Fakultät für Wirtschaftswissenschaften
der Technischen Universität München

Coordination in Multi-Unit Package Auctions

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Vollständiger Abdruck der von der Fakultät für Wirtschaftswissenschaften der Technischen Universität München zur Erlangung des akademischen Grades eines

Doktors der Volkswirtschaft (Dr. oec. publ.)

genehmigten Dissertation.

Vorsitzende:	Prof. Dr. Alwine Mohnen
Prüfer der Dissertation:	1. Prof. Dr. Martin Bichler 2. Prof. Dr. Martin Grunow

Die Dissertation wurde am 11.12.2018 bei der Technischen Universität München eingereicht und durch die Fakultät für Wirtschaftswissenschaften am 15.08.2019 angenommen.

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Per Paulsen

October 9, 2019

To my good friend Nikolaus Johannes Pohl, whom we all miss.

ABSTRACT

Combinatorial multi-object auctions that enable buyers to submit bids on packages of the auctioned items are becoming increasingly popular in practice. These auctions allow for the most general expression of preferences and therefore are likely to be highly efficient. However, although they are known for avoiding the famous exposure problem, only a limited number of studies on optimal bidding behavior in these auction formats exist in the scientific literature. This thesis attempts to close the gap by modeling different combinatorial auctions as Bayesian games and deriving corresponding bidding equilibria in a standard framework in which bidders possess independent private and decreasing marginal values in the number of perfect substitutes obtained. The game-theoretic predictions are then compared with respect to efficiency and seller revenue.

In a restricted setting in which two bidders compete for two units of a homogeneous good, a first-price sealed-bid combinatorial auction weakly dominates a combinatorial ascending second-price format in expected seller revenue. This ranking is reversed in an efficiency comparison. Optimal bidding in both auction formats is characterized by a coordination problem between an efficient and inefficient Bayesian equilibrium for the two bidders. Therefore, revenue and efficiency predictions are not entirely unambiguous. Nevertheless, the dynamic structure of the ascending combinatorial auction format lets the respective equilibrium behavior appear more robust. Generally, the combinatorial ascending second-price format is weakly outcome equivalent to other non-combinatorial second-price auction formats such as the well-known Simultaneous Ascending Auction. The first-price sealed-bid combinatorial format, however, is characterized by peculiar bidding behavior that distinguishes it from any other first- or second-price mechanism and leads to highest seller revenue. Moreover, for more than two bidders, the first-price sealed-bid combinatorial auction still possesses an inefficient equilibrium and only in its efficient equilibrium is strategically equivalent to its ascending-price counterpart and other standard auction formats. Based on these contributions we then also examine the effect of budget constraints in bidding firms (principal-agent relationships) and an application to combinatorial procurement auctions (ex-post split-award auctions).

Principal–agent relationships between the supervisory board and the management of bidding firms are widespread in spectrum auctions, but they can also be observed in other multi-object markets. The management aims at winning the highest valued licenses whereas the board wants to maximize profit and limits exposure in the auction. In environments in which it is efficient for firms to coordinate on allocations with multiple winners, we show that the principals would coordinate on smaller sets of objects while the agents would not and inflate their demand to larger sets. We first analyze multi-unit markets

in which principal and agent have complete information about the valuations, and show that it can be impossible for the principal to implement her equilibrium strategy with only budget constraints in a first-price sealed-bid package auction. With hidden information about the valuations a principal would need to determine contingent budgets and transfers to compensate the agent, which can be impossible with value-maximizing motives of the agent. Contrary, in an ascending package auction this is straightforward even without knowledge of valuations as long as the principals know the efficiency environment. The second-price payment rule of the ascending auction enables the principal to overcome the agency problem more easily and the dynamic mechanism further facilitates coordination for the principals. This result is in strong contrast to a setting in which all units are sold as a single package. Here, the contract does not differ between the two auction formats which is in accordance with prior findings on optimal budget constraints in standard single-object auctions.

Ex-post split-award auctions are a frequently used form of combinatorial procurement auctions in which the demand for some quantity to be procured is split into multiple shares. Markets with diseconomies of scale are wide-spread, but strategically challenging. We show that, unlike in single-object auctions, first-price sealed-bid and the Dutch combinatorial auction formats are not strategically equivalent. While the former exhibits a coordination problem for bidders, the Dutch formats have only efficient equilibria. The price information revealed during the Dutch auction formats avoids equilibrium selection problems and helps bidders coordinate. Also, the theoretical predictions explain important empirical patterns in the data from laboratory experiments.

ACKNOWLEDGEMENTS

I am very grateful to my supervisor Martin Bichler for providing me with the opportunity to conduct scientific research on combinatorial auctions and market design. Thank you Martin for your constant encouragement and guidance on our projects as well as your truly valuable input. I also thank my mentor Paul Karänke for his general support and friendship. In addition, I am highly indebted to my coauthor Gian-Marco Kokott for his exemplary teamwork and comradeship. Thank you Salman Fadaei for the valuable academic discussions and motivational stimuli, and thank you Dirk Bergemann and Vitali Gretschko for your insightful comments. Finally, many thanks to everyone at the chair for Decision Sciences & Sytems, in particular Maria Anna Thanner, for an enjoyable time.

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INTRODUCTION

Strategic bidding behavior of buyers demanding multiple items in combinatorial multi-object auctions is examined in this thesis. Contrary to other multi-object auction formats, combinatorial mechanisms allow the submission of bids on sets (combinations) of objects instead of just on single objects. This greater bidding flexibility allows buyers to better express their valuations for sets of objects and is therefore more likely to avoid inefficient allocations. For example, combinatorial auction mechanisms avoid the well-known exposure problem inherent in most non-combinatorial formats (Milgrom, 2000) by enabling bidders to take into account synergies between the auctioned objects as is explained in Section 1.1.3. Therefore, combinatorial auctions are well-suited to sell almost any type of object and they have extensively been used in practice, for example, to sell the rights to use airport time (landing) slots to competing airlines (Rassenti et al., 1982), to procure bus services from public transport operators for bus routes in London (Cantillon and Pesendorfer, 2006), to assign catering contracts for school meals in Chile (Epstein et al., 2002), to buy raw materials from suppliers at Mars Inc. (Hohner et al., 2003) and, most importantly, as spectrum auctions world-wide (Cramton, 2013). Surprisingly, strategic bidding behavior in combinatorial multi-object auctions has hardly been studied in the theoretic economics literature, which underlines the high academic relevance of this research topic (Klemperer, 1999). Exceptions include Goeree and Lien (2016) and Ausubel and Baranov (2018) amongst others, which will be discussed in detail in Section 1.1.4.

The thesis follows the economic convention and models combinatorial auctions as Bayesian games of incomplete information (Harsanyi, 1967a,b,c) in an independent private values (IPV) setting with ex-ante symmetric buyers. This is the benchmark model to derive Bayesian Nash equilibrium strategies of Bayesian auction games (de Vries and Vohra, 2003; Klemperer, 1999; McAfee and McMillan, 1987) and was first proposed for single-object and multi-unit auctions in the seminal article by Vickrey (1961). In addition, we restrict our attention to a setting in which a risk-neutral and revenue-maximizing auctioneer sells multiple objects to a fixed and commonly known number of profit-maximizing and risk-neutral bidders. In the combinatorial auction formats analyzed, we assume an *XOR* bid language that allows a bidder to specify a bid for each possible package (Nisan, 2000). Throughout the thesis we refer to the above as standard assumptions and deviate only in Chapter 3, in which we examine budget constrained bidding firms. Finally, to keep the model as simple as possible, we focus on a setting in which all auctioned objects are perfect substitutes, which is common practice in the analysis of multi-object auctions such as in Goeree and Lien (2014) and Ausubel et al. (2014) for simultaneous single-object auctions and in Sano (2012) for a combinatorial format, for example.[1] In particular, we assume each buyer's marginal valuations to decrease in the number of perfect substitutes obtained as is also assumed in Ausubel et al. (2014).

Sticking to these standard assumptions we do not contribute to the literature that studies the advantages of auctions compared to other selling mechanisms such as negotiations and posted-price schemes in broader environments, as discussed in Milgrom (1985) and the references cited therein, or embeds auctions in exchange games and standard bargaining models (Milgrom, 1987). However, the uniquely beneficial properties of single-object auctions as single-object market mechanisms as presented in Section 2.3.1, and the increasingly frequent and successful use of auctions in the real world serve as sufficient motivation to examine the theoretically and practically most advantageous formats. For example, Cramton (1997) analyzes spectrum auctions to sell personal communications services (PCS) licenses conducted by the US Federal Communications Commission (FCC) between 1994 and 1996, and finds that the use of auctions lead to a significant improvement compared to other mechanisms such as lotteries or comparative hearings. Auctions have been used since ancient times, for example in Rome commercial trade was carried out via auction-like mechanisms. In modern times common auctioned commodities include livestock, art and construction contracts (Cassady, 1967). Unusual applications comprise the auctioning of the right to a market position as natural monopoly (Williamson, 1976) and the procurement of locations for noxious facilities such as prisons and hazardous waste disposal plants (Kunreuther and Kleindorfer, 1986). Recently, the Internet has led to an explosion in the conduction of auctions, for example on online trading platforms such as eBay and Amazon (David, 2000), and especially by search engines to sell ad spaces (online ad auctions) (Varian, 2009).

By comparing equilibrium bidding behavior in different combinatorial auction formats, the goal of this thesis is to obtain a ranking of combinatorial mechanisms in terms of predicted revenue (sum of expected selling prices) and efficiency (maximum social welfare). Since the pioneering works by Vickrey (1961) and Vickrey (1962), multi-object auction formats have frequently been studied in this way as in Ausubel et al. (2014) and Ausubel and Baranov (2018), for example. It is well-known that the winner determination problem of a combinatorial auction and its approximation is computationally hard (NP-complete) (Rothkopf et al., 1998; Sandholm, 2002). Thus, it can be inferred that the derivation of Bayesian Nash equilibria in multi-object combinatorial auction formats is impossible in polynomial time for general settings with an arbitrary number of buyers and objects as well as any form of buyer valuations. Therefore, attention is restricted to smaller but practically relevant

[1] Many spectrum auctions are for homogeneous goods such as multiple licenses of 5 MHz spectrum in a particular band, for example. While the strategic problems discussed can also be found with heterogeneous goods, markets with homogeneous goods require less notational burden.

environments that allow generalization of results to complex scenarios. In these restricted market settings computational hardness is not an issue anymore. Efficiency and revenue rankings of different combinatorial auction formats may then contribute to policy and managerial advice on selling multiple objects depending on the goals of the auctioneer. If the auctioneer is a state agency representing society's interests, it might focus on maximizing social welfare. However, if the auctioneer is a private company, its shareholders' interests may bind it to maximize revenue. For example, McMillan (1994) states that in the first large spectrum auction to sell PCS licenses in 1994, the US FCC had the primary goal to maximize efficiency and revenue was one of the least concerns. Nevertheless, there is indication that efficient auction formats may also lead to the highest seller revenue as is true for single-object auctions (Myerson, 1981). Efficiency is achieved if the auctioned objects are assigned to those buyers who value them most and, obviously, these bidders are also willing to pay most. As combinatorial auctions are the multi-object auction formats that allow the most general expression of bidder preferences among all multi-object mechanisms they are likely to be most efficient and potentially also maximize seller revenue.

Designing generally efficient and optimal (seller-revenue maximizing) combinatorial mechanisms, as in Myerson (1981) for the sale of a single item for example, is beyond the scope of this thesis. In particular, regarding the optimal auction format, we do not study any form of pre-bundling (of individual objects into item-sets) by the auctioneer before the bidding process starts, as was first examined for a monopolistic seller by (Palfrey, 1983) and in an auction framework amongst others by (Armstrong, 2000). In contrast, we focus on standard post-bundling in which the formation of item-packages results solely from the auctioneer choosing the revenue-maximizing combination of bids submitted by the buyers according to the bidding language of the combinatorial auction, as explained in Section 1.2.1. In this setting the Vickrey-Clarke-Groves (VCG) mechanism (Vickrey, 1961; Clarke, 1971; Groves, 1973), the multi-object generalization of the famous Vickrey (single-object and multi-unit) auction (Vickrey, 1961), is well-known to be the only strategy-proof and efficient direct auction mechanism (Green and Laffont, 1979). Moreover, as shown by Krishna and Perry (1999), it maximizes expected seller-revenue among all efficient, incentive compatible and individually rational mechanisms for allocating multiple objects. Despite this unique feature, the VCG mechanism possesses severe weaknesses that prevent it from being employed in practice, as discussed in more detail in Section 1.1.4, and make it worth considering more practically-relevant combinatorial auction formats and their bidding equilibria.

In the next section we motivate combinatorial auctions as the most advantageous form of multi-object auctions and consecutively summarize our main contributions.

1.1 MOTIVATION

Generally, analyzing the auctioning of many and possibly diverse objects is far more challenging than the sale of a single object as a larger number of more complex auction mechanisms needs to be considered. For example, it is necessary to identify an appropriate bidding language specifying the form of bids the buyers are allowed to submit (Nisan, 2000). Are they allowed to bid on all objects or only a subset and may they submit bids for combinations of items or only bid on individual objects? In the literature it is mainly distinguished between auction mechanisms that only allow for the sale of perfect substitutes and those that also allow for the sale ob objects that are not perfect substitutes (Krishna, 2002). Combinatorial auctions belong to the latter category. We first discuss some practical insights into single-object auctions that help explain why auction mechanisms are so prevalent in the real world in general. Consecutively, we briefly introduce some popular non-combinaotiral multi-object auction formats, those that allow only for the sale of perfect substitutes and those that can also be employed to sell heterogeneous objects. Their shortcomings will then motivate our focus on combinatorial mechanisms.

1.1.1 *Single-Object Auctions*

Single-object auctions probably have been the most widely employed auction mechanisms in practice. For example, the ascending-price (English) auction is used to sell antiques and artwork worldwide, and the descending-price (Dutch) format is traditionally employed for the sale of cut flowers in the Netherlands, fish in Israel and tobacco in Canada. Moreover, real-estate is sold with the first-price sealed-bid (FPSB) auction (McAfee and McMillan, 1987) and mail sails of collectable stamps are conducted via the Vickrey mechanism (Lucking-Reiley, 2000). Myerson (1981) has shown that under our standard assumptions and one (distributional) regularity condition, the optimal market mechanism to sell a single object is a ”standard” second-price sealed-bid (Vickrey) single-object auction with an appropriately chosen reservation price. In a standard single-object auction the buyer with the highest value submits the highest bid and wins the object, which results in the efficient allocation (Krishna, 2002). We will generalize this result to other standard single-object auctions in Section 2.3.1.

An auction with reserve price, however, is not ex-ante efficient as the highest willingness-to-pay among all bidders might be below the threshold, in which case the object remains unsold and social-welfare is not maximized. Another disadvantage of the optimal market mechanism is that the reserve price is determined based on specific buyer parameters. This contrasts with one of the two most fundamental requirements on auction mechanisms, universality, and the other one being anonymity. Universality demands that any type of object can be sold with any auction format and the mechanism does neither depend on object specifics nor on buyer characteristics. Anonymity is achieved if the identities of buyers do not influence the final allocation, including payments, but only the heights of their submitted bids. These two properties are important because they guarantee the implementation of auctions to be relatively easy in practice as object details and buyer specifics need not to be taken into account in the design of the market mechanism. The idea that market mechanisms should adhere to these principles is known as the ”Wilson doctrine” after Wilson (1989). Moreover, Bulow and Klemperer (1996) show that a standard single-object English auction without reserve price but involving one more bidder results in higher expected profit for the seller than the optimal auction with reservation price. Attracting one more buyer to the auction leads to higher expected revenue for the seller than setting an optimal reservation price. Again, this result is generalized to any standard single-object auction format in Section 2.3.1. Based on these arguments, this thesis continues the analysis of optimal auction mechanisms without explicit discussion of reservation prices and the above results serve as the main justification to focus on auctions as ideal market mechanisms. Let us now continue with the examination of non-combinatorial multi-object auction mechanisms.

1.1.2 *Auctions for Perfect Substitutes*

If all objects are perfect substitutes from each buyer’s point of view, as is most likely for multiple units of a homogeneous good, a bidder does not distinguish between individual items. ”Standard” multi-unit auctions (Krishna, 2002), which allow all bidders to simultaneously report a demand function specifying the amount a buyer bids on each additional unit, are particularly suitable to sell multiple units of a homogeneous good. The bids can then be summed over all bidders to obtain an aggregate demand function for the market. Finally, the total number of units is assigned to the respective number of highest bids (Vickrey, 1961, 1962). These auctions are used, for example, in the auction of treasury bills by the U.S. Treasury Department (Weber, 1983). A great disadvantage of multi-unit auctions is that they do not satisfy the fundamental requirement of universality defined in Section 1.1.1 as they cannot be employed to sell objects that are not perfect substitutes. Moreover, as first recognized by Vickrey (1961), all standard multi-unit auctions, except the Vickrey auction, do not generally allocate efficiently.

1.1.3 *Auctions for Heterogeneous Items*

The items are not perfect substitutes from a buyer’s perspective in case he distinguishes between individual objects, which is likely if multiple heterogeneous items are auctioned. Such items can only be sold in multi-object auction mechanisms that allow buyers to bid on distinguished objects or sets of objects and standard multi-unit auctions cannot be employed. In this case one possibility is to sell each item in separate simultaneous or sequential single-object auctions, or the sale of all items simultaneously in one combinatorial multi-object auction.[2] Simultaneous single-object auctions, for example, are employed to sell mineral rights on federal land by the U.S. Department of the Interior and sequential single-object auctions are used as estate auctions for a collection of objects such as stamps, coins and antiques (Weber, 1983). As the modeling of heterogeneous items becomes increasingly notationally complex in the number of items and the general derivation of optimal strategies becomes impossible, the before-mentioned multi-object auctions are often analyzed in a multi-unit (perfect substitutes) framework in the literature. See, for example, Weber (1983), Weber and Milgrom (2000) and Katzman (1999) for sequential single-object auctions, Ausubel et al. (2014) for simultaneous single-object auctions and Sano (2012) for a combinatorial auction. Note, that if auction mechanisms are suitable for the sale of heterogeneous items they can also be used to sell multiple units of a homogeneous good. Thus, in this thesis we focus on the study of combinatorial multi-unit auctions in which only perfect substitutes are sold and bidders have decreasing marginal values. This focus will be further justified in Section 1.1.4.

Similar to standard multi-unit auction formats, sequential and simultaneous single-object auctions are generally inefficient and especially sequential mechanisms are rarely used in practice compared to other multi-object formats. Cramton (1997) lists disadvantages of sequential single-unit auctions with respect to simultaneous formats such as that bidders cannot switch between bidding for different objects and have to build potentially very complex expectations about prices and winning quantities in future auctions when bidding at an early stage. Further theoretical shortcomings are discussed in more detail in Section 2.3.3. chakraborty et al. (1995), McMillan (1994) and Bykowsky et al. (2000) argue that in the practically most relevant form of simultaneous single-unit auctions, the Simultaneous Ascending (multi-unit) Auction (SAA) or Simultaneous Multiple-Round Auction (SMRA), inefficiencies might be smaller than in sequential single-unit auctions. The SMRA auction, which is predominantly used by the US FCC for the sale of PCS licenses, is a dynamic multi-round mechanism in which all objects (licenses) are auctioned simultaneously in single-license auctions.[3] In each round a buyer can raise the bid on any license according to pre-specified bid increments. The auction terminates as soon as no new bids are submitted in one round. Cramton (1995), for example, argues that the ascending-bid (price) format of the SMRA enables buyers to respond to price-information revealed in prior rounds, which then helps the bidders to form efficient aggregations and better express value interdependencies among the auctioned items.

Note that in case a buyer's marginal valuations increase in the items obtained, the well-known "exposure problem" occurs if the bidder may only submit bids on individual objects instead of being allowed to submit a bid for the entire set of complementary objects demanded. The buyer would have to submit

[2] Weber (1983), for example, distinguishes between "simultaneous dependent", "simultaneous independent" (single-object) and "sequential" auctions.

[3] McMillan (1994) describes the FCC's decision process to employ the SAA spectrum auction and more detailed descriptions of the rules of the auction format can be found in Cramton (1995), Milgrom (2000) and Bykowsky et al. (2000).

a bid on each item that exceeds the optimal bid of aiming for a smaller set of objects in expectation of winning the higher valued overall set demanded. However, if any such bidder fails to be assigned the entire set demanded he might end up with overpaid subsets. Anticipating this outcome, any bidder with synergies in items obtained faces the following tradeoff: On the one hand, he can restrict the height of his single-item bids to avoid making losses in case of winning only individual objects but thereby lowering the chances of winning the overall set. On the other hand, he can bid aggressively on separate items to increase his chances of winning the full set with the side effect of risking to make losses when obtaining only a subset. Note, that the first alternative may result in an inefficient allocation whereas the magnitude of the second effect might be so strong that buyers even submit bids that exceed their value for the entire set. The latter phenomenon is known in the literature as "mutually destructive bidding" (Bykowsky et al., 2000). An early, complete information, discussions of the exposure problem in the (SAA) can be found in Milgrom (2000), for example. The exposure problem also occurs in standard multi-unit auctions as well as sequential single-object auctions (Goeree and Lien, 2014; Krishna and Rosenthal, 1996). Goeree and Lien (2014) provide a general Bayesian characterization of the exposure problem in the SMRA for a setting in which "global" bidders with increasing marginal values compete against "local" bidders with single-unit demand who cannot switch between different units (imperfect substitutes). This setting is generally referred to as LLG (local-local-global) model in the literature because in its simplest form it consists of one global and two local bidders. As the SMRA corresponds to the standard ascending-price multi-unit auction for perfect substitutes, it follows that the exposure problem also occurs in standard multi-unit auctions. In an earlier work Krishna and Rosenthal (1996) demonstrate the full trade-off of the exposure problem in the LLG model for two simultaneous as well as two sequential single-unit Vickrey auctions.

Finally, Ausubel et al. (2014) observe that an environment in which small bidders face perfect substitutes in the SMRA resembles more closely the actual setting in spectrum auctions. As strong synergies among licenses are particularly likely in spectrum auctions, giving rise to an exacerbated exposure problem (Milgrom, 2000; Bykowsky et al., 2000), especially Bykowsky et al. (2000) and chakraborty et al. (1995) argue in favor of the use of combinatorial auction formats instead of the SMRA.

1.1.4 *General Combinatorial Auctions*

Combinatorial multi-object auction formats allow the submission of bids on any subset of objects auctioned and therefore eliminate the exposure problem by design. Here, the VCG mechanism stands out as it maximizes expected seller-revenue among all efficient, incentive compatible and individually rational combinatorial formats for allocating multiple objects (Krishna and Perry, 1999). Despite these theoretical merits the VCG auction may result in (un-competitively) low prices for the auctioneer if there are value synergies for at least one bidder (Milgrom, 2007; Ausubel and Milgrom, 2002). In this case revenue is not monotonically increasing in the submitted bids which has further negative effects. The auctioneer may raise revenue by excluding bidders from the auction (Day and Milgrom, 2008), buyers might be able to lower their prices by participating as or collaborating with fake bidders (Yokoo et al., 2004), and losing bidders can jointly deviate to become winning at very low prices. These disadvantages occur if the VCG mechanism selects an allocation that is not in the core (Bikhchandani and Ostroy, 2002). In the core objects and prices are assigned to bidders and the auctioneer, respectively, such that no coalition of buyers and the seller can do better by jointly deviating to a different assignment. The core is always non-empty in our standard auction model (Milgrom, 2007). The described outcomes are likely to be perceived as unfair as there are buyers willing to pay more for certain sets of objects than the respective winners.

Core-selecting combinatorial auctions modify the payment rule of the VCG mechanism to ensure the resulting complete information equilibrium allocation is in the core with respect to reported values as is done amongst others in Day and Raghavan (2007), Day and Milgrom (2008) and Day and Cramton (2012). Frequently applied core-selecting auction formats include the combinatorial FPSB auction (actually modeled as a FPSB menu auction in Bernheim and Whinston (1986)) and the combinatorial clock auction (CCA) (Cramton, 2013; Ausubel et al., 2006) which has been used by Ofcom (the UK communications regulator) to sell spectrum in the UK since 2006, for example. A combinatorial FPSB auction, although not incentive compatible and generally inefficient, by definition does not posses any of the disadvantages of the VCG mechanism mentioned above, because prices are monotonically increasing in bids. Moreover, in a complete information setting Bernheim and Whinston (1986) show that the FPSB mechanism always possesses equilibria in the core. Similarly, the CCA auction selects core allocations with respect to the submitted reports. It is a two-stage mechanism in which an open ascending combinatorial auction is followed by a bidder-optimal, core-selecting and Vickrey nearest (BCV) mechanism. A BCV auction results in a Pareto optimal core allocation for bidders with prices as close as possible to the original Vickrey payments (Goeree and Lien, 2016; Cramton, 2013).

Generally, in core-selecting auctions the VCG payment rule is manipulated (by disregarding general truthfulness) to assure that under complete information the resulting equilibrium allocation is in the core with respect to the reported valuations. However, under incomplete information truthful reporting might not be a Bayesian Nash equilibrium and the resulting allocation is not computed with respect to true values and therefore not necessarily in the true core. Goeree and Lien (2016) demonstrate under incomplete information that if the equilibrium allocation of any auction is in the core then it corresponds to the VCG outcome. Thus, if the VCG mechanism does not allocate within the core, no other auction format does and true core-selecting auctions do not exist. In this case little can yet be said about the exact outcome of (combinatorial) multi-object mechanisms. Moreover, under incomplete information core-selecting auctions suffer especially from the so-called threshold problem (Ausubel and Baranov, 2018; Sano, 2011; Goeree and Lien, 2016; Sano, 2012). In most combinatorial auctions, except the VCG mechanism, the threshold problem occurs if multiple local bidders, each demanding a different subset of a certain set of objects, compete against a global bidder who demands the entire set. For the small bidders to win, the subset bids have to exceed the large buyer's bid for the entire set. In this case the local bidders face the following trade-off: The higher any one buyer bids on his subset the more likely the small bidders win, but also the less payoff this particular bidder makes. Therefore, each local bidder might be tempted to free-ride on the other small buyers by bidding low on his demanded subset and expect the others to bid high enough and beat the global buyer. Early demonstrative accounts of the threshold problem can be found in Cramton (1995), chakraborty et al. (1995), Milgrom (2000) and Bykowsky et al. (2000). However, in particular chakraborty et al. (1995) and Bykowsky et al. (2000) argue that the threshold problem might be of minor practical concern in combinatorial auctions, and it does not occur for ex-ante symmetric bidders anyway.

Similarly, there is no free-rider problem in our model framework as the auctioned objects are perfect substitutes and all ex-ante symmetric bidders have decreasing marginal values in the number of units obtained. Moreover, in this case the allocation of the VCG mechanism is in the core, corresponds to the competitive equilibrium and yields market-clearing prices for the auctioneer (Bikhchandani and Ostroy, 2002). However, it is important to note that seller revenue might still be low due to colluding bidders in the VCG mechanism (Conitzer and Sandholm, 2006) and general equilibrium bidding behavior in other package auction formats is yet unknown.[4] Although revenue might not be the primary goal of the auction designer, it is regarded as a valuable indicator for the social welfare success of an auction. It can be offset by lower taxes and prevents the creation of rents and the related wasteful rent-seeking behavior as long as the auctioned objects are not sold below market value. Additional reasons why the VCG mechanism might not constitute the most suitable package auction format in practice include bidders reluctance to report their true valuations in fear of this information eventually being used against them in other transactions (Rothkopf et al., 1990; Rothkopf, 2007). In addition, although the mechanism may not be too computationally demanding, certainly the formation of its prices are cognitively harder to understand for participants. The above described scenario is exactly the starting point of our analysis: Our standard assumptions guarantee most theoretical advantages of the VCG mechanism which makes it a suitable benchmark and prevent the analytical complexity of discussing the threshold problem in other combinatorial formats. Here, it is of general interest to derive and compare equilibrium bidding behavior in package auction formats such as the FPSB and ascending package auctions in our standard setting. We hope this restricted environment then allows us to derive insights into format-specific

[4] Combinatorial auctions are also referred to as package auctions, in particular if multiple units of a homogeneous good are sold.

bidding peculiarities of different combinatorial multi-object auctions similar to the derivation of demand reduction (Ausubel et al., 2014) and the exposure problem (Goeree and Lien, 2014) in the SMRA, or the threshold problem in the CCA (Sano, 2012), for example.

1.2 CONTRIBUTIONS

The contribution of this thesis to the literature on combinatorial auctions is threefold. First, under our standard assumptions we examine the simplest multi-object market possible in which 2 firms compete for 2 units of a homogeneous good (perfect substitutes) and derive equilibrium bidding strategies for bidders with decreasing marginal values in the FPSB and an ascending combinatorial auction, based on which revenue and efficiency rankings are obtained. Second, in the same 2×2 market environment and for the same combinatorial auction formats we introduce a principal-agent model of bidding firms. In each firm a value-maximizing agent bids on behalf of a profit-maximizing principal and the latter determines budget constraints to restrict the former's bids. Third, we derive equilibrium bidding strategies as well as efficiency and revenue rankings for n firms with increasing marginal costs, competing in ex-post split-award (reverse) procurement auctions for either a 50% share or 100% of a contract. Finally, the theoretical predictions are empirically evaluated in laboratory experiments. These extensions are motivated by practitioners' indications of the prevalence of budget constrained bidding teams as well as agency problems in spectrum auctions, and the employment of multi-object auctions in industrial procurement such as (combinatorial) ex-post split-award auctions, respectively. Let us first motivate combinatorial auctions as the most advantageous form of multi-object auctions.

1.2.1 *Standard Package Auctions*

The results of this section are mainly established for "principals" in Chapter 3 and some insights into larger markets, different first-price auction formats and other possible equilibria are based on our procurement model in Chapter 4. The contributions described in this section are limited to our 2×2 environment from Chapter 3, in which 2 firms compete for 2 units of a homogeneous good (perfect substitutes) in a package auction. In Chapter 4 we then extend some of our contributions for first-price procurement auctions to markets with more bidders. We focus on XOR bidding languages (Nisan, 2000) which allow each bidder to submit one exclusive bid for the package of one unit and one bid for the package of two units. However, for each firm only one package bid can become winning and the auctioneer either sells the package of two units to one firm or a package of one unit to each firm. This restricted scenario captures the central strategic challenges that can also arise in larger markets and we provide corresponding theoretical examples in Chapter 4 and 5, and practical examples in Section 3.9. We primarily focus on the FPSB and an ascending package auction in our analysis due to their relevance in spectrum auctions.[5] The ascending format is a clock auction which, similar to the clock-stage of the CCA, allows for arbitrary package bids but does not consist of separate rounds as prices rise gradually. Our ascending package auction shall represent the most simple generalization of the single-object English auction and the standard ascending-price multi-unit auction to a combinatorial setting. Moreover, we decided to omit the core-selecting stage of the CCA given the negative results on core-selecting auctions by (Goeree and Lien, 2016) and in order to obtain more general bidding insights on ascending combinatorial auctions. We are limiting our attention to markets in which it is always efficient to have two winners (the dual-winner outcome) independent of the bidders' value draws. This efficiency environment has been motivated in a procurement setting by (Anton and Yao, 1992) and we refer to it as *dual-winner efficiency*. In the procurement context of Chapter 4 we use the original name of *dual source efficiency*. Note, that this assumption implies decreasing marginal valuations in the number of units obtained and especially, each buyer's highest possible marginal value for the second unit is below the lowest possible marginal valuation for the first unit. In the procurement setting this translates into an identical form of increasing marginal costs.

This limited setting actually allows us to derive Bayesian Nash equilibrium strategies as one can only analyze combinatorial auctions as Bayesian games in specific environments. The role of this model is similar to that of the stylized LLG model that is often used to analyze the exposure problem in the SMRA (Goeree and Lien, 2014) and the threshold problem in combinatorial auctions Sano (2012), for example. Finally, the described setting models an interesting market environment that is also relevant to business practice. For example, in a spectrum auction two bidders might be interested in multiple homogeneous licenses in a band and it is the efficient solution for bidders to split the spectrum. Remember, that regulators tend to be legally bound to prioritize efficiency.

Similar to (Anton and Yao, 1992) we assume prior information about the efficiency environment, i.e. *dual-winner efficiency*, and show that coordination on the efficient outcome constitutes a Bayesian Nash equilibrium (*dual-winner equilibrium*) for both bidders in the FPSB package auction. In this equilibrium both bidders pool their bids for one unit at the same price and submit equilibrium supporting bids on the package of two units independent of their actual package valuations. Under reasonable distributional assumptions this equilibrium payoff-dominates an entirely inefficient *single-winner equilibrium* in which both bidders use their unilateral "veto-power" to exclude the dual-winner outcome and solely aim for the package of two units. The latter equilibrium is employed as a reference to demonstrate that also inefficient equilibria exist under *dual-winner efficiency* and can be straightforwardly supported by bidders. To further keep the analysis traceable, we do not consider equilibria in which bidders aim for different allocations depending on their

[5] For example, France (2011) and Norway (2013) used a first-price sealed-bid package auction, whereas Romania (2012) used an ascending combinatorial clock auction: https://www.ofcom.org.uk/__data/assets/pdf_file/0021/74109/telefonica_response.pdf.

package valuations as have been examined in a procurement setting similar to that of Anton and Yao (1992) by Anton et al. (2010). Moreover, they simply correspond to a compound of *dual-winner* and *single-winner equilibrium* in which bidders with high values for the single-unit package and low values for the package of two units pool at a constant price for one unit in the dual-winner outcome. The other bidder types use their veto-power against such allocations and implement the single-winner outcome. Similar to the *single-winner equilibrium* such equilibria are inefficient and an unambiguous ranking in bidder payoff is not straightforward. However, in the procurement context of Chapter 4 these "hybrid" equilibria are briefly discussed for the FPSB package auction.

