Energy Conservation in Optical Fibers with Distributed Brick-walls Filters

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- Nonlinear fiber optic causes spectral broadening and channel coupling

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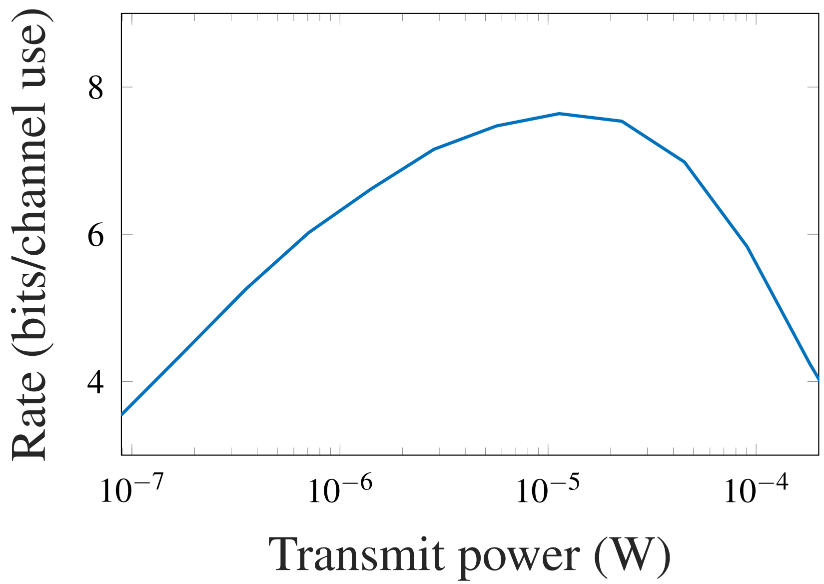

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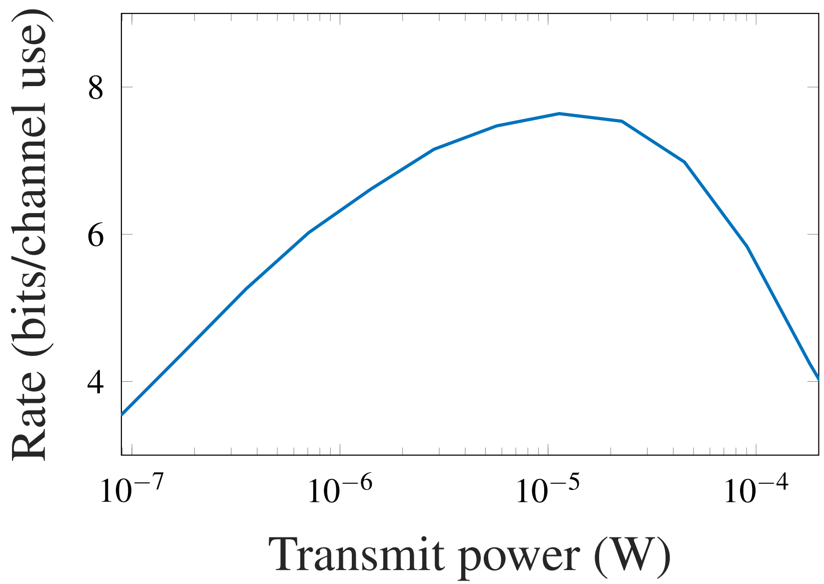

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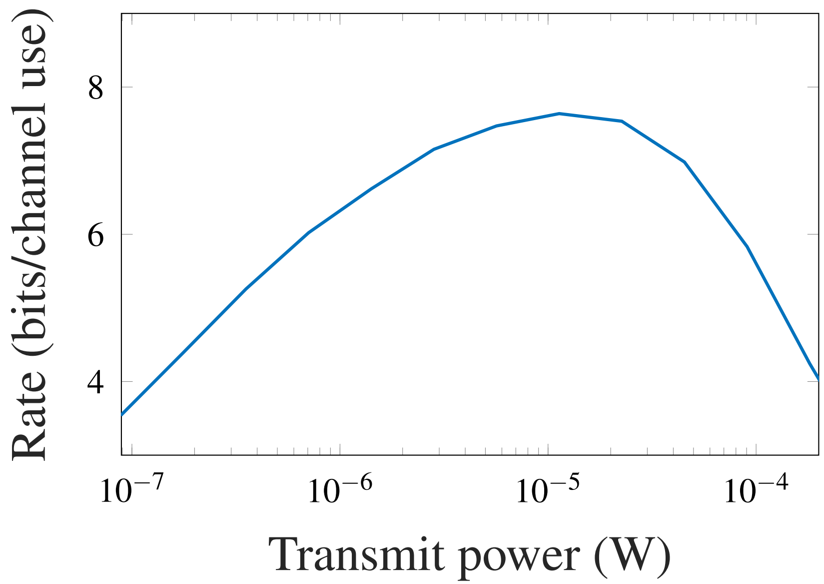

- Achievable rate with current systems reaches a peak at a certain power. What about capacity?
- Periodic or distributed filters seem to prevent broadening and reduce coupling

The Nonlinear Schrödinger Equation (NLSE)

- Frequency-domain wave equation ($\Delta\omega' = \omega' - \omega_0'$)

$$Q\left(z, \Delta\omega\right) = Q(0, \Delta\omega) e^{-\frac{\alpha}{2}z} e^{j\beta z}$$

