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Research Article

Spectral Representation of Uncertainty in Experimental Vibration Modal Data

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It is well known that structures exhibit uncertainty due to various sources, such as manufacturing tolerances and variations in physical properties of individual components. Modeling and accurate representation of these uncertainties are desirable in many practical applications. In this paper, spectral-based method is employed to represent uncertainty in the natural frequencies of fiber-reinforced composite plates. For that, experimental modal analysis using noncontact method employing Laser-Vibrometer is conducted on 100 samples of plates having identical nominal topology. The random frequencies then are represented employing generalized Polynomial Chaos (gPC) expansions having unknown deterministic coefficients. This provides us with major advantage to approximate the random experimental data using closed form functions combining deterministic coefficients and random orthogonal basis. Knowing the orthogonal basis, the statistical moments of the data are used to estimate the unknown coefficients.

1. Introduction

Uncertainty quantification concerns representation and solution of simulation models, e.g., a differential equation or a finite element model, when some levels of modeling such as input parameters are not exactly known. In such conditions, the model is said to be stochastic, i.e., it exhibits some degree of uncertainty. Probabilistic structural dynamics, in particular, endeavor to take into account uncertainties relating various aspects of real structures such as material and geometric parameters, loading terms, and initial/boundary conditions and exploring related impacts on the structure responses. To improve the performance, durability, and efficiency of structures, an exact knowledge of geometrical and material parameters is required. Characterization of the stochastic response due to these uncertainties by stochastic methods has gained interest among researchers in past decades. Stochastic methods in conjunction with finite element method (FEM) have been widely used to quantify uncertainty in structural responses [1–9]. Two major issues have to be addressed regarding stochastic analysis of such structures under uncertainty: first, how the uncertainties can be efficiently identified and modeled in numerical simulations, particularly in finite element models and, secondly, how uncertainties affect the behavior of aforesaid structures.

The former issue requires quantifying the randomness in uncertain parameters. This can be efficiently characterized by the statistical properties of the parameters, e.g., probability density function (PDF). However, identification of the appropriate PDF type characterizing the parameter uncertainties demands to know a priori information which may be collected from experimental tests. Various methods have been developed in past decades for PDF identification from experimental data (cf. [10–13]). The latter issue depends on the availability of an exact model relating inputs to outputs. Construction of such model is not possible due to many common assumptions in modeling of structural dynamics. For that reason, experimental methods are still the most reliable approach for investigation of the uncertainties.

In this paper, uncertainties relating to the experimentally identified natural frequencies of composite plates are investigated. Uncertainties in such materials may have different sources, e.g., manufacturing tolerances, fiber orientations, or physical properties of individual components. To this end, experimental modal analysis using the noncontact method by employing Laser-Vibrometer is conducted on 100 samples of plates having identical nominal topology. The statistical properties of the identified natural frequencies of the plates, in particular, are discussed in detail. The random frequencies then are represented employing generalized Polynomial Chaos (gPC) expansion [14–17]. This provides us with major advantage to approximate the experimental data using closed form functions combining deterministic coefficients and random orthogonal basis. The coefficients then are estimated employing optimization procedure comparing theoretical and experimental values of statistical moments.

This paper is organized as follows: the basic formulation of the spectral-based representation of random parameters is given in the next section. The numerical-experimental simulations are presented in Section 3 and the last section denotes the conclusion.

2. Spectral-Based Representation of Random Parameters

The spectral discretization methods are the key advantage for the efficient stochastic reduced basis representation of uncertain parameters in finite element modeling. This is because these methods provide a similar application of the deterministic Galerkin projection and collocation methods to reduce the order of complex systems. In this way, it is common to employ a truncated expansion to discretize the input random quantities of the structure and system responses. The unknown coefficients of the expansions then can be calculated based on the FE model outputs. Let us consider the uncertain parameter $P(\boldsymbol{\xi})$ where $\boldsymbol{\xi} \in \Omega$ is the vector random variable characterizing the uncertainty in the parameter and Ω denotes the random space. Under the limited variance, i.e., $\sigma^2 < \infty$, the parameter can be approximated by

$$P(\boldsymbol{\xi}) \approx \sum_{i=0}^{N} p_i \Psi_i(\boldsymbol{\xi}) \tag{1}$$

which is known as the truncated generalized Polynomial Chaos (gPC) expansion of the parameter. The deterministic coefficients p_i are calculated employing the stochastic Galerkin projection as [17]

$$p_i = \frac{1}{h_i^2} \int_\Omega P(\boldsymbol{\xi}) \Psi_i(\boldsymbol{\xi}) f(\boldsymbol{\xi}) \, \mathrm{d}\boldsymbol{\xi} \tag{2}$$