Contrary to (Anton and Yao, 1992), and to our procurement model, the proofs are independent of a publicly known efficiency parameter and involve two-dimensional package valuations. We then extend these results to the ascending (second-price) package auction format in which the ex-post *dual-winner equilibrium* always weakly dominates the ex-post *single-winner equilibrium* in payoff for bidders. In the former equilibrium each bidder does not become active on the package of two units but follows the strategy of remaining active on the single-unit package until the respective price reaches her valuation. As an immediate consequence there is no demand for the package, the auction terminates and each bidder receives one unit at a price of zero. The dynamic format of the ascending package auction possesses an additional coordination advantage over other sealed-bid package auctions. Here, buyers can adjust their bidding behavior and switch between equilibrium strategies depending on the observed opponent's bids. To the best of our knowledge this is the first theoretical work to demonstrate the advantage of an ascending package auction to coordinate on efficient allocations in an IPV auction model of incomplete information. The above stated results are further interpreted and integrated into the literature on multi-object auctions in much more detail in Section 2.3.4. There we find a weak form of outcome equivalence between the ascending package auction and standard second-price multi-unit auctions. The VCG mechanism is generally efficient and leads to higher seller prices than the other second-price multi-unit auctions. In its efficient equilibrium the FPSB package auction dominates the mentioned second-price auction mechanisms, including the VCG format, as well as the standard discriminatory multi-unit auction in seller revenue. Moreover, in its inefficient equilibrium it is payoff-equivalent to its ascending counterpart whereas the latter appears to be generally more efficient. Further combinatorial (ascending) first-price reverse procurement auctions are shown to solely posses efficient equilibria that are outcome-equivalent to the *dual-winner equilibrium* of the FPSB format in a procurement setting.[6].

The insights into combinatorial bidding in the 2×2 market under our standard assumptions depend to a large extend on each buyer's power to unilaterally veto the dual-winner outcome. For an increasing number of bidders this veto power is likely to vanish. In Chapter 4 we demonstrate within

[6] It is important to highlight that all Bayesian Nash equilibria derived in the procurement context in Chapter 4 are "linear" equilibria as specified in Section 1.2.3

the procurement context that for $n > 2$ bidders the FPSB package auction still possesses an efficient *dual-winner equilibrium* and an inefficient *single-winner equilibrium*. However, by the RET the former equilibrium is outcome equivalent to the unique *dual-winner equilibrium* of all other combinatorial reverse mechanisms, such as a descending-price (English) auction, as well as to all standard multi-unit auctions with single-unit demand from Section 2.3.2.

1.2.2 *Principal-Agent Relations in Package Auctions*

The contributions in this section are derived in Chapter 3. Agency problems between the board of directors and the management of a telecommunication corporation are commonly observed in spectrum auctions (chakraborty et al., 1995; Shapiro et al., 2013) but also occur in other auction markets (Engelbrecht-Wiggans, 1987). Schmidt (2004) argues that they are likely to cause inefficiencies in the resulting allocations. We introduce a principal–agent model of bidding firms within our 2×2 package auction setting from Section 1.2.1. Here, a principal who is uninformed about the firms package valuations provides an informed agent with budget constraints to either implement the *single-winner* or the *dual-winner equilibrium*. The inherent agency bias between profit-maximizing principal and value-maximizing agent predicts inefficient outcomes in multi-object auctions under *dual-winner efficiency* in which bidders inflate their demand to larger sets of objects instead to coordinate on efficient smaller sets.

Determining an optimal budget constraint that cannot be overbid by an informed agent corresponds to an optimal delegation problem. Within a single-object auction framework Burkett (2015) has to be given credit for introducing optimal delegation in principal-agent relationships. He showed how the fact that budget constraints are endogenously set by the principal to mitigate the agency problem does not affect the revenue comparisons between standard FPSB and Vickrey single-item auctions. Later, Burkett (2016) studies a principal's optimal choice of the budget constraint for an agent participating in an auction-like direct-revelation mechanism. Principal and agent are assumed to be equity holders in the firm, interested in maximizing the firm's expected return at the auction, but the bidder receives an additional private payoff when the firm wins the good. In contrast, in Chapter 3 we model a complementary environment with multiple objects in which agents are no equity holders. The types of manipulations possible for agents in such multi-object markets are quite different from single-object auctions. We show that the information asymmetry as well as the different preferences of principal and agent result in an agency dilemma that is difficult to resolve even in a symmetric information environment in which the principal is fully informed about the firm's valuations. Therefore, we also consider optimal contracts that consist of a menu of contingent budgets and transfer payments from uninformed principal to agent in the asymmetric information setting.

If two units are sold as a single package to both bidding principal-agent pairs in a single-package auction (1×2 market), there is no difference in the optimal contract between the FPSB and the ascending auction. This result is reminiscent of the findings in Burkett (2015, 2016) for an optimal delegation setting in a single-object market. The analysis indicates how to overcome the agency problem in case the principals wish to implement the *single-winner equilibrium* in the 2×2 market. Nevertheless, if the agency bias is too strong, an optimal contract does not exist in the asymmetric information environment. Moreover, the first finding contrasts with the difference in overcoming the principal-agent problem between the FPSB and the ascending package auction in the 2×2 market.

In the FPSB package auction the agency bias cannot even generally be solved in the symmetric information environment if principals try to implement the *dual-winner equilibrium*. Moreover, the same impossibility result holds in the asymmetric information setting if the agent's value-maximization motive is too strong. However, even if the agent's motive is sufficiently moderate, the distributional conditions that need to be met for the principal to prefer the *dual-winner equilibrium* to the *single-winner equilibrium* are harder to satisfy in comparison to the setting without agents in Section 3.6.1. Contrary, in the ascending package auction the agency problem is straightforward to overcome in the symmetric as well as in the asymmetric information setting if the principal's goal is to implement the *dual-winner equilibrium*. The second-price payment rule of the ascending package auction allows for an easy solution of the principal-agent problematic as is demonstrated by deriving outcome equivalence in solving the agency problem to the VCG mechanism in the same setting. Finally, by the same methods it is possible to solve the agency problem in the non-combinatorial SMRA and even extend our finding to larger markets. In summary, we find evidence in favor of (ascending) second-price auction formats in efficiently overcoming agency problems within bidding firms in mutli-object (package) auctions. Given the general coordination advantage for bidders in the ascending package auction within a standard IPV setting as described in Section 2.3.4, we now possess additional reasons to argue towards a use of this package auction format if the auctioneer's primary goal is to maximize social welfare.

1.2.3 *Procurement Package Auctions*

This section introduces the contributions of Chapter 4. Split-award auctions are often used for multi-sourcing in industrial procurement. Companies such as Sun Microsystems (Oracle Corporation) and HP Inc., for example, procure products worth hundreds of millions of dollars using different types of multiple sourcing auctions (Elmaghraby, 2007; Donohue et al., 2017). For risk considerations, such firms often want to have more than one supplier, especially in case the dual source solution is the cost-minimal outcome. In split-award auctions a procurement manager splits his demand for a larger quantity of a contract into two (or more) shares (aka. lots), such as a 30% and a 70% share or two 50% shares.

There are two classes of split-award auctions, which allow for different allocations. Ex-ante split-award auctions always implement an outcome with multiple winners, as no single supplier is allowed to win the 100% share only (Bichler et al., 2015; Chaturvedi et al.).[7] Hence, such formats are apt, when the procurement manager commits to multi-sourcing a priori, for example due to diseconomies of scale in the market. Contrary, ex-post split-award auctions decide endogenously, whether there are multiple winners or only one supplier wins the entire demand. Here, suppliers (bidders) can bid on all shares and the cost-minimizing allocation is selected by the revenue-maximizing buyer. Especially in settings in which the buyer is unsure about the exact scale economies in the market ex-post split-award auctions allow to determine the cost-minimal outcome for different environments. In this thesis we focus mainly on combinatorial ex-post split-award auctions and for the sake of simplicity refer to them as split-award auctions throughout. The ex-ante and the non-combinatorial (typically sequential) ex-post formats, in particular discussed in the conclusion in Chapter 5, are denoted explicitly.[8]

The majority of (reverse) procurement auctions are FPSB auctions (Bogaschewsky, 2015) but one can think of a number of possible first-price split-award auction formats combining sealed-bid and ascending-price (Dutch) mechanisms. We focus on the types of auction designs used by one of the largest European electronics and manufacturing multinationals, i.e. auctions with a total purchasing volume of between 250 thousand and 175 million Euros each, within one year (April 2015 to March 2016). We concentrate on first-price auctions only, as nearly all of the split-award auctions in the mentioned period were first-price auctions (FPSB and versions of a Dutch auction). Only a small share were descending-price (English) auctions, and the company never organized a VCG mechanism. There are two interesting observations from the empirical analysis which motivate this research.

1. About every third first-price auction was a split-award auction, most of which included two shares only. About 81% were ex-ante and 19% are ex-post split-award auctions.

2. Only 5% of the split-award auctions were organized as combinatorial auctions allowing bidders to submit a package bid. The majority was organized as ex-ante or non-combinatorial (sequential) ex-post split-award auctions.

Observation (i) shows the importance of split-award auctions in the procurement practices of the electronics multinational. Most of them involved only two predefined shares. The frequent application of ex-ante split-award auctions arises from the unwillingness of a buyer to allow single-winner outcomes, in case he wants to implement a dual sourcing strategy for sure. The motivation

[7] These auction formats are similar to standard multi-unit auctions in which each buyer demands one unit only.

[8] In these non-combinatorial (sequential) ex-post split-award auctions the winner of the first share(s) is also allowed to bid for subsequent shares, if he is interested in winning the package, which distinguishes them from standard multi-unit auctions with unit-demand (Demange et al., 1986).

is to keep up competition for future auctions or to have a second source in case the primary supplier defaults. Nevertheless, in about every fifth split-award auction the buyer of the electronics multinational delegated the decision about the sourcing strategy to the market mechanism by applying an ex-post format. The most surprising observation (ii), however, is that the majority of these ex-post split-award auctions did not allow for package bids, but were non-combinatorial auctions. Sequential split-award auctions were employed by the procurement managers, amongst others, in the hope of achieving lower prices in the second stage as competition is more transparent.

Nevertheless, we focus almost exclusively on the analysis of combinatorial ex-post split-award auctions in this thesis. One simple reason for the relatively rare use might be the still lacking understanding of equilibrium strategies as well as welfare and revenue properties of these mechanisms. Thus, our goal is to promote these universal procurement mechanisms by making them more accessible to procurement managers and by demonstrating their benefits. In particular, in the conclusion in Chapter 5 combinatorial ex-post split-award auctions are compared to their non-combinatorial (sequential) counterparts in an environment of economies of scale and decreasing marginal costs.

Our basis for the study of combinatorial ex-post split-award auctions is the work by Anton and Yao (1992) which provides a Bayesian Nash equilibrium analysis for a FPSB split-award auction in a market in which two suppliers compete for either a 50% share or 100% of a contract under diseconomies of scale. We extend this analysis to $n \geq 2$ bidders and different ascending first-price (Dutch) auction formats in Chapter 4 that are used in the field. The $2 \times n$ notation denotes n bidders competing for 2 homogeneous 50% shares of the sold contract. Most importantly, in this procurement context we study reverse auctions in which the suppliers submit bids to the auctioneer that represent prices at which the latter may purchase their services or products. The buyer then chooses his cost-minimizing allocation. The bidders incur costs of conducting the order or the purchase which are their private information. Within the independent private values model, the private values are simply replaced by private costs but for sake of simplicity it is nevertheless referred to as IPV model.

We are still focusing on a market in which it is always efficient to have two winners (the split outcome) independent of the bidders' costs for 50% or 100% of the contract, i.e. *dual source efficiency*. This efficiency setting implies increasing marginal costs in the size of the share purchased in which the lowest possible marginal cost for the second 50% share strictly exceeds the highest possible marginal cost for the first 50% share as well as strictly separated cost ranges for 50% or 100% of the order. In contrast to the 2×2 forward auction market, a bidder's costs are based on the one-dimensional private cost draw for 100% of the contract and the costs for the 50% share correspond to a constant fraction determined by an efficiency parameter. Be aware that this linear cost structure limits us to the derivation of linear equilibria, similar to Ausubel et al. (2014), but contrary to the 2×2 market, allows to analyze $n \geq 2$ bidding firms. We assume the *dual source efficiency* environment, i.e.

the efficiency parameter, is commonly known by all suppliers but not by the auctioneer. The latter assumptions helps modeling a scenario in which the auctioneer is unaware of the exact economies of scale in the market and thus cannot simply employ an ex-ante split-award auction to achieve the efficient allocation but relies on auction mechanisms that endogenously allocate social-welfare maximizing.

According to Anton and Yao (1992) it is well-known that for $n = 2$ bidders the FPSB split-award auction possesses efficient σ *equilibria* and an inefficient *winner-takes-all (WTA)* that correspond to the *dual-winner equilibria* and the *single-winner equilibrium* of Section 3.6.1, respectively. Moreover, we show that it also contains inefficient *hybrid equilibria* as defined in Anton et al. (2010). For $n > 2$ bidders the latter equilibrium ceases to exist and the σ *equilibrium* does not involve a pooling price anymore. Contrary, the Dutch split-award auctions only contain efficient σ *equilibria* independent of the number of bidders. If the σ *equilibrium* is the outcome of the FPSB auction then all considered split-award auctions are fully efficient and cost equivalent. Obviously, this result does not extend to the *WTA equilibrium* of the FPSB format.

In consecutive laboratory experiments we evaluate our theoretical results on the three combinatorial ex-post split-award auction formats. For $n = 2$ bidders we find that the Dutch auction on-average is more efficient than the FPSB format which, nonetheless, is cheaper for the buyer. A (compound) Dutch-FPSB mechanism combines the merits of both formats because it is as efficient as the Dutch auction and as cheap as the FPSB auction. For $n = 3$ bidders all differences between the three auction formats vanish and they are fully efficient. Not surprisingly, the increased competition causes a sharp drop in procurement costs for the auctioneer as a relatively high pooling price cannot be maintained by the bidders anymore.

1.3 OUTLINE OF THE THESIS

This thesis consists of five main chapters in which the first chapter introduced the topic of bidding behavior in combinatorial auctions as well as the contributions, the second chapter sets the formal stage and updates the reader on recent relevant findings, the third and fourth chapters contain applications on optimal budget constraints and procurement auctions, respectively, and the fifth chapter concludes with an outlook. All contributions are mainly based on the three works Bichler and Paulsen (2018), Kokott et al. (2018a) and Kokott et al. (2018b), each of which was co-authored by the author of the thesis.

The introduction in the present Chapter 1 motivated the study of optimal bidding behavior in combinatorial auctions by outlining the unique properties of auctions as optimal single-object market mechanisms and successively depicting the shortcoming of non-combinatorial auction formats in multi-object markets. Finally, the detailed evaluation of combinatorial auctions, including the contributions of this thesis, constitutes the completion of valid policy advice on optimal multi-object sales mechanisms. Chapter 2 contains definitions of all game- and contract-theoretic concepts made use of to establish the results of this thesis. Moreover, it formalizes the standard Bayesian IPV framework in which our contributions are derived and compares them to the existing literature on single- and multi-object auctions within the same environment. In Chapter 3 we introduce a principal-agent model of bidding firms into our model of equilibrium bidding behavior in combinatorial auctions and investigate the effect of optimal budget constraints on revenue and efficiency comparisons. All proofs of this chapter and detailed descriptions of the discussed auction formats can be found in Appendix A. The chapter is based on Bichler and Paulsen (2018). Another application, following Kokott et al. (2018a), is contained in Chapter 4 in which we examine the ranking of first-price ex-post split-award auctions in our standard model. These are reverse combinatorial procurement auctions employed with increasing frequency in the industry. Our theoretical findings are evaluated with an empirical analysis of laboratory experiments. The relevant proofs and elaborate statistical summaries of the experimental evaluation are provided in Appendix B. Finally, Chapter 5 summarizes the contributions of this thesis, presents potential shortcomings as well as their remedies and extensions as in Kokott et al. (2018b), and points towards complementary future research.

2

THEORETICAL BACKGROUND

In this chapter we will first provide an introduction into the concepts of game theory that allow us to analyze combinatorial auctions strategically. Second, we formulate auctions as games of incomplete information in our standard IPV framework with risk-neutral and ex-ante symmetric bidders. Third, we present the basics of hidden information principal-agent relationships used to model agency problems in bidding firms in a contract theory setting. Finally, an overview of the most important results in auction theory within the specified IPV environment is provided.

2.1 BAYESIAN GAME THEORY

In this section we define all game-theoretic concepts that are employed to establish our theoretical contributions in Chapters 3 to 5. The summary is mainly based on Mas-Colell et al. (1995), Leyton-Brown and Shoham (2008) and Fudenberg and Tirole (1991). Game theory analyzes decision situations (games) in which the outcome depends on the decisions of several interacting self-interested decision makers, i.e., for each decision maker the outcome (payoff) depends on her own decisions and those of the other players. In a standard single-object auction, for example, each player submits a bid and any one player only wins the object if she outbids all the other decision makers. In the game theoretic analysis of such decision situations it is generally assumed that each subject behaves rationally, i.e., within a given environment each player aims at maximizing utility and does not succumb to any intellectual restrictions. This fundamental assumption can be interpreted normatively and positively. In the first interpretation it is not claimed that all decisions makers actually behave fully rational in a real world decision situation but the assumption demonstrates the rational outcome which then might serve as a guideline. The positive interpretation argues that at least in the long run only rational behavior will be enforceable in practice. In Chapters 4 to 5 we confirm the positive interpretation experimentally in a procurement context.

As is convention in the game-theoretic analysis of auctions, we restrict our attention to non-cooperative games as opposed to cooperative game theory. In the former, binding cooperative agreements between subgroups of self-interested decision makers (enforced by an external authority) are not possible and thus, only self-enforcing alliances (coalitions) may result in any game. Moreover, we focus on games of incomplete information in which each player is not completely informed about the structure of the game and, in particular, assume that she does not know her opponents' exact payoff functions. Alternatively, incomplete information may refer/extend to the opponents' strategy or information spaces. In his benchmark works Harsanyi (1967a,b,c) proposes an approach that allows to elegantly model all possible forms of incomplete information. Here, each player's entire "private" information (not known to the other players) is summarized in her "type" which is drawn "by nature" from a commonly known probability distribution function prior to the start of the game. Each player's payoff function then depends not only on her strategy choices and those of the opponents but also on her own type. As each opponent's strategy choice depends on his private type, every player in the game has to build beliefs about the other players' types given her own type draw. As these beliefs correspond to conditional probability distributions derived from the common prior distribution via Bayes' rule, games of incomplete information are referred to as Bayesian games and the corresponding analysis is called Bayesian game Theory. Analyzing auctions game-theoretically in a complete information context would prohibit to capture and outline the essential strategic trade-offs present in real-world applications Engelbrecht-Wiggans (1980).

In the next two sections we stick to convention and discuss static and dynamic games of incomplete information separately. In static games, also called simultaneous move games, such as (closed) sealed-bid auction formats for example, the players move (make their decisions) simultaneously without knowing their opponents' choices. In dynamic games, like (open) ascending- or descending-price auction formats, players move sequentially and at each stage of the game some of the opponents' prior moves might have been observed. Finally, we introduce the reader to a formal game-theoretic definition of the standard IPV model.

2.1.1 *Static Games of Incomplete Information*

We begin with a definition of all components of a Bayesian game, its normal (strategic) form, which consecutively allows us to define strategies and various equilibrium concepts.

Definition I. *(Leyton-Brown and Shoham, 2008, Definition 7.1.2) (Normal form of a Bayesian game): The normal form of a Bayesian game* $\mathbb{G} = \{I, S, U, T, F\}$ *specifies:*

- *the set of players* $I = \{1, ..., n\}$ *in which any single player is denoted by* $i \in I$*;*

- *the strategy space* $S = S_1 \times ... \times S_n$ *in which* $S_i \subset S$ *is the space of strategies available to player i and possibly includes mixed strategies (statistical distributions over the set of all deterministic strategies);*

- *the space of utility functions* $U = U_1 \times ... \times U_n$ *in which* $U_i \subset U$ *is the utility function of player i;*

- *the type space $T = T_1 \times ... \times T_n$ in which $T_i \subset T$ is the type space of player i;*
- *and the common prior probability distribution of types $F(\cdot)$.*

The information in the normal form game $\mathbb{G}$ is common knowledge among all players except each player i's type, $t_i \in T_i$, which is private information to the corresponding agent. Before the game starts, each player is assigned (by "nature") this, potentially multi-dimensional, private type such that the vector of all types corresponds to $t = (t_1, ..., t_n) \in T$. The commonly known joint cumulative distribution function of types $t \in T$ is $F(t)$ with mapping $F : T \to [0, 1]$ and marginal distribution $F_i(t_i)$ with mapping $F_i : T_i \to [0, 1]$ for any player i's type, $t_i \in T_i$. Let us next define the type draws as $t = (t_i, t_{-i})$ in which $t_{-i} \in T_{-i}$ represents the types of all players other than i. Naturally, $T = (T_i, T_{-i})$ and, given player i's type draw $t_i \in T_i$, the ex-interim conditional distribution of the other players' types $F(t_{-i}|t_i) = F(t) / F_i(t_i)$ can be computed. Now, we are able to define each agent i's strategy space S_i. Player i chooses a strategy $s_i(t_i) \in S_i$ which is a mapping of the form $s_i : T_i \to S_i$. Let the set of strategies chosen by all players be defined as $s = (s_i, s_{-i})$ in which $s_{-i} \in S_{-i}$ corresponds to the subset of strategies chosen by all players other than i. Also, $s \in S = (S_i, S_{-i})$. Although the strategy space S allows for mixed strategies, any strategy $s_i(t_i) \in S_i$ may also denote a pure (deterministic) strategy as it simply corresponds to a special case of a mixed strategy (probability distribution over all possible deterministic strategies) in which the statistical distribution is degenerate. Finally, any player i's von Neumann-Morgenstern utility function (von Neumann and Morgenstern, 1944) maps from the strategy and type spaces into utility payoff such that for given type draws $t \in T$ and given set of strategies employed $s \in S$ the utility is denoted as $U_i = u_i(s, t)$ in which $u_i : S \times T \to \mathbb{R}$. The above definitions allow us to introduce three different forms of expected utility that will facilitate the exposition of solution concepts to the normal form Bayesian game $\mathbb{G}$.

Definition II. *(Leyton-Brown and Shoham, 2008, Definition 7.2.1) (Ex-Post Expected Utility): In a Bayesian game $\mathbb{G}$ in which the players' actual type draws $(t_i, t_{-i}) \in T$ are commonly known by all players and the unknown set of strategies employed is given by $(s_i, s_{-i}) \in S$, agent i's ex-post expected utility is defined as*

$$EU_i(s_i, s_{-i}, t_i, t_{-i}) = E_{s_{-i} \in S_{-i}}\Big(u_i(s_i, s_{-i}, t_i, t_{-i}) \qquad \text{(EXP)}$$

Note that, although the actual vector of all players' type draws t is known, there is uncertainty about ex-post utility from agent i's point of view as she does not know which set of potentially mixed strategies $s_{-i} \in S_{-i}$ are played by the other players. Next, suppose the vector of type draws $t \in T$ is not commonly known (as is standard in any Bayesian game $\mathbb{G}$) but each player i's actual type draw t_i is private information.

Definition III. *(Leyton-Brown and Shoham, 2008, Definition 7.2.2) (Ex-Interim Expected Utility): In a Bayesian game $\mathbb{G}$ in which player i's type draw is given by t_i and the agents' set of strategies by $(s_i, s_{-i}) \in S$, agent i's ex-interim expected utility is defined as*

$$EU_i\left(s_i, s_{-i}, t_i; F\left(t_{-i}|t_i\right)\right) = E_{t_{-i}\in T_{-i}}\left(EU_i\left(s_i, s_{-i}, t_i, t_{-i}\right), F\left(t_{-i}|t_i\right)\right). \quad \text{(EXI)}$$

The ex-interim expected utility of agent i with type draw t_i corresponds to the expected ex-post expected utility in Definition II over all possible type draws $t_{-i} \in T_{-i}$ of the other players build with the conditional common prior $F\left(t_{-i}|t_i\right)$. In the final expected utility definition we assume that no agent i does yet known his actual type draw $t_i \in T_i$.

Definition IV. *(Leyton-Brown and Shoham, 2008, Definition 7.2.3) (Ex-Ante Expected Utility): In a Bayesian game $\mathbb{G}$ in which the agents' set of strategies is given by $(s_i, s_{-i}) \in S$, agent i's ex-ante expected utility is defined as*

$$EU_i\left(s_i, s_{-i}; F(t)\right) = E_{t_i\in T_i}\left(EU_i\left(s_i, s_{-i}, t_i; F\left(t_{-i}|t_i\right), F_i\left(t_i\right)\right)\right). \quad \text{(EXA)}$$

Agent i's ex-ante expected utility equals the expected ex-interim expected utility from equation (EXI) in Definition III over all her possible type draws $t_i \in T_i$ based on the marginal common prior $F_i\left(t_i\right)$.[9] In the next step we can now define an agent i's best response.

Definition V. *(Leyton-Brown and Shoham, 2008, Definition 7.2.4) (Best Response): In a Bayesian game $\mathbb{G}$ player i's best response $s_i \in S_i$ to the other agents' set of strategies $s_{-i} \in S_{-i}$ is defined by*

$$BR_i\left(s_{-i}\right) = \underset{s'_i\in S_i}{argmax}\, EU_i\left(s'_i, s_{-i}; F(t)\right). \quad \text{(BR)}$$

Any strategy from agent i's set of best responses $s_i \in BR_i \subseteq S_i$ maximizes her ex-ante expected utility from equation (EXA) in Definition IV over all possible strategies $s'_i \in S_i$ given the other players employ strategy profile $s_{-i} \in S_{-i}$. Be aware that agent i's set of best responses in equation (BR) also maximizes his ex-interim expected utility. This becomes clear by considering equation (EXA) in Definition IV and noticing that the expression for ex-interim expected utility conditioned on type t_i does not depend on strategies that i would chose if he were not of that type. Thus, by maximizing ex-ante expected utility any element of BR_i is in fact also maximizing ex-interim expected utility conditioned on each possible type $t_i \in T_i$. Definition V enables us to present the most general solution concept for a Bayesian game.

[9] Alternatively, ex-ante expected utility can be defined as corresponding to expected ex-post expected utility from Definition (II) over all possible type draws $t \in T$ based on the common prior $F(t)$, i.e.,

$$EU_i\left(s_i, s_{-i}; F(t)\right) = E_{t\in T}\left(EU_i\left(s_i, s_{-i}, t_i, t_{-i}\right), F(t)\right).$$

Definition VI. *(Leyton-Brown and Shoham, 2008, Definition 7.2.5) (Bayesian Nash Equilibrium): A set of best responses* (s_i, s_{-i}) *that satisfies* $s_i \in BR_i(s_{-i})$ *from Definition V for all* $i \in I$ *constitutes a Bayesian Nash equilibrium of the Bayesian game* $\mathbb{G}$.

A Bayesian Nash equilibrium consists of a profile of mutually best responses defined via ex-ante expected utility. The next notion of equilibrium represents a special case of a Bayesian Nash equilibrium.

Definition VII. *(Leyton-Brown and Shoham, 2008, Definition 7.4.1) (Ex-Post Equilibrium): An ex-post equilibrium of a Bayesian game* $\mathbb{G}$ *is identified by a strategy set* (s_i, s_{-i}) *that solved the following equality for all* $t \in T$ *and all* $i \in I$

$$s_i = \underset{s_i' \in S_i}{argmax}\, EU_i\left(s_i', s_{-i}, t\right). \tag{EXPEQU}$$

Intuitively, an ex-post equilibrium comprises a set of mutually best responses defined for ex-post expected utility. In this equilibrium no agent would want to deviate from the equilibrium strategy even if knowing all players' types. Thus, this equilibrium is stronger than a Bayesian Nash equilibrium in the sense of being robust against perturbations in the common prior $F(t)$. An ex-post equilibrium does not require any agent to believe that the other players possess precise beliefs about his type. Before introducing the final solution concept, let us first demonstrate how each player i's strategy space S_i can be reduced "rationally". Suppose, any player i's strategies that are never a best response, in the sense of Definition V, are removed iteratively from her set of strategies S_i. We then end up with the following subset.

Definition VIII. *(Mas-Colell et al., 1995, Definition 8.C.2) (Set of Rationalizable Strategies): For any player i in a Bayesian game* $\mathbb{G}$*, the strategies that survive the iterated elimination of strategies that are never a best response constitute her set of rationalizable strategies.*

A "rational" player should never employ a strategy which does not belong to the set of rationalizable strategies. Special instances of this set are strictly dominant strategies and the set of strategies that survive iterated elimination of strictly dominated strategies.

Definition IX. *(Mas-Colell et al., 1995, Definition 8.B.1) (Dominant Strategy): In a Bayesian game* $\mathbb{G}$ *player i's strategy* $s_i \in S_i$ *is dominant iff, for all possible type draws* $t \in T$ *and all strategy profiles of the other players* $s_{-i} \in S_{-i}$,

$$s_i = \underset{s_i' \in S_i}{argmax}\, EU_i\left(s_i', s_{-i}, t\right). \tag{DTS}$$

A dominant strategy is a unique best response (based on ex-post expected utility from Definition II) against all other players' strategy profiles $s_{-i} \in S_{-i}$ for all possible type draws $t \in T$. Closely related to the concept of dominant strategies are dominated strategies.

Definition X. *(Mas-Colell et al., 1995, Definition 8.B.2) (Dominated Strategy): Player i's strategy $s_i' \in S_i$ is dominated in a Bayesian game $\mathbb{G}$ iff there exists another strategy $s_i \in S_i$ with $s_i' \neq s_i$ such that for all possible type draws $t \in T$ and all strategy profiles of the other players $s_{-i} \in S_{-i}$,*

$$EU_i\left(s_i, s_{-i}, t\right) \geq EU_i\left(s_i', s_{-i}, t\right). \tag{DDS}$$

Here, strategy s_i also dominates s_i'. Moreover, a strategy s_i that dominates every other strategy $s_i' \in S_i$ is a dominant strategy. By iteratively deleting player i's strictly dominated strategies from her strategy space S_i in a Bayesian game $\mathbb{G}$, we end up with the set of strategies that survive iterated elimination of strictly dominated strategies. Strategies within this set cannot be strictly dominated and always the same strategies will result independent of the order of deletion. The same is not true for iterated elimination of weakly dominated strategies. Moreover, the set of strategies surviving iterated elimination of strictly dominated strategies cannot be smaller than the set of rationalizable strategies. Based on Definition IX, we can present the last solution concept.

Definition XI. *(Equilibrium in Dominant Strategies): An equilibrium in dominant strategies of a Bayesian game $\mathbb{G}$ occurs if each player $i \in I$ employs a dominant strategy according to Definition IX.*

An equilibrium in dominant strategies is the strongest possible solution concept to any game as it does not require a player to build beliefs and expectations about his opponents' types and strategies. Moreover, an agent does not have to believe the other players to act rationally. Although every equilibrium in dominant strategies is a Bayesian Nash equilibrium, there is no one-to-one relationship between dominant strategy and ex-post equilibria. Moreover, both equilibria are rather unlikely to exist in any Bayesian game $\mathbb{G}$. The general existence of any Bayesian Nash equilibrium is an immediate consequence of the existence of a Nash equilibrium in a game of complete information. The relevant existence conditions are summarized in the following definition.

Definition XII. *(Mas-Colell et al., 1995, Proposition 8.D.3) (Existence of Bayesian Nash Equilibrium): In a Bayesian game $\mathbb{G}$ a Bayesian Nash equilibrium exists in pure strategies if for all players $i \in I$,*

- *the strategy space S_i is a nonempty, convex and compact subset of some Euclidean space $\mathbb{R}^n$;*
- *and the utility function U_i is continuous in $s \in S$ and quasiconcave in $s_i \in S_i$.*

Definition XII is satisfied in all our settings as will be demonstrated in Sections 3.3 and 4.3. Let us continue with the discussion of dynamic games of incomplete information in the next section.

2.1.2 *Dynamic Games of Incomplete Information*

Similar to a static game from Section 2.1.1, a dynamic game of incomplete information can be described by its normal form $\mathbb{G}$ from Definition I. Thus, Bayesian Nash equilibria, as defined in Definition VI, are computable for dynamic games in the same way as for static games of incomplete information. However, per definition of the normal form, these equilibria do not take into account the sequential structure (time and information) of the dynamic game and therefore, might be based on incredible strategies. Strategies are incredible if they contain empty threats of the form that player i specifies to make a certain move at a specific stage of the game (threat) which, however, is not rational to carry out in case this stage and information set is actually reached. To rule out such incredible behavior we require each strategy to comply with the concept of sequential rationality. Sequentially rational strategies specify optimal behavior from any point in the actual sequential structure of a dynamic game onwards. The current section introduces an equilibrium refinement that is based on sequentially rational strategies, though not in full formality, taking into account the actual sequential time- and information-structure of a dynamic game of incomplete information.

Nature first draws all players' types and consecutively they participate in a continuation game (dynamic subgame of the actual game). Note that as each agent is uninformed about the other players' type draws, she does not know in which exact continuation game she finds herself and has to build corresponding believes based on the common prior $F(t)$. However, in the variety of dynamic games of incomplete information studied in this thesis, in each continuation game the sequential moves by all players are fully visible and any player i can iteratively update her beliefs accordingly. In this scenario sequential rationality requires strategies to be optimal in every continuation game of the original game given the respective beliefs. Based on these concepts we can now introduce an equilibrium refinement of a Bayesian Nash equilibrium that satisfies sequential rationality given a system of consistent beliefs.

Definition XIII. *(Mas-Colell et al., 1995, Definition 9.C.3) (Perfect Bayesian Equilibrium): A Perfect Bayesian equilibrium of a dynamic game of incomplete information consists of a profile of strategies and beliefs such that,*

- *all players' strategies are sequentially rational given the system of beliefs;*
- *and the beliefs are derived from the players equilibrium strategies with Bayes' Rule whenever possible.*

It is important to note that every Perfect Bayesian equilibrium of a dynamic game of incomplete information is also a Bayesian Nash equilibrium of the same game. However, obtaining existence results is more challenging and we have to content ourselves with the following observation.

