Reliability analysis for runway overrun using subset simulation

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ABSTRACT: Runway overrun is one of the main accident types in airline operations. Nevertheless, due to the high safety levels in the aviation industry, the probability of a runway overrun is small. This motivates the use of structural reliability concepts to estimate this probability. We apply the physically-based model for the landing process of Drees and Holzapfel (2012) in combination with a probabilistic model of the input parameters. Subset simulation is used to estimate the probability of runway overrun for different runway conditions. We also carry out a sensitivity analysis to estimate the influence of each input random variable on the probability of an overrun. Importance measures and parameter sensitivities are estimated based on the samples from subset simulation and concepts of the first-order reliability method (FORM).

1 INTRODUCTION

The airlines organized in the International Air Transport Association (IATA) carried about 3.6 billion passengers in 2012. Among these, 414 where killed in an aviation accident (IATA, 2013). Of the many accident types, runway excursion is the most common one. There are several types of runway excursions; one of the most critical one is runway overrun (RWO) of a landing aircraft, which is investigated in this paper.

The high safety requirements within the civil aviation industry make a quantitative estimation of the probability of a runway excursion important. However, the already high reliability standards within the industry hinder an estimation of accident probabilities with purely statistical methods. In particular, it is difficult to investigate the effect of individual factors on the probability of specific accidents. This motivates the use of structural reliability concepts. This is demonstrated in this paper, where we estimate the probability of runway overrun using a previously proposed physically-based model for the landing process (Drees and Holzapfel, 2012).

A few authors proposed RWO models based on regression e.g. (Kirkland, 2001, Hall et al., 2008, Ayres Jr. et al., 2011). However, to the best of our knowledge, structural reliability methods in combination with a physically-based model have not been applied previously.

In the next section a short outline to structural reliability in general and subset simulation in particular is given. This is followed by a summary of the RWO model and the related probabilistic model. Thereafter, the results of the reliability analysis are presented for different runway conditions. The influence of the individual input random variables on the probability of a RWO is investigated through different importance and sensitivity measures. In this context, FORM is shortly explained, as some of its theory is applied. The paper concludes with a discussion of the results.

2 STRUCTURAL RELIABILITY

In reliability analysis, $\mathbf{X}$ denotes a vector of random variables that are the input to a model; $\mathbf{X}$ has the joint probability density function $f(\mathbf{x})$. The response of the physical model as a function of $\mathbf{X}$ is described by means of limit state functions (LSF) $g(\mathbf{x})$. For RWO, $g(\mathbf{x})$ corresponds to the stop margin of the aircraft, i.e. the length of the runway minus the operational landing distance (the distance actually needed to stop the aircraft).

A runway overrun corresponds to the event $g(\mathbf{x}) \leq 0$ and its probability can be written as:

$$\Pr(RWO) = \int_{g(\mathbf{x})\leq 0} f(\mathbf{x})\, d\mathbf{x} \quad (1)$$

This type of problem is well known within the field of structural reliability, which is concerned with estimating probabilities of failure of engineering structures (Rackwitz, 2001). As the integral in Equation (1) can in general not be calculated analytically, a number of methods for efficiently solving this kind of problems have been developed in the field of structural reliability. The most prominent among these are the first- and second-order reliability methods (FORM and SORM) e.g. (Der Kiureghian, 2005), the standard Monte Carlo simulation (MCS), and advanced sampling methods like importance sampling or subset simulation (SuS) (Au and Beck, 2001).

Both FORM and SORM rely on finding the most likely failure point (MLFP), i.e. the most likely realization of $\mathbf{X}$ that leads to failure, here RWO. These methods are based on optimization for finding the MLFP. This may be critical if the method is to be applied for the computation of the probability of RWO in near-real time. For this reason, we apply SuS, which is still relatively efficient, but does not necessitate optimization.