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- **But**, the wave number β depends on both **frequency** and **power**:

$$\beta(\Delta\omega) \approx \beta_0 + \beta_1 \Delta\omega + \frac{\beta_2}{2}\Delta\omega^2 + \gamma |q(z,t)|^2$$

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- Taking $\frac{\partial}{\partial z}$ and going back to time domain:

$$\frac{\partial q(z,t)}{\partial z} = -\frac{\alpha}{2} q(z,t) - j\frac{\beta_2}{2}\frac{\partial^2 q(z,t)}{\partial t^2} + j\gamma |q(z,t)|^2 q(z,t) + n(z,t)$$

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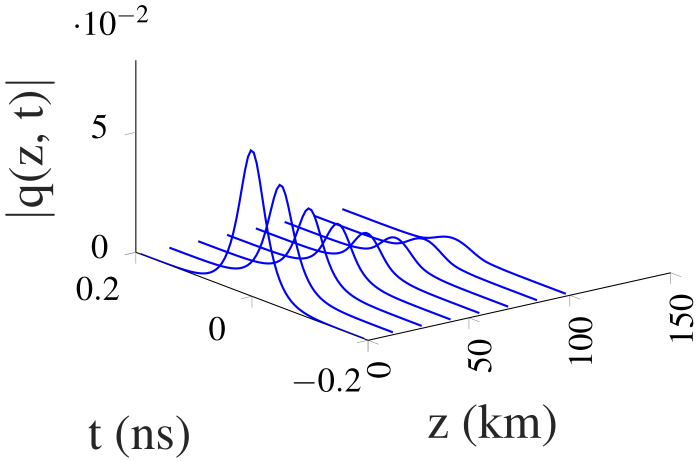
Attenuation

- Exponential decay in power

$$P(z) = P(0)e^{-\alpha z}$$



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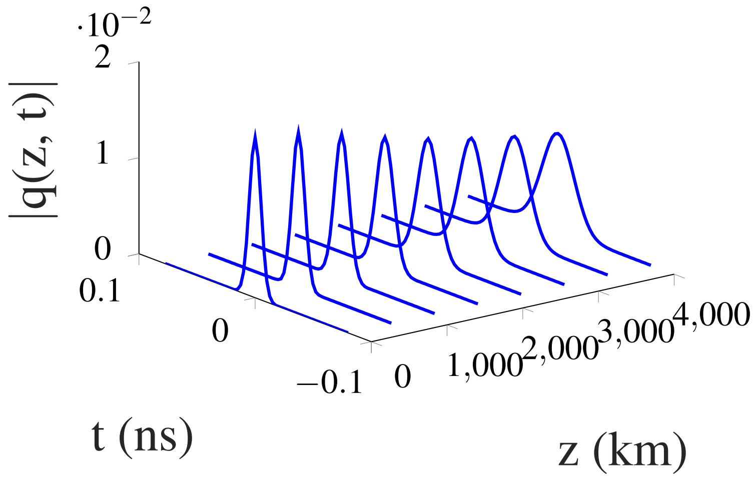
Dispersion

- Linear term
- Causes pulse broadening in time



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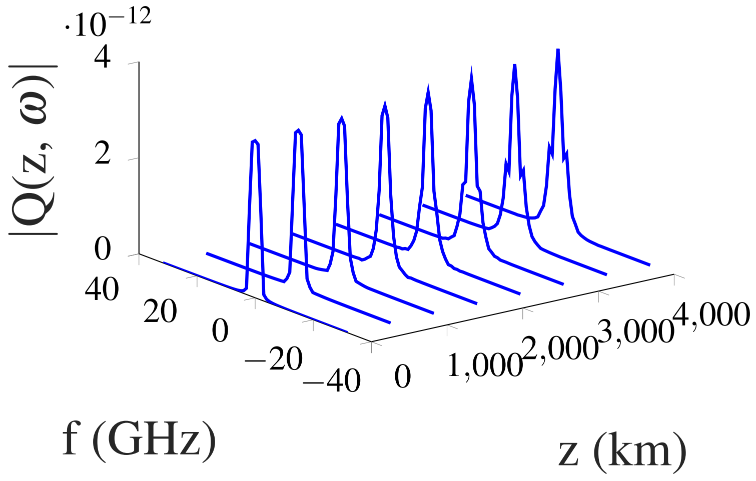
Nonlinearity

- Causes frequency mixing (spectral broadening, SPM, XPM, FWM)