Definition XIV. *(Fudenberg and Tirole, 1991, Theorem 8.5) (Existence of Perfect Bayesian Equilibrium): In a finite dynamic game of incomplete information a Perfect Bayesian equilibrium exists (possibly in mixed strategies).*

There are even further equilibrium refinements, such as Sequential and Perfect equilibria, that put additional requirements on consistency of beliefs off the equilibrium path. Nevertheless, these equilibrium concepts place restrictions beyond the needs of this thesis and thus, we continue with the definition of our standard IPV model as a Bayesian game.

2.1.3 *Standard IPV Auction Model*

In this section we derive a general IPV normal form for (forward) auction games, in which a single object is sold by a risk-neutral and revenue maximizing auctioneer to multiple ex-ante symmetric and risk-neutral buyers, based on Krishna (2002) and the survey of Engelbrecht-Wiggans (1980). This framework has been introduced to analyze auctions as strategic games of incomplete information by Vickrey (1961) and then generalized by Riley and Samuleson (1981). Let us consider each component of the normal form game $\mathbb{G} = \{I, S, U, T, F\}$ from Definition I in Section 2.1.1 successively. In an auction game the set of players I corresponds to the fixed set of bidders participating in the auction. Any bidder i's strategy space defines the range of allowed bids for the single object in the auction and we determine $S_i = [0, \overline{s}] \subseteq \mathbb{R}_{\geq 0}$ with upper bound $\overline{s} \in (0, +\infty)$. Next, we assume identical non-empty value spaces for all bidders such that $T_i = \left[\underline{t}, \bar{t}\right]$ with $\underline{t}, \bar{t} \in (0, +\infty)$, $\underline{t} < \bar{t}$ and $E(t_i) < +\infty$ for all $i \in I$. Moreover, each bidder i's private valuation t_i is drawn independently from an identical monotonically increasing cumulative distribution function $F(t_i)$ according to the mapping $F : \left[\underline{t}, \bar{t}\right] \to [0, 1]$. In this setting with independent values, the conditional distribution then corresponds to $F(t_{-i}|t_i) = \prod_{t_j \in T_{-i}} F\left(t_j\right)$ and ex-interim expected utility can be denoted as $EU_i\left(s_i, s_{-i}, t_i; \prod_{t_j \in T_{-i}} F\left(t_j\right)\right)$. We assume that $F(t_i)$ admits a continuous and strictly positive density function $f(t_i)$ on the full support for all $i \in I$. The last two assumptions ensure ex-ante symmetric bidders. Finally, buyer i's strategy $s_i(t_i) \in [0, \overline{s}]$ determines the height of his submitted bid as a function of his valuation space $s_i : \left[\underline{t}, \bar{t}\right] \to [0, \overline{s}]$.

We focus on a quasilinear von Neumann-Morgenstern utility function of the form $U_i = t_i - p_i$. Utility equals the difference (payoff) between a bidder i's type $t_i \in T_i$, his value (maximum willingness to pay) for the object, and p_i, the price paid if the object is won. If bidder i does not obtain the item, her reservation utility is normalized to be zero. Note, that the final price of the object $p_i = p(s_i, s_{-i})$ depends on the pricing rule of the auction format used based on the bids submitted by all buyers. We assume the pricing function to be continuous in the submitted bids, identical for all buyers and to never charge a bidder more than her bid. The assumptions are captured in the mapping of $p : [0, \overline{s}]^n \to [0, \overline{s}]$ with $p_i \leq s_i$. Based on the above assumptions a bidder i's utility can be expressed as a function of her type draw and all chosen strategies such that $U_i = t_i - p(s_i, s_{-i})$, and the quasilinear specification has three fundamental implications. First, it implies risk-neutrality, i.e., for each bidder the expected utility equals the utility of the expected payoff. Second, buyers maximize expected profit of participating in the auction. Third, it implies independence of buyers' valuations in the sense that the value any bidder i assigns to the object does not depend on the height of any other buyer's willingness to pay. Within the specified environment the strategy space $S_i = [0, \bar{s}] \subseteq \mathbb{R}_{\geq 0}$ is a nonempty, convex and compact subset of an Euclidean space. Moreover, the quasilinear utility function U_i is per definition continuous in $s \in S$ and quasiconcave in $s_i \in S_i$. Therefore, according to Definition XII, Bayesian Nash equilibria are guaranteed to exist in pure strategies in our standard IPV setting with risk-neutral and ex-ante symmetric bidders. Also, if a finite dynamic game of incomplete information is analyzed within this setting, Perfect Bayesian equilibria (possibly in mixed strategies) exist with certainty as stated in Definition XIV.

Finally, there are numerous ways to extend the above model to a multi-object setting. Suppose for example each buyer demands one item out of a set of auctioned objects. In this case a bidder i's type draw t_i determines his value for the item demanded and the above specification can be transferred almost one to one. See the works by Vickrey (1961), Ortega-Reichert (1968), Harris and Raviv (1981), Weber (1983) and Milgrom (1985), for example, for applications of this setting to standard multi-unit and sequential auctions (perfect substitutes). Contrary, if a buyer demands more than one item she possesses different marginal valuations for the different objects that might be related with each other via a specific functional form such as satisfying certain scale economies. These valuations need to be taken into account in any bidder's strategy space. For example, each bidder i's type draw t_i might correspond to a multi-dimensional vector and a strategy has to specify multi-dimensional decisions based on these multi-valued type draws. Versions of our standard model are implemented for standard multi-unit auctions with multi-unit demand by Noussair (1995), Engelbrecht-Wiggans and Kahn (1998a), Engelbrecht-Wiggans and Kahn (1998b) and Ausubel et al. (2014). The implementation of the described IPV model in our combinatorial auction setting is carried out in Sections 3.3 and 4.3. Note, that the more complex the auction formats, the more challenging becomes the application of our standard model and an unambiguous game-theoretic analysis of auctions as Bayesian games. In some of these settings the proof of existence of Bayesian Nash equilibria might be much more complicated and needs to be established by case.

Before reviewing the most important findings for single- and multi-object auctions within the above defined standard setting in the literature, let us next provide the reader with a brief introduction into contract theory that sets the stage for our analysis of agency problems in bidding firms in Chapter 3.

2.2 CONTRACT THEORY

The introduction to contract theory in this section is largely based on Mas-Colell et al. (1995), Fudenberg and Tirole (1991) and Dewatripont and Bolton (2005). The rather informal excursion into optimal delegation follows Alonso and Matouschek (2008), Holmström (1977) and Holmstrom (1984). Contract theory is concerned with designing incentive schemes (contracts) in strategic situations in which groups of individuals may maximize social welfare if they cooperate but some members of the group have an incentive to deviate. The contract is written in order to prevent or minimize this deviating behavior. Only environments with asymmetric information between members of the group, and between the group and an all-enforcing external authority are relevant to contract theory. Without these informational asymmetries, a complete contingent contract that controls the behavior of each individual by severely punishing deviating moves could be implemented. However, in situations of asymmetric information or in settings in which a complete contingent contract cannot be written it is often impossible to implement the social welfare maximizing outcome. Here, contract theory aims at specifying an incentive system that enforces the ”second best” outcome which achieves social welfare as close as possible to full efficiency. Within a company, for example, the management might have a misguided incentive to maximize short run revenue or profit in order to obtain as large bonus payments as possible whereas the board of directors would like the corporation to operate efficiently, sustainable and achieve long run profit goals. Therefore, the company has to enter an employment contract with each member of the management that sets the right incentives from the firm’s point of view. However, as the management is usually much better informed about the company’s operations and performance potentials than the board, the latter party is unable to write a complete contingent contract that implements the desired behavior precisely. Approximating incentive schemes have to be installed instead.

Contrary to game theory, that takes the decision situation as given and derives the optimal strategic behavior within, contract theory tries to alter the strategic environment in order to achieve a desired outcome. Intuitively, contract theory relies on many concepts from game theory but has a different perspective. Subdisciplines include mechanism design, principal-agent theory, new institutional economics and so on. Therefore, most concepts introduced in this section are also relevant in applications of mechanism design to auction theory, such as the use of the revelation principle in the design of the optimal single-object auction mechanism (Myerson, 1981) or efficient generalized VCG mechanism (Vickrey, 1961; Clarke, 1971; Groves, 1973). In this thesis we focus on a certain class of contractual problem within the principal-agent theory, adverse selection, defined by information asymmetries between a fully informed agent and a non-informed principal that arise before a contract is negotiated. The agent is supposed to act on behalf of the principal’s interest but has an incentive to deviate from the allocation that maximizes both parties’ welfare. Therefore, the principal must design an optimal contingent contract that aligns the incentives of the agent and implements the ”second best” outcome. Applications include the study of how to regulate the pricing policy of a monopoly with unknown costs in order to maximize social welfare (Baron and Myerson, 1982), optimal monopolistic price discrimination (Maskin and Riley, 1984) and famously optimal taxation (Mirrlees, 1971), for example. Amador and Bagwell (2013) examine trade agreements between governments in an optimal delegation framework. Let us now formulate a general adverse selection principal-agent relationship that closely resembles a Bayesian game of normal form $\mathbb{G} = \{I, S, U, T, F\}$ from Definition I in Section 2.1.1. Throughout this section we refer to these strategic situations as Bayesian principal-agent relationships. Consecutively, we formulate the principal's optimization problem and derive a general form of the optimal "second-best" contract.

The set of players corresponds to $I = \{P, A\}$ in which $i = P$ and $i = A$ denote the principal and agent, respectively. Note that the principal does not have any private information but the agent's private type is $t \in T = \left[\underline{t}, \bar{t}\right]$. The type is drawn from the monotonically increasing cumulative distribution function $F(t)$ with mapping $F : \left[\underline{t}, \bar{t}\right] \to [0, 1]$, and continuous as well as strictly positive density function $f(t)$ on full support. After the contract is negotiated the agent chooses a strategy $s(t) \in S = [0, \overline{s}] \subseteq \mathbb{R}_{\geq 0}$ in which the upper bound satisfies $\overline{s} \in (0, +\infty)$. The strategy is a mapping of the form $s : T \to S$. Finally, the quasilinear von Neumann-Morgenstern utility functions of principal and agent for a given type draw t and a chosen strategy s are $U_P = u_P(s, t) - r$ and $U_A = u_A(s, t) + r$, respectively. Here, the mapping is $u_i : S \times T \to \mathbb{R}$ for all $i \in I = \{P, A\}$, and $r \in \mathbb{R}$ represents a transfer payment from principal to agent as specified in the contract. The transfer $r(t)$ is modeled as a function of the form $r : T \to \mathbb{R}$. Note that the form of the utility function implies risk-neutral and profit-maximizing principal and agent. We also assume that the principal can fully commit to the terms of the contract.

2.2.1 *Mechanism Design*

Given the described information asymmetry, the general structure of the uninformed principal's optimal contract is determined by the Revelation Principle.

Definition XV. *(Mas-Colell et al., 1995, Proposition 14.C.6) (Revelation Principle): In a Bayesian principal-agent relationship, an allocation which assigns to every possible agent's type $t \in T$ a menu of strategy and transfer $\left(s(t), r(t)\right)$, can be implemented by any mechanism iff it can be implemented by a (direct) revelation mechanism.*

Thus, any allocation in a Bayesian principal-agent relationship can be implemented by a direct revelation mechanism. In a (direct) revelation mechanism of a Bayesian principal-agent relationship, for every possible of the agent's reported type $\hat{t} \in T$ a set of actual strategy and transfer $\left(s\left(\hat{t}\right), r\left(\hat{t}\right)\right)$ is assigned. Additionally, the revelation mechanism must be incentive compatible which implies that it is optimal for the agent to truthfully report his actual type draw $\hat{t} = t$ for all $t \in T$. From the Revelation Principle it follows that we can focus on (direct) revelation contracts $\left(s(t), r(t)\right)$ to solve the principal's second best optimization problem.

2.2.2 *Second-Best Problem*

This section is based on Section 3.3.3 in Dewatripont and Bolton (2005). To ensure the existence of an optimal contract as specified above, the following additional assumptions need to be made.

Definition XVI. *(Assumptions for Existence of Optimal Contract): The existence of an optimal "first best" and "second best" contract in the general adverse selection principal-agent problem requires the following assumptions:*

- *A1: The utility function u_A is monotonic in t,* $\frac{\partial u_A(s,t)}{\partial t} > 0$.
- *A2: The "single-crossing property" is satisfied for utility function u_A,* $\frac{\partial^2 u_A(s,t)}{\partial s \partial t} > 0$.
- *A3: The optimization problem is concave as the utility function u_i is concave in s for all $i \in I$,* $\frac{\partial^2 u_i(s,t)}{\partial s^2} < 0$.
- *A4: The following technical assumptions are satisfied to guarantee that the second-order condition of the optimization problem hold,* $\frac{\partial^2 u_P(s,t)}{\partial s \partial t} \geq 0$, $\frac{\partial^3 u_A(s,t)}{\partial s^2 \partial t} \geq 0$ *and* $\frac{\partial^3 u_A(s,t)}{\partial s \partial t^2} \leq 0$.
- *A5: The "monotone hazard rate" is satisfied,* $\frac{\partial}{\partial t}\left(\frac{f(t)}{1-F(t)}\right) \geq 0$.
- *A6: To guarantee that the "firs best" problem has an interior solution it is assumed that for any $t \in T$ there is an $s < \bar{s}$ such that $s \in$ argmax$_{s' \in S_i}$* $\left\{u_P(s,t) + u_A(s,t)\right\}$.

Note that assumption A1 restricts the analysis to one-dimensional type spaces. Nevertheless, most assumptions can be relaxed with complicating effects on the structure of the optimal contract. For example, if assumption A5 is not given, then the optimal second best contract might consist of pooling ("bunching") regions in which the optimal strategy is the same for different type draws. See amongst others Baron and Myerson (1982) and Guesnerie and Laffont (1984) on this topic.

Let us now introduce the principal's optimization problem in the asymmetric information setting in which she is not informed about the agent's type draw $t \in T$.

Definition XVII. *(Second Best Optimization Problem): The principal's optimization problem if she is not informed about the agent's type draw $t \in T$ corresponds to,*

$$\max_{s(t),r(t)} \quad \int_{\underline{t}}^{\bar{t}} \left(u_P\left(s(t),t\right) - r(t)\right) dF(t) \tag{SB}$$

$$\textit{subject to} \quad t = \underset{\hat{t} \in T}{\operatorname{argmax}} \left\{u_A\left(s(t),t\right) + r(t)\right\} \textit{ for all } t, \hat{t} \in T; \tag{SB-IC}$$

$$\textit{and } u_A\left(s(t),t\right) + r(t) \geq 0 \textit{ for all } t \in T. \tag{SB-IR}$$

The principal maximizes ex-ante expected profit subject to the agent's incentive compatibility condition that ensures truthfulness (as part of focusing on direct revelation mechanisms) in (SB-IC) and the agent's individual rationality constraint in (SB-IR). The solution to the optimization problem in Definition XVII is provided in the next definition.

Definition XVIII. *(Second Best Contract): The optimal second best contract* $\left(s^{SB}(t), r^{SB}(t)\right)$ *in the asymmetric information environment for all type draws* $t \in T$ *of the agent corresponds to*

$$\frac{\partial u_P\left(s^{SB}(t),t\right)}{\partial s}+\frac{\partial u_A\left(s^{SB}(t),t\right)}{\partial s}-\left(\frac{1-F(t)}{f(t)}\right)\frac{\partial^2 u_A\left(s^{SB}(t),t\right)}{\partial s \partial t}=0; \tag{SB-S}$$

and corresponding transfer of

$$r^{SB}(t)=-u_A\left(s^{SB}(t),t\right)+\int_{\underline{t}}^{t}\frac{\partial u_A\left(s^{SB}(y),y\right)}{\partial y}dy. \tag{SB-R}$$

Analyzing the optimal second best contract, i.e. the first order conditions in (SB-S), we find that there is "underperformance" for all types $t < \bar{t}$ except for the highest type $t = \bar{t}$ (no distortion at the top) in comparison to a "first-best" social welfare maximizing allocation. Moreover, the agent's information rent $\int_{\underline{t}}^{t}\frac{\partial u_A\left(s^{SB}(y),y\right)}{\partial y}dy$ in equation (SB-R) is increasing in t and there is a general trade-off between maximizing social surplus and minimizing the agent's information rent.

2.2.3 *Optimal Delegation*

Contrary to standard contract theory, in which a second best contract consists of a menu of contingent strategies and transfer payments $\left(s(t), r(t)\right)$, in optimal delegation the principal can only influence the strategy space but is unable to employ transfers. In this setting the principal's optimization problem in the asymmetric information setting corresponds to (Alonso and Matouschek, 2008; Holmstrom, 1984),

Definition XIX. *(Delegation Problem): The uninformed principal's optimal delegation problem is,*

$$\max_{D \subseteq S} \quad \int_{\underline{t}}^{\bar{t}}\left(u_P\left(s(t),t\right)\right)dF(t) \tag{D}$$

$$\text{subject to} \quad s(t) \in \underset{s \in D}{\operatorname{argmax}}\left\{u_A(s,t)\right\}, \tag{D-IC}$$

in which the delegation set D is a subset of the strategy space S.[10]

The agent's incentive compatibility constraint in (D-IC) induces the agent to choose an optimal strategy from the delegation set such that an appropriate

[10] In an alternative formulation of the delegation problem, the delegation set D is defined as one subset from the collection of compact sets N of the strategy space S, i.e., $D \in N \subseteq S$.

definition of D and the resulting strategy-choice maximizes the principal ex-ante expected profit in (D). Following Holmstrom (1984), a solution to the delegation problem in Definition XIX is guaranteed to exist. However, as the form of the optimal delegation set is strongly case-dependent and any solution is not necessarily unique, we do not provide a general characterization but simply refer to the particular delegation problem of Section 3.3.2 and its solutions.

Let us now summarize the major findings in auction theory within our standard IPV model, also with respect to optimal delegation, that set the stage for an integration of our contributions into the academic discourse.

2.3 STATE OF RESEARCH

In this section we summarize the most important findings in auction theory for our standard IPV model with ex-ante symmetric and risk-neutral bidders who have decreasing marginal values in the number of units of the homogeneous good (perfect substitutes) obtained. Despite the modeling complexity of multi-object auction formats we focus on auctions as candidates for most beneficial multi-object market mechanisms and begin with a small positive result in which Engelbrecht-Wiggans (1988), based on Vickrey (1961) and Vickrey (1962), and similar to Myerson (1981) for single-object auctions, generalizes the RET under our standard assumptions and one stronger regularity condition: Any two multi-object auction formats that assign the same probabilities to the same allocations and deliver identical payoffs to some benchmark type bidder with lowest value for each possible allocation result in the same expected revenue for the seller. Unfortunately, equilibrium bidding strategies are much more difficult to characterize and differ substantially for specific auction formats. Therefore, the RET generally cannot be applied and unambiguous revenue comparisons are hard to obtain except in special cases. Moreover, note that multi-object auctions may possess more than one equilibrium in bidding strategies. Thus, in examining optimal strategies we assume each bidder to follow exactly one of the existing equilibria. Also, given our standard IPV assumptions with ex-ante symmetric bidders, we focus exclusively on symmetric equilibria in pure strategies in which each buyer follows the same strategy as a function of his possible type draws. This generally excludes (hybrid) equilibria in mixed strategies although we make an exception in Chapter 4. Nevertheless, we may have to compare efficiency and expected seller revenue between different equilibria of different auction formats and introduce additional equilibrium selection criteria such as payoff-dominance.

Let us next summarize the most important findings on single-object auctions in the literature which will then guide us through the discussion of multi-object formats.

2.3.1 *Single-Object Auctions*

The results on optimal single-object auctions by Myerson (1981) and Bulow and Klemperer (1996) as stated in Section 1.1.1 can be generalized to any standard single-object auction format as follows: By the Revenue Equivalence Theorem (RET), in a first version expressed by Vickrey (1961) and formalized by Myerson (1981) and Riley and Samuleson (1981), under the same assumptions, any two standard single-object auctions result in the same expected revenue for the seller. Moreover, the Vickrey auction and the English auction are outcome equivalent whereas the FPSB and the Dutch format are strategically equivalent (Vickrey, 1961). Any standard auction achieves the strongest competition among buyers and therefore is efficient and implements a higher expected market price than any other mechanism that could be employed instead, such as a simple posted price, a lottery or any form of negotiation.

However, note that the results stated so far heavily rely on our standard assumptions, especially ex-ante symmetric buyers whose object values are drawn from the same distribution function. Maskin and Riley (2000) demonstrates that with asymmetric buyers, whose values are drawn from different distribution functions, no general ranking in expected revenue between first- and second-price single-object auctions is possible. Moreover, a second-price auction is ex-ante efficient whereas a first-price auction is not, as first noted by Vickrey (1961). As will be discussed in Section 2.3.2, even if buyers' are assumed to be ex-ante symmetric in the sense of demanding the same set of objects and their valuations for subsets of objects being drawn from the same distribution functions, multi-object auctions result in ex-interim asymmetric information among bidders with similar consequences than in single-object auctions with ex-ante asymmetric buyers. This complicates the comparison and ranking of multi-object mechanisms dramatically.

In the following two sections we first consider auction formats that only allow for the sale of perfect substitutes and then discuss those which also allow to sell heterogeneous items that are not perfect substitutes, in particular combinatorial auctions. We stick to our standard model assumptions for a market in which multiple objects are sold and each buyer's marginal value for every demanded item can be computed.

2.3.2 *Auctions for Perfect Substitutes*

Similar to Engelbrecht-Wiggans (1988) for multi-object auctions, Maskin and Riley (1989) generalize earlier findings by Vickrey (1961), Ortega-Reichert (1968), Weber (1983), Milgrom (1985) and Harris and Raviv (1981), and demonstrates that the RET extends to all standard multi-unit auction formats for a setting in which each buyer demands at most one unit (single-unit demand).[11] This result follows already from Myerson (1981) if the analysis of single-object auctions is interpreted through marginal values. However, this

[11] Single-unit (single-object) demand is a special case of a single-minded bidder who demand only one specific set of objects as defined in Lehmann et al. (2002).

efficiency result does not extend to settings in which buyers demand more than one unit (multi-unit demand). Bolle (1997) establishes general necessary conditions for efficient equilibria in multi-unit auctions and shows that the standard discriminatory and uniform-price (highest losing bid) formats are inefficient. In a standard multi-unit auction with multi-unit demand, any buyer's bid on a certain unit, for example his first, competes with other buyers' bids on different units, e.g. their second or third. As the distributions of marginal values for distinct units differ, the market environment has close analogy to standard single-object auctions with ex-ante asymmetric bidders, which we know to be inefficient from Section 1.1.1 except the Vickrey auction. Intuitively, the asymmetries across bidders in the single-object setting are replaced by asymmetries across units in multi-unit auctions for ex-ante symmetric buyers with multi-unit demand.

Although closed-form expressions for bidding strategies cannot generally be formulated, characterizations of equilibrium bidding strategies for bidders with decreasing marginal valuations in the uniform-price (highest losing bid) and the discriminatory auction are derived by Engelbrecht-Wiggans and Kahn (1998a) based on Noussair (1995), and Engelbrecht-Wiggans and Kahn (1998b), respectively. Ausubel et al. (2014) formalize differential shading of bids for consecutive units between the discriminatory and uniform-price standard multi-unit auction formats independent of the total number of units demanded. In the latter auction mechanism bids fall stronger with successive units as lower bids, except on the first unit, lower the expected price paid for all units. This effect is termed "demand reduction". In fact, the differences in equilibrium bidding strategies between standard auction formats are so fundamental that the mechanisms may lead to different allocations and, based on Maskin and Riley (1989), the RET cannot be applied. Therefore, comparisons in efficiency and expected revenue for the seller remain ambiguous.

2.3.3 *Auctions for Heterogeneous Items*

Let us begin with the examination of sequential single-object auctions. In contrast to standard multi-unit auctions in which all units are sold at one go, each consecutive unit is sold to the buyer with the highest bid in the respective single-unit auction and the total number of units is not necessarily allocated to the highest bids. However, in case of bidders with single-unit demand sequential single-unit auctions are efficient and the RET applies as separately demonstrated for various sequential multi-unit auction formats by Weber (1983) and Weber and Milgrom (2000). For buyers with multi-unit demand it is even more difficult to characterize equilibrium strategies in sequential multi-unit auctions than in standard formats as further asymmetries are introduced. The winner of the first unit, for example, bids in the consecutive single-unit auction for his second unit whereas all other buyers compete for their first unit. In addition to their marginal values being drawn from a different distribution, the other buyers have to build believes about the marginal value of the first unit to the winner which the latter has to take into account ex-ante and so on.

This might be one of the main reasons why sequential auctions are rarely used in practice and other multi-object mechanisms are preferred. An exception is found in Katzman (1999) who analysis the sequential sale of two units in two consecutive second-price auctions for bidders demanding up to two units. Here, it is each bidder's dominant strategy to bid truthfully in the second auction and in the first auction all buyers are symmetric per definition. Therefore, equilibrium strategies can be derived despite bidder asymmetries and the two sequential second-price auctions turn out to be efficient. Another instance in which it is possible to derive linear Bayesian Nash equilibria in sequential auctions under our standard IPV assumptions is provided in Chapter 5 based on Kokott et al. (2018b).

In simultaneous single-unit auctions the highest bidding buyer in each auction wins the corresponding unit and thus, opposite to standard multi-unit auctions, the total number of units is not necessarily allocated to the respective number of highest bids. Even in a restricted environment in which bidders have single-unit demand, simultaneous single-unit auctions can be inefficient. For simultaneous Vickrey single-unit auctions Engelbrecht-Wiggans and Weber (1979) finds that in equilibrium (potentially in mixed strategies) buyers submit bids of different height at more than one single-unit auction to increase the chances of winning at least one unit. Here, the optimal strategy in any one of the simultaneous single-unit auctions differs from the equilibrium strategy in a standard single-object auction.[12] We can conclude that the bidding behavior in simultaneous single-unit auctions might differ strongly from the one in standard multi-unit auctions. Moreover, to the author's knowledge there are very limited characterizations of equilibrium bidding strategies for simultaneous single-unit auctions if bidders with multi-unit demand compete.

However, it is known that for perfect substitutes, the SMRA corresponds to the standard ascending-price multi-unit auction (Ausubel et al., 2014; Goeree and Lien, 2014), which is the outcome-equivalent pendant to the standard (closed) uniform-price (highest losing bid) format. Thus, the SMRA is efficient for bidders with single-unit demand but not for bidders with multi-unit demand. In fact Ausubel et al. (2014) argue that for bidders with multi-unit demand and decreasing marginal valuations, demand reduction and the resulting inefficiency is likely to be aggravated in the SMRA in comparison to its closed-form counterpart because bidders can "propose divisions" of the units already from an early stage. Nevertheless, the asymmetries and resulting inefficiencies might still be smaller than in sequential single-unit auctions.[13]

[12] At this point it is worth illustrating that optimal bidding behavior might differ significantly between the sale of perfect substitutes and objects that are not perfect substitutes in non-standard multi-object auctions. In the model by Engelbrecht-Wiggans and Weber (1979), if objects are not perfect substitutes, buyers will only participate in the single-object auction that contains their item of interest. Nevertheless, the analysis of perfect substitutes might still allow to demonstrate the different fundamental strategic trade-offs of different multi-object auction formats.

[13] Milgrom (2000) and Gul and Stacchetti (1999) show that if the auctioned objects are substitutes for bidders and they bid "straightforward", i.e. in each round a bidder submits bids on those objects that provide the highest profit at current prices, then prices converge to competitive equilibrium and the outcome is efficient. The allocation is in the core. If licenses are

2.3.4 *Combinatorial Auctions*

This section on equilibrium bidding strategies in combinatorial auctions is mostly based on the contributions of Bichler and Paulsen (2018) but also on Kokott et al. (2018a) whose main findings have already been introduced in Section 1.2.1. We now discuss their results in more detail and place them in a greater context by comparing the equilibrium bidding predictions with those of other non-combinatorial auction formats in the same multi-unit setting. Remember, we are in our standard IPV setting in which ex-ante symmetric and risk-neutral bidders with decreasing marginal valuations in the number of perfect substitutes obtained compete.

The *dual-winner equilibrium* in the ascending package auction is reminiscent of the strong demand-reduction equilibrium in the standard ascending-price multi-unit auction in the same setting with decreasing marginal values, but not necessarily *dual-winner efficiency*. Here, each bidder remains active on one unit until her valuation is reached but does not become active on a second unit. As a result there is no excess demand and each buyer obtains one unit at a price of zero (Engelbrecht-Wiggans and Kahn, 1998a).[14] Remember, in the standard uniform-price multi-unit auction there is a tradeoff of raising the bid on the second unit: It increases the chances of winning two units but also the price paid for both units. Intuitively, in the described equilibrium the second effect outweighs the first, which is exactly the same reason for the demand reduction equilibrium of the ascending package auction. Any positive bid on the package of two units can only increase the price paid for the single unit but does not raise the chances of winning the large package given *dual-winner efficiency*. Moreover, this specific efficiency environment is a sufficient condition for each bidder not to become active on the second unit in the standard ascending-price multi-unit auction. Two units can only be won by outbidding the opponent's truthful bid on one unit which, by definition of *dual-winner efficiency*, would then result in a loss for the winner of both units.[15]

Based on the above results we are able to establish outcome equivalence between the efficient equilibria of the ascending package auction and the standard ascending-price multi-unit auction in our model under *dual-winner efficiency*. Furthermore, as the standard multi-unit format is generally outcome equivalent to the SMRA we can further extend this form of equivalence

not perfect substitutes and bidders cannot freely switch between units, there is no equivalence between the SMRA and the standard ascending-price multi-unit auction (Goeree and Lien, 2014). However, in case of bidders with single-object demand it is a weakly dominant strategy for any buyer to remain active on the unit of interest until the price reaches his valuation. Note, though, that contrary to the standard ascending-price multi-unit auction, the SMRA is not generally efficient for buyers with single-object demand.

[14] Engelbrecht-Wiggans and Kahn (1998a) derive conditions under which this "single unit bid" constitutes an equilibrium in the standard uniform-price (highest losing bid) multi-unit auction which is outcome equivalent to the ascending-price multi-unit auction.

[15] Due to the above reasoning we assume bidders do not even start being active on both units. Equivalently, letting the price for one unit only rise so far that it cannot become winning is treated as not actively bidding for this unit.

between efficient equilibria.[16] Despite the above results we cannot conclude general outcome equivalence between the package auction and the standard multi-unit format in our setting, as there is also a *single-winner equilibrium* in the former mechanism which does not exist in the latter. This inefficient equilibrium is the result of any bidder's unilateral veto power to exclude the dual-winner outcome in the 2×2 setting of the ascending package auction. Respective strategic possibilities neither exist in the standard ascending-price multi-unit auction nor in the SMRA. Thus, we are able to present a strategic peculiarity of the ascending package auction that cannot be found in other standard multi-unit auctions even in our restricted setting. Moreover, the VCG mechanism is not outcome equivalent to the ascending second-price auction formats in our model, as it involves positive payments for both bidders. Not surprisingly, this hints at the revenue-maximizing properties of the VCG mechanism amongst all efficient, incentive compatible and individually rational mechanisms to allocate multiple objects (Krishna and Perry, 1999).

Continuing, it might at least be possible to argue towards a weak form of general outcome equivalence between the ascending auction formats in our model. First, the *dual-winner equilibrium* strictly dominates the *single-winner equilibrium* in payoff. Second, the ascending format allows both buyers to observe each others' actions and adjust accordingly. Suppose, for example a bidder starts trying to coordinate on the dual-winner outcome whereas her opponent aims for the single-winner outcome. In this case the bidder still has the possibility of switching to her strategy in the *single-winner equilibrium*. The ascending package auction appears to be robust against an equilibrium selection problem which might serve as another reason to start coordinating on the dual-winner outcome. Moreover, as the number of buyers rises, bidders lose their unilateral veto power and it becomes increasingly difficult to support a *single-winner equilibrium* although not impossible as is demonstrated in Chapter 4. Therefore, it might additionally be possible to conjecture some form of approximate outcome equivalence between the ascending auction formats for a large enough number of buyers. Strong forms of demand reduction, not involving package bids, are unlikely in such a setting and expected seller revenue might then even approach the VCG profit.

In our model with diminishing marginal values, but not necessarily *dual-winner efficiency*, Engelbrecht-Wiggans and Kahn (1998b) derive conditions under which an equilibrium of the standard discriminatory multi-unit auction involves pooling of bids for different units at the same price (by the same bidder). This occurs if marginal values for the first and second unit are close enough (bearing in mind that the marginal value for the second unit is strictly lower than for the first unit) and because bidders bid more aggressively on the second unit as they face more competition than in case of just aiming for one unit. However, contrary to the FPSB package auction under *dual-winner efficiency* in which two different bidders pool their single-unit bids at the same

[16] However, this outcome equivalence of efficient equilibria under *dual-winner efficiency* does not generally carry over to less restricted environments of decreasing marginal valuations. The necessary and sufficient conditions for the demand reduction equilibrium depend on the auction format, especially the bidding language, and therefore differ.

price in order to successfully and most profitably coordinate on a *dual-winner equilibrium*, the same form of pooling will only occur by pure chance in the standard discriminatory auction. These are two entirely distinct forms of pooling, with different underlying motives, indicating that there is no sort of strategic or outcome equivalence between the two first-price auction formats. Moreover, based on the work of Engelbrecht-Wiggans and Kahn (1998b) it can be shown that the package format leads to weakly higher seller revenue in its efficient equilibrium than the standard discriminatory multi-unit auction in our model under *dual-winner efficiency*. In the latter format, submitting a bid on the first unit in height of the optimal pooling price from the efficient equilibrium in the FPSB package auction results in bidder profit of at least the same amount as in the *dual-winner equilibrium*. A strictly lower bid earns the bidder even weakly higher expected profits. Intuitively, the coordination problem between dual-winner and single-winner outcome in the package auction prohibits bidders to make higher profits. In the standard discriminatory auction there is no coordination problem and the bidding language forces each buyer to submit a relative low bid on her second unit (below the marginal value) which then also allows lower winning prices for each bidder's first unit. It appears that the strategic complexities of the FPSB package auction are even more peculiar with respect to non-combinatorial mechanisms than in its ascending counterpart.