2.1 *Subset simulation (SuS)*

The classical MCS approach provides an unbiased estimate $\hat{P}_F$ of Equation (1) by generating n_S samples from the joint distribution $f(\mathbf{x})$ and evaluating $g(\mathbf{x})$ for each of the samples. The estimated probability of failure is then:

$$\hat{P}_F = \frac{1}{n_S}\sum_{i=1}^{n_S} I_{g(\mathbf{x}_\mathrm{i})\leq 0}(\mathbf{x}_\mathrm{i}) \quad (2)$$

where $I_{g(\mathbf{x}_\mathrm{i})\leq 0}(\mathbf{x}_\mathrm{i})$ is an indicator function, which is 1 for $g(\mathbf{x}_\mathrm{i}) \leq 0$ and 0 otherwise. This MCS approach becomes computationally unfeasible for small probabilities of failure. For a desired coefficient of variation of the estimate $\delta_{\hat{P}_F}$ the required number of samples is:

$$n_S = \frac{1-\Pr(F)}{\delta^2_{\hat{P}_F}\Pr(F)} \quad (3)$$

For a small probability of failure of $\Pr(F) = 10^{-8}$ and a desired coefficient of variation of $\delta_{\hat{P}_F} = 0.1$ approximately 10^{10} LSF evaluations would be necessary. SuS proposed by (Au and Beck, 2001) overcomes this problem by expressing the failure event F as a sequence of nested events F_i.

$$F_1 \subset F_2 \subset \cdots \subset F_n = F$$

With these nested events the probability of failure can be rewritten as:

$$\Pr(F) = \Pr(F_1)\prod_{i=2}^{n}\Pr(F_i|F_{i-1}) \quad (4)$$

The probability of the first intermediate event, $\Pr(F_1)$, is not conditional on a previous intermediate event and can therefore be estimated with a standard MCS. All other probabilities $\Pr(F_i|F_{i-1})$ in Equation (4) are conditional on a previous intermediate event, these conditional probabilities are estimated using Markov Chain Monte Carlo (MCMC) simulation procedures. In Figure the MCMC approach is schematically shown for estimating $\Pr(F_2|F_1)$. The grey points represent the samples generated from $f(\mathbf{x})$ in the initial MCS step. Those samples $\mathbf{x}_\mathrm{i}$, which are above the LSF (shown here as a black line) are said to be in the failure domain F_1, i.e. for them $\mathbf{x}_\mathrm{i} \in F_1$. From each of those m intermediate failure samples, a Markov Chain of length n_S/m is generated (black samples). In these chains a new candidate state is generated conditional on the previous state of the chain. Furthermore if the newly generated candidate state $\mathbf{x}'$ fulfills the condition $\mathbf{x}' \in F_1$ it becomes the new state of the chain otherwise the previous state is repeated. SuS is performed in the uncorrelated standard normal space (U-Space). The component wise Metropolis Hastings algorithm, which was proposed by (Au and Beck, 2001) as a MCMC algorithm for SuS makes use of this by generating the candidate states independently for each dimension. The performance of SuS thus becomes independent of the number of dimensions of the problem.

In this paper we apply a MCMC algorithm proposed by (Papaioannou et al., 2014) for use in SuS. Like the algorithm of (Au and Beck, 2001) this algorithm works in a component-wise fashion and its performance is therefore also independent of the dimensionality of the problem. However, the generation of the pre-candidate states is done, such that every pre-candidate state is accepted.

Typically the intermediate failure events F_i are chosen adaptively, such that $\Pr(F_1) = \Pr(F_2|F_1) = \cdots = \Pr(F_{n-1}|F_{n-2}) = p_0$. Often a value of 0.1 is chosen for p_0. The final failure event $F_n = F$ is fixed and therefore its conditional probability cannot be chosen a priori. Equation (4) is then:

$$\Pr(F) = {p_0}^{n-1}\Pr(F|F_{n-1}) \tag{5}$$

As SuS is applied in U-space the random variables **X** have to be transformed to U-space. Typically the Nataf or the Rosenblatt transformations are applied for this purpose (Hohenbichler and Rackwitz, 1981). In the problem at hand one of the random variables is defined conditional on other random variables. For that reason the Rosenblatt transformation appears to be suitable. This transformation is based on expressing the distribution of every random variable conditional on the previous random variables.