where f is the joint probability density function (PDF) of random vector $\boldsymbol{\xi}$ which for independent random variables ξ_i can be written as the multiplication of the individual PDF for each variable $f_i(\xi_i)$; i.e.,

$$f(\boldsymbol{\xi}) \, d\boldsymbol{\xi} = f_1(\xi_1) f_2(\xi_2) \cdots f_n(\xi_n) \, \mathrm{d}\xi_1 \mathrm{d}\xi_2 \cdots \mathrm{d}\xi_n \tag{3}$$

and h_i denotes the norm of polynomials defined as

$$h_i^2 = \langle \Psi_i(\boldsymbol{\xi}), \Psi_i(\boldsymbol{\xi}) \rangle = \int_\Omega \Psi_i^2(\boldsymbol{\xi}) f(\boldsymbol{\xi}) \, \mathrm{d}\boldsymbol{\xi} \tag{4}$$

For the sake of simplicity, we will focus on monodimensional random input in this work; i.e., $\boldsymbol{\xi} = \{\xi\}$.

2.1. Estimation of the gPC Coefficients from Experimental Data. Calculation of the coefficients using (2) requires prior information on the PDF of uncertain parameters which may not be available. Statistical moments, in contrast, exhibit adequate implicit information on the probability properties of random quantities. Once the experimental data on the uncertain parameters are available, the estimation of the gPC coefficients from statistical moments [11, 17] is possible. Here, the statistical moments derived analytically from the gPC expansion are compared to those calculated from the data. For the truncated gPC expansion, only the first few orders of moments are required to calculate the coefficients. The major benefit is that information on the probability distribution of uncertain parameters does not have to be known *a priori*. The coefficients are then estimated comparing statistical moments constructed from the gPC expansion and experimental data via an optimization procedure. The kth-oder statistical moment, m_k, of uncertain P having the gPC expansion given in (1) is calculated as

$$m_k = \mathrm{E}\left[P^k\right] = \int_\Omega \left[\sum_{i=0}^{N} p_i \Psi_i(\xi)\right]^k f(\xi) \, \mathrm{d}\xi, \qquad k = 0, 1, \dots \tag{5}$$

with $m_0 = 1$ and $m_1 = p_0$. In this paper, we use the probabilistic orthonormal Hermite polynomials for $\Psi_i(\xi)$ and, accordingly, $f(\xi) = (1/\sqrt{2\pi})\exp(-\xi^2/2)$. The kth-order central statistical moment is then calculated as

$$\mu_k = \mathrm{E}\left[(P - \mathrm{E}[P])^k\right] = \sum_{i=0}^{k} \binom{k}{i} (-1)^{k-i} m_i m_1^{k-i}, \qquad k = 2, 3, \dots \tag{6}$$

This leads imminently to the following expression for the second central statistical moment (variance) μ_2 of the uncertain parameter P as

$$\mu_2 = m_2 - m_1^2 = \sum_{i=1}^{N} p_i^2 h_i^2 \tag{7}$$

in which $h_i^2 = i!$ for the Hermite polynomials. Similar expressions can be derived for the higher order moments as functions of the gPC coefficients. The calculated moments form the gPC expansion can be compared to the corresponding values obtained from experimental data for an uncertain parameter. In such a way, one can attempt to estimate the unknown coefficients from the available experimental data. That is, for a given set of experimental data $\{P_1, P_2, \dots, P_M\}$

Table 1: The first three statistical moments of the first nine measured natural frequencies; mean value, μ_f, the standard deviation σ_f, and the skewness γ_3.

	f_1	f_2	f_3	f_4	f_5	f_6	f_7	f_8	f_9
μ_f [Hz]	115	145	275	396	504	530	556	704	781
σ_f [Hz]	4.60	5.90	8.93	15.22	11.92	15.92	15.35	17.98	25.95
γ_{3_f} [−]	0.485	0.366	0.895	0.292	0.166	1.086	1.312	0.622	1.419

on P, the experimental central moments, μ_k^{expri}, are calculated as

$$\mu_k^{\text{expri}} = \frac{1}{M}\sum_{j=1}^{M}\left[P_j - \mathrm{E}(P)\right]^k, \quad k = 2, 3, \ldots \tag{8}$$

in which $\mathrm{E}(P) = (1/M)\sum_{j=1}^{M} P_j$ is the mean value of samples. An error function based on the least-square criterion corresponding to the difference between the moments derived from (5) and (8) can be used to estimate the optimal coefficients p_i. This leads to a minimization problem as follows:

$$\begin{aligned} \underset{p_i}{\text{minimize}} \quad & \sum_{n=1}^{k} f_n^2(p_i) \\ \text{s.t.} \quad & f_0(p_i) = \mu_1^{\text{expri}} - \mu_1 = 0 \\ & f_n(p_i) = \mu_k^{\text{expri}} - \mu_k, \quad n \geq 2 \end{aligned} \tag{9}$$