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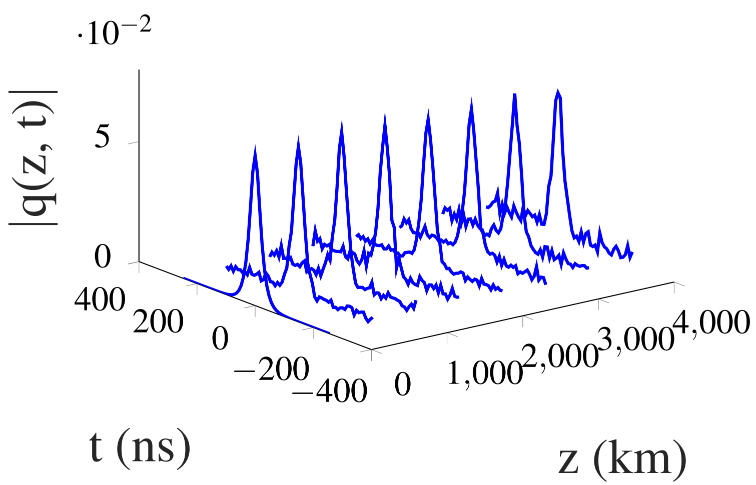
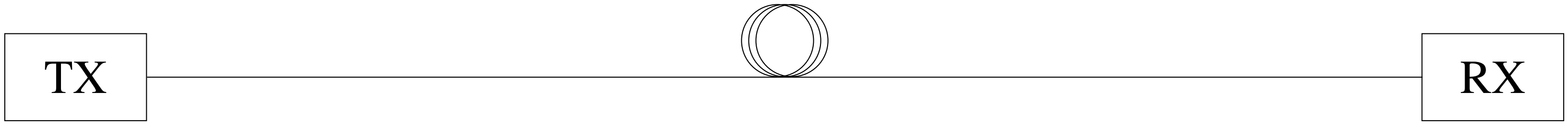
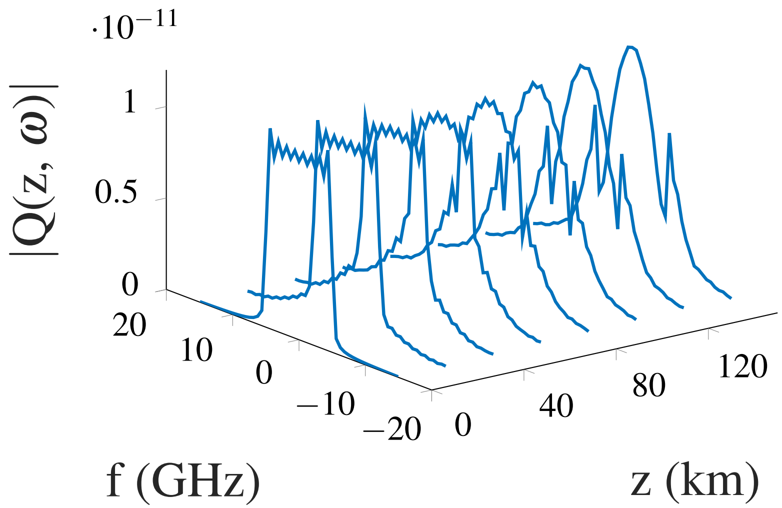
Noise

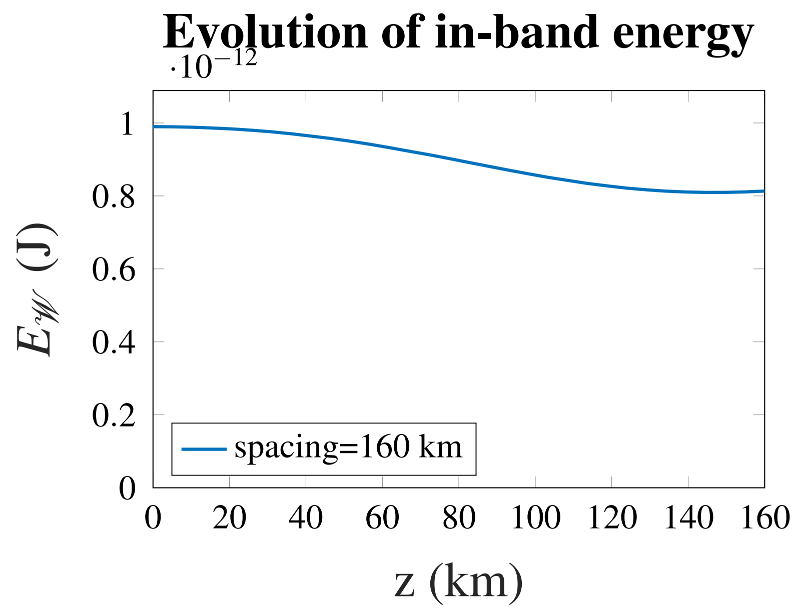
- Distributed along the fiber
- Mixes nonlinearly with signal!



Periodically filtered NLSE channel







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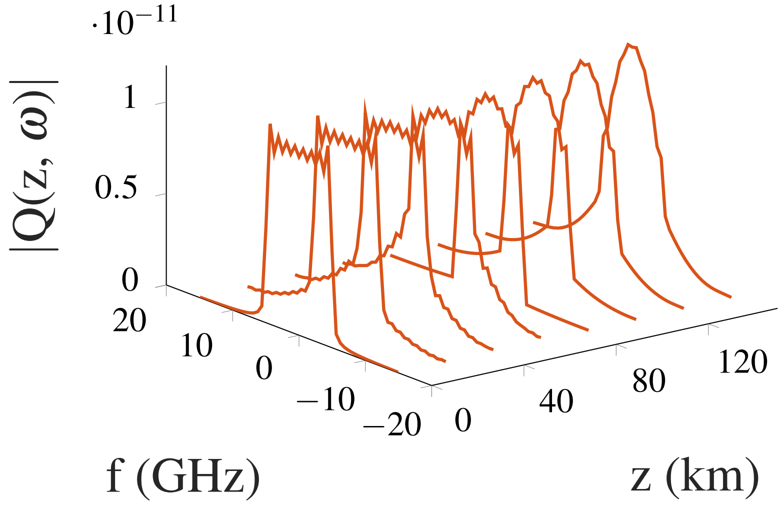