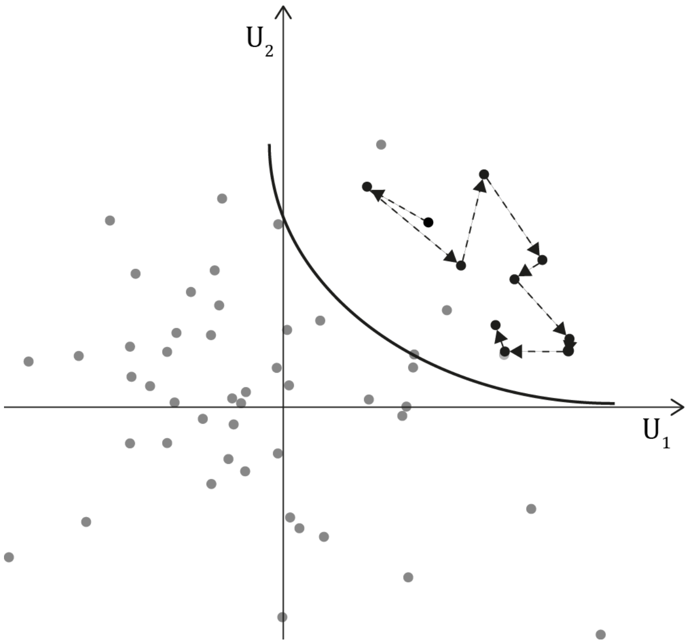

Figure 1 Generation of samples conditional on a first intermediate failure event. The new samples (black) are sampled starting from a seed sample using a MCMC algorithm. The grey samples were obtained by MCS in a previous step.

3 PHYSICAL MODEL OF RUNWAY OVERRUN (RWO)

In this section, we *briefly* present the applied RWO model. For a more detailed description, the reader is referred to the original paper by (Drees and Holzapfel, 2012). The RWO event can be defined as the operational landing distance exceeding the runway length (cf. Figure 2). The LSF can therefore be written as:

$$g(\mathbf{x}) = Stop\ margin(\mathbf{x})$$

The stop margin is the deterministic runway length minus the operational landing distance, which depends on a number of factors that are modeled as random variables. The model is derived from the equations of motion. The acceleration of the aircraft in x-direction is:

$$\dot{V} = \frac{1}{m}[T - D - mg \cdot \sin\gamma - \mu_F(mg \cdot \cos\gamma - L)] \tag{6}$$

where m is the mass of the aircraft, T is the propulsion force from the aircraft engines and D is the aerodynamic drag. The term $mg \cdot \sin\gamma$ – with g being the constant of gravitation and γ the flight path angle – represents the contribution of the runway slope to the acceleration. Finally, the last term describes the influence of the friction and brake forces. L denotes the aerodynamic lift and μ_F the friction coefficient, which depends on the runway condition, on the brake force and on the velocity of the aircraft.

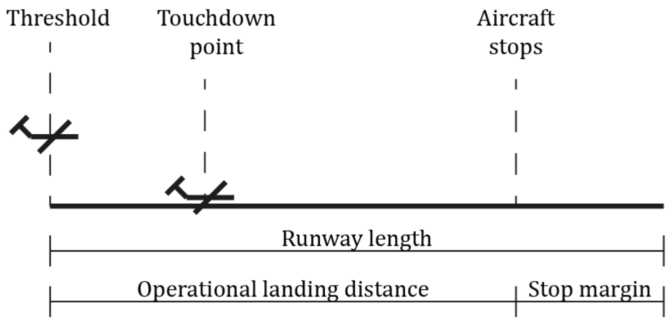

Figure 2 Definitions

The drag force can be written as:

$$D = \frac{\rho}{2}(V_K - V_W)^2 S \cdot C_D \tag{7}$$

and the lift force as:

$$L = \frac{\rho}{2}(V_K - V_W)^2 S \cdot C_L \tag{8}$$

ρ is the air density, V_K is the speed of the aircraft and V_W the speed of the surrounding air, such that $V_K - V_W$ is the speed of the aircraft relative to the wind speed. Furthermore, S is the reference area of the wings and C_D respectively C_L are the drag and the lift coefficients.