Although unambiguous distributional conditions for ex-ante payoff dominance of the *dual-winner equilibrium* with respect to the *single-winner equilibrium* can be established (from point of view of the buyers), the FPSB package auction is generally not efficient as each bidder can always unilaterally veto the dual-winner outcome. Nevertheless, it results in weakly higher seller revenue than the VCG mechanism in its efficient *dual-winner equilibrium*. Here, the constant revenue from pooling bidders corresponds to the maximum possible seller profit in the VCG mechanism in which this only occurs by pure chance. Moreover, seller revenue in the inefficient equilibrium corresponds to the expected second highest valuation for two units and is identical to the respective price in the ascending package auction. Thus, the FPSB package auction weakly dominates its ascending counterpart (as well as all standard multi-unit auctions and the VCG mechanism) in expected seller revenue whereas the latter is more likely to be generally efficient. Within the set of second-price multi-unit auctions the VCG is efficient and maximizes seller revenue.

In the procurement context of Chapter 4 and for linear equilibria we extend our results for the 2×2 market to a setting with $n > 2$ bidders. In case the *dual-winner equilibrium* is the outcome of the FPSB package auction, we can employ the RET to show that all combinatorial auction formats, including the VCG mechanism, are outcome equivalent to standard multi-unit auctions with single-unit demand. The last result complements our findings on outcome equivalence between second-price package auctions and standard second-price multi-unit auctions with multi-unit demand in the 2×2 forward auction market. Remember, there is still no general outcome equivalence between the FPSB package and other formats because the *single-winner equilibrium* always exists. This demonstrates that the peculiarity of the FPSB package auction persists to some extend even for $n > 2$ bidders and that the outcome of all second-price package auctions does indeed correspond to the VCG allocation in the $2 \times n$ market.

3

PRINCIPAL-AGENT RELATIONS IN PACKAGE AUCTIONS*

3.1 INTRODUCTION

Agency problems in bidding teams are pervasive in many auction markets. For example, it is well-known that the relationship between the supervisory board (principal) and the management (agent) of a telecom company play an important role in determining the firm's bidding strategy in spectrum auctions (chakraborty et al., 1995; Shapiro et al., 2013). Such principal–agent relationships have been put forward as one possible cause of allocative inefficiencies in spectrum auctions (Schmidt, 2004). In this chapter, we introduce a principal–agent model of a bidding firm in which the principal provides the agent with an upper limit on the amount to be spent in an auction (budget constraint). This model helps explain inefficient outcomes in multi-object auctions that are characterized by bidders inflating their demand to larger sets of objects (demand inflation) instead of efficiently coordinating on smaller sets. One of our central results is that principals might be more likely to overcome the agency problem if second-price auction formats are adopted, in particular ascending mechanisms.

Our motivation is a wide-spread hidden information problem in auctions in which the agent knows the valuation of different objects or packages, but the principal does not. The agent has limited liability and the principal determines upper limits on exposure in the auction. Any residual money is re-invested in the firm and the agent is unlikely to be induced to maximize profit. In this environment the agent tries to maximize the value of objects won given the budget constraint. Let us motivate our model by looking at spectrum auctions that are an important economic activity (generating billions of dollars worldwide) and that have been a catalyst for theoretical research in auctions. For example, principal–agent relationships arise between the management of a multinational telecom and the management of a national subsidiary bidding in the auction. In spectrum auctions, firms have preferences

*This chapter is based on the publication *A principal-agent model of bidding firms in multi-unit auctions* published in *Games and Economic Behavior (2018), Vol. 111, 20-40, co-authored with Martin Bichler*. Compared to the publication, we extend our principal-agent relationship to single-object auctions and generalize the findings to a setting with biased profit-maximizing agent. Moreover, the derived profit-maximizing principal's equilibrium strategies and the resulting rankings of auction formats in terms of revenue and efficiency are discussed with respect to the existing literature already in Sections 1.2 and 2.3.

over different packages of spectrum licenses. Each of these packages can be assigned a business case with a net present value. The management knows the market best, it knows the technology, the competition, and the end consumer market, and so they can compute business cases that allow for a good estimate of the net present value of each package. The board of directors does not have this information, and the management has no incentive to reveal it truthfully. Principals often need to rely on analyst estimates that typically have an enormous variance.[17] The principal will also not learn the true valuations of the licenses after the auction, as the future profits of the firm depend on many other decisions.

The payments made by telecommunication firms in spectrum auctions are often billions of dollars, and thus the management cannot cover the cost of the auction. This means, the agent in these markets has limited liability and the principal has to pay in the auction. The budgets that need to be reserved for such an auction by the board are also such that they cannot just be transferred in total to the agent in order to induce a profit-maximizing motive. The residual budget after the auction might be in the billions, and there can be much more efficient investments elsewhere. The different incentives of principal and agent are nicely summarized in a report by a consulting firm in this field (Friend, 2015):

"The amount of money spent by mobile operators at auction is often staggering. The money needed to pay for spectrum cannot usually be funded from the agreed capital expenditure budget of the business. As a result, spectrum payments are usually treated as a separate amount that does not impact the Key Performance Indicators of the business upon which the management team's bonuses are often based. However, the management team of a mobile business usually prefers to have more spectrum rather than less. The Chief Marketing Officer prefers more spectrum than his competitors as it allows him to advertise a bigger, faster and better network which helps him achieve his sales target. The Chief Technical Officer prefers more spectrum as it means she needs to build fewer sites to provide the same capacity and helps him achieve his capex to sales targets. The CEO is happy because the business is hitting its targets. So the management team will typically prefer to win more rather than less spectrum at auction."

Empire-building motives are a widespread reason for such value-maximizing behavior of agents in the principal–agent literature (Jensen, 1986). Note that spectrum auctions are only one example of value maximizing agents. Engelbrecht-Wiggans (1987) writes *"... in bidding for mineral leases, a firm may wish to maximize expected profits while its bidder feels it should maximize the firm's proven reserves."* In addition, he discusses auctions for

[17] Bulow et al. (2009) writes that *"Prior to the AWS auction, analyst estimates of auction revenue ranged from \$7.00 to \$15.00 billion. For the recent 700 MHz auction, they varied over an enormous range from \$10.00 to \$30.00 billion."* This is by no means an exception and estimates of investment banks and other external observers can be quite different from the actual revenue of the auction. Prior to the German spectrum auction in 2010, most analysts expected low revenue (Berenberg Bank estimated €1.67 bn., the LBBW bank estimated €2.10 bn.). The actual revenue from the auction was €5.00 bn.

defense systems and construction contracts. Payoff maximization is hard to defend for an agent in such relationships, and agents typically try to ”win within budget”. In contrast, *value maximization* is a good approximation of such agent motives. In this setting it is important to understand the impact of the agent’s bias on the firm’s bidding behavior and means for the principal to implement an optimal strategy.

3.2 CONTRIBUTIONS AND OUTLINE

Based on the description on Bayesian equilibrium bidding in combinatorial auction formats in Sections 1.2.1 and 2.3.4, we introduce a principal-agent model of bidding firms in the same standard model and 2×2 market environment. In each of the two firms an agent bids on behalf of a principal. The agent has private information about the valuations of the packages and wants to win the package with the highest valuation whereas the uninformed principal aims at maximizing profit. The latter determines budget constraints that restrict the agent’s bids. The auction itself corresponds to a Bayesian game in which the risk-neutral auctioneer selects the revenue maximizing set of package bids submitted by the privately informed firms (the privately informed agent within each firm).

This principal-agent problem can be seen as a form of optimal delegation in which an uninformed principal delegates decision rights to an informed but biased agent. Holmström (1977), Holmstrom (1984) and Alonso and Matouschek (2008) showed that the optimal mechanism for the principal if utility is not transferable consists of choosing a subset of actions, from among which the agent is allowed to pick the most desired one. A budget constraint that cannot be overbid corresponds to a restriction on the actions of the bidding agent. Budget constraints are widely used as a means to discipline the bidding agent in spectrum auctions and other high-stakes auctions (Engelbrecht-Wiggans, 1987; Shapiro et al., 2013). Whether the principal can implement an optimal bidding strategy with such a constraint crucially depends on the level of information she has about the valuations. We show that even if the principal had full information (symmetric information environment), she could not always optimally align the agent’s incentives via budget constraints only. Therefore, we also analyze contracts in which the principal can employ contingent transfers to optimally incentive align the agent in an asymmetric information environment. Principal-agent relationships within bidding firms have only recently become a topic of interest in auction theory, but prior models focus on delegation in single-object auctions (Burkett, 2015, 2016). We demonstrate that multi-object auctions are characterized by additional strategic considerations on which the agency problem has an impact that do not appear in single-object auctions.

First, we describe the environment formally as the *principal-agent* 2×2 *package auction model* in which 2 firms compete for 2 units of a homogeneous good (perfect substitutes) in Section 3.3. The 2×2 model captures the central strategic challenge that can also arise in larger markets as we demonstrate in

Chapters 4 and 5, and provide practical examples in the conclusion of this chapter. We primarily focus on the FPSB package auction and the ascending package auction, however, in Section 3.8 we also discuss the SMRA as it has frequently been employed to sell spectrum. It is still a topic of debate among regulators which auction format to use. Ideally, a mechanism would be strategy-proof and welfare-maximizing for the agent and the principal. Unfortunately, there is no general strategy-proof mechanism for value-maximizing agents (Fadaei and Bichler, 2017) and therefore there cannot be a strategy-proof mechanism for both. We briefly illustrate this result in our 2×2 model for the VCG mechanism. In the *principal-agent* 2×2 *package auction model* we are limiting our attention to *dual-winner efficiency* as introduced in Section 1.2.1. This context enables us to demonstrate strategic difficulties that can arise with principal–agent relationships in bidding firms as principals need to coordinate in the efficient equilibrium and there is a conflict of interest with the agent. Interestingly, one can often observe demand inflation in such situations in the field even though payoff-maximizing bidders would reduce demand from the start. Remember, that regulators tend to be legally bound to aim for efficiency.

In Section 3.5, we begin with the discussion of the peculiar *principal-agent* 1×2 *package auction model* in which two units are sold as the unique package to both bidding principal-agent pairs in a single-package auction. Analogue to Burkett (2016) we model this setting by the use of a general incentive-compatible direct-revelation mechanism, the *principal-agent* 1×2 *direct-revelation mechanism model*, that incorporates the *principal-agent* 1×2 *FPSB package auction model* as well as the *principal-agent* 1×2 *ascending package auction model* as special cases. The derivation of the solution is also similar to the discussion of contracts with perfect commitment in Krishna and Morgan (2008). The examination of the single-package market serves as an introduction into the *principal-agent* 2×2 *package auction model* and facilitates our main analysis. Similar to Burkett (2015, 2016), the optimal contract between principal and agent does not differ between different auction formats. Moreover, in the principal can easily overcome the agency problem in the symmetric information setting. However, if agency costs are too high, the optimal contract in the asymmetric information environment cannot be feasibly implemented which foreshadows the result for the *principal-agent* 2×2 *FPSB package auction model*.

In Section 3.6, we focus on the *principal-agent* 2×2 *FPSB package auction model* and analyze the agency problem that arises. First, in Section 3.6.1 we analyze the equilibrium bidding strategies of profit-maximizing principals in case they had full information about the valuations of the firm and were participating in the auction without their agents. The results of the analysis have already been introduced within our standard IPV framework for combinatorial auctions in Sections 1.2.1 and 2.3.4. In Section 3.6.2 we then analyze strategies of the agents if they were to participate in the auction without their principals, but with a budget constraint for one and two units. In this section, the constraints are considered exogenous and drawn from a random distribution. We show that it is the unique Bayesian Nash equilibrium for the agent not to bid on a single unit but only on the package of two units. The analysis highlights the bias of the agent and shows how his equilibrium bidding strategy differs from that of the principal leading to inefficiency. Consecutively, we focus on the implementation of the principal's original efficient equilibrium using budget constraints in the symmetric information environment in Section 3.6.3. We show that even if the principal has complete information, there cannot always be budget constraints that set the right incentives for the agents and at the same time constitute the equilibrium for the principal in the FPSB package auction. This results contrasts severely with the *principal-agent* 2×2 *package auction model* in the same information setting. Finally, in Section 3.6.4 we study the implementation of the optimal second-best contract with menus of contingent transfers and budget constraints in an asymmetric information setting. We focus on a strategically interesting situation in which there is uncertainty about the package valuations as well as their corresponding ranges, but not about the efficiency environment. With knowledge about the efficiency in the market the principal can align the incentives of the agent by paying wages that compensate the latter for not aiming to win the package of two units. However, theses wages might have to be very high making it impossible to implement the coordination equilibrium. Finally, under a regularity condition and with the agent's value-maximization motive being too high it can be shown that for the principal to prefer the implementation of the *dual-winner equilibrium* to the *single-winner equilibrium* the distributional assumptions become harder to satisfy in comparison to the setting without agents. Given the direction of the agency bias, the *dual-winner equilibrium* is more expensive to implement compared to the *single-winner equilibrium*.

In Section 3.7, we extend our analysis to the *principal-agent* 2×2 *ascending package auction model*. The same bidding behavior of agents as in the FPSB package auction continues to be optimal in the ascending format as shown in Section 3.7.2. This finding indicates that agents' bidding behavior is entirely independent of the rules of the standard package auction format. As agents do not internalize prices in their utility function, pricing rules are irrelevant and only allocation rules (the highest combination of bids wins) matter. Moreover, this bidding behavior is in stark contrast to the principals' equilibrium strategy which involves demand-reduction on one unit each as already described in Sections 1.2.1 and 2.3.4. Nevertheless, if the principal knows the efficiency environment in the market she could set the budget for the package to null and implement her equilibrium strategy in the symmetric as well as asymmetric information environment. Remember, that as an additional advantage over the FPSB package auction the ascending mechanism allows bidders to observe their opponents strategies and adjust accordingly. For profit-maximizing bidders this reduces the burden of starting with coordination as the strategy can be altered anytime if the opponent does not cooperate. Interestingly, the major advantage of the ascending package auction over its FPSB counterpart in solving the *principal-agent* 2×2 *package auction model* more easily is a feature of the second-price payment rule. We demonstrate this result by establishing outcome equivalence between the *principal-agent* 2×2 *ascending package auction model* and the *principal-agent* 2×2 *VCG mechanism model* with respect to solving the agency dilemma in Section 3.7.5.

Based on this major insight we are able to demonstrate that there is also a solvable agency problem in non-combinatorial mechanisms in Section 3.8, such as the well-known SMRA, for example. Finally, our findings are relevant for a setting in which the agent is value-maximizing to some extent but also internalizes package prices similar to Burkett (2015, 2016), too. The proofs for all above findings are contained in Appendix A.1.

3.3 MODEL

In our model we consider 2 ex-ante symmetric firms $i, j \in I$ with $|I| = 2$, competing in a multi-unit package auction for 2 units of a homogeneous good. The number of units within a package is denoted by $l \in L = \{1, 2\}$. From now on we mostly refer to the package of one unit as "one unit" and to the package of two units as "package". The revenue-maximizing auctioneer either sells one package of two units to one firm or a package of one unit to each of the two firms. He always allocates all units. We assume an *XOR* bid language that allows a bidder to specify a bid for each possible package, i.e., the package of one unit and the package of two units, but only one of the bids can win (Nisan, 2000).

Each firm i has a private value for the package of l units of $v_i(l) \in V(l) = \left[\underline{v}(l), \overline{v}(l)\right]$. We define the vector of package values as $v_i = \left(v_i(1), v_i(2)\right) \in V = V(1) \times V(2)$ for all $i \in I$. All package valuations in our model can then be summarized within $v = \left(v_i, v_j\right) \in V^2$ with $i, j \in I$. Let us normalize the reservation utility $v_i(0) = 0$ and assume $v_i(1) < v_i(2)$ for each $i \in I$. Note that the latter assumption implies $\overline{v}(2) > \overline{v}(1)$ and $\underline{v}(2) > \underline{v}(1)$. Moreover, we restrict our analysis to *dual-winner efficiency*, an environment in which it is efficient for both firms to obtain one unit each, independent of their package valuation draws. The condition $2 \cdot \underline{v}(1) > \overline{v}(2)$ ensures *dual-winner efficiency* for all $v_i, v_j \in V$. The above condition corresponds to a setting with decreasing marginal values and an extra assumption that the highest possible marginal value of the second unit is less than the lowest possible value for the first unit, i.e. $\overline{v}(2) - \underline{v}(1) < \underline{v}(1)$. *Dual-winner efficiency* and the assumption of strictly separated package-valuation ranges, $\underline{v}(2) > \overline{v}(1)$, provide an interesting and practically relevant market environment that actually allows us to derive firms' Bayesian Nash equilibria in various combinatorial auction mechanisms.

Throughout the entire discussion each firm i has private information about its package valuations and does not know the other firm's vector of valuations. Thus, we assume each firm i's vector of valuation draws, v_i, to be a priori distributed according to a monotonically increasing joint cumulative distribution function $F(v_i)|_{v_i(1)<v_i(2)}$ with $F|_{v_i(1)<v_i(2)} : V \to [0, 1]$. The marginal distribution function for the package-value of l units is of the form $F_l\left(v_i(l)\right)$ with $F_l : V(l) \to [0, 1]$ and strictly positive density function of $f_l\left(v_i(l)\right) > 0$.

The distribution functions $F(\cdot)|_{v_i(1)<v_i(2)}$, $F_l(\cdot)$ and $f_l(\cdot)$ are assumed to be common knowledge within each firm i and between both firms i and j.

Before any other decisions are made, the auctioneer chooses the multi-unit package auction format. Let each firm's report submitted to the auction mechanism for the package of l units be denoted by $b_i(l) \in B(l) = \left[0, \overline{b}(l)\right] \subseteq \mathbb{R}_{\geq 0} \cup O$ for all $l \in L$. The report $b_i(l) = O$ specifies that firm i does not compete in the auction for the package of l units. We can then define the vector of package reports as $b_i = \left(b_i(1), b_i(2)\right) \in B = B(1) \times B(2)$ for all $i \in I$. All reports in the auction are then contained within $b = \left(b_i, b_j\right) \in B^2$ with $i, j \in I$. The exact functional form of $b_i(l)$ depends on the setting and will be defined correspondingly below.

Within each firm i the principal maximizes expected profit with the profit of winning a package of l units given by the function $\pi_i(l) = v_i(l) - p_i(l)$ and the profit of not winning a package is zero. In the former expression $p_i(l)$ denotes the price paid for a package of l units and is defined as a function of the vector of reports $p_i(l) = p_l\left(b_i, b_j\right)$ with mapping $p_l : B^2 \to \mathbb{R}_{\geq 0}$. The precise functional relationship between the reports and the prices depends on the specific multi-unit package auction format.

The agent's gross utility consists of his value-maximizing motives and is denoted as $u_i(l) = w\left(v_i(l)\right)$ in case of winning a package of l units. The function $w : V(l) \to \mathbb{R}_{\geq 0}$ is strictly increasing in package value, $v_i(l)$, and thus, an agent always prefers winning two units to one unit. Moreover, the function $w(\cdot)$ is assumed to be commonly known among both firms i and j, and within each firm i. As long as the price for a bundle of l units is weakly lower than his respective budget constraint of $a_i(l) \in A(l) = \left[\underline{a}(l), \overline{a}(l)\right] \subseteq \mathbb{R}_{\geq 0}$ for all $l \in L$, the agent obtains a utility of $u_i(l)$. Any report $b_i(l) > a_i(l)$ is not allowed as it may result in the firm making losses. For agent i, the budget constraint $a_i(l)$ reduces the range of permissible reports to $B_i(l) = \left[0, a_i(l)\right]$ for all $l \in L$.[18] We refer to the vector of all package budget constraints as $a_i = \left(a_i(1), a_i(2)\right)$ with $a_i \in A = A(1) \times A(2)$.

In the settings in Section 3.3.1 and Section 3.3.2 the agent's report for the package of l units is a function of the vector of package values as well as the vector of budget constraints $b_i(l) = \beta_l(v_i, a_i)$ with $\beta_l : V \times A \to B(l)$. The vector of reports $b_i = \beta(v_i, a_i)$ corresponds to the function $\beta : V \times A \to B$.

Finally, in our model the strategy space $B \subseteq \mathbb{R}^2_{\geq 0}$ is a nonempty, convex and compact subset of an Euclidean space. Also, the quasilinear utility functions $\pi_i(l)$ and $u_i(l)$ for all $l \in L$ of principal and agent, respectively, are continuous in $b \in B^2$ and quasiconcave in $b_i \in B$. Therefore, according to Definition XII in Section 2.1.1, Bayesian Nash equilibria are guaranteed to exist in pure strategies in our standard IPV setting with risk-neutral and ex-ante symmetric principals and agents. Also, Perfect Bayesian equilibria

[18] The principal may not specify a lower-bound on the report for the package of l units such that the function $a_i(l)$ constitutes a standard budget constraint. This budget constraint then corresponds to a special form of delegation in which, for example, dictation of bids is not permitted.

(possibly in mixed strategies) are guaranteed to exist in finite dynamic game of incomplete information as stated in Definition XIV of the Section 2.1.2.

3.3.1 *Independent Optimization*

To better understand the bias between principal and agent within each firm i we first abstract from the underlying principal-agent relationship and derive the principal's as well as the agent's Bayesian Nash equilibria if they were to participate on their own in a multi-unit package auction. To do so we assume full information about the firm's package valuations and their ranges for principal and agent. However, there is still information asymmetry between the two different firms i and j.

Let the principal's reporting function for l units be defined as $b_i(l) = \beta_l(v_i)$ and $\beta_l : V \to B(l)$. The vector of package reports is a function of the vector of package valuations such that $b_i = \beta(v_i)$ with mapping $\beta : V \to B$ for all $i \in I$. Now, according to Definition III in Section 2.1.1 and given vector of valuations v_i we can denote the principal's ex-interim expected profit of participating in the auction as $\Pi(\cdot)$ with corresponding maximization problem of

$$\max_{b_i \in B_i} \quad \Pi\left(b_i, b_{-i}, v_i; F\left(v_j\right)|_{v_j(1)<v_j(2)}\right) \quad \text{for all } v_i \in V. \tag{P-EXI}$$

The ex-interim expected profit is a function of the vectors of valuations v_i and reports b, taking into account the distribution function of the opponent's valuations $F\left(v_j\right)|_{v_j(1)<v_j(2)}$.

In our initial analysis of the agents' strategies without the principals we assume the package budget constraint for l units, $a_i(l)$, to be an exogenous random variable drawn from $A(l)$. Further, we assume $a_i(1) \leq a_i(2)$. The vector of budget constraints a_i is a priori distributed according to a monotonically increasing joint cumulative distribution function $Q(a_i)|_{a_i(1)\leq a_i(2)}$ with mapping of $Q|_{a_i(1)\leq a_i(2)} : A \to [0, 1]$ with marginal distribution function of $Q_l\left(a_i(l)\right)$ with $Q_l : A(l) \to [0, 1]$ and corresponding strictly positive density function $q_l\left(a_i(l)\right) > 0$ for all $a_i(l) \in A(l)$. The distribution functions $Q(\cdot)|_{a_i(1)\leq a_i(2)}$, $Q_l(\cdot)$ and $q_l(\cdot)$ are assumed to be common knowledge between agent i and j.

Analogue to Definition III in Section 2.1.1, we denote the agent's ex-interim expected utility of participating in the multi-unit package auction as $EU(\cdot)$, given valuation and budget vectors of v_i and a_i, respectively, and his optimization problem corresponds to

$$\max_{b_i(l)\leq a_i(l) \forall l \in L} \quad EU\left(b_i, b_j, v_i; Q(a_j)|_{a_j(1)\leq a_j(2)}, F\left(v_j\right)|_{v_j(1)<v_j(2)}\right) \quad \text{for all } v_i \in V \text{ and } a_i \in A. \tag{A-EXI}$$

The expected utility is determined by the valuations v_i and reports b, taking into account the distribution functions of the other agent's budgets and values, $Q\left(a_j\right)|_{a_j(1)\leq a_j(2)}$ and $F\left(v_j\right)|_{v_j(1)<v_j(2)}$, respectively. Ex-interim expected utility in (A-EXI) is maximized for all possible valuation $v_i \in V$ and budgets $a_i \in A$ given the reports do not exceed the respective budgets $b_i(l) \leq a_i(l)$. We demonstrate in Section 3.6.2 and 3.7.2 that the above optimization problem is actually independent of the vector of valuation draws $v \in V^2$ and the respective distribution functions.

Remember, according to in Definition VI of Section 2.1.1, if each principal (agent) $i \in I$ solves the problem in (P-EXI) ((A-EXI)) for all possible vector of valuations $v_i \in V$ (and budgets $a_i \in A$) a Bayesian Nash equilibrium is obtained. In the two next sections the principal's optimal contracting problems in the symmetric and asymmetric information settings are formulated according to Sections 2.2.3 and 2.2.2, respectively.

3.3.2 *Symmetric Information Environment*

In the symmetric principal-agent information setting in Sections 3.5, 3.6 and 3.7 we analyze an environment in which the agent does not posses private information and there is no information asymmetry within each firm i. The package value $v_i(l)$ as well as its corresponding range $V(l)$ is known to principal and agent for all $l \in L$. There is, however, information asymmetry between both firms i and j.

After the auctioneer announces the auction format, principal and agent learn their firm's vector of package valuations $v_i \in V$. The principal then provides her agent with the vector of budget constraints a_i derived according to a function of the firm's true vector of valuations $a_i = \alpha(v_i)$ with $\alpha : V \to A$. Similar, the budget for the package of l units, $a_i(l)$, is defined as the function $a_i(l) = \alpha_l(v_i)$, in which $\alpha_l : V \to A(l)$ for all $l \in L$.

The definition of the agent's reporting functions remains unaltered to Section 3.3.1, however, as the budget constraints are endogenously determined by the principal as functions of the vector of package valuations we employ $F(\cdot)|_{v_i(1)<v_i(2)}$ as the relevant distribution function for the agent. The principal's optimization problem in the symmetric information environment of the *principal-agent* 2×2 *package auction model* is based on Definition XIX of the optimal delegation problem in Section 2.2.3 and corresponds to

$$\max_{a_i(l)\forall l\in L} \quad \Pi\left(\beta(v_i,a_i), b_j, v_i; F(v_j)|_{v_j(1)<v_j(2)}\right) \tag{PA-D}$$

$$\text{subject to} \quad \beta(v_i,a_i) \in \operatorname*{argmax}_{b_i(l)\leq a_i(l)\forall l\in L} EU\left(b_i, b_j, v_i; F(v_j)|_{v_j(1)<v_j(2)}\right); \tag{PA-D-IC}$$

$$\textit{and } a_i(l) \leq v_i(l) \text{ for all } l \in L \text{ in the equilibrium allocation.} \tag{PA-D-NL}$$

The principal determines a vector of budget constraints a_i (the delegation set) which, given the agent's incentive compatibility condition in (PA-D-IC), induce the agent to choose a vector of reports $\beta(v_i, a_i)$ that maximizes the principal's ex-interim expected utility in (PA-D). Again, the distribution function of the opposing firm's values, $F(v_j)|_{v_j(1)<v_j(2)}$, is taken into account in the principal's (agent's) ex-interim expected profit (utility). Finally, it cannot be optimal for the principal to let the value-maximizing agent, who does not internalize prices, report higher than the valuation for any package of l units that is part of the equilibrium allocation. This additional restriction is ensured in the no-loss condition (PA-D-NL). Note that the principal does not have to take an individual rationality constraint of the agent into account as the agent cannot incur negative utility in this setting.

Be aware that the definition of the delegation problem in Section 2.2.3 refers to an asymmetric information setting in which the principal does not know the package valuations which is not the case in our symmetric information environment. Therefore, the above optimization problem is a non-standard special case that bears some similarity to a "first best" delegation problem in which only an upper bound on the delegation space is set (and no lower bound) and simple dictation of strategies by the principal is not permitted. However, in the next section we formulate the principal's optimization problem in an asymmetric environment that corresponds to the standard second best contracting problem as formulated in Definition XVII in Section 2.2.2. We do not specify a delegation problem in the asymmetric information setting as our analysis shows that optimal delegation is not even possible in the symmetric information environment.

3.3.3 *Asymmetric Information Environment*

In the asymmetric information setting in Sections 3.5.4, 3.6.4 and 3.7.4 each firm i's principal is informed about *dual-winner efficiency* but she neither knows the package value draw $v_i(l)$ nor its exact range $V(l)$. However, we assume that she knows supports for the range bounds: the support for the lower bound $\underline{v}(l)$ is $\underline{V}(l) = \left[\underline{\underline{v}(l)}, \overline{\underline{v}(l)}\right]$ and the support for the upper bound $\overline{v}(l)$ is $\overline{V}(l) = \left[\underline{\overline{v}(l)}, \overline{\overline{v}(l)}\right]$. The agent has exactly the same information as the principal and in addition knows the precise package value draw $v_i(l)$. However, he also does not know the true range $V(l)$.

As the principal does not know the true range $V(l)$, from her point of view, any package value $v_i(l)$ can be drawn from the largest possible value range $Z(l) = \left[\underline{\underline{v}(l)}, \overline{\overline{v}(l)}\right]$. The principal knows $Z(l)$ because she is informed about $\underline{V}(l)$ and $\overline{V}(l)$. We must then specify the monotonically increasing joint cumulative distribution function $G(v_i)|_{v_i(1)<v_i(2)} : Z \to [0,1]$ in which $Z = Z(1) \times Z(2)$ with corresponding marginal distribution function of the form $G_l\left(v_i(l)\right)$ with $G_l : Z(l) \to [0,1]$ and strictly positive density function $g_l\left(v_i(l)\right) > 0$ for all $v_i(l) \in Z(l)$ and $l \in L$. The distribution functions

$G(\cdot)|_{v_i(1)<v_i(2)}$, $G_l(\cdot)$ and $g_l(\cdot)$ are common knowledge within a firm i and between both firms i and j.

In the asymmetric information setting, the contract proposed by the principal after the auctioneer announces the auction format specifies a menu of as many as four functions (all functions of v_i): The budget constraint for the package of l units is $a_i(l) = \alpha_l(v_i)$ with $\alpha_l : Z \to A(l)$. The vector of budgets $a_i = \alpha(v_i)$ is a function with $\alpha : Z \to A$. The transfer from principal to agent for the package of l units is $m_i(l) = \mu_l(v_i)$ with $m_i(l) \in M(l) \subseteq \mathbb{R}_{\geq 0}$ and $\mu_l : Z \to M(l)$. The vector $m_i = \big(m_i(1), m_i(2)\big) \in M = M(1) \times M(2)$ summarizes the package transfers and we define $m_i = \mu(v_i)$ with $\mu : Z \to M$. Finally, we restrict our analysis to positive transfers from principal to agent with $m_i(l) \geq 0$.[19] We denote the full contract by (α_i, m_i).[20]

The agent's reports to the contract $\hat{v}_i \in Z$ are translated into budgets $\hat{a}_i = \alpha(\hat{v}_i) \in A$ and corresponding transfer payments $\hat{m}_i = \mu(\hat{v}_i) \in M$ according to the above specified functions. He then reports to the auction mechanism bids of the form $b_i(l) = \beta_l(v_i)$ with $\beta_l : Z \to B(l)$ and the vector of bids is $b_i = \beta(v_i)$ in which $\beta : Z \to B$.

In the optimum the principal's use of transfer payments ensures that the agent reports the true valuation, $\hat{v}_i = v_i$, (via the direct revelation mechanism) to the contract and that the latter's reports to the auction mechanism for one unit and the package will always correspond to the principal's optimal budget constraints, i.e., $b_i = a_i$. Moreover, as the agent's reports to the contract, v_i, are directly translated into reports to the auction mechanism, $b_i = \beta(v_i)$, the contract can be rewritten as (b_i, m_i). According to general formulation of the second best optimization problem in Definition XVII of Section 2.2.2, any principal i's optimization problem in the asymmetric information setting of the *principal-agent* 2×2 *package auction model* is then denoted as

$$\max_{\left(\beta(v_i),\mu(v_i)\right)} \quad E_{v_i \in Z}\Big(\Pi\big(\beta(v_i), \mu(v_i), b_j, v_i; G(v_j)|_{v_j(1)<v_j(2)}\big)\Big) \tag{PA-SB}$$

$$\text{subject to} \quad v_i \in \underset{\hat{v}_i \in V}{\operatorname{argmax}}\, EU\Big(\beta(\hat{v}_i), \mu(\hat{v}_i), b_j, v_i; G(v_j)|_{v_j(1)<v_j(2)}\Big) \tag{PA-SB-IC}$$

$$b_i(l) \leq v_i(l) \text{ for all } v_i(l) \in V(l) \tag{PA-SB-NL}$$

and for all $l \in L$ in the equilibrium allocation.

As the principal does not know the firm's vector of valuation, v_i, she has to maximize the ex-ante expected profit (expected ex-interim expected profit), which is denoted by $E_{v_i}(\cdot)$ in (PA-SB). Ex-ante expected profit is determined by the valuations v_i, reports b_i and transfer payments m_i, taking into account the other firm's valuations distribution function $G(v_j)|_{v_j(1)<v_j(2)}$. The agent's incentive compatibility constraint, which ensures truthfulness of the direct

[19]This limited liability constraint on the principal is similar to Regime 2 in Burkett (2016).

[20]Contrary to the symmetric information environment in Section 3.3.2 in which the agent is provided with fixed values as package budgets, the principal now allows the agent to choose from sets of contingent package budget constraints defined as functions of v_i.

revelation mechanism, has to be taken into account in (PA-SB-IC) as well as condition (PA-SB-NL) that prevents loss within the equilibrium allocation. Again, the latter additional assumption is needed because a value-maximizing agent does not internalize prices. As the agent receives weakly positive transfers only, the individual rationality constraint does not apply for the principal.