The first condition of the process denotes that the expected value of the data represents the first coefficient of the gPC expansion. Since the calculated moments for the gPC expansion are nonlinear functions of the coefficients, one has to employ nonlinear optimization procedure. The optimization leads to unique solution under the convergence condition for coefficients of one-dimensional gPC; i.e., $\|p_{i+1}\| < \|p_i\|$.

3. Experimental and Numerical Study

As a case study, in this section, the natural frequencies of fiber-reinforced composite (FRC) plates are represented as random parameters. The experimental modal analysis has been performed on 100 sample plates with nominal identical topology of $a = 250$ mm, $b = 125$ mm, and thickness of 2 mm; see Figure 1. The plates were suspended by very thin elastic bands to simulate free boundary conditions. The Laser-Vibrometer has been employed to collect the vibration responses due to the excitation force from impulse hammer with tip force transducer at some predefined points of the plates. The average of 5 impacts at each point was recorded. The sample test has returned to rest before the next impact is taken. A standard data acquisition facility along with a postprocessing modal analysis software has been used to extract modal data, e.g., natural frequencies. The measured first nine natural frequencies of plates are given in Figure 2. As shown, a considerable range of uncertainty is observed in frequencies. Spectral representation of the measured random frequencies requires estimating the statistical moments. The

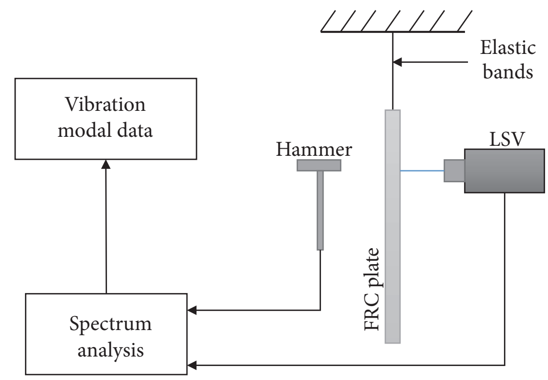

Figure 1: Experiment setup for modal analysis of FRC plates.

second-order gPC expansion is employed to approximate the frequencies. This leads to three unknown coefficients and, consequently, estimation of the first three statistical moments as given in Table 1. The third statistical moment is given by the skewness, γ_{3_f}, defined as $\mu_3^{\text{expri}}/\sigma_f^3$. The random natural frequencies are represented using second-order gPC expansions having random Hermite polynomials $H(\xi)$ as basis; i.e.,

$$f_n = \sum_{i=0}^{2} p_{n_i} H_i(\xi) = p_{n_0} + p_{n_1}\xi + p_{n_2}\left(\xi^2 - 1\right), \quad n = 1, 2, \ldots, 9 \tag{10}$$

The unknown deterministic coefficients p_{n_i} are calculated by equality of the statistical moments of the gPC expansions given in (6) and the experimental estimations given in Table 1. The optimization Application *optimtool* in MATLAB® for constrained nonlinear problem is employed for this purpose. This leads immediately to the following expressions for optimization problem defined in (9):

$$\begin{aligned} f_1\left(p_{n_0}\right) &= \mu_f - p_{n_0} = 0 \\ f_2\left(p_{n_1}, p_{n_2}\right) &= \sigma_f^2 - p_{n_1}^2 + 2p_{n_2}^2 \\ f_3\left(p_{n_1}, p_{n_2}\right) &= \gamma_{3_f} - \frac{1}{\sigma_f^3}\left(6p_{n_1}^2 p_{n_2} + 8p_{n_2}^3\right) \end{aligned} \tag{11}$$

The nonlinear optimization problem is then solved to estimate p_{n_i} as given in Table 2. The second-order coefficients are very small compared to the first two coefficients. This denotes