Integrating equation (6) twice with respect to time will yield the operational landing distance. For practical reasons the operational landing distance is split in three parts. First the touchdown distance, which is directly modeled as a random variable; second the distance from the touchdown point to the point where the pilot deactivates the auto-brake system, which, in addition to the environmental and technical factors, depends on a number of human factors, e.g. the point in time until the pilot starts braking and the point in time when the spoiler is deployed. The third part is the distance the aircraft travels from the point of auto-brake deactivation to the final stopping position. During a normal landing, the aircraft will not stop completely on the runway but exit the runway with slow velocity to the taxiways. However, it is assumed that in the case of a critical landing, which is likely to result in a RWO, this distance is traveled with maximum deceleration efforts i.e. maximal braking forces.

3.1 *Technical and human factors*

The quantities in the differential Equation (6) are influenced by environmental factors on the one hand, and technical as well as human factors on the other hand. The technical aspects are captured in the RWO model by allowing certain technical components like the spoiler, the auto-brake system or the engines to be either operative or inoperative with a certain probability. The brake system can be operative, inoperative or degraded. In the scope of this paper we do not consider any technical failures, thus all components are considered to be fully operative. Furthermore, we assumed the flaps and slats to be in configuration full, which is used in 96% of approaches (Drees and Holzapfel, 2012).

The friction coefficient between the tire and the runway μ_F is a major factor influencing the deceleration of the aircraft. This friction coefficient is mainly determined by the runway condition; we differ between a dry and a wet runway in our calculations.

The auto-brake system automatically applies a brake pressure when the landing gear touches ground. The magnitude of the applied pressure depends on the setting of the auto-brake system. For the scope of this paper, we compute RWO probabilities for the auto-brake settings set to medium.

In operational aviation, the times after touchdown at which the spoiler and thrust reversers are deployed and the times at which the braking starts and ends can be obtained from operational data. On the basis of this measured data, distribution models are selected. These models are summarized in Table 1. Another factor, whose variability is influenced mainly by human actions, is the touchdown point. A normal distribution, with fixed standard deviation and a mean value (μ_{TDP}) that is a function of the environmental conditions is used for this. It was found from flight operation data that pilots change their touchdown behavior according to the required landing distance estimate, which they calculate during the approach. The required landing distance is thereby calculated from aircraft and runway specific characteristics as well as from the flap setting, the head wind, the temperature and the landing weight.

4 PROBABILITY OF A RWO

As a case study, we calculate the RWO probabilities for an aircraft of type Airbus A320 landing on runway EDDM 26L at the Munich airport (MUC). The runway is 4000m long with a concrete surface. We apply subset simulation using the MCMC algorithm proposed in (Papaioannou et al., 2014). The inter-sample correlation is initially chosen as 0.8. The intermediate thresholds were chosen adaptively, such that at each simulation level 10% of the samples would lie in the intermediate failure region. At each of the simulation levels, 1000 samples were generated such that consider-

Table 1 Random variables of the physical model. The pressure is the air pressure adjusted to the sea level. Approach speed deviation is the deviation from the target approach speed. The mean value of the touchdown point depends on the calculated required landing distance, which again is a function among others of the headwind, the landing weight and the temperature. Abbreviations: weibull distribution (WBL), normal distribution (N) and generalized extreme value distribution (GEV)

Random variable	**Distribution model**	**Mean**	**Std. dev.**
Landing weight [t]	$WBL(60.0,44.3)$	59.3	1.69
Head wind [kts]	$N(5.4,5.8)$	5.4	5.8
Temperature [°C]	$GEV(-0.26,7.9,6.5)$	9.4	8.0
Pressure [hPa]	$N(1016,8.1)$	1016	8.1
Appr. speed dev. [kts]	$GEV(-0.20,4.0,3.0)$	4.7	4.2
Rev. deployment [s]	$Gamma(9.0,0.55)$	5.0	1.7
Start braking [s]	$GEV(0.15,3.9,10.2)$	13.1	6.4
Splr. Deployment [s]	$GEV(0.11,0.89,3.7)$	4.3	1.3
End braking [s]	$N(25,5)$	25	5
Touchdown point [m]	$N(\mu_{TDP},121.9)$	μ_{TDP}	121.9

ing the presented results in Table 2 7000 respectively 8000 LSF evaluations were necessary for each simulation run.