Next, we also define the random draw $d \in D = \left[\underline{d}, \overline{d}\right]$ with $d \equiv \overline{v}(2) - \underline{v}(1)$ as the agent's private information that is unknown to the principal and helps us solve the latter's optimization problem (PA-SB) for Section 3.6.4. However, given the assumptions of the asymmetric information setting, the agent only knows the value of d and not the values of $\overline{v}(2)$ and $\underline{v}(1)$. From the agent's point of view d can be the result of various $\overline{v}(2) \in \overline{V}(2)$, $\underline{v}(1) \in \underline{V}(1)$ combinations. Note that d corresponds to the true entire range out of which valuations for one unit and the package can be drawn. Again, as the principal knows $\underline{V}(l)$ and $\overline{V}(l)$ she also knows D and its bounds with $\underline{d} \equiv \underline{\overline{v}(2)} - \overline{\underline{v}(1)}$, the smallest possible range out of which all values can be drawn, and $\overline{d} = \overline{\overline{v}(2)} - \underline{\underline{v}(1)}$, the largest possible range. We assume $\overline{\overline{v}(2)} > \overline{\underline{v}(1)}$ and $\underline{\overline{v}(2)} > \underline{\underline{v}(1)}$, and that d is commonly known, within and between firms, to satisfy *dual-winner efficiency* such that $2 \cdot \underline{v}(1) > \overline{v}(2)$ always holds.

In this setting the agent reports $\hat{d} \in D$ to the contract which are then translated into reports of $b_i(l) = \beta_l(d)$ with $\beta_l : D \to B(l)$ and transfers $m_i(l) = \mu_l(d)$ with $\mu_l : D \to M(l)$ to the auction. The vector of reports $b_i = \beta(d)$ is a function with $\beta : D \to B$ and the vector of transfers $m_i = \mu(d)$ is a function $\mu : D \to M$.

Unlike standard single-object auctions, combinatorial package auctions may possess multiple Bayesian Nash equilibria for profit-maximizing principals. We solve the principal's optimization problems (PA-D) and (PA-SB) subject to the respective constraints by implementing specific Bayesian Nash equilibria. Finally, we compare the different solutions to the optimization problems in terms of payoff-dominance.

3.4 AUCTION FORMATS

We first analyze the simplest multi-unit package market in which both units are sold as a single package. This setting is modeled by letting the auctioneer employ an incentive-compatible 1×2 *direct-revelation mechanism* for profit-maximizing bidders to sell the package. The mechanism contains the 1×2 *FPSB package auction* as well as the 1×2 *ascending package auction* as standard single-package auction applications.

In settings in which the items are not sold as a single package, it is straightforward to see that the 2×2 *VCG mechanism* is not incentive-compatible for agents, who do not internalize payments in their utility function. Moreover, there cannot be a strategy-proof and deterministic selling mechanism for value-maximizing agents (Fadaei and Bichler, 2017). We then focus on the 2×2 *FPSB package auction*, the 2×2 *ascending package auction* and a 2×2

ascending uniform-price auction. These auction formats are being used in spectrum sales and other high-stakes auctions. In multi-unit package auctions, each bidder i submits an all-or-nothing bid for every package. Here, each package is identified by the number of units it contains $l \in L$. In these package auctions, we assume an XOR bid language, because it is the most general bidding language allowing the expression of complements and substitutes (Nisan, 2000), and it is also typically used in spectrum auctions. An XOR bid language allows a bidder to specify a bid for all possible packages, but only one of the bids submitted can become winning. Finally, we assume that in each auction format the auctioneer always sells all units.

3.4.1 *The* 1×2 *Direct-Revelation Mechanism*

The analysis of the 1×2 *direct-revelation mechanism* is based on the model by (Burkett, 2016) and helps us to derive the implementation of the *single-winner equilibrium* in the *principal-agent* 2×2 *package auction model* as well as to formulate our main findings.

Let us first define the mechanism to sell the package for the setting in which the profit-maximizing principal participates in the auction independently of her value-maximizing agent as described in Section 2.2.1. The direct revelation mechanism, that is based on Proposition XV in Section 2.2.1, is proposed by the auctioneer before any actions are taken and is composed of two functions, $F_2\left(b_i(2)\right)$ and $T_2\left(b_i(2)\right)$ with $T_2 : B(2) \to \mathbb{R}_{\geq 0}$. These functions specify the probability that the package is awarded as a function of the report $b_i(2)$ and the expected transfer made to the auctioneer, respectively. For a fully informed and profit-maximizing principal who employs reporting function $b_i(2) = \beta_2\left(v_i(2)\right)$, the mechanism $\left(F_2\left(\beta_2\left(v_i(2)\right)\right), T_2\left(\beta_2\left(v_i(2)\right)\right)\right)$ is assumed to be incentive compatible. Thus, the the principal reports $\beta_2\left(v_i(2)\right) = v_i(2)$, and incentive compatibility of the mechanism $\left(F_2\left(v_i(2)\right), T_2\left(v_i(2)\right)\right)$ implies via standard arguments that $F_2\left(v_i(2)\right)$ is non-decreasing and that $T_2\left(v_i(2)\right)$ can be expressed as a function of $F_2\left(v_i(2)\right)$ and $T_2\left(\underline{v}(2)\right)$. We also assume the incentive-compatible 1×2 *direct-revelation mechanism* to be individually rational, which requires that the payment made by the principal with the lowest value does not exceed her valuation for two units, i.e., $T_2\left(\underline{v}(2)\right) \leq \underline{v}(2)$.

In the setting in which agents compete without their principals in the auction, the distribution function $F_2(\cdot)$ is replaced by $Q_2(\cdot)$ and $T_2\left(\beta_2\left(a_i(2)\right)\right)$ maps according to $T_2 : A(2) \to \mathbb{R}_{\geq 0}$. Moreover, in the asymmetric information environment $F_2\left(v_i(2)\right)$ is replaced by $G_2\left(v_i(2)\right)$ and $T_2\left(v_i(2)\right)$ is defined via $T_2 : Z(2) \to \mathbb{R}_{\geq 0}$. Individual rationality then implies $T_2\left(\underline{\underline{v}(2)}\right) \leq \underline{\underline{v}(2)}$.

3.4.2 *The* 2×2 *VCG mechanism*

The VCG mechanism, as formulated by (Groves, 1973), is the generalization of the well-known second-price sealed-bid (Vickrey) auction Vickrey (1961) to combinatorial package auctions. The VCG mechanism is composed of the Clarke Pivot payment function, as introduced by (Clarke, 1971), that is strategy proof for profit-maximizing bidders (principals) and they truthfully report their package valuations. The VCG mechanism further contains a welfare-maximizing social choice function that allocates the set of items with the sum of highest package reports to the corresponding bidders and is therefore efficient.

Budgets in the *principal-agent* 2×2 *VCG mechanism model* can optimally be set as in the *principal-agent* 2×2 *ascending package auction* with sufficient information about the efficient allocation as we show in Section 3.7.5. Therefore, the VCG mechanism and the ascending package auction are outcome equivalent with respect to the agency dilemma. Nevertheless, we mainly focus on the ascending format as it is practically relevant.

3.4.3 *The* 2×2 *FPSB Package Auction*

In the 2×2 *FPSB package auction*, both bidders i and j simultaneously submit their bids b_i and b_j to the auctioneer without knowing the opponent's bids. Consecutively, a risk-neutral auctioneer selects the revenue-maximizing combination of package bids. This can either be an allocation in which each bidder i and j gets one unit, $b_i(1) + b_j(1) \geq \max\left\{b_i(2), b_j(2)\right\}$, or an allocation in which one bidder wins the package of both units, $\max\left\{b_i(2), b_j(2)\right\} > b_i(1) + b_j(1)$. In case of a tie between an allocation with two winners or a single winner, the auctioneer allocates one unit to each of the bidders. In case of a tie between two package bids, the auctioneer randomizes. Any firm i that wins a package of l units pays a corresponding price of $p_i(l) = b_i(l)$ to the auctioneer.

3.4.4 *The* 2×2 *Ascending Package Auction*

In this section we formulate the 2×2 *ascending package auction* as a multi-unit clock auction with continuously increasing package prices. There is one clock to indicate the current single-unit price $p_c(1)$ and another clock for the current package price $p_c(2)$. One clock per package guarantees anonymity of the auction format. Each bidder i can individually decide if and when (at which price) to become active on (demanding) any one of the packages. Moreover, each bidder i specifies with his vector of bids $b_i = \left(b_i(1), b_i(2)\right)$ until which current price to remain active on the package of l units, $p_c(l) = b_i(l)$, for all $l \in L$. Any current package price $p_c(l)$ increases automatically as long as the package of l units belongs to a currently demanded but non-winning allocation, or to an allocation for which there is excess demand. Prices rise continuously (marginally) just until an allocation is winning and not over-demanded anymore. This process guarantees the second-price character of the ascending pricing rule as will be demonstrated next.

Suppose, for example, each of the two bidders is currently active on the single-unit and on the package of two units. In this setting there is over-demand for the single-winner outcome but never for the dual-winner outcome.[21] In case twice the single-unit price marginally exceeds the package price at $p_c(1) = p_c(2)/2 + \epsilon$ with $\epsilon \to 0$, the latter rises whereas the dual-winner outcome is currently winning and the single-unit price remains constant. If, however, the package price marginally exceeds twice the single-unit price at $p_c(2) = 2p_c(1) + \epsilon$ with $\epsilon \to 0$, both prices, for one unit and for the package, rise. Again, there is excess demand for the package of two units and the demanded dual-winner outcome is non-winning. Obviously, only one bidder can win the package of two units in the end. As soon as all clocks stop increasing, the auction terminates with the highest priced allocation being sold to the corresponding bidders. The corresponding current prices then become the final winning prices. If the dual-winner outcome wins, each bidder i's final price is $p(1) = p_c(2)/2$, and if the single-winner outcome results, the highest bidder i on the package pays either $p(2) = 2p_c(1)$ or $p(2) = p_c(2)$. A participating bidder needs to become active at least on one of the two packages at the start and once she stops bidding on one of both packages, she cannot become active on this package again. Moreover, she cannot win a package for which she is not active anymore. Note, this implies that if a bidder i stops being active for the single unit but remains active for the package, the remaining active bidder j on one unit actually competes for two units as she cannot win a single unit on her own and the auctioneer always sells both units. Any bidder j prefers or is indifferent to winning two units at a price at which she demands one unit. Finally, as soon as a buyer quits being active on both packages she cannot reenter the auction again. It is common knowledge among both bidders and the auctioneer which buyer is active on which package.

3.4.5 *The* 2×2 *Ascending Uniform-Price Auction*

We also discuss a standard (non-package) 2×2 *ascending uniform-price auction* that corresponds to the SMRA in a setting with perfect substitutes. In this auction, there is one continuously increasing price $p_c(1)$ for one unit. Each bidder can individually specify when (at which current price) to demand which number of units. Once a bidder has decreased the number of units demanded, it cannot be raised again. Moreover, by reducing demand to zero units a bidder ultimately drops out of the auction. The price stops increasing as soon as there is no excess demand and the bidders receive the number of units for which they are still active. With slight abuse of notation, in this setting the vector of bids $b_i = \big(b_i(1), b_i(2)\big)$ defines until which price bidder i demands two units $p_c(1) = b_i(2)$ and up to which price she is active on one unit $p_c(1) = b_i(1)$.

[21] With two bidders and two units, there cannot be excess demand for the dual-winner outcome.

Appendix A.2 provides a detailed discussion of the possible outcomes of the 2×2 *ascending package auction* and the 2×2 *ascending uniform-price auction*.

3.5 SINGLE-PACKAGE AUCTIONS

In this section we derive the solution to the *principal-agent* 1×2 *package auction model* if an incentive-compatible direct-revelation mechanism is employed by the auctioneer to sell two units as a single package. It is well known that by the revenue equivalence theorem (Myerson, 1981) the solution then also holds for the *principal-agent* 1×2 *FPSB package auction model* and the *principal-agent* 1×2 *ascending package auction model* in the symmetric and the asymmetric information environment. We begin with the analysis if principals were to bid without agents.

3.5.1 *Principals' Strategies*

The principal's ex-interim expected payoff (P-EXI) in Section 3.3.1 is maximized by a report of $\beta_2\left(v_i(2)\right) = v_i(2)$ for all $v_i(2) \in V(2)$ in case of participating in the incentive compatible 1×2 *direct-revelation mechanism* $\left(F_2\left(v_i(2)\right), T_2\left(v_i(2)\right)\right)$. The same is true for opponent j who truthfully reports $\beta_2\left(v_j(2)\right) = v_j(2)$. Here, the corresponding expected profit in indirect notation then corresponds to $\Pi\left(v_i(2), v_j(2), v_i(2); F_2\left(v_i(2)\right)\right) = F_2\left(v_i(2)\right) \cdot v_i(2) - T\left(v_i(2)\right)$. In this expression $T\left(v_i(2)\right) = F_2\left(v_i(2)\right) \cdot v_i(2) - \int_{\underline{v}(2)}^{v_i(2)} F_2(x)dx + T\left(\underline{v}(2)\right)$ is the ex-interim expected payment to the auctioneer given a report of $\beta_i(2) = v_i(2)$ and $F_2(\cdot)$ takes into account the probability that bidder i's value draw for the package $v_i(2)$ exceeds the opponent's draw. Following Mirrlees (1971), ex-interim expected profit can be expressed as,

$$\Pi\left(v_i(2), v_j(2), v_i(2); F_2(\cdot)\right) = \int_{\underline{v}(2)}^{v_i(2)} F_2(x)dx - T\left(\underline{v}(2)\right). \quad \text{(SPA-P-EXI)}$$

In the 1×2 *ascending package auction* the principal's equilibrium strategy is to stay active on the package of two units until the price reaches her valuation, i.e., $\beta_2\left(v_i(2)\right) = v_i(2)$. The ex-interim expected payoff then corresponds to (SPA-P-EXI). Similar, the equilibrium bidding function in the 1×2 *FPSB package auction* is well-known to be $\beta_2\left(v_i(2)\right) = v_i(2) - F_2\left(v_i(2)\right)^{-1} \cdot \int_{\underline{v}(2)}^{v_i(2)} F_2(x)dx$. Thus, reporting a valuation of $v_i(2)$ to this function results in the same ex-interim expected profit of (SPA-P-EXI).

Example 1. *Let us demonstrate these relations with an example in which the package valuation is uniformly distributed with* $v_i(2) \sim U[160, 190]$ *and* $F_2\left(v_i(2)\right) = \frac{v_i(2)-160}{30}$. *In this setting the principal's equilibrium bidding strategy in the* 1×2 *FPSB package auction is* $\beta_2\left(v_i(2)\right) = v_i(2)/2 + 80$

and $\beta_2\left(v_i(2)\right) = v_i(2)$ *in the* 1×2 *ascending package auction. The ex-interim expected payment under both auction formats is* $T\left(v_i(2)\right) = \frac{160}{30} \cdot \left(v_i(2) - 160\right)$. *Moreover, ex-interim expected and ex-ante expected profit correspond to* $\Pi\left(v_i(2), v_i(2)|F_2\left(v_i(2)\right)\right) = \frac{1}{30} \cdot \left(v_i(2) - 160\right)^2$ *and* $\int_{160}^{190} \Pi\left(v_i(2), v_i(2)|F_2\left(v_i(2)\right)\right) \cdot f_2\left(v_i(2)\right) dv_i(2) = 5$*, respectively.*

3.5.2 *Agents' Strategies*

With (exogenous) budget constraints the agent would simply submit a bid of his entire budget for two units on the package, i.e., $\beta_2\left(a_i(2)\right) = a_i(2)$ if facing the 1×2 *direct-revelation mechanism* $\left(Q_2\left(a_i(2)\right), T_2\left(a_i(2)\right)\right)$. This maximizes his chances of winning as the probability of being assigned the package is non-decreasing in the report and the ex-interim expected utility (A-EXI) in Sub 3.3.1 corresponds to $EU\left(a_i(2), a_j(2), v_i(2); Q_2\left(\cdot\right)\right) = Q_2\left(a_i(2)\right) \cdot w\left(v_i(2)\right)$. Here, $Q_2\left(a_i(2)\right)$ denotes the probability that agent i's package budget draw $a_i(2)$ exceeds the opponent's draw and $w\left(v_i(2)\right)$ is the resulting winning-payoff.

3.5.3 *Symmetric Information Principal-Agent Model*

In the symmetric information environment of the *principal-agent* 1×2 *direct-revelation mechanism model* the principal's optimization problem from Section 3.3.2 corresponds to the principal maximizing her expected profit (SPA-P-EXI) from Section 3.3.2 by simply assigning the agent a budget for two units in height of her equilibrium report $\alpha_2\left(v_i(2)\right) = v_i(2)$. The agent will report exactly this budget, $\beta_2\left(v_i(2)\right) = v_i(2)$, to the mechanism, $\left(F_2\left(v_i(2)\right), T_2\left(v_i(2)\right)\right)$, as it maximizes his expected utility as expressed in Section 3.5.2. Analogue, in the *principal-agent* 1×2 *ascending package auction model* the principal implements her equilibrium strategy by allowing the agent to stay active on the package until the price reaches her valuation, $\alpha_2\left(v_i(2)\right) = v_i(2)$. Similarly, in the *principal-agent* 1×2 *FPSB package auction model* the principal provides her agent with a budget in height of her optimal bid of $\alpha_2\left(v_i(2)\right) = v_i(2) - F_2\left(v_i(2)\right)^{-1} \cdot \int_{\underline{v}(2)}^{v_i(2)} F_2\left(v_j(2)\right) dv_j(2)$ to implement the equilibrium.

3.5.4 *Asymmetric Information Principal-Agent Model*

Now, we determine the optimal contract of contingent budget constraints and transfers between principal and agent in the asymmetric information setting of the *principal-agent* 1×2 *direct-revelation mechanism model*. The principal's optimization problem (PA-SB) in the asymmetric information setting in Section 3.3.3 if facing the incentive compatible direct revelation mechanism $\left(F_2\left(\beta_2\left(v_i(2)\right)\right), T_2\left(\beta_2\left(v_i(2)\right)\right)\right)$ then corresponds to,

$$\max_{\left(\beta_2\left(v_i(2)\right),\mu\left(v_i(2)\right)\right)} \quad E_{v_i(2)\in Z(2)}\Bigg(G_2\left(\beta_2\left(v_i(2)\right)\right)\cdot\left(v_i(2)-\beta_2\left(v_i(2)\right)\right)+ + \int_{\underline{v}(2)}^{\beta_2\left(v_i(2)\right)} G_2(x)dx - T\left(\underline{v}(2)\right) - \mu_2\left(v_i(2)\right)\Bigg) \tag{SPA-SB}$$

$$\text{subject to} \quad v_i(2) \in \underset{\hat{v}_i(2)\in Z(2)}{\operatorname{argmax}}\ G_2\left(\beta_2\left(\hat{v}_i(2)\right)\right)\cdot w\left(v_i(2)\right) + \mu_2\left(\hat{v}_i(2)\right) \tag{SPA-SB-IC}$$

$$\beta_2\left(v_i(2)\right) \leq v_i(2) \text{ for all } v_i(2) \in Z(2). \tag{SPA-SB-NL}$$

The principal maximizes her ex-ante expected profit for the package in (SPA-SB)[22] given the agent's incentive compatibility constraint (SPA-SB-IC) subject to no-loss condition (SPA-SB-NL) for the value-maximizing agent who does not internalize the package price. Be aware, in equilibrium we assume the opposing firm j to truthfully report $\beta_2\left(v_j(2)\right) = v_j(2)$ to the incentive compatible direct revelation mechanism. The solution to the principal's optimization problem above is provided in the next theorem.

Proposition 1. *The principal's optimal contract $\left(\beta_2\left(v_i(2)\right), \mu_2\left(v_i(2)\right)\right)$ for the principal-agent 1×2 direct-revelation mechanism model in the asymmetric information setting corresponds to a report of*

$$\beta_2\left(v_i(2)\right) = v_i(2) \tag{SPA-SB-R}$$

with corresponding transfer

$$\mu_2\left(v_i(2)\right) = w\left(\overline{v}(2)\right) - \int_{v_i(2)}^{\overline{v}(2)} G_2(x)\cdot\frac{\partial w(x)}{\partial x}dx - G_2\left(v_i(2)\right)\cdot w\left(v_i(2)\right) \tag{SPA-SB-T}$$

[22] Note that, based on the indirect notation of ex-interim expected profit in Section 3.5.1, $\Pi_i\left(\beta_i(2), v_j(2), \beta_i(2); Q_2\left(\beta_i(2)\right)\right) = Q_2\left(\beta_i(2)\right)\cdot\beta_i(2) - T\left(\beta_i(2)\right)$, which again can be expressed as $\Pi_i\left(\beta_i(2), v_j(2), \beta_i(2); Q_2(\cdot)\right) = \int_{\underline{v}(2)}^{\beta_i(2)} G_2(x)dx - T\left(\underline{v}(2)\right)$. It follows that $T\left(\beta_i(2)\right) = G_2\left(\beta_i(2)\right)\cdot\beta_i(2) - \int_{\underline{v}(2)}^{\beta_i(2)} G_2(x)dx + T\left(\underline{v}(2)\right)$. Substituting the last equation into $\Pi_i\left(\beta_i(2), v_j(2), \beta_i(2); Q_2\left(\beta_i(2)\right)\right) = Q_2\left(\beta_i(2)\right)\cdot\beta_i(2) - T\left(\beta_i(2)\right)$ and writing $\beta_i(2) = \beta_2\left(v_i(2)\right)$ as well as $m_i(2) = \mu_2\left(v_i(2)\right)$ results in the stated expression for ex-interim expected payoff.

for all $v_i(2) \in Z(2)$ *and ex-ante participation constraint*

$$\int_{\underline{v}(2)}^{\overline{v}(2)} \left(G_2\left(v_i(2)\right) \cdot \left(w\left(v_i(2)\right) + \frac{\partial w\left(v_i(2)\right)}{\partial v_i(2)} \cdot \frac{G_2\left(v_i(2)\right)}{g_2\left(v_i(2)\right)} \right) + \int_{\underline{v}(2)}^{v_i(2)} G_2(x)dx \right) \cdot$$
$$\cdot g_2\left(v_i(2)\right) dv_i(2)$$
$$\geq \tag{SPA-SB-PC}$$
$$T\left(\underline{v}(2)\right) + w\left(\overline{v}(2)\right).$$

In the proof of Proposition 1 we reformulate the principal's maximization problem (SPA-SB) subject to the constraints (SPA-SB-IC) and (SPA-SB-NL) to an information setting in which the principal does not know the package valuation but its exact support $V(2)$ as well as *dual-winner efficiency*. The derivation of the solution is then based on Burkett (2016) and similar to the discussion of contracts with perfect commitment in Krishna and Morgan (2008). Consecutively, we extend the optimal contract to the asymmetric information environment with unknown package valuation and uncertain boundaries, but common knowledge of *dual-winner efficiency*, as described in Section 3.3.3.

The transfer structure in Theorem 1 ensures that no agent with true package value $v_i(2) \in V(2)$ has an incentive to misreport and exceed his designated truthful report. As neither principal nor agent know the precise package valuation range $V(2)$, the optimal reporting function $\beta_2\left(v_i(2)\right) = v_i(2)$ cannot be implemented cheaper. The optimal contract $\left(\beta_2\left(v_i(2)\right), \mu_2\left(v_i(2)\right)\right)$ is the same for the *principal-agent* 1×2 *FPSB package auction model* and the *principal-agent* 1×2 *ascending package auction model*. This insight is similar to the findings in Burkett (2015) and Burkett (2016) that the optimal endogenous budget constraint contract is independent of the standard single-object auction format employed by the auctioneer. Let us continue our example from Section 3.5.1 to illustrate these results.

Example 1 (Continued). *We now assume* $v_i(2) \sim U[150, 200]$ *and* $G_2\left(v_i(2)\right) = \frac{v_i(2)-150}{50}$. *Moreover, suppose* $w\left(v_i(2)\right) = \rho \cdot v_i(2)$ *with* $0 < \rho < 1$ *such that the agent values the package less than the value to the firm, which could be in the billions. The optimal report to the mechanism is* $\beta\left(v_i(2)\right) = v_i(2)$ *with corresponding transfer of* $m\left(v_i(2)\right) = \frac{1}{100} \cdot \rho \cdot \left(40000 - v_i(2)^2\right)$. *As* $T\left(\underline{v}(2)\right) = 0$ *for our relevant auction formats, the principal's ex-ante expected profit is weakly greater than zero for* $\rho \leq \frac{1}{11}$. *The optimal contract restricts the report to truth-telling which may be very expensive to implement.*

If the magnitude of the agency-bias, $w(\cdot)$, in our model is too strong, participation in the 1×2 *direct-revelation mechanism* is ex-ante unprofitable for the firms. This result continues to hold in the asymmetric information environment of the *principal-agent* 2×2 *FPSB package auction model* in the next section.

3.6 COMBINATORIAL FPSB PACKAGE AUCTION

In this section, we first analyze the equilibrium bidding strategy of the principal (without an agent) if she had full information about the valuations in the 2×2 *FPSB package auction*. Then, we discuss equilibrium bidding strategy of the agent (without principals) and show the bias with respect to the principal's equilibrium bidding strategy and the resulting inefficiencies. Next, we examine the principal's options to align the incentives of the agent with budget constraints in the symmetric information environment of the *principal-agent* 2×2 *FPSB package auction model*. Finally, we discuss the optimal contract in the asymmetric information setting.

3.6.1 *Principals' Strategies*

We will first derive necessary and sufficient conditions for the *dual-winner equilibrium* in Propositions 2 and 3, respectively. This is a Bayesian Nash equilibrium that solves the principal's maximization problem (P-EXI) in Section 3.3.1 for the 2×2 *FPSB package auction* in which the efficient dual-winner outcome results for all possible valuations $v_i, v_j \in V$. Proposition 4 states conditions under which the *single-winner equilibrium* solves the principal's maximization problem. This equilibrium is an adaptation of the principal's Bayesian Nash equilibrium for the 1×2 *FPSB package auction* from Section 3.5.1. Finally, Proposition 5 establishes a condition for which the efficient *dual-winner equilibrium* is payoff-dominant. We will first start with the necessary conditions for the *dual-winner equilibrium*.

Proposition 2. *Given principal i with full information about her vector of valuations $v_i \in V$, suppose the bids b_i constitute a dual-winner equilibrium in the 2×2 FPSB package auction. Then it must be true that*

1) $b_i(1) = \beta_1$ *is constant over* $v_i(1) \in V(1)$

2) $\beta_1 \in \left[\overline{v}(2) - \underline{v}(1), \underline{v}(1)\right]$

3) $2 \cdot \beta_1 = sup_{v_i(2)}\left\{\beta_2\left(v_i(2)\right)\right\}.$

In the *dual-winner equilibrium* both bidders must pool at a constant single-unit bid of β_1 from condition 1) within the range in condition 2). Moreover, the single-unit bid bounds in condition 2) show that *dual-winner efficiency* is necessary for the existence of a *dual-winner equilibrium*. Condition 3) ensures that the auctioneer always selects the dual-winner outcome in equilibrium. Let us now derive sufficient conditions for the *dual-winner equilibrium* in the next theorem.

Proposition 3. *Assume dual-winner efficiency is given, then for a principal i with full information about her valuations $v_i \in V$, the vector of bids $b_i = \left(\beta_1, \beta_2\left(v_i(2)\right)\right)$ is a dual-winner equilibrium in the 2×2 FPSB package auction if the following conditions hold:*

1) $\beta_1 \in \left[\overline{v}(2) - \underline{v}(1), \underline{v}(1)\right]$

2) $\beta_2\left(v_i(2)\right)$ *is continuous and strictly increasing on its support* $V(2)$

3) $\beta_2\left(\overline{v}(2)\right) = 2 \cdot \beta_1$

4) $G\left(v_i(2), \beta_1\right) \leq \beta_2\left(v_i(2)\right)$ *for all* $v_i(2) \in V(2)$.

The lower bound $G(.)$ *is defined as:*

$$G\left(v_i(2), \beta_1\right) \equiv \beta_1 + \frac{\beta_1 - \underline{v}(1) \cdot \left(1 - F_2\left(v_i(2)\right)\right)}{F_2\left(v_i(2)\right)}. \qquad \text{(CFPA-S-G)}$$

Conditions 2) and 4) restrain any bidder i's equilibrium bidding function for two units. It is not allowed to fall below the lower bound of $G\left(v_i(2), \beta_1\right)$ in order to support the pooling bid for one unit. This condition ensures that winning the package is less profitable in expectation than obtaining a single unit in equilibrium. For our analysis we focus on the lowest pooling bid for one unit of $\beta_1 = \overline{v}(2) - \underline{v}(1)$. This maximizes the utility of both bidders and therefore serves as a natural focal point for implicit coordination in the *dual-winner equilibrium*. Let us extend our earlier example 1 from the 1×2 *FPSB package auction* in Section 3.5.1 to illustrate the *dual-winner equilibrium*.

Example 2. *Suppose* $v_i(1) \in [110, 150]$ *and as before* $v_i(2) \in [160, 190]$, *both uniformly distributed. The distribution assumptions on the package correspond to the assumptions made in the earlier example in Section 3.5.1. The payoff-dominant pooling bid in the dual-winner equilibrium for both bidders is* $\beta_1 = \overline{v}(2) - \underline{v}(1) = 190 - 110 = 80$. *The upper bound for* $b_i(2)$ *is* 160. *A higher bid would make the auctioneer select the package bid. Remember, in case of a tie, the auctioneer selects the dual-winner outcome. The bidder with the lowest type for the single unit,* $v_i(1) = 110$, *and the highest type for the package,* $v_i(2) = 190$, *has the strongest incentive to deviate. With equilibrium bids* $b_i = (80, 160)$, *her payoff for one unit as well as for the package is 30. The lower bound for* $\beta_2\left(v_i(2)\right)$ *is defined by the function* $G\left(v_i(2), 80\right)$. *For a low value draw of* $v_i(2) = 165$ *the lower bound is* $G(165, 80) = 10$, *and an equilibrium bid for the package is* $b_i(2) \in [10, 160]$. *Each bidder type has to follow this lower bound function in equilibrium to ensure the opponent has no incentive to profitably deviate on the single-winner award, i.e., the payoff in the dual-winner equilibrium of 30 for the lowest type on one unit always needs to be higher or equal to the expected payoff of the package bid. In addition, suppose opponent j bids low, say zero, on the package. Then bidder i could also bid low on the package in an attempt to be able to bid* $\beta_1 < 80$ *on a single object and make a higher profit on one unit in a dual-winner outcome. The lower bound* $G\left(v_i(2), 80\right)$ *avoids such deviations from equilibrium, too.*

The *dual-winner equilibrium* is not the only equilibrium for payoff-maximizing principals in our model, and there is also a *single-winner equilibrium*.

Proposition 4. *For a principal i with full information about her vector of values, $v_i \in V$, the vector of bids b_i is a single-winner equilibrium in the 2×2 FPSB package auction under dual-winner efficiency if the following conditions hold:*

1) $\beta_2\left(v_i(2)\right) = v_i(2) - F_2\left(v_i(2)\right)^{-1} \cdot \int_{\underline{v}(2)}^{v_i(2)} F_2(x)\,dx$

2) $b_i(1) \in \left[0, \underline{v}(2) - \overline{v}(1)\right)$.

The equilibrium bid on the package in the *single-winner equilibrium* of the 2×2 *FPSB package auction* from condition 1) corresponds to the equilibrium strategy of the 1×2 *FPSB package auction* from Section 3.5.1. Condition 2) ensures any bidder can enforce the *single-winner equilibrium* by making the dual-winner outcome unprofitable for the opponent. Let us include this equilibrium in our example.

Example 2 (Continued)**.** *Remember from our ongoing example that the equilibrium bid on the package is* $\beta_2\left(v_i(2)\right) = v_i(2)/2 + 80$. *The equilibrium bid on one unit must be low enough to veto the dual-winner award for all possible bidder types, i.e.,* $b_i(1) \in [0, 10)$. *If principal i with type* $v_i = (130, 165)$ *bids* $b_i(1) = 9$ *and* $b_i(2) = 162.5$*, even opponent j with the highest value draw for one unit,* $v_j(1) = 150$*, cannot profitably implement the dual-winner outcome as the sum of single-unit bids is strictly smaller than the lowest equilibrium bid on two units, i.e.,* $150 + 9 < 160$.

Using payoff-dominance, we can show that for specific distributional properties, payoff-maximizing bidders prefer to coordinate on the *dual-winner equilibrium* rather than select the *single-winner equilibrium*.

Proposition 5. *Any principal i with full information about the values support V ex-ante prefers the dual-winner equilibrium to the single-winner equilibrium of the 2×2 FPSB package auction model for all $v_i \in V$ under dual-winner efficiency iff the expected value of two units exceeds two times the bidder-optimal dual-winner equilibrium pooling bid* $\beta_1 = \overline{v}(2) - \underline{v}(1)$*, i.e.,* $2 \cdot \left(\overline{v}(2) - \underline{v}(1)\right) < \int_{\underline{v}(2)}^{\overline{v}(2)} f_2(x) \cdot x dx$.

Intuitively, the expected package valuation exceeds twice the pooling bid if the probability for large package value draws is high. If high value draws for the package are likely, however, any bidder prefers the *dual-winner equilibrium* to the *single-winner equilibrium* because, for her given vector of valuations v_i, she is likely to lose in the latter equilibrium. Given the payoff-dominance condition from Proposition 5 is satisfied, we analyze the principal's possibilities to implement the *dual-winner equilibrium* in the symmetric and asymmetric information environments in Sections 3.6.3 and 3.6.4, respectively. Let us finally examine if the condition is satisfied in our earlier example.

Example 2 (Continued)**.** *The condition is satisfied as* $2 \cdot 80 < 175$. *In this case the principal prefers the dual-winner equilibrium as it yields higher expected payoff.*

Next, we turn to the analysis of the agent's equilibrium strategy in the 2×2 *FPSB package auction*.

3.6.2 *Agents' Strategies*

We analyze the agent's strategy that solves the maximization problem (A-EXI) in Section 3.3.1 for the 2×2 *FPSB package auction* assuming the budget constraints to be random variables. This assumption is sufficient to highlight the bias of the agent and the resulting inefficiency of the auction. In Section 3.6.3 we will then analyze if a principal can set budget constraints such that the agents implement her equilibrium bidding strategy. Let us first provide a few useful lemmas for the 2×2 *FPSB package auction* that eliminate weakly dominated strategies. These lemmas allow for a succinct analysis of equilibrium bidding strategies.