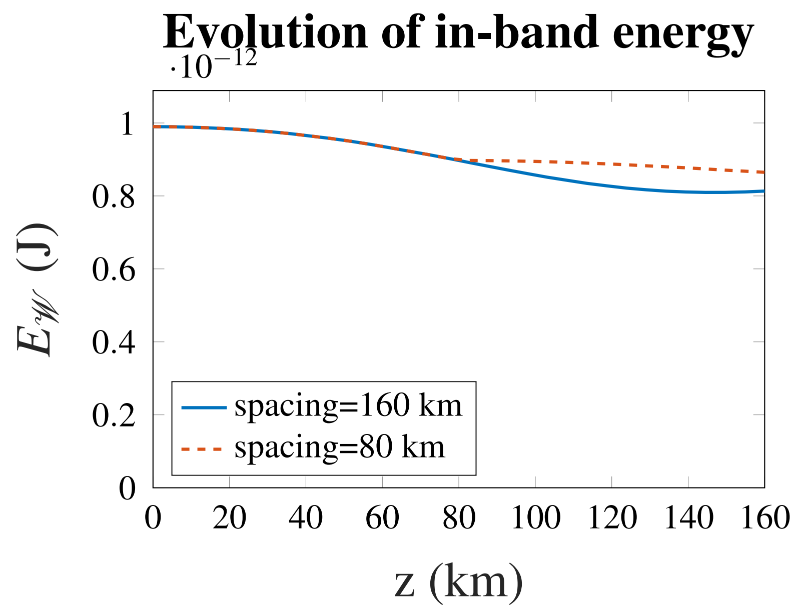

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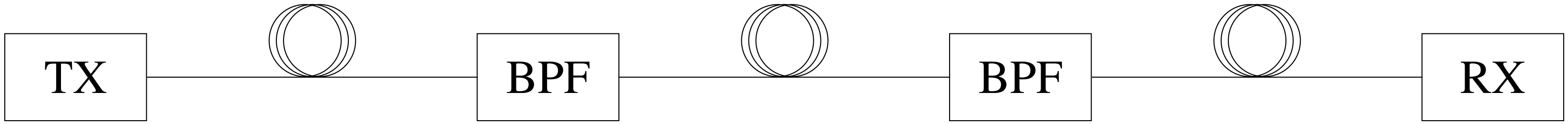

