{\mathrm{d}t_0}{\mathrm{d}Z}=\int_{-\infty}^{\infty}\Im\left\{F_0(z,t)\right\}\cdot t\,\mathrm{sech}(2\eta t)\,\mathrm{d}t$$

$$\frac{\mathrm{d}\phi_0}{\mathrm{d}Z}=2\xi\frac{\mathrm{d}t_0}{\mathrm{d}Z}-\int_{-\infty}^{\infty}\Re\left\{F_0(z,t)\right\}\cdot \mathrm{sech}(2\eta t)\left[1-2\eta t\tanh\left(2\eta t\right)\right]\,\mathrm{d}t$$

J. Yang, “Soliton Perturbation Theories and Applications,” *Nonlinear Waves in Integrable and Nonintegrable Systems*, ch. 4, pp. 119–162, 2010