Lemma 1. *With full knowledge about his vector of values $v_i \in V$ and exogenously determined budgets of $a_i \in A$ with which agent i is provided, any strategy that involves package-bids of $\hat{b}_i(2) < a_i(2)$, is weakly dominated by any strategy including a bid on two units of $b_i(2) = \alpha_i(2)$ independent of his bid on one unit in the 2×2 FPSB package auction.*

As agents are value-maximizing and strictly prefer the package to one unit, there is no reason for them to not fully bid their budget on two units.

Lemma 2. *For agent i with full information about valuations of $v_i \in V$ and exogenously determined vector of budgets $a_i \in A$, the set of strategies $\hat{b}_i = \big(\hat{b}_i(1), a_i(2)\big)$ with $\hat{b}_i(1) \in \big(0, a_i(1)\big)$ is weakly dominated by the strategy-set $b_i = \big(b_i(1), a_i(2)\big)$ with $b_i(1) \in \big\{0, a_i(1)\big\}$ in the 2×2 FPSB package auction.*

A value-maximizing agent either expects his budget for the package to be high enough to win the package, and he vetoes the dual-winner outcome with a bid of zero on one unit, or he beliefs coordination on the dual-winner outcome allows his to win at least one unit, then he maximizes his chances by bidding his full budget for one unit. Bids in the interval $b_i(1) \in \big(0, a_i(1)\big)$ are always weakly dominated. With Lemmas 1 and 2, we can derive the agent's equilibrium strategy in both 2×2 *package auction* formats. Our first observation of the 2×2 *FPSB package auction* is an ex-post equilibrium in which agents do not coordinate.

Proposition 6. *It is an ex-post equilibrium for agent i with full knowledge about his vector of values $v_i \in V$ and exogenously determined budget constraints of $a_i \in A$ to submit a vector of bids $b_i = \big(0, a_i(2)\big)$ in the 2×2 FPSB package auction.*

Intuitively, any agent i's opponent j would only be willing to coordinate on one unit if his valuation for two packages was low. In this case, however, it would be a best response for bidder i not to coordinate, but try to win the package of both units independent of both agents' actual values. In the ex post equilibrium arbitrary risk-averse bidders cannot coordinate on winning one unit with certainty.[23] Interestingly, $b_i = \big(0, a_i(2)\big)$ is also the unique Bayesian Nash equilibrium as we will show next.

Proposition 7. *The strategy $b_i = \big(0, a_i(2)\big)$ is the unique Bayesian Nash equilibrium strategy for agent i with full information about his values $v_i \in V$ and vector of exogenously determined package budgets $a_i \in A$ in the 2×2 FPSB package auction.*

Note that the agent does not respond to the valuations of the firm v_i, but only to the budgets a_i that he is given as long as $v_i(1) < v_i(2)$ and $a_i(1) \leq a_i(2)$. Therefore, the agent's unique equilibrium bidding strategy is independent of the efficiency environment considered. More importantly, the unique equilibrium of the agent in the 2×2 *FPSB package auction* is in conflict with the efficient *dual-winner equilibrium* of the principal. In the next Section 3.6.3 we analyze the possibilities of the principal to implement her dual-winner equilibrium bidding strategy by constraining the agent with budgets in the symmetric information environment.

3.6.3 *Symmetric Information Principal-Agent Model*

In this section, we assume the symmetric information environment from Section 3.3.2 in which the supports of the prior distributions and the value draws are known to principal and agent. Given the principal's payoff-dominance condition from Proposition 5 is satisfied, the *dual-winner equilibrium* achieves a higher value of the principal's objective function (PA-D) in Section 3.3.1 than the *single-winner equilibrium* for the 2×2 *FPSB package auction*. We show that even in this symmetric information setting budget constraints can be insufficient to implement the principal's *dual-winner equilibrium* as a solution to her optimization problem (PA-D) in Section 3.3.2 for the *principal-agent* 2×2 *FPSB package auction model*.

In Section 3.6.2, we assumed agent i's budget constraints, $a_i \in A$ with $a_i(1) \leq a_i(2)$, to be exogenous random draws from the distribution $Q(a_i)|_{a_i(1) \leq a_i(2)}$. As we have shown, agents would not bid on a single unit in the unique equilibrium with these types of budget constraints. There is always a set of possible values $v_i \in V$ and budgets $a_i \in A$ with a high enough package budget constraint such that only bidding on two units yields higher expected utility for the agent i, independent of the opponent's strategy. Therefore, the incentive compatibility constraint (CFPA-D-IC) in Section 3.3.2 would not be satisfied for the 2×2 *FPSB package auction*. The only option to counteract these incentives is to assign a relatively low budget constraint for two units for all package valuations. In particular, budget constraints $a_i = \big(a_i(1), a_i(2)\big)$ with upper bounds for the set of single-unit and package constraints of $A(1)$

[23] We refer to risk aversion as is implied by a concave utility function $w(\cdot)$ over possible outcomes of the auction (lottery) $\big\{0, v_i(1), v_i(2)\big\}$ for some agent i. Regarding the agent's ex-post equilibrium in Proposition 7 one could expect a risk averse agent to prefer the certain dual-winner outcome with utility of $w\big(v_i(1)\big)$ over the lottery of winning the single-winner outcome with expected utility of $w\big(v_i(2)\big) \cdot F_2\big(v_i(2)\big)$ and not winning at all.

and $A(2)$, respectively, of the form $\overline{a}(2) < \overline{a}(1)$ are required. Given opponent j wants to coordinate on the dual-winner outcome, even an agent i with the highest possible package budget of $\overline{a}(2)$ cannot win two units with certainty by not bidding on the small package. With the help of the next lemma we can reformulate the incentive compatibility constraint (CFPA-D-IC).

Lemma 3. *A principal i with full knowledge of her valuations $v_i \in V$ can direct her agent on the dual-winner outcome in the symmetric information environment of the principal-agent 2×2 FPSB package auction model by assigning her package-dependent budget constraints a_i that satisfy*

1) $\overline{a}(2) < \overline{a}(1)$

2) $\alpha_1\left(v_i(1)\right) + \underline{a}(1) \geq \alpha_2\left(v_i(2)\right)$ *for all* $v_i \in V$

3) $u_i(1) \geq u_i(2) \cdot P\left(\alpha_2\left(v_i(2)\right) \geq \alpha_2\left(v_j(2)\right) \cap \alpha_2\left(v_i(2)\right) \geq \alpha_1\left(v_j(1)\right)\right)$ *for all* $v_i \in V$.

In Lemma 3 condition (1)) implements budgets that ensure the agents are able to coordinate on the dual-winner outcome and (2)) lets the auctioneer select the respective outcome for all budgets. The expression $P(\cdot)$ denotes the probability of agent i's package budget constraint exceeding opponent j's budget constraints for one and two units. In order of keeping the lemma traceable we do not express the corresponding probability in terms of the joint distribution function $F\left(v_j\right)|_{v_j(1)<v_j(2)}$. Formally, the principal's optimization problem (PA-D) from Section 3.3.2 for the *principal-agent* 2×2 *FPSB package auction model* can then be denoted as

$$\begin{aligned} &\text{Implement} && a_i \text{ that correspond to the bidding functions in Proposition 3} && \text{(CFPA-D)} \\ &\text{subject to} && a_i \in A \text{ satisfy Lemma 3} && \text{(CFPA-D-IC)} \\ &&& \alpha_i(1) \leq v_i(1). && \text{(CFPA-D-NL)} \end{aligned}$$

The conditions in Lemma 3 now replace the agent's incentive compatibility constraint. Note that in the above optimization problem, in particular in (CFPA-D), we assume the opposing firm j to successfully implement the *dual-winner equilibrium* from Proposition 3. Nevertheless, in Proposition 8 we derive a distributional condition under which the principals cannot implement the *dual-winner equilibrium* as a solution to the optimization problem (CFPA-D) with constraints (CFPA-D-IC) and (CFPA-D-NL).

Proposition 8. *There is no vector of budget constraints a_i with which principal i with full information about her valuations of $v_i \in V$ can implement the dual-winner equilibrium under dual-winner efficiency in the symmetric information environment of the principal-agent 2×2 FPSB package auction model if the following inequality is true:* $2 \cdot \left(\overline{v}(2) - \underline{v}(1)\right) > \overline{v}(1)$.

It follows that a solution in which principal and agent aim for the dual-winner outcome cannot exist under reasonable ranges of valuations. Let us illustrate this scenario within our example.

Example 2 (Continued)**.** *In our leading example with $v_i(1) \in [110, 150]$ and $v_i(2) \in [160, 190]$, both uniformly distributed, it is easy to verify that the condition in Proposition 8 is satisfied as $2 \cdot 80 > 150$. Therefore, the principal cannot implement her dual-winner equilibrium.*

Budget constraints are not always sufficient to align agent strategies in the *principal-agent* 2×2 *FPSB package auction model*, even if the principal knows the valuations. Intuitively, any firm faces the following trade-off: On the one hand, the principal has to bid high enough on two units in equilibrium to prohibit the opponent from making a profit by deviating from the *dual-winner equilibrium*. On the other hand, the agent can only be directed on bidding for one unit if his budget constraint on the package is low enough. Both requirements cannot always be met simultaneously.

Moreover, note that the impossibility result in Proposition 8 highlights consequences of the principal-agent problem that are specific to the multi-unit package auction environment. In the standard single-package auctions in Section 3.5, for example, the principal can simply provide the agent with a budget in height of her optimal bid for the package and solve the agency problem under symmetric information within our *principal-agent* 1×2 *FPSB package auction model* as shown in Section 3.5.3. As a direct consequence the principal can easily implement the *single-winner equilibrium* of the 2×2 *FPSB package auction* by assigning the agent no budget for one unit and the same budget for the package as in the symmetric information setting of the *principal-agent* 1×2 *FPSB package auction model* in Section 3.5.3. Therefore, whenever the implementation of the *dual-winner equilibrium* in the *principal-agent* 2×2 *FPSB package auction model* is not possible, the *single-winner equilibrium* can always be enforced, which supports demand inflation.

3.6.4 *Asymmetric Information Principal-Agent Model*

In this section we derive the optimal contract that solves the principal's optimization problem PA-SB in Section 3.3.3 for the asymmetric information environment of the *principal-agent* 2×2 *FPSB package auction model* and implements the principal's *dual-winner equilibrium* from Proposition 3. Taking into account the transfer payments principal i has to direct her agent to submit the following reports to the auction,[24]

$$\begin{aligned} b_i(1) &= \overline{v}(2) - \underline{v}(1) + m_i(1); \\ b_i(2) &= 2 \cdot \left(\overline{v}(2) - \underline{v}(1) + m_i(1)\right). \end{aligned} \tag{CFPA-SB-bids}$$

It is straightforward to verify that the above bids satisfy the sufficient conditions for the principal's *dual-winner equilibrium* in Proposition 3 as long as $m_i(1)$ is not too large. Moreover, the reports can be expressed as functions of the random variable d as defined in Section 3.3.3, i.e., $b_i(1) = \beta_1(d)$ and $b_i(2) = \beta_2(d)$ with $\beta_1(d) = d + m_i(1)$ and $\beta_2(d) = 2 \cdot \left(d + m_i(1)\right)$. The

[24] Incorporating the transfer payments, m_i, into the principal's profit function, $\pi_i(l) = v_i(l) - p_i(l) - m_i(l)$, the *dual-winner equilibrium* bids from Proposition 3 adjust accordingly.

optimal contract needs to specify four functions which are solely determined by the parameter $d \in D$: $b_i(l) = \beta_l(d)$ and $m_i(l) = \mu_l(d)$ for all $l \in L$. The principal's optimization problem (PA-SB) in the asymmetric information setting from Section 3.3.3 for the *principal-agent* 2×2 *FPSB package auction model* in this case corresponds to

$$\begin{aligned} &\text{Implement} && b_i \text{ and } m_i \text{ that satisfy (CFPA-SB-bids) given } G(v_i)|_{v_i(1)<v_i(2)} \\ & && \text{for all} v_i \in Z \qquad \text{(CFPA-SB)} \\ &\text{subject to} && d \in \operatorname*{argmax}_{\hat{d} \in D} EU\left(\beta\left(\hat{d}\right), \mu\left(\hat{d}\right), v_i; G(v_j)|_{v_j(1)<v_j(2)}\right). \\ & && \text{(CFPA-SB-IC)} \end{aligned}$$

The bidding functions (CFPA-SB-bids) satisfy $\beta_i(1) \leq v_i(1)$ as long as $m_i(1)$ is not too large and we can ignore the no-loss constraint (PA-SB-NL) of the principal's optimization problem (PA-SB) in the asymmetric information setting in Section 3.3.3. Note that the incentive compatibility constraint (CFPA-SB-IC) can always be met with transfers and does not need to be reformulated. Moreover, we assume, explicitly in (CFPA-SB), that principal j implements the bidding function from (CFPA-SB-bids). The optimal contract that solves the optimization problem (CFPA-SB) is summarized in the next proposition.

Proposition 9. *The principal's optimal contract $(\beta(d), \mu(d))$ to implement the dual-winner equilibrium in the asymmetric information environment of the principal-agent 2×2 FPSB package auction model given dual-winner efficiency corresponds to reports of*

$$\beta_1(d) = d \text{ and } \beta_2(d) = 2 \cdot d \text{ for all } d \in D \qquad \text{(CFPA-SB-R)}$$

with transfer payments of

$$\mu_1 = w\left(\overline{v}(2)\right) - w\left(\underline{v}(1)\right) \text{ and } \mu_2 = 0 \text{ for all } d \in D \qquad \text{(CFPA-SB-T)}$$

and ex-ante participation constraint

$$E_{v_i}\left(v_i(1) - d - w\left(\overline{v}(2)\right) + w\left(\underline{v}(1)\right)\right) \geq 0 \text{ for all } v_i \in Z \text{ and } d \in D. \qquad \text{(CFPA-SB-PC)}$$

The principal does not need to set incentives for bidding on the package but needs to pay a constant amount that compensates the agent for not winning two units. This transfer payment is solely a function of the agent's utility and does not depend on the implemented pooling price. Therefore, it is always optimal to condition on the lowest possible pooling price as higher prices cannot be achieved with lower transfer costs. Unfortunately, the agency costs caused by wages in the *principal-agent* 2×2 *FPSB package auction model* can be very high as the proposition shows. However, if any principal i now sets the optimal menu of budget constraints and payments, the set of implementable *dual-winner equilibria* becomes a subset of the original set.

This becomes clear by looking at the range of implementable pooling prices: $\beta_1 \in \left[\overline{v}(2) - \underline{v}(1) + \mu_1, \underline{v}(1) - \mu_1\right]$.

Note, that for the given upper bound, $\overline{d} = \overline{\overline{v}(2) - \underline{v}(1)}$, of the random draw d, the optimal transfer, $\mu_1 = w\left(\overline{\overline{v}(2)}\right) - w\left(\underline{\underline{v}(1)}\right)$, increases as the magnitude of the agent's value-maximizing motive, $w(.)$, rises. With larger μ_1 the lower bound of supportable pooling prices, $\overline{v}(2) - \underline{v}(1) + \mu_1$, increases whereas the upper bound, $\underline{v}(1) - \mu_1$, decreases. Therefore, it can happen that there is no range of supportable pooling prices $\left[\overline{v}(2) - \underline{v}(1) + \mu_1, \underline{v}(1) - \mu_1\right]$. Also, the ex-ante participation constraint (CFPA-SB-PC) of Proposition 9 will not be satisfied anymore. In this case the principal cannot implement an efficient *dual-winner equilibrium* even with contingent transfers in the asymmetric information setting.

As in the asymmetric information environment of the *principal-agent* 1×2 *FPSB package auction model* in Section 3.5.4, if the agency bias is too high, the principals cannot implement their desired equilibrium. Furthermore, note that in the implementation of the *single-winner equilibrium* the principal can simply provide her agent with no budget and no transfer payment for the single unit but with the same contract for the package as in Corollary 1 of the *principal-agent* 1×2 *FPSB package auction model* under asymmetric information.[25] Unlike the implementation of the *single-winner equilibrium* in the symmetric information environment, the agency-bias might be too high for

[25] Considering transfer payments to implement the *dual-winner equilibrium* and the *single-winner equilibrium* in the asymmetric information environment, the payoff-dominance condition for the principal from Proposition 5 changes. Suppose agency costs are not too high such that both equilibria can be implemented and the regularity condition of $\partial\left(\frac{w'\left(v_i(2)\right)}{g_2\left(v_i(2)\right)}\right) / \partial v_i(2) > 0$ is satisfied. Then the difference in ex-interim expected profit between the *dual-winner equilibrium* and the *single-winner equilibrium* for each $v_i \in V$ is:

$$\begin{aligned}\Delta_T\left(v_i(1), v_i(2)\right) = \quad & v_i(1) - d + w\left(\underline{v}(1)\right) - \\ & - \int_{\underline{v}(2)}^{v_i(2)} G_2(x)\, dx - G_2\left(v_i(2)\right) \cdot \left(w\left(v_i(2)\right) + w'\left(v_i(2)\right) \cdot \frac{G_2\left(v_i(2)\right)}{g_2\left(v_i(2)\right)}\right).\end{aligned} \tag{3.6.1}$$

Function (3.6.1) is strictly increasing in its first argument and strictly decreasing in its second argument, and therefore takes its maximum value at $v_i = \left(\overline{v}(1), \underline{v}(2)\right)$ and its minimum value at $v_i = \left(\underline{v}(1), \overline{v}(2)\right)$. The minimum is strictly positive and therefore the *dual-winner equilibrium* strictly preferred to the *single-winner equilibrium* for all $v_i \in Z$ whenever the following condition is satisfied:

$$2 \cdot \overline{d} - w\left(\underline{v}(1)\right) + w\left(\overline{v}(2)\right) + \frac{w'\overline{v}(2)}{g_2\left(\overline{v}(2)\right)} < \int_{\underline{v}(2)}^{\overline{v}(2)} x \cdot g_2(x)\, dx. \tag{3.6.2}$$

The LHS of (3.6.2) strictly exceeds the LHS of the principal's payoff-dominance condition in Proposition 5. This implies that expected value draws for the package need to be even higher for the *dual-winner equilibrium* to be preferred to the *single-winner equilibrium*. In the implementation of the *dual-winner equilibrium* the principal incurs relatively higher agency costs than in the contract for the *single-winner equilibrium* making the payoff-dominance condition harder to satisfy.

an ex-ante profitable contract to exist and each firm i might not participate in the auction at all. In the next section we demonstrate that such negative results do not occur in the *principal-agent* 2×2 *ascending package auction model*.

3.7 COMBINATORIAL SECOND-PRICE PACKAGE MECHANISMS

In what follows we will first extend our analysis to the *principal-agent* 2×2 *ascending package auction model*. The structure of this section follows the same logic as the previous Section 3.6 with an additional section at the end in which we also analyze the benchmark *principal-agent* 2×2 *VCG mechanism model*. The solution and its comparison to the *principal-agent* 2×2 *FPSB package auction model* and the *principal-agent* 2×2 *ascending package auction model* enable us to derive our main insights and generalize the findings to non-combinatorial auction formats in Section 3.8, for example.

3.7.1 *Principals' Strategies*

We start with the characterization of the principal's *dual-winner equilibrium* that solves the maximization problem (P-EXI) in Section 3.3.1 for the 2×2 *ascending package auction* if the principal had full information and were to bid alone in the auction.

Proposition 10. *In the ex-post dual-winner equilibrium of the* 2×2 *ascending package auction under dual-winner efficiency, a principal i with full information about her vector of valuations* $v_i \in V$ *starts bidding only on one unit until the respective price reaches her valuation of* $b_i(1) = v_i(1)$. *If the opposing bidder j starts bidding on two units, bidder i becomes active on the package and remains until the opponent drops out or until her respective valuation is reached,* $b_i(2) = v_i(2)$.

In the equilibrium described above there is no demand for the package in the beginning and the auction terminates immediately at zero price for the bidders. Becoming active on the package constitutes an off-equilibrium threat that is not carried out in equilibrium. The last fact distinguishes the *dual-winner equilibrium* of the 2×2 *ascending package auction* from the one of the 2×2 *FPSB package auction* in which the "'threat-bid"' on the package is actually submitted in equilibrium, although it does not become part of the winning allocation. Similar to the 2×2 *FPSB package auction*, there is also a *single-winner equilibrium*.

Proposition 11. *In an ex-post single-winner equilibrium of the* 2×2 *ascending package auction under dual-winner efficiency, any principal i with full knowledge about her values* $v_i \in V$ *remains active on the single unit before its price reaches* $p_c(1) = \underline{v}(2)/2$ *and simultaneously bids on the package of two units until the respective price reaches her valuation of* $b_i(2) = v_i(2)$.

The bid on the package in the *single-winner equilibrium* of the 2×2 *ascending package auction* corresponds to the equilibrium bid in the 1×2 *ascending* *package auction* as shown in Section 3.5.1. Any bidder can enforce the *single-winner equilibrium* by simply dropping out from the dual-winner outcome before the allocation becomes winning. However, the *dual-winner equilibrium* always strictly dominates the *single-winner equilibrium* in payoff as the next corollary demonstrates.

Corollary 1. *Any principal i with full information about the valuations $v_i \in V$ ex-ante strictly prefers the dual-winner equilibrium to the single-winner equilibrium of the 2×2 ascending package auction for all $v_i \in V$ under dual-winner efficiency.*

Be aware though, that the *single-winner equilibrium* is not based on weakly-dominated strategies as is straightforward to show.[26] Although the 2×2 *ascending package auction* is characterized by a similar equilibrium selection problem than the 2×2 *FPSB package auction*, the former auction format possesses two advantages. First, the *dual-winner equilibrium* strictly dominates the *single-winner equilibrium* in profit and therefore serves as a natural focal point for the bidders to choose. Second, the ascending combinatorial mechanism possesses coordination advantages in the sense that bidders can observe their opponents' equilibrium choices and adjust accordingly. This means they can see if the opponent wants to coordinate on a *dual-winner equilibrium*. If this is not the case, they can switch and still aim for the *single-winner equilibrium*.

To illustrate the last point, suppose bidder i plays the *dual-winner equilibrium,* defined in Theorem 3, and let opponent j chose the *single-winner equilibrium* from Theorem 4. As all package prices are publicly observable, principal i is able to recognize that her opponent is playing a different equilibrium and can adjust her own equilibrium strategy to that of the *single-winner equilibrium*. Note that this robustness of the 2×2 *ascending package auction* against the equilibrium selection problem might serve as an additional reason for each bidder to start trying to coordinate on the *dual-winner equilibrium*.

3.7.2 *Agents' Strategies*

In the analysis of the agent's bidding behavior without principal in the 2×2 *ascending package auction*, let us first introduce an adapted definition of *straightforward bidding*:

Definition 1. *(Straightforward bidding of agents in the 2×2 ascending package auction): For any vector of exogenously determined budget constraints $a_i \in A$, agent i with full knowledge about his valuations of $v_i \in V$ begins to bid on the most valuable package of two units and remains active until the*

[26] Suppose the opposing bidder j starts bidding on both packages until her respective valuations are reached. With the *single-winner equilibrium* strategy buyer i obtains profit of $v_i(2) - v_j(2)$ if $v_i(2) > v_j(2)$. In this case, if she follows the *dual-winner equilibrium* strategy, i receives profit of $v_i(1) - v_j(2)/2$. Note that both profits cannot be unambiguously ranked and therefore, the *single-winner equilibrium* strategy is not weakly dominated. Moreover, j's initially assumed bidding behavior may be part of a strategy aimed at enforcing the single-winner outcome and therefore not dominated either as is demonstrated in the proof to Proposition 10.

package price reaches his corresponding budget of $p_c(2) = \alpha_i(2)$. As long as he is winning, he does not bid for the single unit. If he is overbid, he starts to bid for the less valuable package and again remains active until the price equals his respective budget of $p_c(1) = \alpha_i(1)$.

Remember that Lemmas 1 and 2 describing the set of the agent's non weakly-dominated strategies for the 2×2 *FPSB package auction* also hold for the 2×2 *ascending package auction*: An agent i's strategies that are not weakly dominated are to remain active on the package until the price equals his corresponding budget and either to quit directly on one unit or to remain active until the respective single-unit budget is reached. With these insights we are able to derive the solution to the agent's maximization Problem (A-EXI) in Section 3.3.1 for the 2×2 *ascending package auction*.

Proposition 12. *In the 2×2 ascending package auction straightforward bidding of agent i with full information about the vector of values $v_i \in V$ and any vector of exogenously determined budgets $a_i \in A$ constitutes an ex-post equilibrium. In this equilibrium the agent with the highest budget for two units does not get active on one unit.*

Similar to Theorem 7 of the 2×2 *FPSB package auction*, Theorem 12 describes an ex-post equilibrium that is robust against risk aversion. In both auction formats, agents never coordinate on winning one unit each, independent of the efficiency environment. The analysis shows that once a bidder does not care about profit, its bidding problem in both 2×2 *package auction formats* is the same: agents will only bid on the large package in equilibrium. Moreover, any bidder i's equilibrium strategy is entirely independent of his vector of valuations $v_i \in V$ and the efficiency setting, but only depends on the height of his budget for the package of $a_i(2) \in A(2)$. In general, the agent's equilibrium behavior of not bidding on one unit leads to a conflict of interest with the principal's *dual-winner equilibrium*. In the next section, we discuss how to overcome this conflict via budget constraints in the *principal-agent* 2×2 *ascending package auction model*.

3.7.3 *Symmetric Information Principal-Agent Model*

Unlike the *principal-agent* 2×2 *FPSB package auction model*, the principal's *dual-winner equilibrium* can be implemented as a solution to her optimization problem (PA-D) in the symmetric information environment from Section 3.3.2 for the *principal-agent* 2×2 *ascending package auction model*, as long as *dual-winner efficiency* is known. Moreover, according to strict payoff dominance in Corollary 1 the *dual-winner equilibrium* achieves higher expected profit than the *single-winner equilibrium* for all $v_i \in V$ and therefore could be interpreted as the unique solution to (PA-D).

Proposition 13. *Any principal i with full knowledge about the valuations of $v_i \in V$ can implement the dual-winner equilibrium with budget constraints of $a_i(1) = v_i(1)$ and $a_i(2) = 0$ in the symmetric information environment of the* *principal-agent* 2×2 *ascending package auction model under dual-winner efficiency.*

This proposition assumes that the opposing principal uses the same budgets to implement the *dual-winner equilibrium*. Furthermore, it is very important to understand that the principal does not have to use a budget for two units of $a_i(2) = v_i(2)$ to implement the double-unit threat from Proposition 10 as this is never realized in equilibrium anyways. Given the agent of firm i only bids on one unit, firm j has no incentive to deviate on two units anyway and vice versa as is made clear in the proof of Proposition 10.

The budget constraint in this theorem requires complete information about the valuation for a single unit or at least a good estimate of $v_i(1)$. Alternatively, the principal could just set a budget at $a_i(1) = \overline{v}(1)$. In our model, from the beginning there is no demand for the package and the auction would end immediately anyway. As the principal can easily implement her payoff-dominant *dual-winner equilibrium* we do not consider the *single-winner equilibrium*. Unlike the symmetric information setting of the *principal-agent* 2×2 *FPSB package auction model* there is no demand inflation in the *principal-agent* 2×2 *ascending package auction model*. Moreover, remember that if one principal does not start to coordinate on the dual-winner outcome, the auction continues and the other principal might be able to adjust budgets and also pursue the single-winner award. Again, this observation might constitute an additional reason why successful implementation of the *dual-winner equilibrium* is easier in the *principal-agent* 2×2 *ascending package auction model* than in the *principal-agent* 2×2 *FPSB package auction model*. The result continuous to hold for the asymmetric information environment as we show in the next section.

3.7.4 *Asymmetric Information Principal-Agent Model*

The implementation of the *dual-winner equilibrium* as a solution to the principal's optimization problem (PA-SB) in the asymmetric information setting from Section 3.3.3 for the *principal-agent* 2×2 *ascending package auction model* remains as simple as in the symmetric information setting in Section 3.7.3 with the only difference of assigning a budget constraint weakly below $\overline{v}(1)$ for one unit. Thus, principals do not incur any agency costs and the *dual-winner equilibrium* remains strictly preferred to the *single-winner equilibrium* for all $v_i \in V$. Furthermore, unlike the asymmetric information environment of the *principal-agent* 2×2 *FPSB package auction model* there is no risk of a market breakdown as participation is always ex-ante profitable.

3.7.5 *The Principal-Agent* 2×2 *VCG Mechanism Model*

Ignoring the agents, the dual-winner outcome results for the principals in the 2×2 *VCG mechanism* under *dual-winner efficiency*. The payment function forces the principals to truthfully reveal their package valuations and *dual-winner efficiency* guarantees the social choice function to select the dual-winner outcome. Note, that given firm j reports zero on two units, principal i is indifferent between reporting truthfully and fully hiding her valuation on the package. Hiding the valuation on one unit is weakly dominated. The agents are not affected by the generalized second-price payment function and their reporting behavior corresponds to the one in Theorem 6 of the 2×2 *FPSB package auction*.

With precise information about the valuation for a single unit in the symmetric information setting, the principal can apply the same budgets as in Theorem 13 of the *principal-agent* 2×2 *ascending package auction* to implement the efficient allocation in the *principal-agent* 2×2 *VCG mechanism model*. This outcome equivalence is reminiscent of the outcome equivalence between the standard single-package English and Vickrey auctions and suggests that the payment function causes the different results for the implementation of the principal's *dual-winner equilibrium* in Theorem 8 and Theorem 13 for the *principal-agent* 2×2 *FPSB package auction model* and the *principal-agent* 2×2 *ascending package auction model*, respectively. Moreover, the outcome equivalence allows to focus on a comparison between the *principal-agent* 2×2 *VCG mechanism model* and the *principal-agent* 2×2 *FPSB package auction model* for the remaining discussion in this section.

Remember, in any 2×2 *package auction format* each bidder is able to unilaterally veto the dual-winner outcome with a low enough report on the single unit. Thus, under the first-price payment function the principal has to submit a high package bid to make deviations from the dual-winner outcome by her opponent unprofitable in equilibrium. This prevents a solution of the agency dilemma as the budget for two units is too high for agents to coordinate. Contrary, in the strategy-proof VCG mechanism the principal's report on the package does not affect the competing principal's bidding behavior who is indifferent between truthfully reporting her valuation for the package and hiding this value. Vice versa, this reasoning is true for both principals and therefore, the agency dilemma is straightforward to solve if the principals know that the solution with two winners is efficient. This is because the principal can also set the budget for the package bid to zero as in the *principal-agent* 2×2 *ascending package auction model* without altering the truthful report on one unit.

As we have shown in Section 3.7.4, even under asymmetric information coordination is straightforward to implement in the *principal-agent* 2×2 *ascending package auction model* and therefore also in the *principal-agent* 2×2 *VCG mechanism model* as long as the efficiency setting is known. This is a direct consequence of the independence of the principals' bidding behaviors in a strategy-proof second-price mechanism. The social choice function still selects the efficient outcome even if bidders hide their valuations for packages that are not part of the welfare-maximizing allocation. Therefore, principals do not have to implement precise bids on all packages in the *principal-agent* 2×2 *VCG mechanism model*. In more complex environments, however, this reasoning might not apply. If, for example, the principals neither know the values nor the efficiency environment, they are not able to set the budget constraints such that the payoff-dominant equilibrium is selected in our model. The bidding firms compete although they would not if the principals had the same information as the agents.

Moreover, the insights of the *principal-agent* 2×2 *package auction model* are also helpful for the analysis of non-combinatorial auction formats and less severe agency bias.

3.8 GENERALIZATION OF RESULTS

In this section, we first analyze the 2×2 *ascending uniform-price auction* to establish the existence of the agency-dilemma in non-combinatorial mechanisms. We also show that our findings hold for principal-agent relationships in which the agent internalizes prices but has value-maximization motive, too.

3.8.1 *Ascending Uniform-Price Auction*

Similar to Ausubel et al. (2014), who focus on an ascending uniform-price multi-unit auction, in this section we discuss the 2×2 *ascending uniform-price auction* as a model for the SMRA, which is used worldwide to sell spectrum licenses. Unlike the 2×2 *ascending package auction* there is only a *dual-winner equilibrium* in which both principals immediately reduce demand to one unit, but no *single-winner equilibrium*. To analyze the agent's bidding strategy in the 2×2 *ascending uniform-price auction* we need an adapted definition of straightforward bidding.

Definition 2. *(Straightforward bidding of agents in the 2×2 ascending uniform-price auction): Suppose agent i with full information about his vector of valuations $v_i \in V$ is provided with exogenously determined budget constraints of the form $a_i(1) \geq a_i(2)/2$, then he remains active on two units until the unit price reaches half his budget for the package, $a_i(2)/2$. He then reduces his demand to one unit and remains active until the price reaches his single-unit budget constraint of $a_i(1)$. If agent i is provided with budgets of $a_i(1) < a_i(2)/2$, he will bid up to $a_i(1)$ for two units, then indicate demand for one unit and immediately drop out completely.*

Any agent *i* must not bid beyond his constraint for one unit. If the agent did bid beyond $a_i(1)$ for two units and first dropped out at $a_i(2)/2$ from both units, then he could be assigned a single unit at a price of $a_i(2)/2$ beyond budget, which is unacceptable. Moreover, the agent distributes his double-unit budget constraint evenly over both units, because he needs to beat agent *i* on both single units to obtain the double-unit package. An uneven distribution would favor one single unit and therefore cannot be optimal. It is straightforward to show an equilibrium in which both agents engage in straightforward bidding that may result in the dual-winner outcome or in the single-winner outcome depending on the budget constraints.

Proposition 14. *In the 2×2 ascending uniform-price auction straightforward bidding of agent i with full knowledge of his values of $v_i \in V$ and any exoge-*

nously determined budget constraints of $a_i \in A$ constitutes a Bayesian Nash equilibrium. The equilibrium results in the following outcomes depending on the sizes of the budget constraints:
Agent i is provided with budget constraints of $a_i(1) \geq a_i(2)/2$ and wins

A) One unit if $a_i(1) \geq \min\left\{a_j(2)/2, a_j(1)\right\}$ and $a_i(2)/2 \leq a_j(1)$

B) Two units if $a_i(2)/2 > a_j(1)$.