Table 2 Simulation results for different runway conditions (dry and wet). For each simulation 1000 samples were used per simulation level.

Runway condition	$Pr(RWO)$
Dry	$1.06E-7$
Wet	$7.03E-8$

The resulting probabilities are in the same order of magnitude for the different runway conditions. The fact that the RWO probabilities are smaller for a wet runway than for a dry one may be counterintuitive, as a wet runway reduces the friction between the tire and the runway and thus also the effectiveness of the braking procedure.
The explanation for this result is that the pilots calculate the required landing distance according to an approximate formula when approaching an airport and adapt their landing behavior to the result. In the model this is accounted for through the mean value of the touchdown point. The inferior grip of the tires is therefore compensated or even overcompensated by an adapted touchdown behavior, at least according to the model.

5 SENSITIVITY ANALYSIS

We conduct a sensitivity analysis to find the influence of the individual random variables on the probability of a RWO. First importance measures based on FORM are calculated. For the most important random variables, parameter sensitivities are then calculated based on the samples from subset simulation. In the following the concepts of FORM, which are of major importance for our purpose are introduced. For a deeper introduction the reader is again referred to the literature e.g. (Der Kiureghian, 2005, Ditlevsen and Madsen, 2007). In a later section parameter sensitivities are calculated using the samples from SuS.

5.1 *Importance measures with FORM*

FORM gives an approximation of Equation (1) by substituting the LSF in U-space with a linear surrogate model $g_L(\mathbf{u})$. The linearization of the LSF is done in the design point i.e. the most likely failure point $\mathbf{u}^*$ in U-space. The probability of failure $\Pr(F)$ can then be approximated through the probability of failure corresponding to the linear surrogate model. With Φ being the standard normal CDF and β_{FORM} the reliability index $\Pr(F)$ can be approximated as:

$$\hat{P}_F = \Phi(-\beta_{FORM}) \tag{9}$$

With the reliability index being defined as:

$$\beta_{FORM} = |\mathbf{u}^*| \tag{10}$$

The task of calculating the probability of failure reduces thus to finding the design point $\mathbf{u}^*$. In general the convergence of the optimization algorithms, applied to find $\mathbf{u}^*$ is the bottleneck in FORM analysis.
A useful byproduct of the FORM analysis is the normal vector of the linear approximation of the LSF $\boldsymbol{\alpha}$ (Der Kiureghian, 2005). The elements of this vector can be interpreted as importance measures of the standard normal random variables U_i (Figure 3):

$$\alpha_i = \frac{u_i^*}{\beta_{FORM}} \tag{11}$$

If a random variable U_i is the only one influencing $\hat{P}_F$ its corresponding α_i will have an absolute value of 1. If on the other hand U_i has no influence on $\hat{P}_F$ its α_i is 0. We can further distinguish between random variables of load type, which have a positive α_i and those, which are of capacity type that correspond to a negative α_i.
If the random variables in $\mathbf{X}$ are independent the importance measures $\boldsymbol{\alpha}$ of the transformed variables $\mathbf{U}$ are readily valid also in the original space otherwise a transformation as described in (Der Kiureghian, 2005) needs to be applied.
In the scope of this paper the FORM importance measures are determined for a linear LSF fitted to sample points as proposed in (Melchers and Ahammed, 2004). Thus the optimization usually necessary for FORM, which is a bottleneck especially in near-real time applications, is circumvented. Fitting a linear hyper-plane to a set of points is a robust way to estimate the design point according to (Melchers and Ahammed, 2004). As the linear LSF $g_L(\mathbf{u})$ should approximate the actual LSF as accurately as possible mainly in the proximity of the design point/limit state surface it is suggested to only use those sample points $\mathbf{u}$, for which $g_L(\mathbf{u}) \approx 0$ or $g_L(\mathbf{u}) \leq 0$.
The sensitivity analysis is done in this paper for a dry runway. A simulation run with 1000 samples per simulation level was done to this end and the samples from the last simulation step were used to fit the linear LSF with the method of least squares.