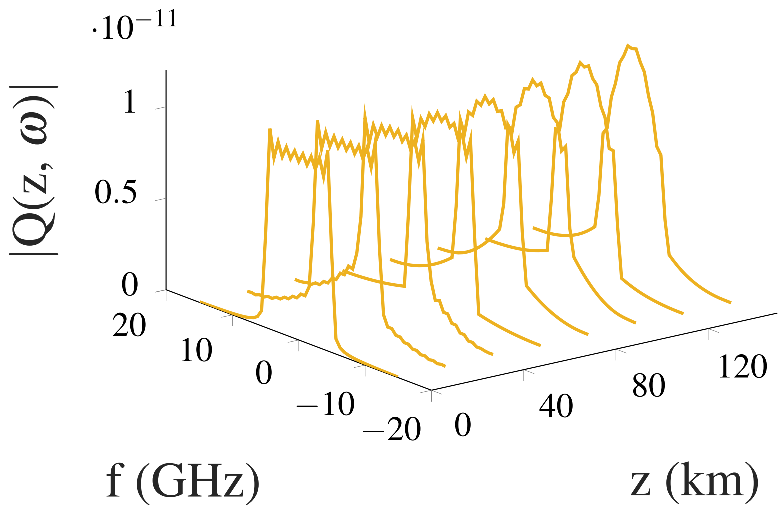

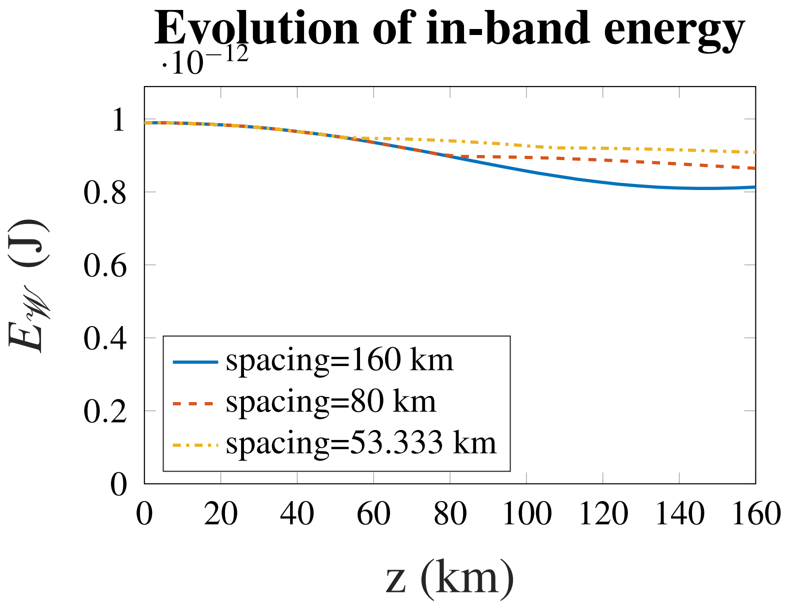

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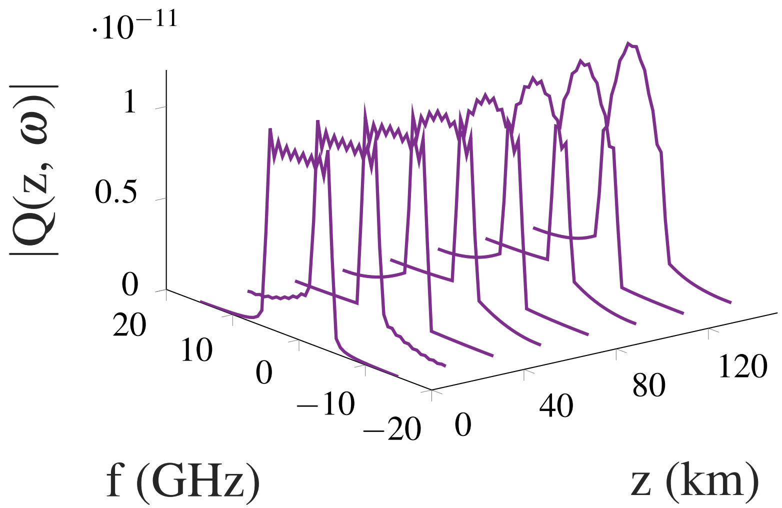

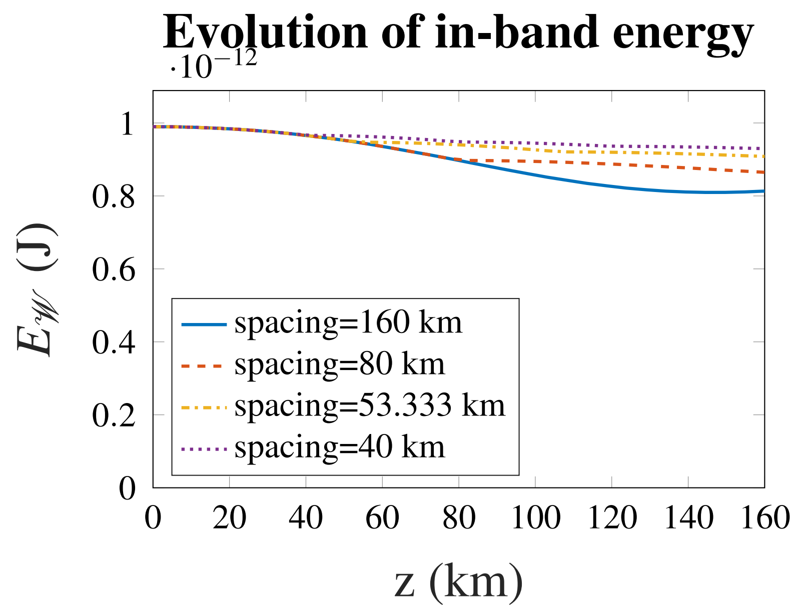

Limiting case: distributed filter

- Frequency-domain NLSE:

$$\frac{\partial}{\partial z}Q(z,\omega) = -\frac{\alpha(\omega)}{2}Q(z,\omega) + j\frac{\beta_2}{2}\omega^2 Q(z,\omega) + j\frac{\gamma}{4\pi^2}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} Q(z,\omega_1)Q^*(z,\omega_2)Q(z,\omega-\omega_1+\omega_2)\,\mathrm{d}\omega_1\,\mathrm{d}\omega_2$$

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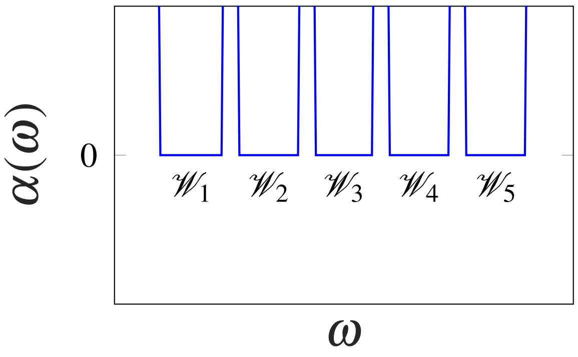
- Model distributed filters as frequency-dependent attenuation profile:



$$\alpha(\omega) = \begin{cases} 0, & \text{in the passband,} \\ \infty, & \text{in the stopband.} \end{cases}$$

Energy conservation with distributed filters

- With distributed filters, **the total energy is conserved**:

$$E(z) = \frac{1}{2\pi} \int_{\mathscr{W}} |Q(z,\omega)|^2 \, \mathrm{d}\omega = E(0) \quad \forall z$$