Agent i faces budget constraints of $a_i(1) < a_i(2)/2$ and wins

C) One unit if $a_i(1) = a_j(1)$ in case $a_j(1) < a_j(2)/2$ or if $a_i(1) > a_j(2)/2$ in case $a_j(1) \geq a_j(2)/2$

D) Two units if $a_i(1) > a_j(1)$.

The derived equilibrium is also valid ex-post as knowing the opponent's budget constraints does not lead to an improvement in straightforward bidding. Similar to the 2×2 *ascending package auction*, principals can set a zero budget constraint for two units to implement the *dual-winner equilibrium* with their agents, and the auction would stop immediately in our model in which the principals know that there is *dual-winner efficiency*. In all multi-unit auction formats an agency dilemma occurs because principals would like to coordinate on the welfare-maximizing dual-winner outcome in *dual-winner efficiency* whereas agents would never do so independent of the efficiency environment in the 2×2 *package auction formats*. Even in the 2×2 *ascending uniform-price auction* agents might not coordinate. For different efficiency settings this divergence in preferred equilibrium might be less severe. Nevertheless, we show in the following section that the agency-bias with respect to underlying preferences might still prevent the implementation of an optimal contract.

In our model the agency bias occurs because the principal maximizes expected profit whereas the agent maximizes value. In the next section we show that the main findings of our model carry over to a setting in which the agency bias is more moderate. We demonstrate that the principal might not always be able to implement her *dual-winner equilibrium* with budget constraints only (not even always with transfer payments) in the asymmetric information environment of the *principal-agent* 2×2 *FPSB package auction model* even if the agent maximizes profit to some extent.

3.8.2 *Biased Profit-Maximizing Agent*

In this section we derive the optimal contract to implement the *dual-winner equilibrium* of the *principal-agent* 2×2 *FPSB package auction model* in the asymmetric information environment as defined in Section 3.3.3 in case principal and agent have preferences as in the model by Burkett (2015, 2016). In this setting the agent's payoff corresponds to $u_i(l) = v_i(l) - p_i(l) + m_i(l)$ and the principal maximizes ex-interim expected profit, in which her profit of winning a package of l units is given by $\pi_i(l) = v_i(l) - \eta\left(v_i(l)\right) - p_i(l) +$

$m_i(l)$. The function $\eta : V(l) \to \mathbb{R}$ represents the bias between both parties for the package of l units and the principal's payoff is assumed to be positive and smaller than the agent's utility. Otherwise the model remains as described in Section 3.3.

As the agency bias is not as large as with the utility functions described in Section 3.3, we are able to derive necessary and sufficient conditions for the optimality of a pure budget constraint contract without transfers. This result is an extension of Burkett (2016)'s characterization of an optimal budget constraint for single-package auction-like mechanisms to combinatorial auctions. Note that the implementation of the optimal budget constraint contract for the *principal-agent* 2×2 *ascending package auction model* remains as simple as in Section 3.7.4.

The proof for the *principal-agent* 2×2 *FPSB package auction model* is based on the following observations: First, note that the agent internalizes the price and has a profit-maximizing motive. The agents can therefore coordinate on a *dual-winner equilibrium* which corresponds to the one defined in Proposition 3 in Section 3.6.1 for the 2×2 *FPSB package auction model*. Let us from now on denote the agent's equilibrium bids by $b_i^a = \left(\beta_1^a, \beta_2^a\left(v_i(2)\right)\right)$ with corresponding lower bound for the package of $\beta_2^a\left(v_i(2)\right) \geq G^a\left(v_i(2), \beta_1^a\right)$ satisfying Proposition 3. Second, we can modify the sufficient conditions for the *dual-winner equilibrium* in Proposition 3 for the adapted principal's utility function as follows.

Corollary 2. *Assume dual-winner efficiency is given,* $\overline{v}(2) - \eta\left(\overline{v}(2)\right) < 2 \cdot \left(\underline{v}(1) - \eta\left(\underline{v}(1)\right)\right)$*, then for any principal i with full information about her valuations $v_i \in V$, the vector of bids* $b_i^p = \left(\beta_1^p, \beta_2^p\left(v_i(2)\right)\right)$ *is a dual-winner equilibrium of the* 2×2 *FPSB package auction, if the following conditions hold:*

1) $\beta_1^p \in \left[\overline{v}(2) - \eta\left(\overline{v}(2)\right) - \underline{v}(1) + \eta\left(\underline{v}(1)\right),\ \underline{v}(1) - \eta\left(\underline{v}(1)\right)\right]$

2) $\beta_2^p\left(v_i(2)\right)$ *is continuous and strictly increasing on support* $V(2)$

3) $\beta_2^p\left(\overline{v}(2)\right) = 2 \cdot \beta_1^p$

4) $G^p\left(v_i(2), \beta_1^p\right) \leq \beta_2^p\left(v_i(2)\right)$ *for all* $v_i(2) \in V(2)$.

The lower bound $G^p(.)$ *is defined as:*

$$G^p\left(v_i(2), \beta_1^p\right) \equiv \beta_1^p + \frac{\beta_1^p - \left(\underline{v}(1) - \eta\left(\underline{v}(1)\right)\right) \cdot \left(1 - F_2\left(v_i(2)\right)\right)}{F_2\left(v_i(2)\right)}.$$

Third, in the asymmetric information setting with unknown value ranges $V \subseteq Z$, the principal's and the agent's lowest pooling prices can be expressed as functions of the random variable d : $b_i^p(1) = \beta_1^p(d)$ and $b_i^a(1) = \beta_1^a(d)$, respectively, with $\beta_1^p(d) = d - \eta\left(\overline{v}(2)\right) + \eta\left(\underline{v}(1)\right)$ and $\beta_1^a(d) = d$. Therefore, the final reports of the *principal-agent* 2×2 *FPSB package auction model* to the auction can be expressed as functions of d, too: $b_i = \beta(d)$. The principal's optimization problem for the asymmetric information environment (PA-SB) from Section 3.3.3 can be specified for the adjusted utility functions from this section as follows:

$$\text{Implement} \quad b_i \text{ that satisfy Corollary 2 given } G(v_i)|_{v_i(1)<v_i(2)} \tag{B-CFPA-SB}$$

$$\text{subject to} \quad b_i = \beta(d) \text{ fulfill Proposition 3} \tag{B-CFPA-SB-IC}$$

$$\beta_i(1) \leq v_i(1) \text{ for all } v_i(1) \in V(1). \tag{B-CFPA-SB-NL}$$

Although the agent maximizes profit, he is biased and the no-loss condition in (B-CFPA-SB-NL) is required in our setting. We can now state necessary and sufficient conditions for optimality of a budget constraint contract that solves (B-CFPA-SB) subject to (B-CFPA-SB-IC) and (B-CFPA-SB-NL).

Proposition 15. *If the necessary condition $2\cdot \underline{v}(1) \geq \overline{v}(2)+\eta\left(\underline{v}(1)\right)$ is fulfilled, a principal i can ex-ante optimally implement her dual-winner equilibrium with reports to the auction of*

$$\beta_1^p(d) = \begin{cases} d & \text{if } d \leq \overline{\beta_1^p}(d) \\ \overline{\beta_1^p}(d) & \text{if } d > \overline{\beta_1^p}(d) \end{cases} \quad \text{and } \beta_2^p(d) = 2\cdot\beta_1^p(d) \tag{B-CFPA-SB-R}$$

in which $\beta_1^p\overline{(d)} = \bar{d} - \eta\left(\overline{v}(2)\right) + \eta\left(\underline{v}(1)\right)$, in the asymmetric information environment of the principal-agent 2×2 FPSB package auction model given dual-winner efficiency, $\overline{v}(2)-\eta\left(\overline{v}(2)\right) < 2\cdot\left(\underline{v}(1)-\eta\left(\underline{v}(1)\right)\right)$. The ex-ante participation constraint is given by

$$E_{v_i}\left(v_i(1)-\eta\left(v_i(1)\right)-d\right) \geq 0 \textit{ for all } v_i \in Z \textit{ if } d \leq \overline{\beta_1^p}(d)$$
$$E_{v_i}\left(v_i(1)-\eta\left(v_i(1)\right)-\overline{\beta_1^p}(d)\right) \geq 0 \textit{ for all } v_i \in Z \textit{ if } d > \overline{\beta_1^p}(d). \tag{B-CFPA-SB-PC}$$

Note that the necessary condition in Proposition 15 implies that *dual-winner efficiency* for the agent's utility function as defined in the model in Section 3.3 is satisfied. Similar to Burkett (2016) in the optimal budget constraint contract any principal lets her agent truthfully report the latter's equilibrium strategy $\beta_1^a(d) = d$ until a certain upper-bound $\overline{\beta_1^p}(d)$ is reached. However, unlike Burkett (2016) in which the principal optimally determines the upper-bound to solve a trade-off between capturing the information rent and mitigating the agency-bias to maximize ex-ante expected profit, in Proposition 15 it is simply the nature of the principal's *dual-winner equilibrium* as specified in Corollary 2 that does not allow him to implement a lower upper-bound than $\overline{\beta_1^p}(d)$ on the highest report by the agent to maximize ex-ante expected profit. For all $d \leq \overline{\beta_1^p}(d)$ the agency-bias carries through the optimal reporting functions.

The principal cannot implement a more profitable contract with transfer payments as the optimal constant transfer for one unit, $m(1) = \eta\left(\overline{\overline{v}(2)}\right) - \eta\left(\underline{\underline{v}(1)}\right)$, that sets incentives for the agent to submit the principal's optimal reporting function of $\beta_1^p(d)$ to the auction is relatively high independent of the agent's type. Nevertheless, if the necessary condition, $2 \cdot \underline{v}(1) \geq \overline{v}(2) + \eta\left(\underline{v}(1)\right)$, in Proposition 15 is not satisfied a pure budget constraint contract cannot be implemented and the principal must employ the costly transfer payment for one unit to let the agent submit reports of $\beta_1^p(d) = d - \eta\left(\overline{v}(2)\right) + \eta\left(\underline{v}(1)\right)$ and $\beta_2^p(d) = 2 \cdot \beta_1^p(d)$ to the auction. In this case the principal's ex-interim expected profit for given $d \in D$ and $v_i \in Z$ is $v_i(1) - \eta\left(v_i(1)\right) - \beta_1^p - \eta\left(\overline{\overline{v}(2)}\right) + \eta\left(\underline{\underline{v}(1)}\right)$. Similar to our prior findings for the *principal-agent* 2×2 *FPSB package auction model* in the asymmetric information environment in Section 3.6.4, as the magnitude of the agency bias $\eta(\cdot)$ becomes too large, profit becomes negative and implementation of the *dual-winner equilibrium* unprofitable even if the agent possesses some profit-maximizing motives.

3.9 CONCLUSION

We analyze a hidden information model in which the agent bids on behalf of the principal in a multi-unit auction. He knows the goods valuations but has limited liability and the principal has to pay. In this model, there is no reason for the agent to maximize payoff, but he tries to win the most valuable allocation within budget. We show that there is a conflict of interest between principal and agent in efficiency settings in which it is payoff dominant for the principals to coordinate. The types of manipulation discussed in this chapter are specific to multi-object auctions, and differ from the problems in single-object auctions (Burkett, 2015, 2016).

If the auctioneer is concerned about efficiency and is aware of agency problems among bidding firms he should favor ascending package auctions. First, if the principals understand that there is *dual-winner efficiency*, the ascending package auction and the VCG mechanism allow to implement the efficient equilibrium that is also payoff-dominant in our model. Second, ascending package auctions further alleviate coordination problems in multi-object markets and might even have advantages in environments in which the principals do not know the efficiency environment ex ante because they can learn new information about the competition during the auction.

However, note that in our principal–agent package auction model, the principal would need to know that there is *dual-winner efficiency* efficiency to set budget constraints appropriately in an ascending package auction or the VCG mechanism. In a FPSB package auction, this might not even be possible in equilibrium with budget constraints only, even if the principal had precise information about the valuations. The wide use of budget constraints in bidding firms might be due to the fact that there is often considerable uncertainty about the valuations, the efficiency environment and the prior type distributions for the principal in the field and only the agent has detailed information. With uncertainty about the efficiency environment, the equilibrium bidding strategies in our model are unknown. In such an environment, principals often try to at least limit the risk of the agent overbidding substantially via budget constraints. However, overall budget constraints are insufficient to make the agent bid payoff-maximizing in general as we have shown.

Let us leverage the insights from our model and revisit some well-known examples of spectrum auctions. In the German spectrum auction in 1999 the two strong players Mannesmann and T-Mobile reduced demand to five blocks each in the initial rounds of the ascending auction (Grimm et al., 2004). One could assume that in this simple environment with 10 homogeneous objects it was clear to the principal that the dual-winner outcome would be payoff-dominant. The auction was criticized for low revenue, but it can well have been the efficient allocation.[27]

While the 1999 auction result is compatible with enforcing coordination on a dual winner outcome, other auctions show that demand inflation is also observed, which is compatible with the agency problem generally described in this chapter. An example is the German auction in 2015 (Bichler and Goeree, 2017).[28] The bidders did not coordinate and several observers reported demand inflation rather than demand reduction.[29] The analysts' estimates before the auction differed substantially and it is likely that the principals had little information about the value of a package making it very hard to set appropriate budget constraints.[30]

There is not much public information about FPSB combinatorial auctions. France used this auction format for selling spectrum in the 800 MHz auction and the 2.6 GHz auction in 2011 and the average prices in these auctions were among the highest in Europe.[31] Of course, one must not over-interpret these observations. The comparison of prices in spectrum auctions in the field is far from trivial and a number of factors influence the final prices. Apart from the specifics of the auction format, the competitive situation, the reservation prices and spectrum caps, the types of bands in the auction, and country-level idiosyncrasies matter to name just a few. In summary, the traditional assumption in which bidders are modeled as payoff-maximizing individuals might be too simple and the presence of principal–agent relationships in the bidding firms can have significant negative impact on the efficiency of auctions as a means to allocate scarce resources.

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[27] We leave revenue maximization by the auctioneer as a topic for future research and focus on efficient auction design, which is the primary goal of regulators.

[28] http://www.bundesnetzagentur.de/DE/Sachgebiete/Telekommunikation/Unternehmen_Institutionen/Frequenzen/OeffentlicheNetze/Mobilfunknetze/Projekt2016/projekt2016-node.html

[29] http://telecoms.com/opinion/the-german-spectrum-auction-failure-to-negotiate/

[30] The strategic analysis of the wide-spread two-stage combinatorial clock auction is more involved, which is why we do not discuss it in this context (Bichler and Goeree, 2017).

[31] https://www.ofcom.org.uk/__data/assets/pdf_file/0021/74109/telefonica_response.pdf

4

OPTIMAL BIDDING IN EX-POST SPLIT-AWARD AUCTIONS†

4.1 INTRODUCTION

Following Section 1.2.3, we focus on (combinatorial) ex-post split-award auctions in an $n \times 2$ market under strong diseconomies of scale, i.e. *dual source efficiency*. Furthermore, we assume that the suppliers know the scale economies in the market, but the buyer does not. The choice of a proper auction design depends on the prevailing scale economies in the market: with economies of scale (and no risk-premium), it is efficient to select a single supplier (sole source award), and the procurement manager should employ a single-object auction. However, the solution with two suppliers (split award) is efficient with diseconomies of scale and in case a buyer expects savings from an ex-ante split-award auction. Knowing the scale economies in the market, a buyer could use the appropriate efficient auction design for each efficiency scenario. However, although it is reasonable to assume that bidders know the scale efficiencies of their product, this is often not true for the buyer. Thus, an auctioneer prefers to employ a (combinatorial) ex-post split-award auction in which both outcomes, a sole source and a split award, are possible. In these auctions, the suppliers can submit bids on shares as well as on the whole business, i.e., the package of both shares. The buyer then selects the cost minimizing combination of bids and therefore, the decision whether to split the award or to select a single supplier is endogenous.

Markets with diseconomies of scale are interesting for a number of reasons. First, in the efficient σ equilibrium bidders need to coordinate, which is strategically challenging. Second, environments with diseconomies of scale are relevant for a large number of procurement events, for example if suppliers face capacity limits or stepwise fixed costs. Anton and Yao (1992) motivate the environment with two bidders by a defense procurement example, but settings with two shares and two bidders are also common in the electronics industry

†This chapter is based on the working paper *Combinatorial first-price auctions: Theory and experiments* co-authored by *Martin Bichler and Gian-Marco Kokott, Working-Paper Technical University of Munich, 2018, Munich*. In comparison with the working paper, the obtained equilibrium strategies and the resulting rankings of auction formats in terms of revenue and efficiency are already discussed with respect to the existing literature in detail in Sections 1.2 and 2.3. Moreover, Section B.2 contains further outputs of regression analyses and statistical summary plots to allow for a more extensive analysis than in a forthcoming space-restricted publication.

due to the high specificity of the goods procured. It is unclear, however, whether the two-bidder case in Anton and Yao (1992) generalizes and therefore, we extend the analysis to markets with more than two bidders and different types of first-price combinatorial auctions. We provide a comprehensive theoretical analysis of FPSB and Dutch auction formats and also report results of lab experiments, which provide evidence that different versions of the Dutch combinatorial auction formats have remarkable properties with respect to expected buyer revenue and efficiency.

4.2 CONTRIBUTIONS

Our contributions to the literature on (combinatorial) ex-post split-award auctions are twofold. First, we derive Bayesian Nash equilibrium bidding strategies for various first-price split-award auction formats in which $n \geq 2$ suppliers compete for a contract.

Besides the simple combinatorial ex-post FPSB split-award auction, we focus on two practically relevant ascending-price formats. The first format, the Dutch split-award auction, consists of two phases. In the first phase sellers can accept ascending prices for the 100% share and the 50% share of the contract. If the entire contract is accepted, the auction ends with a single-winner outcome. However, in case the 50% share is accepted first, the second 50% share is auctioned in a consecutive second stage, again via an ascending-price mechanism among all sellers. Finally, in the Dutch-FPSB split-award auction the ascending-price in the second phase is replaced by a simple simultaneous sealed-bid mechanism. We are not aware of a game-theoretical treatment of the two latter Dutch auctions in spite of their wide-spread use in procurement practice, nor do we know of an analysis with more than two bidders. We show that both auction formats reduce the strategic complexity for bidders considerably, because only efficient σ *equilibria* exist, whereas there is a coordination problem for the bidders in the FPSB auction as there is also an inefficient *WTA equilibrium* and an inefficient hybrid equilibrium as defined in Anton et al. (2010). In contrast to the case with two bidders, σ equilibria do not comprise pooling prices for the split, but prices are increasing with costs.

While the Dutch and the Dutch-FPSB split-award auctions are cost equivalent and fully efficient, this outcome only extends to the FPSB split-award auction if bidders choose the payoff-dominant σ *equilibrium*. This result contrasts with the well-known strategic and costs equivalence (RET) of the Dutch and FPSB mechanisms in standard single-object auctions. Overall, the fine differences among the information revealed to bidders during the FPSB, Dutch-FPSB, and Dutch formats lead to interesting and non-obvious insights into the equilibrium bidding strategies of multi-object auctions. In particular we show that in the symmetric σ equilibrium, $n > 2$ bidders and identical 50% shares, all auctions are strategically equivalent to standard multi-unit auctions with single-unit demand as presented in Section 2.3.2 such that we can draw on the RET for this environment. However, for the FPSB split-award auction this is only true if the efficient σ *equilibrium* is chosen by the bidders. All above listed results are proven in Appendix B.1.

Second, we provide an experimental analysis of the three first-price auctions for two- and three-bidder settings, and find that the theoretical models explain important empirical regularities in the lab. As predicted, the two Dutch formats are on average more efficient than their sealed-bid counterpart. In the two-bidder environment, the Dutch auction is much more efficient than the FPSB auction although at a higher cost. The price information from the first phase of the Dutch auction formats provides a signal that facilitates subjects to coordinate on the efficient split award. In contrast, the equilibrium selection problem makes it very hard to coordinate in the FPSB auction with two suppliers. Here, the experimental results demonstrate that bidders select both types of outcomes (split and sole source awards), and we find 55% inefficient allocations. The Dutch-FPSB auction appears as an interesting and simple alternative that yields the highest share of efficient allocations (82%) of all three mechanisms as well as low procurement cost. The sealed-bid auction in the second phase of the Dutch-FPSB format allows for a broader set of equilibrium bids and leads to lower prices in the first phase of the experiments. This avoids deviation incentives that arise in the Dutch auction in which subjects sometimes overbid the unique predicted equilibrium price in an attempt to achieve a higher payoff, making it attractive to win the 100% share. This phenomenon actually leads to a higher number of inefficient sole source awards in the Dutch, compared to the Dutch-FPSB auction. Furthermore, we find evidence for pooling and tacit collusion in all three mechanisms, as subjects, who succeed to coordinate on the split award, achieved high profits in all three auction formats.

Interestingly, the addition of just one more bidder levels the differences among the three first-price mechanisms and almost always results in the selection of an efficient split award. This is also the case for FPSB split-award auction, although this format still possesses an inefficient *WTA equilibrium*. Raising the number of bidders to three, also has substantial impact on total procurement costs, which drop on average by roughly 42% in the FPSB and Dutch-FPSB auctions, and by 49% in the Dutch auction. Here, a high pooling price cannot be maintained in equilibrium anymore. Furthermore, in this more competitive three-supplier setting, the bidding behavior in the Dutch auction does not significantly differ from the equilibrium strategy. This is somewhat remarkable as bidding in standard first-price single-object auctions typically deviates substantially from the risk-neutral Bayesian Nash equilibrium strategy (Cox et al., 1983; Filiz-Ozbay and Ozbay, 2007; Kirchkamp and Reiß, 2011; Bichler et al., 2015). Overall, we find surprisingly high levels of efficiency in simple combinatorial first-price auctions. An exception is the FPSB format with two bidders, in which the equilibrium selection problem and the power of bidders to unilaterally veto a split award leads to a high share of inefficient allocations.

4.3 MODEL

Before describing the auctions discussed in this chapter, we first provide some necessary notation and terminology.

A buyer conducts a split-award auction in order to award a business among $n \geq 2$ ex-ante symmetric, risk-neutral, and profit-maximizing suppliers.[32] We focus on a simple setting in which bidders can win either a contract for 50 or 100% of the business, which makes it technically a combinatorial (reverse) auction with two identical units and the package up for auction. The possibility to submit all-or-nothing package bids makes this type of auction different from multi-unit auctions with multi-unit demand as discussed by Chakraborty (2006). Bidder i's costs for 100% of the business, k_i^s, are determined by θ_i (with $i \in \{A, B, ...\}$ and $n = |\{A, B, ...\}|$) which is independently and identically distributed on the interval $\left[\underline{\Theta}, \overline{\Theta}\right]$, $0 < \underline{\Theta} < \overline{\Theta}$, according to a distribution function $F(\cdot)$. The density f is positive and continuous. The cost draw Θ_i of θ_i is private, i.e. every supplier only knows his own costs which are not affected by the cost draws of the opponent(s). Hereafter, the j-th lowest order statistic out of n different cost types is denoted by $\Theta_{j:n}$ (with $j \in \{1, 2, ..., n\}$). A constant efficiency parameter $C \in (0, 1)$, which is equivalent for and known to all suppliers, determines the costs for 50% of the business, $k_i^\sigma = C\Theta_i$. Costs for no award are zero. Furthermore, the buyer does not know the efficiency parameter C.

We discuss both static and dynamic formats in this chapter. In the static mechanism, an offer of bidder i comprises prices for 100% and\or 50% of the business, $p^s(\Theta_i) : \left[\underline{\Theta}, \overline{\Theta}\right] \longrightarrow \mathbb{R}$ and $p^\sigma(\Theta_i) : \left[\underline{\Theta}, \overline{\Theta}\right] \longrightarrow \mathbb{R}$ respectively.

Multi-stage games with observed actions are used to model the dynamic mechanisms. The function $p^{s1}\left(\Theta_i, h^0\right)$ defines the price level at which a bidder i accepts the sole source award in phase 1. When a bidder i accepts the split award in phase 1, the respective price level is denoted by $p^{\sigma 1}\left(\Theta_i, h^0\right)$. Both price functions include the history $h^0 = \{\}$. The bidding strategies of phase 2 depend on the history of the game $h^1 = \left\{p^{\sigma 1}\left(\Theta_w, h^0\right)\right\}$: a winner of phase 1 with cost type Θ_w plays a bidding strategy $p^{\sigma 2w}\left(\Theta_w, h^1\right)$ and the loser (s) with cost type (s) Θ_l with ($l \neq w$ and $\Theta_l, \Theta_w \in \left[\underline{\Theta}, \overline{\Theta}\right]$) follow a strategy $p^{\sigma 2l}\left(\Theta_l, h^1\right)$. In phase 1 and phase 2, all price functions map from $\left[\underline{\Theta}, \overline{\Theta}\right] \times \mathbb{R}$ to $\mathbb{R}$. All bidding functions are non-decreasing and continuous.

Each bidder i maximizes expected profit with the profit of winning the 50% share and 100% given by $\pi_i^\sigma = p^\sigma(\Theta_i) - C\Theta_i$ and $\pi_i^s = p^s(\Theta_i) - \Theta_i$, respectively, in the static auction. In the dynamic format the 50% share can be won in phase 1 with profit of $\pi_i^{\sigma 1} = p^{\sigma 1}\left(\Theta_i, h^0\right) - C\Theta_i$ or in the second phase with profit $\pi_i^{\sigma 2l} = p^{\sigma 2l}\left(\Theta_l, h^1\right) - C\Theta_i$. The 100% share can be won directly in phase 1 and profit of $\pi_i^{s1} = p^{s1}\left(\Theta_i, h^0\right) - \Theta_i$ or consecutively with profit

[32] The terms auctioneer and buyer as well as bidder and supplier are used interchangeably.

$\pi_i^{s1} = p^{\sigma 1}\left(\Theta_i, h^0\right) + p^{\sigma 2w}\left(\Theta_w, h^1\right) - \Theta_i$. Finally, the profit of not winning at all is zero.

Bidders are assumed to be individually rational, which means that all submitted bids must be at least as high as the supplier's costs for the respective allocation. The auctioneer is ex-ante indifferent between awarding 100% of the business to a single supplier (sole source award) and awarding 50% of the business each to two different suppliers (split award). Hence, the winner determination in a split-award auction must satisfy the auctioneer's indifference condition. We focus on markets with strong diseconomies of scale in which suppliers must coordinate in the efficient solution. *Dual source efficiency (DSE)* describes a setting in which it is always efficient for the buyer to award 50% of the business to each of two different suppliers. An ex-ante defined risk-premium by the procurement manager extends the scope of *dual source efficiency*. The same types of equilibria emerge in a setting with a constant risk premium r for the sole source award and with $C < \frac{\underline{\Theta}+r}{\underline{\Theta}+\overline{\Theta}}$, which also allows values for C of greater than 0.5, e.g., a setting with $C = 0.52$, $\Theta \in [100, 140]$, and $r = 25$ in which an equilibrium with pooling prices and split awards exists even though suppliers have economies of scale.

Let us denote the vector of any bidder i's bids as $p_i = \left(p^s\left(\Theta_i\right), p^\sigma\left(\Theta_i\right)\right)$ and summarize all sellers' bids in the vector $p = \left(p_i, p_{-i}\right)$ in which p_{-i} denotes all bids other than those of i. In this model environment the strategy space, defined by the pair of bids $\left(p^s\left(\Theta_i\right), p^\sigma\left(\Theta_i\right)\right) \subseteq \mathbb{R}^2_{\geq 0}$ for all Θ_i and all i, is a nonempty, convex and compact subset of an Euclidean space. In addition, the profit functions $\pi_i^{\cdot}$ are continuous in p and quasiconcave in p_i. Thus, based on Definition XII in Section 2.1.1, the existence of Bayesian Nash equilibria in pure strategies is guaranteed in our standard IPV setting. Perfect Bayesian equilibria (possibly in mixed strategies) will exist in our finite dynamic auction games of incomplete information as stated in Definition XIV of Section 2.1.2.

Based on Definition III in Section 2.1.1 and given costs Θ_i, we can denote a bidder's ex-interim expected profit of participating in the auction as $E\left[\Pi\left(\Theta_i\right)\right]$ for each Θ_i with corresponding maximization problem of

$$\max_{p_i} \quad E\left[\Pi\left(p_i, p_{-i}, \Theta_i\right); F\left(\cdot\right)\right] \quad \text{for all } \Theta_i \in \left[\underline{\Theta}, \overline{\Theta}\right]. \tag{S-EXI}$$

For any bidder i, the ex-interim expected profit depends on the costs Θ_i and reports p, taking into account the distribution function of the opponents' costs $F\left(\cdot\right)$. Remember that combinatorial package auctions, including ex-post split-award auctions, generally possess multiple Bayesian Nash equilibria for profit-maximizing bidders, whether in a forward or in a reverse auction setting. Therefore, we solve each bidder's problem of minimizing ex-ante expected utility in (S-EXI) with respect to specific Bayesian Nash equilibria. Finally, we compare different equilibria in terms of payoff-dominance.

4.4 AUCTION FORMATS

We next describe the auction formats analyzed in this chapter. As the FPSB split-award auction is simple and identical to the mechanism in Anton and Yao (1992), we only introduce the Dutch and Dutch-FPSB auction.

4.4.1 *The Dutch Split-Award Auction*

The Dutch split-award auction can be divided into two stages or phases. In the first phase, bidders simultaneously compete for the split as well as the sole source award. After one of the bidders accepts the price for 100% of the business, the auction ends. In the case in which one of the bidders approves a counteroffer for the 50% share, phase 2 starts.

PHASE 1: In each round r of the auction, bidders simultaneously receive counteroffers[33] for 50%, c_r^σ, and 100% respectively, of the business, c_r^s. The starting prices for both counteroffers should be at least lower than or equal to the minimal costs for each share, i.e. $c_1^\sigma \leq C\underline{\Theta} \text{and} c_1^s \leq \underline{\Theta}$. The auctioneer can also start close to zero. Subsequently, both price functions are raised continuously by the buyer such that $c_r^s = 2c_r^\sigma$ is true in each round r. The auctioneer must stick to this pricing rule in every round to assure that the outcome of the auction satisfies his indifference condition.[34] In each round, a bidder i has three options: he can approve the counteroffer for 50 or 100% of the business, or he can reject both. The following three scenarios are possible:

1. If bidder i is first to accept a counteroffer for the 50% share in round r, the split is awarded to supplier i at a price of c_r^σ and phase 1 is over;
2. If bidder i is first to accept c_r^s in round r, this supplier i wins the sole source award and the auction terminates immediately;
3. If a bidder rejects both counteroffers in round r, he risks losing the whole or at least a share of the business.

PHASE 2: The second phase is only relevant, when the split award has been awarded to a single supplier in phase 1. In this case, the remaining 50% of the business is auctioned off to all suppliers in a regular single-unit Dutch auction. Regardless of the price for the 50% share in phase 1, the starting price in phase 2 is c_1^σ (the same as in phase 1); this is necessary to allow efficient

[33] Typically, there is a request for quotation (RFQ) before the final awarding, in which suppliers are asked by the buyer to submit first offers for the business. Hence, the auctioneer's offers in the final Dutch are normally called counteroffers in procurement practice.

[34] Suppose that the buyer would choose a pricing rule with $c_r^s < 2c_r^\sigma$. If one bidder accepts the counteroffer for 50% in round t of phase 1 and a different bidder is willing to pay the same price in phase 2, the auction ends with the buyer awarding the split at a price $2c_t^\sigma$. However, the buyer does not know, if a supplier would have accepted 100% of the business for a counteroffer c_r^s, such that $c_t^s < c_r^s < 2c_t^\sigma$. With such a deviation from the pricing rule proposed above, a buyer would risk higher purchasing costs. Similar reasoning applies if the buyer commits to a pricing rule such that $c_r^s > 2c_r^\sigma$.

equilibrium bidding strategies (Gretschko et al., 2014). The first bidder to approve a counteroffer wins the remaining half of the business and the auction is over.

The following tie-breaking rules apply for the split-award auctions discussed in this chapter: First, the split award is always chosen by the auctioneer, if the procurement costs of the sole source and split allocation are equally high; second, a lottery with equal chances for each involved supplier defines the winning supplier if there is a tie between two or more bids for the same award.

4.4.2 *The Dutch-FPSB Split-Award Auction*

The Dutch-FPSB split-award auction is a hybrid format containing elements from both the Dutch and the FPSB split-award auction formats. It can also be divided into two phases, with phase 1 following the same rules as in the Dutch format. Phase 2 becomes relevant if the split is awarded to a bidder in phase 1. However, the remaining 50% of the business is auctioned off by an FPSB in phase 2. All bidders including the winner of phase 1 are submitting bids for the remaining 50% share, and the supplier with the lowest price wins.

4.5 EQUILIBRIUM BIDDING IN THE 2-BIDDER MODEL

First, we analyze equilibrium bidding behavior in split-award auctions with only two bidders, for which bidders can veto the split outcome. This is a specific environment, which needs to be analyzed differently. However, it provides a basis for our analysis of markets with more bidders. We start with equilibria in the FPSB auction. Subsequently, we derive equilibria in the Dutch and the Dutch-FPSB split-award auction.

Within a given auction format, each equilibrium solves seller i's maximization problem (S-EXI) in Section 4.3 for all possible costs draws $\Theta_i \in \left[\underline{\Theta}, \overline{\Theta}\right]$. Consecutively, we compare the different equilibria via payoff-dominance to determine the equilibrium that globally maximizes (S-EXI).