In Table 3 the FORM importance measures for the problem at hand are summarized. The numbers in brackets give the values for a linear LSF that was fitted to sample points from a different simulation run.

'Head wind', 'Start braking', 'Touchdown point', 'Approach speed deviation' and 'Landing weight' appear to be the random variable, having the most influence. All of them are of load type except head wind, which is of capacity type meaning that a large head wind velocity reduces the probability of a RWO.

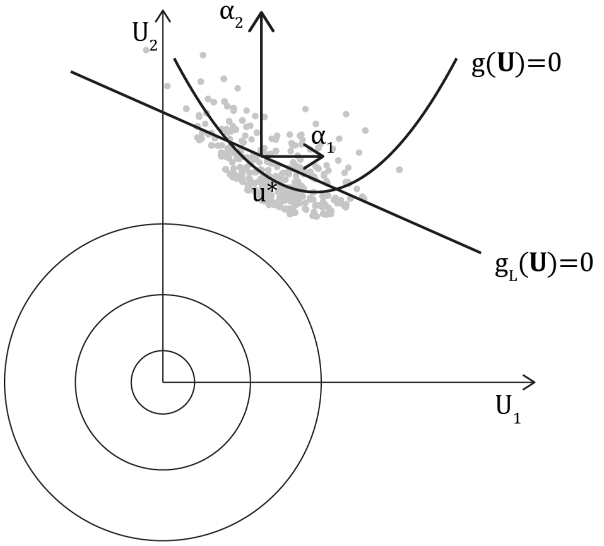

Figure 1 Importance measures **α** of linear LSF approximation fitted to samples from a SuS run.

From Table 3 it becomes obvious that the linear LSF depends on the points used for fitting. However the results show that the measures for the variables with a large influence (i.e. $> |0.1|$) are fairly similar in both runs while the variables that have only a minor influence are in one case even of different type (i.e. load or capacity) in the two runs.

Table 3 FORM importance measures for the dry runway conditions and a flap setting medium.

Random variable	FORM measures
Landing weight [t]	0.116 (0.138)
Head wind [kts]	−0.693 (−0.663)
Temperature [°C]	0.020 (0.010)
Pressure [hPa]	0.010 (0.002)
Appr. speed dev. [kts]	0.202 (0.179)
Rev. deployment [s]	0.037 (0.042)
Start braking [s]	0.650 (0.680)
Splr. Deployment [s]	0.019 (−0.013)
End braking [s]	0.004 (0.013)
Touchdown point [m]	0.204 (0.213)

Although the 'Touchdown point' is one of the main influencing factors according to the FORM measures one may expect it to have an even larger influence. The reason for that is, that it was modeled as a function with fairly low variability and a mean, which is shifted according to the outcome of other random variables.

In general it should be noted that the FORM importance measures in Table 3 are dependent on the distributions of the random variables. For example from the fact that the pressure appears to have only a minor influence in the case study at hand it cannot be concluded that the pressure may not be an important factor if one considers different airports, where some of them are located at high altitudes.

5.2 *Sensitivity measures*

For the random variables, which were found to be important according to the FORM importance measures, parameter sensitivities are calculated. We calculate sensitivities of the probability of failure with respect to the parameters of $f(\mathbf{x})$, $\boldsymbol{\theta}$. These sensitivities can be defined as:

$$\frac{\partial \hat{P}_F}{\partial \theta_i} = \int_{g(\mathbf{x})\leq 0} \frac{\partial f_{\mathbf{X}}(\mathbf{x}, \boldsymbol{\theta})}{\partial \theta_i} d\mathbf{x} \tag{12}$$

where $f_{\mathbf{X}}(\mathbf{x}, \boldsymbol{\theta})$ is the probability density function of X with the parameters $\boldsymbol{\theta}$. θ_i denotes the parameter with respect to which the sensitivity is calculated. In the scope of this paper we consider sensitivities with respect to the means μ_i and standard deviations σ_i of the random variables X_i. Based on the calculated FORM approximation of the LSF, sensitivities of the form $\frac{\partial \Pr(F)}{\partial \boldsymbol{\theta}}$ can be calculated as described in (Der Kiureghian, 2005). These approximate sensitivities can be calculated based on a design point, which was obtained using an optimization algorithm. In a near real-time context, where convergence of the optimization algorithms may be critical this can be done also based on a linear LSF fitted to samples, as already described. Here we calculate the sensitivities with respect to the mean values μ_i and standard deviations σ_i directly from the samples obtained from SuS. The parameter sensitivities can be written as:

$$\frac{\partial \hat{P}_F}{\partial \theta} = \sum_{i=1}^{n} \frac{\hat{P}_F}{P_i} \frac{\partial P_i}{\partial \theta} \tag{13}$$