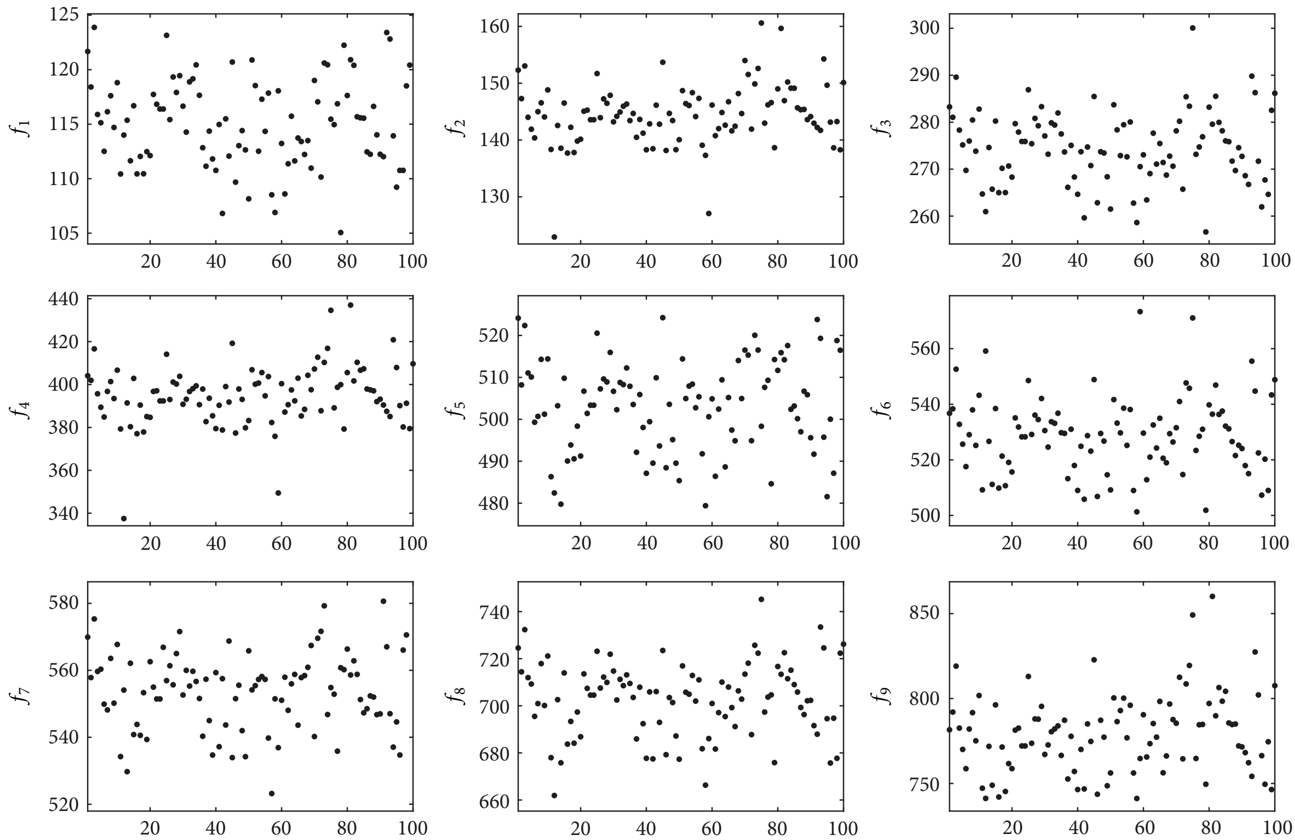

Figure 2: Measured first nine natural frequencies of 100 FRC plates (all in Hz).

Table 2: The coefficients of the gPC expansions approximating the measured uncertain natural frequencies.

	f_1	f_2	f_3	f_4	f_5	f_6	f_7	f_8	f_9
p_{n_0}	115	145	275	396	504	530	566	704	781
p_{n_1}	4.53	5.87	8.72	15.18	11.90	15.36	14.53	17.78	24.32
p_{n_2}	0.37	0.36	1.35	0.74	0.33	2.95	3.48	1.88	6.40

that the second-order gPC expansion has enough accuracy to represent uncertainty in the random frequencies. Once the unknown coefficients are known, the statistical properties of the measured data can be calculated using the constructed gPC expansions.

4. Conclusion

Natural frequencies of composite plates have been considered as random parameters. The generalized Polynomial Chaos expansions has been employed to approximate the uncertainty in the measured natural frequencies. The method offers the major advantage that the unknown deterministic coefficients of the expansions have to be calculated instead of random parameters. This has been performed by comparing the statistical moments of the experimental results for 100 identical plates and from the expansions via the minimization of least-square based error. The results have been given for the first nine natural frequencies using second-order expansions.

Data Availability

The measured data used to support this study were supplied from project no. DFG-KR 1713/18-1 funded by the German Research Foundation (Deutsche Forschungsgemeinschaft-DFG). There is no restrictions to use the officially published data.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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