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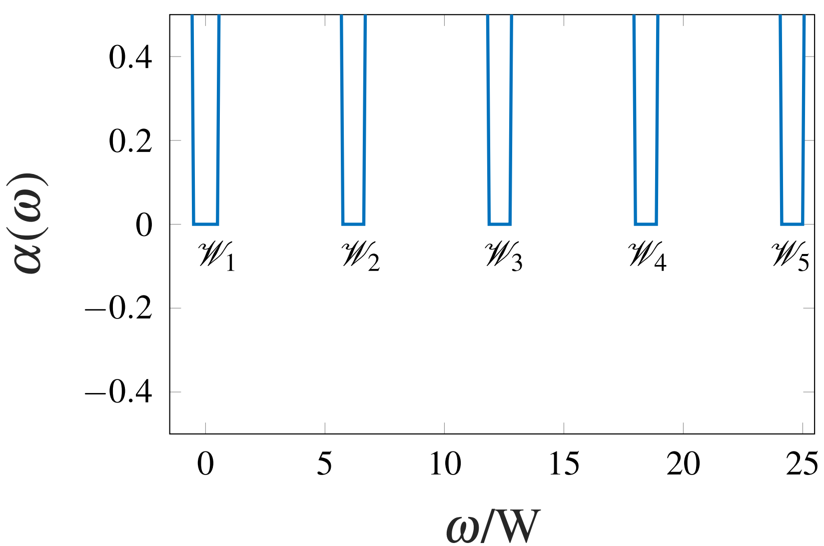
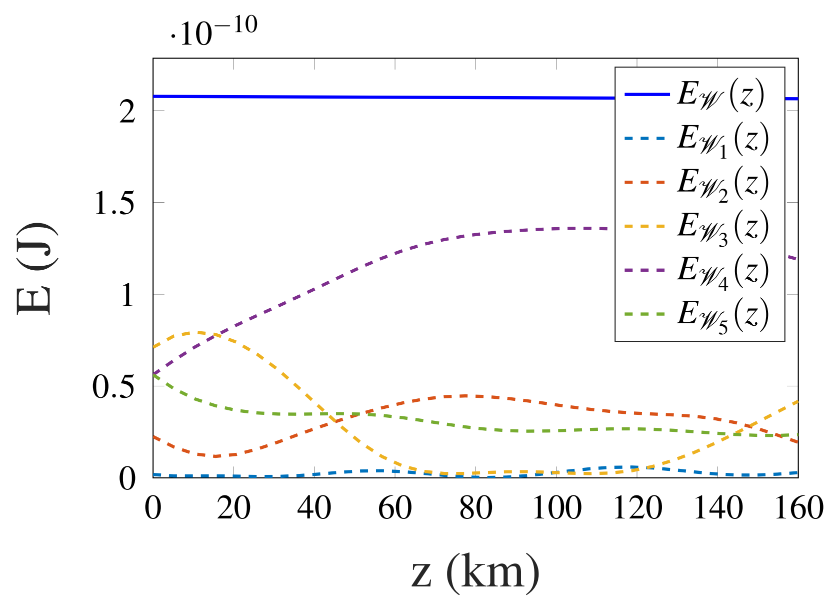
- However, the **energy of each channel** is not conserved (energy coupling):





Design of an energy-decoupled system

- The following condition ensures the absence of **energy** coupling between channels:

$$\left(\mathscr{W}_{n_1} + \mathscr{W}_{n_2}\right) \cap \left(\mathscr{W}_{n} + \mathscr{W}_{n_3}\right) = \emptyset,\ \forall \{n_1, n_2\} \neq \{n, n_3\}, \tag{1}$$

where $+$ denotes the *sum of intervals*:

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$$\omega_n = 2m_nW$$

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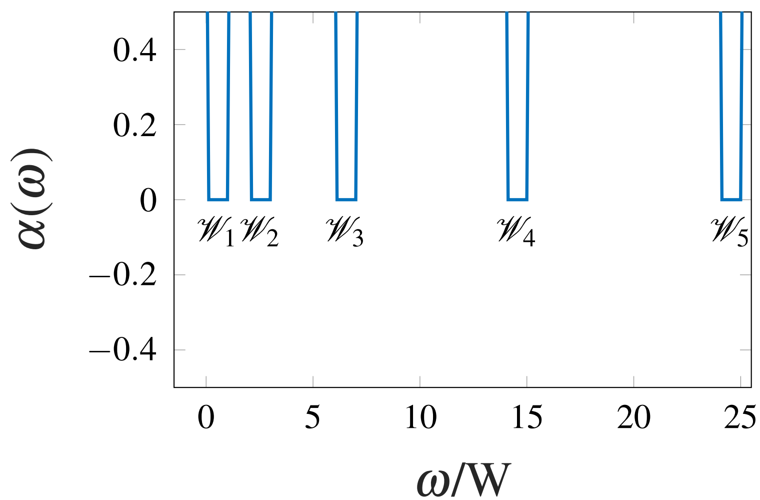
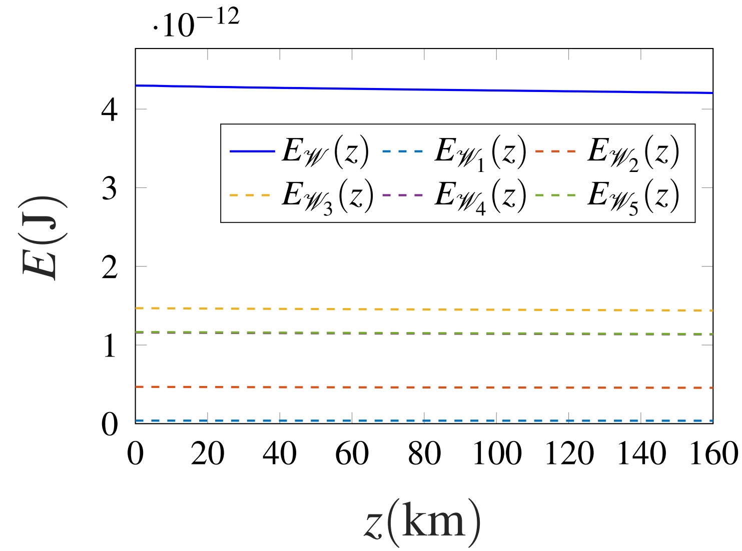
i. e. an energy-decoupled N-channel system can asymptotically fill **at most** a fraction $1/N$ of the spectrum.