Table 4 Elasticities of $\hat{P}_F$ with respect to μ_i and σ_i, calculated following Equation (13) and (15).

Random variable	$\frac{\mu_i}{\hat{P}_F}\frac{\partial \hat{P}_F}{\partial \mu_i}$	$\frac{\sigma_i}{\hat{P}_F}\frac{\partial \hat{P}_F}{\partial \sigma_i}$
Landing weight [t]	53.4	1.4
Head wind [kts]	−3.2	10.6
Appr. speed dev. [kts]	0.8	0.04
Start breakinng [s]	1.1	56.0
Touchdown point [m]	–	1.0

where P_i are the (un)conditional failure probabilities corresponding to the intermediate failure events i.e. $P_1 = \Pr(F_1)$ and $P_i = \Pr(F_i|F_{i-1})$ for $i > 1$. The derivative of the intermediate failure probabilities can be estimated using the samples obtained in subset simulation (Song et al., 2009):

$$\frac{\partial P_i}{\partial \theta} \approx \frac{1}{N}\sum_{k=1}^{N}\left[\mathrm{I}_{F_i}(\mathbf{x}_k)\left(\frac{1}{f_{\mathbf{X}}(\mathbf{x}_k)}\frac{\partial f_{\mathbf{X}}(\mathbf{x}_k)}{\partial \theta} - \sum_{j=1}^{i-1}\frac{1}{P_i}\frac{\partial P_j}{\partial \theta}\right)\right] \quad (14)$$

where $\mathrm{I}_{F_i}(\mathbf{x}_k)$ denotes the indicator function, which is 1 if $\mathbf{x}_k \in F_i$ and 0 otherwise.

In Table 4 the elasticities $\varepsilon_{\hat{P}_F,\theta}$ are shown for the five most important random variables according to the FORM importance measures from Table 3. Elasticities are defined as:

$$\varepsilon_{\hat{P}_F,\theta} = \frac{\theta}{\hat{P}_F}\frac{\partial \hat{P}_F}{\partial \theta} \quad (15)$$

They describe the relative change of probability of failure due to a relative change in the parameter θ and are in general easier to interpret than sensitivities.

6 CONCLUDING REMARKS

Structural reliability methods have been applied to estimate runway-overrun probabilities and were shown to be suitable for this purpose. In particular, the calculated sensitivities and FORM importance measures support the interpretation and further development of the model. With the help of the FORM importance measures, one can simplify the model by modeling the quantities of minor importance deterministically. We have found that the parameters 'Temperature', 'Pressure', 'Reverser deployment', 'Spoiler deployment' and 'End breaking' may be modeled deterministically without inducing a large error. This does, however, not imply that the influence of these quantities is small. It can only be concluded that the uncertainty associated with the quantity is not significant. The parameter sensitivities were in this paper calculated based on the samples obtained with subset simulation. These parameter sensitivities describe the effect of a change of the mean or standard deviation on the probability of failure. We presented the parameter sensitivities in the form of elasticities, which are typically easier to interpret. From the results it can be seen that the variables, which are of main importance according to the FORM importance measures, are the most sensitive ones with respect to a change in the standard deviation (i.e. the variability).

A physically-based RWO model in combination with robust methods for reliability analysis could be applied in real time or near-real time risk assessment. A possible scenario would be an aircraft approaching an airport and getting gradually better information on uncertain factors like landing weight, head wind at the destination airport etc. Based on the calculated RWO probabilities a pilot could decide on whether it is save to land at the destination airport or to approach an alternate airport.

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