Energy evolution in a Sidon system





Parameter	**Symbol**	**Value**
Filter spacing	Δz	10 km
Channel bandwidth	$W/(2\pi)$	1 GHz
Power per channel	P	0.86 mW
Dispersion parameter	β_2	-21.67 ps^2/km
Nonlinear parameter	γ	1.26 W^{-1}km^{-1}

Distributed noise

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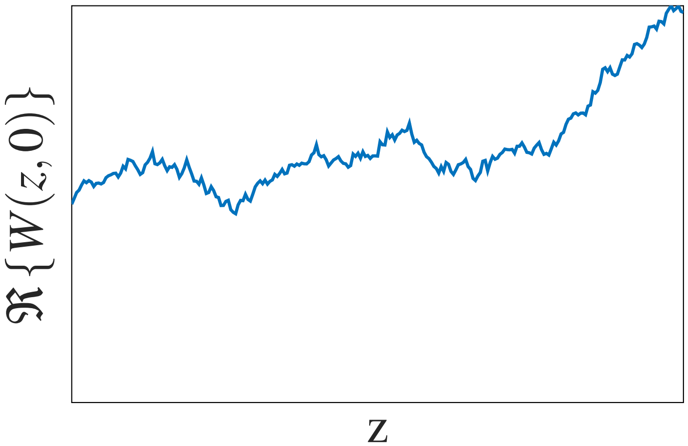

$$W(z,t) = \lim_{L\to\infty} \frac{1}{\sqrt{L}} \sum_{\ell=1}^{\lfloor Lz \rfloor} v_\ell(t)$$

$v_\ell(t)$ Gaussian i.i.d. in ℓ, bandwidth B.

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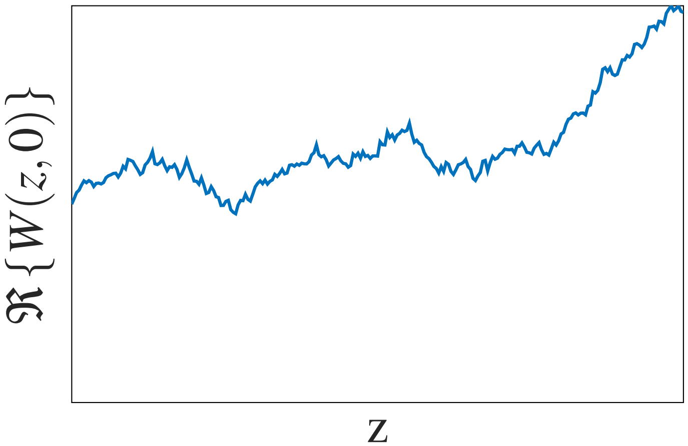

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$n(z,t)$ is formally the derivative of $W(z,t)$

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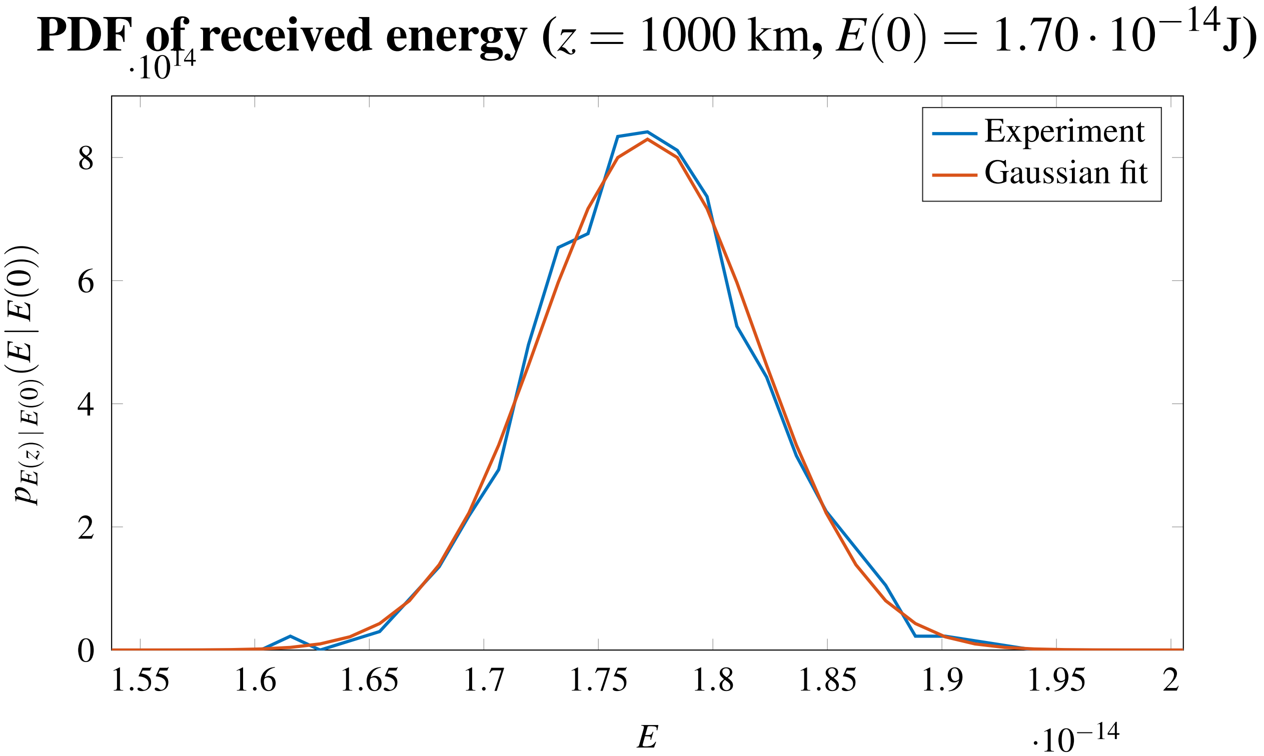
- With nonlinearity and dispersion, $E(z) \mid E(0)$ *looks reasonably Gaussian*



Lower bound on capacity

- The energy channel is conditionally Gaussian:

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- The capacity for this channel (power constraint $\mathbb{E}\left[E(0)\right] \leq PT$) is lower bounded by:

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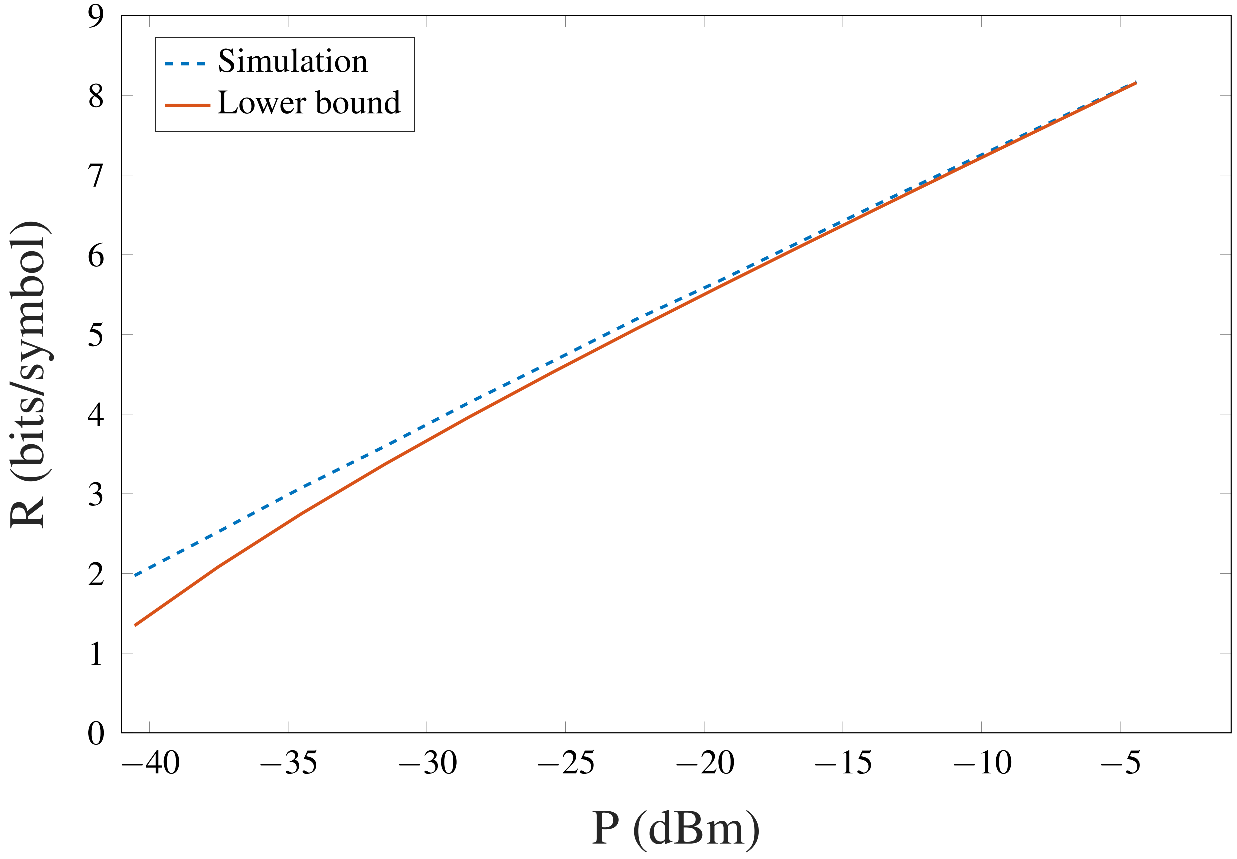
- For large power P, the capacity **grows unboundedly** with power:

$$C_{\text{lb}}(P) \underset{P\to\infty}{\sim} \frac{1}{2}\log P$$

Lower bound on capacity



RRC pulses, $N_{\text{ase}}Bz = -44.77$ dBm, $z = 3000$ km, $B = 2$ GHz

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- This allows calculation of a lower bound on capacity that grows unboundedly with power
- Disclaimer:
  - Proved only for the memoryless case!