4.5.1 *The FPSB Split-Award Auction*

Anton and Yao (1992) analyze equilibrium bidding behavior in a FPSB split-award auction with two bidders and *dual source efficiency*, demonstrating both WTA equilibria and σ equilibria. One result of this work is that constant pooling prices for 50% of the business are necessary in order to derive a σ equilibrium; they show that various σ equilibria with pooling prices $p_e^\sigma \in \left[\overline{\Theta}C, (1-C)\,\underline{\Theta}\right]$ can exist. In such cases, bidders submit high sole source prices that support the equilibrium and must not be higher than a given boundary $G\left(\Theta_i, p_e^\sigma\right) = p_e^\sigma + \frac{p_e^\sigma - \overline{\Theta}CF(\Theta_i)}{1-F(\Theta_i)}$ with $\Theta_i \in \left[\underline{\Theta}, \overline{\Theta}\right]$ and $p_e^\sigma \geq \overline{\Theta}C$ in order to avoid profitable deviations for the sole source award. As bidders are individually rational, σ equilibria can only exist, when the boundary $G\left(\Theta_i, p_e^\sigma\right)$ allows for sole source prices above costs for 100% of the business. We briefly revisit the main propositions of Anton and Yao (1992), in order to allow for a simple comparison of equilibrium bidding strategies in other auction formats and more general environments.

Proposition 16. *(Anton and Yao, 1992, Proposition 2) Consider the dual source efficiency split-award auction model including $n = 2$ bidders with cost types Θ_i. In the FPSB split-award auction, a σ equilibrium S_e^{BNE} is given by*

$$p_e^s(\Theta_i) \geq \Theta_i \text{ strictly increasing and continuous and}$$
$$p_e^\sigma(\Theta_i) = p_e^\sigma \in \left[\overline{\Theta}C, \underline{\Theta}(1-C)\right].$$

Additionally, $p_e^s\left(\underline{\Theta}\right) = 2p_e^\sigma$ is true and $p_e^s(\Theta_i) \leq G\left(\Theta_i, p_e^\sigma\right)$ applies for all $\Theta_i \in \left[\underline{\Theta}, \overline{\Theta}\right]$.

An inefficient WTA equilibrium exists as well, as a bidder i can strategically veto the split allocation with a high bid-to-lose price for 50% of the business, for example, with $p_e^\sigma(\Theta_i) = p_e^s(\Theta_i) - C\underline{\Theta}$. The sole source price $p_e^s(\Theta_i)$ of such a strategy equals the price in a single-unit auction, as this is the profit-maximizing strategy of a bidder i, when the probability to win the split award is zero in equilibrium.

Proposition 17. *(Anton and Yao, 1992, Proposition 4) Consider the dual source efficiency split-award auction model including $n = 2$ bidders with cost types Θ_i. In the FPSB split-award auction, a WTA equilibrium S_e^{BNE} with*

$$p_e^s(\Theta_i) = \Theta_i + \frac{\int_{\Theta_i}^{\overline{\Theta}} \left(1 - F(t)\right)^{n-1} dt}{\left(1 - F(\Theta_i)\right)^{n-1}}$$
$$p_e^\sigma(\Theta_i) = p_e^s(\Theta_i) - C\underline{\Theta}$$

exists.

Hybrid equilibria are of interest for settings with uncertain economies of scale (Anton et al., 2010). These type of equilibria are described by a strategic cost type τ, for which bidders change their equilibrium bidding strategy: low-cost bidders with $\Theta_i < \tau$ focus on winning the sole source award, whereas bidders with high cost types try to win the split award. It is interesting to note that the same type of hybrid equilibria also exist in settings with *dual source efficiency*. Because the split is the efficient award for all cost draws under *dual source efficiency*, τ is not restricted to a specific interval as with uncertain economies of scale. Hence, hybrid equilibria with $\tau \in \left(\underline{\Theta}, \overline{\Theta}\right)$ can exist as long as individual rationality is given by condition 4.5.1. The proof of Corollary 3 is a straightforward extension.

Corollary 3. *Consider the dual source efficiency split-award auction model including $n = 2$ bidders with cost types Θ_i, as well as a constant parameter $\tau \in \left(\underline{\Theta}, \overline{\Theta}\right)$. In the FPSB split-award auction, a hybrid equilibrium S_e^{BNE} with*

$$\left(p_e^s(\Theta_i), p_e^\sigma(\Theta_i)\right) =$$
$$\begin{cases} \left(\min\left\{2\left[\tau - C\tau\right], \Theta_i\right\}, \ \tau(1-C)\right) & \text{if } \Theta_i \geq \tau \\ \left(\Theta_i + \tau(1-2C)\frac{1-F(\tau)}{1-F(\Theta_i)} + \int_{\Theta_i}^{\tau}\frac{1-F(x)}{1-F(\Theta_i)}dx, \ \tau(1-C)\right) & \text{if } \Theta_i < \tau \end{cases},$$

exists, if

$$2\tau(1-C) > \overline{\Theta} \tag{4.5.1}$$

applies.

The fact that beyond the WTA and the σ equilibrium also hybrid equilibria exist, underscores that there is a veritable equilibrium selection problem. This is discussed further in Section 4.7.1, in which we analyze conditions for payoff dominance of the efficient σ equilibrium.

4.5.2 *The Dutch Split-Award Auction*

In this section, we analyze bidding behavior in the Dutch split-award auction, for which perfect Bayesian equilibria are applied as solution concept. Thus, an equilibrium strategy in the Dutch split-award auction defines prices, for which a supplier accepts either the split or the sole source award, as well as a system of beliefs μ. Unlike the FPSB split-award auction, only the σ equilibrium with the highest pooling price is possible in the Dutch split-award auction.

Corollary 4. *Consider the dual source efficiency split-award auction model including $n = 2$ bidders with cost types Θ_i. Then, if a σ equilibrium (S_e^{PBE2}, μ) exists in the Dutch split-award auction, the split prices $p_e^{\sigma 1}\left(\Theta_i, h^0\right)$ and $p_e^{\sigma 2l}\left(\Theta_l, h^1\right)$ must be constant and equal to $p_e^\sigma = \underline{\Theta}(1-C)$, i.e.*

$$p_e^{\sigma 1}\left(\Theta_i, h^0\right) = p_e^{\sigma 2l}\left(\Theta_l, h^1\right) = p_e^\sigma = \underline{\Theta}(1-C)$$

for all bidders with cost types $\Theta_i, \Theta_w, \Theta_l \in \left[\underline{\Theta}, \overline{\Theta}\right]$.

All proofs can be found in the Appendix B.1. As in the FPSB auction, the split price in a σ equilibrium must be constant in the Dutch split-award auction. Otherwise, it would be always more profitable for the supplier with the lower price for 50% to accept the same counteroffer as his opponent. However, only a σ equilibrium with constant split prices $p_e^{\sigma 1}\left(\Theta_i, h^0\right) = p_e^{\sigma 2l}\left(\Theta_i, h^1\right) = \underline{\Theta}(1-C)$ can emerge, not multiple efficient equilibria as in the FPSB format.

The main difference between the FPSB and the Dutch split-award auction is the information provided about the opponent's behavior. Whereas the Dutch split-award auction is a two-stage game, in which the winner immediately observes a deviation from a σ equilibrium, this information is provided ex-post in the FPSB split-award auction. Consider a setting with two bidders A and B. If bidder A is the winner of 50% of the business for a price p^σ in phase 1 of a Dutch split-award auction then it must be a possible threat for A to accept the offer for the remaining share at a price of $p_e^{\sigma 2w}\left(\Theta_A, h^1\right) \geq p^\sigma$ when it becomes obvious that his opponent deviates from equilibrium.

In a two-stage game such a threat is only credible if bidder A makes at least as much payoff as already achieved in phase 1, i.e., if at least bidder A's additional costs for providing 100% of the business, $(1-C)\,\Theta_A$, are covered. Therefore, $(1-C)\,\underline{\Theta}$ remains as the only possible split price for both bidders because a profitable split deviation as described above cannot be prevented by the winner of phase 1 for lower split prices. In the next proposition, we provide conditions for which a pure σ equilibrium exists.

Proposition 18. *Consider the dual source efficiency split-award auction model including $n = 2$ bidders with cost types Θ_i. In the Dutch split-award auction, there is a unique and efficient σ equilibrium with $\left(S_e^{PBE2}, \mu\right)$ involving strategies*

$$p_e^{\sigma 1}\left(\Theta_i, h^0\right) = \underline{\Theta}\,(1-C)$$
$$p_e^{\sigma 2w}\left(\Theta_w, h^1\right) = \Theta_w\,(1-C)$$
$$p_e^{\sigma 2l}\left(\Theta_l, h^1\right) = \underline{\Theta}\,(1-C)$$

and beliefs

$$\mu_{-i}^1\left(\Theta_i \mid h^0\right) = F\left(\Theta\right)$$
$$\mu_l^2\left(\Theta_w \mid h^1\right) = F\left(\Theta\right)$$
$$\mu_w^2\left(\Theta_l \mid h^1\right) = F\left(\Theta\right)$$

with $\Theta, \Theta_i, \Theta_w, \Theta_l \in \left[\underline{\Theta}, \overline{\Theta}\right]$, if for all $x \in \left(\underline{\Theta}, \overline{\Theta}\right]$

$$\Delta^{\Pi}\left(x, \overline{\Theta}\right) = \left(x\,(1-C) - C\overline{\Theta}\right)\left(1 - F\left(x\right)\right) - \underline{\Theta}\,(1-C) + C\overline{\Theta} < 0 \quad (4.5.2)$$

applies.

Sole source deviations can be ignored, as for all possible cost types Θ_i the payoff for winning the the split award is higher than the payoff for winning the sole source award in every round $q < r$ with counteroffers $c_q^s < c_r^s = 2\underline{\Theta}\,(1-C)$. Because the buyer sticks to his indifference condition, such a deviation cannot be realized unilaterally. The proof of Corollary 4 shows that split deviations for the remaining share are difficult to exclude, as the threat to prevent such deviations by the winner of phase 1 has to be credible. Therefore, condition (4.5.2) assures that a split deviation that tries to win the remaining share in phase 2 for a higher split price than $\underline{\Theta}\,(1-C)$ yields a lower expected payoff than the equilibrium payoff. Because of the pooling, there is no additional information about the cost type of the winner in phase 1, which is why updating of beliefs is not critical for the derivation of the equilibrium strategy. Next, we show that, unlike the FPSB split-award auction, the efficient σ equilibrium is unique in a Dutch split-award auction.

Proposition 19. *Consider the dual source efficiency split-award auction model including $n = 2$ bidders with cost types Θ_i. In the Dutch split-award auction, there is neither a WTA nor a hybrid equilibrium.*

In the FPSB split-award auction, bidders are able to play a WTA strategy, because they can unilaterally exclude the split by submitting high bid-to-lose prices for the 50% share. However, this is not possible in the Dutch auction, because there is always a profitable split deviation for high cost types playing a potential WTA equilibrium. Hence, such a strategy cannot be an equilibrium. All possible types of hybrid equilibria can be excluded as well. Thus, if a σ equilibrium exists in the Dutch split-award auction with $n = 2$ bidders, it is the unique equilibrium.

4.5.3 *The Dutch-FPSB Split-Award Auction*

Next, we analyze equilibrium bidding in the Dutch-FPSB split-award auction. The auction format combines the two first-price mechanisms and the bidding behavior contains elements from the equilibrium strategies of both auction formats. The necessary conditions for a split price in a σ equilibrium are summarized in Corollary 5.

Corollary 5. *Consider the dual source efficiency split-award auction model including $n = 2$ bidders with cost types Θ_i. Then, if a σ equilibrium with (S_e^{PBE2}, μ) exists in the Dutch-FPSB split-award auction, the split prices must be constant with $p_e^\sigma \in \left[\overline{\Theta}C, \underline{\Theta}\left(1 - C\right)\right]$, i.e.*

$$p_e^{\sigma 1}\left(\Theta_i, h^0\right) = p_e^{\sigma 2l}\left(\Theta_l, h^1\right) = p_e^\sigma$$

for all bidders with cost types $\Theta_i, \Theta_l \in \left[\underline{\Theta}, \overline{\Theta}\right]$.

Multiple constant split prices in a given range are possible in a σ equilibrium. As in the FPSB split-award auction, a bidder can only observe ex-post, whether or not his opponent played a σ equilibrium or not. Hence, it is easier for the bidders to implement a σ equilibrium strategy. As described above, a threat must be realized, and becomes payoff-relevant, when the opponent deviates in a Dutch split-award auction. When phase 2 is a sealed-bid stage, this problem disappears and it suffices that the threat prevents the opponent from deviating. When this is fulfilled, the threat never becomes effective and the expected payoff of the winner of phase 1 remains the same. Proposition 20 summarizes the results for the existence of pure σ equilibria:

Proposition 20. *Consider the dual source efficiency split-award auction model including $n = 2$ bidders with cost types Θ_i. In the Dutch-FPSB split-award auction, there are multiple efficient σ equilibria (S_e^{PBE2}, μ) involving strategies*

$$p_e^{\sigma 1}\left(\Theta_i, h^0\right) = p_e^{\sigma}$$
$$p_e^{\sigma 2w}\left(\Theta_w, h^1\right) = \max\left\{p_e^{\sigma}, \Theta_w - p_e^{\sigma}\right\}$$
$$p_e^{\sigma 2l}\left(\Theta_l, h^1\right) = p_e^{\sigma}$$

and beliefs

$$\mu_{-i}^1\left(\Theta_i \mid h^0\right) = F\left(\Theta\right)$$
$$\mu_l^2\left(\Theta_w \mid h^1\right) = F\left(\Theta\right)$$
$$\mu_w^2\left(\Theta_l \mid h^1\right) = F\left(\Theta\right)$$

with $\Theta, \Theta_i, \Theta_w, \Theta_l \in \left[\underline{\Theta}, \overline{\Theta}\right]$ *and* $p_e^{\sigma} \in \left[\overline{\Theta}C, (1 - C)\,\underline{\Theta}\right]$, *if*

$$\Theta_i \leq G\left(\Theta_i, p_e^{\sigma}\right) = p_e^{\sigma} + \frac{p_e^{\sigma} - C\overline{\Theta}F\left(\Theta_i\right)}{1 - F\left(\Theta_i\right)} \text{ for all } \Theta_i \in \left[\underline{\Theta}, \overline{\Theta}\right] \tag{4.5.3}$$

applies.

The reasoning here is similar to that in Proposition 18. As the auctioneer offers the shares according to his indifference condition in phase 1, sole source deviations are not possible in equilibrium. Condition (4.5.3) is important to assure that the credible threat of the winner of phase 1 is possible without violating the assumption of individual rationality. Note that the function $G\left(\cdot,\cdot\right)$ is the same as in Anton and Yao (1992). This means that exactly the same σ equilibria as in the FPSB split-award auction can emerge. Furthermore, if condition (4.5.3) applies, all split deviations in phase 2 can be excluded in a σ equilibrium with split price p_e^{σ}. As in the Dutch split-award auction, only efficient σ equilibria can emerge.

Proposition 21. *Consider the dual source efficiency split-award auction model including $n = 2$ bidders with cost types Θ_i. There is neither a WTA nor a hybrid equilibrium in the Dutch-FPSB split-award auction.*

We omit the proof for the Proposition 21 as it follows that of Proposition 19. The same efficient equilibria as in the FPSB auction emerge without additional restrictions. Furthermore, it can be shown that WTA and hybrid equilibria are excluded as equilibrium bidding strategies, which reduces the coordination problem to efficient equilibria. As we see in the welfare analysis below, such a coordination problem can be solved via payoff dominance.

The characteristics of both first-price mechanisms influence the equilibrium bidding behavior in the Dutch-FPSB split-award auction. The combinatorial Dutch auction in phase 1 is sufficient to exclude inefficient equilibria. The sealed-bid mechanism in phase 2 also allows for σ equilibria with various split prices in the same range as in the FPSB auction, as the winner in phase 1 has a credible threat to punish deviations from a σ equilibrium.

4.6 EQUILIBRIUM BIDDING IN THE $n > 2$-BIDDER MODEL

Next, we analyze the bidding behavior with more than two suppliers. The n-bidder case leads to differences in how the equilibrium strategies are derived and in the outcome compared to the 2-bidder case. In particular, a pooling equilibrium at high prices that exists in all first-price auction formats with two bidders cannot be maintained anymore.

4.6.1 *The FPSB Split-Award Auction*

We start with analyzing bidding behavior in the FPSB split-award auction. First, it is interesting to see that there is a WTA equilibrium in *dual source efficiency* with $n > 2$ suppliers, even though bidders have less power to veto a split award for their opponents:

Proposition 22. *Consider the dual source efficiency split-award auction model including $n > 2$ bidders with cost types Θ_i. In the FPSB split-award auction, there is a WTA equilibrium S_e^{BNE} with*

$$p_e^s(\Theta_i) = \Theta_i + \frac{\int_{\Theta_i}^{\overline{\Theta}} \left(1 - F(t)\right)^{n-1} dt}{\left(1 - F(\Theta_i)\right)^{n-1}} \tag{4.6.1}$$

$$p_e^\sigma(\Theta_i) = p_e^s(\Theta_i) - \underline{\Theta} C. \tag{4.6.2}$$

By following such an equilibrium strategy, the split-award auction is reduced to a single-object auction for 100% of the business, because the split is excluded for all bidders (and the auctioneer) due to sufficiently high split prices. The expected payoffs of all possible (unilateral) split deviations are zero with probability 1. Therefore, the sole source price must be equal to the equilibrium strategy in a single-object auction in order to maximize the expected profit for winning the whole business.

In addition to the WTA equilibrium, a σ equilibrium exists with *dual source efficiency* as well. This equilibrium always results in the efficient allocation, the split award.

Proposition 23. *Consider the dual source efficiency split-award auction model including $n > 2$ bidders with cost types Θ_i. In the FPSB split-award auction, a σ equilibrium S_e^{BNE} with*

$$p_e^s(\Theta_i) = \max\left\{\overline{\Theta} C + p_e^\sigma(\Theta_i), \Theta_i\right\} \tag{4.6.3}$$

$$p_e^\sigma(\Theta_i) = \Theta_i C + C \frac{\int_{\Theta_i}^{\overline{\Theta}} \left(1 - F(t)\right)^{n-1} + (n-1) F(t) \left(1 - F(t)\right)^{n-2} dt}{\left(1 - F(\Theta_i)\right)^{n-1} + (n-1) F(\Theta_i) \left(1 - F(\Theta_i)\right)^{n-2}}, \tag{4.6.4}$$

exists, if either

$$C < \frac{\underline{\Theta}}{2\overline{\Theta}} \tag{4.6.5}$$

or

$$\begin{aligned} E\left[\Pi_e^\sigma\left(\Theta_i\right)\right] > &\left(p_e^\sigma\left(x_1\right) + p_e^\sigma\left(x_2\right) - \Theta_i\right)\cdot \\ &\cdot P\Big(p_e^\sigma\left(x_1\right) + p_e^\sigma\left(x_2\right) < \min\Big\{p_e^\sigma\left(\Theta_{1:n-1}\right) + \\ &+ \min\left\{p_e^\sigma\left(\Theta_{2:n-1}\right), p_e^\sigma\left(x_2\right)\right\}, \\ &\max\left\{\Theta_{1:n-1}, p_e^\sigma\left(\Theta_{1:n-1}\right) + \overline{\Theta}C\right\}\Big\}\Big) + \\ &+ \left(p_e^\sigma\left(x_2\right) - C\Theta_i\right) P\Big(p_e^\sigma\left(x_2\right) < p_e^\sigma\left(\Theta_{2:n-1}\right) \wedge \\ &\wedge p_e^\sigma\left(x_1\right) \geq p_e^\sigma\left(\Theta_{1:n-1}\right)\Big) \end{aligned} \tag{4.6.6}$$

applies for all $\Theta_i \in [\underline{\Theta}, 2C\overline{\Theta})$ *and* $x_1 < x_2$ *with* $x_1 \in \left[\underline{\Theta}, \overline{\Theta}\right]$, $x_2 \in (x_1, \overline{\Theta}]$.

The split price is derived by maximizing the expected payoff of being amongst the two suppliers winning the split award in order to rule out split deviations. In contrast to the setting with two bidders, in which the suppliers' bids for the split award are constant, split prices are increasing with costs and the highest cost type $\overline{\Theta}$ makes a payoff of zero in equilibrium. Bidders who concentrate on winning the split award, submit sole source prices at least as high as the buyer's maximal purchasing costs in the σ equilibrium, $2\overline{\Theta}C$. However, sole source or even hybrid deviations are nevertheless possible, if the efficiency parameter is not too small, i.e. $\frac{\underline{\Theta}}{2\overline{\Theta}} < C < \frac{\underline{\Theta}}{\underline{\Theta}+\overline{\Theta}}$.

In contrast to a split deviation, a deviating bidder is not dependent on another competitive bid for the same share. Thus, there are sole source and hybrid deviations with positive expected payoff, which is why the proof of excluding both types of deviations is the most challenging part of the proof. While sole source deviations can be excluded in general for all possible settings within the model assumptions, the additional condition (4.6.6) as stated in the proposition is needed to assure the exclusion of all possible hybrid deviations. When condition (4.6.6) is verified, Proposition 23 provides a closed-form solution for an efficient Bayesian Nash equilibrium strategy in a sealed-bid combinatorial first-price auction and a general number of bidders n. Condition (4.6.6) can be approximated by a stricter condition that is simple to evaluate and numerical experiments yield that it holds for a wide range of distributions.

Furthermore, the following result can be derived.

Proposition 24. *Consider the dual source efficiency split-award auction model including* $n > 2$ *bidders with cost types* Θ_i*. There is no hybrid equilibrium in the FPSB split-award auction.*

With a hybrid equilibrium, there must be at least one definite cost type τ, for which the winning allocation stays the same provided that all cost draws of the $n > 2$ bidders are higher than τ. It can be shown that there is no such potential cost type that fulfills all the required conditions with *dual source efficiency*, which has as a consequence that such equilibria do not exist. Therefore, a bidder faces a coordination problem in a FPSB split-award auction, for which he has to decide whether to play a WTA or σ equilibrium. We discuss this coordination problem in Section 4.8.1.

4.6.2 *The Dutch Split-Award Auction*

As presented in Section 4.4, the Dutch split-award auction comprises two phases, if a single bidder has accepted a counteroffer for the 50% share in phase 1. Therefore, an efficient equilibrium strategy has to maximize the bidders' expected payoff in both phases and has to consider the asymmetric cost structure of the suppliers in phase 2. After the result of phase 1 is observed, there are two different types of suppliers, one winner w and $n-1$ losers l of phase 1. Whereas all suppliers have the chance to win the whole business in phase 1, this only applies for one supplier in phase 2. When the winner of phase 1 accepts a counteroffer for the split in phase 2, he is the winner of the whole business.[35] Hence, we have to define the equilibrium strategies for both of these different types of bidders, because suppliers are not symmetric anymore in phase 2.

Proposition 25. *Consider the dual source efficiency split-award auction model including $n > 2$ bidders with cost types Θ_i. In the Dutch split-award auction, the unique σ equilibrium (S_e^{PBE2}, μ) is given by*

$$p_e^{\sigma 1}\left(\Theta_i, h^0\right) = \frac{\int_{\Theta_i}^{\overline{\Theta}} p_e^{\sigma 2l}\left(t\right)\left(n-1\right)\left(1-F\left(t\right)\right)^{n-2} f\left(t\right) dt}{\left(1-F\left(\Theta_i\right)\right)^{n-1}}$$

$$p_e^{\sigma 2w}\left(\Theta_w, h^1\right) = \Theta_w\left(1-C\right)$$

$$p_e^{\sigma 2l}\left(\Theta_l, h^1\right) = \Theta_l C + C\frac{\int_{\Theta_l}^{\overline{\Theta}}\left(1-F\left(t\right)\right)^{n-2} dt}{\left(1-F\left(\Theta_l\right)\right)^{n-2}}$$

[35]The buyer can also exclude the winning supplier from the auction in phase 2. All the results apply for both auction variants with *dual source efficiency*. The format without the possibility of a requote is easier tractable with other efficiency settings.

and beliefs

$$\mu^1_{-i}\left(\Theta_i \mid h^0\right) = F\left(\Theta\right)$$

$$\mu^2_l\left(\Theta_w \mid h^1\right) = \begin{cases} 0 & \text{if } \Theta < \Theta_w \\ 1 & \text{if } \Theta \geq \Theta_w \end{cases}$$

$$\mu^2_w\left(\Theta_l \mid h^1\right) = \begin{cases} 0 & \text{if } \Theta < \Theta_w \\ \frac{F(\Theta)-F(\Theta_w)}{\left(1-F(\Theta_w)\right)} & \text{if } \Theta \geq \Theta_w \end{cases}$$

with $\Theta, \Theta_i, \Theta_w, \Theta_l \in \left[\underline{\Theta}, \overline{\Theta}\right]$.

In order to exclude split deviations in phase 1 and 2, we take the equilibrium strategy of an ex-ante split-award auction, in which the 50% share is awarded sequentially to two different suppliers. This strategy maximizes the expected payoff for the split award in both phases. Hence, it only remains to be shown that there is no sole source deviation, which is more profitable than the σ equilibrium. An assessment of the expected payoff of such deviations yields the desired result for phase 1. Additionally, it can be shown that the winner of phase 1 has no chance to win the remaining 50% share in phase 2. One of his opponents secures himself the remaining 50% share before this award becomes attractive for him. Because this would be the only possible sole source deviation in phase 2, the proof is complete.

In the FPSB split-award auction, bidders are able to play a WTA strategy, as they can exclude the split by submitting high bid-to-lose prices in equilibrium. However, this is not possible in the Dutch auction, as there is always a profitable split deviation for high cost types playing a potential WTA equilibrium. Hence, such a strategy cannot be an equilibrium. Furthermore, similar deliberations as in the proof of Proposition 24 show that no hybrid equilibrium exists with *dual source efficiency* in the Dutch split-award auction.

4.6.3 *The Dutch-FPSB Split-Award Auction*

The equilibrium analysis for the Dutch-FPSB split-award auction is identical to the Dutch auction with more than 2 bidders. All the equilibrium strategies of Section 4.6.2 are equivalent.

4.7 WELFARE ANALYSIS IN THE 2-BIDDER MODEL

In this section we first study the efficiency of all three auction formats and then discuss differences in the procurement costs for the auctioneer. As in the equilibrium analysis, the setting with only two suppliers needs to be analyzed separately from the setting with $n > 2$ suppliers.

4.7.1 *Efficiency Analysis*

There is a unique and efficient σ equilibrium strategy in the Dutch split-award auction. The Dutch-FPSB split-award auction, in which multiple σ equilibria exist, always results in the efficient allocation. The σ equilibrium with the highest possible split price $p_e^\sigma = \underline{\Theta}(1-C)$ is payoff-dominant over all other efficient equilibria; obviously, a σ equilibrium with a lower split price yields less payoff, because the probability to win does not increase when the split price decreases.

There is a coordination problem in the FPSB split-award auction, because a WTA, multiple σ equilibria with split prices $p_e^\sigma \in \left[\overline{\Theta}C, \underline{\Theta}(1-C)\right]$, and multiple hybrid equilibria with different strategic parameters $\tau \in \left(\underline{\Theta}, \overline{\Theta}\right)$ can exist in this auction format. Proposition 5 in Anton and Yao (1992) describes a setting in which the σ equilibrium is payoff-dominant over the WTA equilibrium for the bidders. We extend these results by considering hybrid equilibria as well; Corollary 6 gives conditions for which an efficient σ equilibrium is payoff-dominant over all other types of equilibria.

Corollary 6. *Consider the dual source efficiency split-award auction model including $n = 2$ bidders with cost types Θ_i and a constant parameter $\tau \in \left(\underline{\Theta}, \overline{\Theta}\right)$. In the FPSB split-award auction, a σ equilibrium is payoff-dominant for all bidders if the following conditions apply:*

1. $p_e^\sigma = \underline{\Theta}(1-C)$
2. $E\left[\Pi_e^{hybrid}(\Theta_i)\right] \leq \underline{\Theta}(1-C) - \Theta_i C \ \forall\ \Theta_i \in \left[\underline{\Theta}, \overline{\Theta}\right]$
3. $E(\theta_i) < 2\underline{\Theta}(1-C)$

The expected payoff of a supplier i in a hybrid equilibrium with strategic parameter τ is

$$E\left[\Pi_e^{hybrid}(\Theta_i)\right] = \begin{cases} \tau(1-2C)\left(1-F(\tau)\right) + \int_{\Theta_i}^{\tau}\left(1-F(x)\right)dx & \text{if } \Theta_i < \tau \\ \left(\tau(1-C) - \Theta_i C\right)\left(1-F(\tau)\right) & \text{if } \Theta_i \geq \tau \end{cases}.$$

As mentioned above, only the σ equilibrium with the highest pooling price $p_e^\sigma = \underline{\Theta}(1-C)$ can be payoff-dominant over all other equilibria. When condition (ii) applies, there is no cost type with a higher equilibrium payoff in a hybrid than in a σ equilibrium. Condition (iii) assures that the σ equilibrium is payoff-dominant over its WTA counterpart. Corollary 6 can be interpreted as a sign for strategic complexity of bidding in a FPSB split-award auction. The conditions are very restrictive and it is hard to find a setting, for which these conditions are fulfilled simultaneously. Furthermore, the strategic parameter τ, for which bidders change the strategy must be known to all bidders and to the buyer when hybrid equilibria are possible. This additional assumption is hard to motivate in procurement practice.

4.7.2 *Comparison of Purchasing Costs*

We use payoff dominance to overcome the bidders' coordination problem. A prediction on the expected procurement costs of a buyer can only be done in settings with a payoff-dominant equilibrium. This applies for the Dutch as well as the Dutch-FPSB split-award auction, although not always for the FPSB auction.

Corollary 7. *Consider the dual source efficiency split-award auction model including $n = 2$ bidders with cost types Θ_i. Then, there is cost equivalence between the Dutch and the Dutch-FPSB auction. The buyer's expected procurement costs are $E\left[p_b^\sigma(\cdot,\cdot)\right] = 2\underline{\Theta}(1-C)$ in these auctions. This applies for the FPSB auction if the conditions of Corollary 6 are valid.*

We omit the proof, as it is trivial and follows directly from the equilibrium analysis. Because the split is awarded with probability 1 in a σ equilibrium, the purchasing costs in the Dutch and Dutch-FPSB split-award auctions equal twice the highest possible split price $p_e^\sigma = \underline{\Theta}(1-C)$. This only applies for the FPSB split-award auction when all conditions of Corollary 6 are fulfilled and there is a payoff-dominant σ equilibrium. Otherwise, cost equivalence between the (partly) ascending auction formats and the sealed-bid variant fails. According to the equilibrium analysis, bidders are able to coordinate on very high split prices in a σ equilibrium, although they face lower average costs for 50% than for 100% of the business. In settings in which the expected costs of a σ equilibrium are lower than in a single-unit auction, a buyer should prefer one of the two (partly) ascending split-award auctions (Dutch and Dutch-FPSB) in order to achieve higher efficiency at lower costs in equilibrium.

Furthermore, for all possible cost draws of the two suppliers, costs of the auctioneer in an efficient σ equilibrium of the analyzed first-price auctions are always lower than the VCG costs. The reason for this is that the VCG payments for a supplier depend on the opponent's sole source price, which is high in *dual source efficiency*. This does not apply for $n > 2$, which will be explained in more detail below.

Corollary 8. *Consider the dual source efficiency split-award auction model including $n = 2$ bidders with cost types Θ_i. The costs for the auctioneer, $p_b^\sigma(\cdot,\cdot)$, in a first-price split-award auction, for which a payoff-dominant σ equilibrium exists, are lower than the VCG costs, $p_b^{VCG}(\cdot,\cdot)$, independent of the cost draws of the suppliers.*

In other words, cost equivalence not only fails between the different first-price split-award auctions, but also does not hold between first- and second-price split-award auctions in the 2-Bidder-Model. By applying a Dutch or Dutch-FPSB split-award auction instead of a VCG mechanism, the auctioneer achieves full efficiency and lower expected procurement costs.

4.8 WELFARE ANALYSIS IN THE $n > 2$-BIDDER MODEL

In what follows, the efficiency and procurement costs with more than two bidders are analyzed.

4.8.1 *Efficiency Analysis*

Bidding behavior in the Dutch or Dutch-FPSB split-award auction is straightforward for suppliers, as there is a unique and efficient equilibrium strategy. However, in the FPSB split-award auction, bidders face a coordination problem between the WTA and the σ equilibrium. Payoff dominance can be a remedy in coordination problems, but it does not help in the FPSB auction.

Proposition 26. *Consider the dual source efficiency split-award auction model including $n > 2$ bidders with cost types Θ_i. In the FPSB split-award auction, the WTA equilibrium cannot be payoff-dominant over the σ equilibrium for all cost types $\Theta_i \in \left[\underline{\Theta}, \overline{\Theta}\right]$.*

In the proof, we show that bidders with a high cost draw always prefer a σ equilibrium due to higher expected profits regardless of the parameters $n, C, F(\cdot)$ or the support $\left[\underline{\Theta}, \overline{\Theta}\right]$. Therefore, only the σ equilibrium can be payoff-dominant with *dual source efficiency*. However, if there is at least a single cost type $\Theta_i \in \left[\underline{\Theta}, \overline{\Theta}\right]$, whose expected profits in a WTA are higher than in a σ equilibrium, the coordination problem cannot be solved by payoff dominance. The proof of this proposition is omitted, because it follows directly from the comparison of equilibrium payoffs of a σ and WTA equilibrium.

Proposition 27. *Consider the dual source efficiency split-award auction model including $n > 2$ bidders with cost types Θ_i. Neither the σ nor the WTA equilibrium is payoff-dominant in a FPSB split-award auction, if for at least one cost type $\Theta_i \in \left[\underline{\Theta}, \overline{\Theta}\right]$*

$$C < \frac{\int_{\Theta_i}^{\overline{\Theta}} \left(1 - F(x)\right)^{n-1} dx}{\int_{\Theta_i}^{\overline{\Theta}} \left(1 - F(x)\right)^{n-1} + (n-1) F(x) \left(1 - F(x)\right)^{n-2} dx} \tag{4.8.1}$$

is true.

Condition (4.8.1) applies in many environments, e.g. whenever the cost parameters are uniformly distributed over any support $\left[\underline{\Theta}, \overline{\Theta}\right]$.

Corollary 9. *Consider the dual source efficiency split-award auction model including $n > 2$ bidders with cost types Θ_i. There is no setting, for which either the WTA equilibrium or the σ equilibrium is payoff-dominant for all cost types in a FPSB split-award auction, if $\theta_i \sim U\left[\underline{\Theta}, \overline{\Theta}\right]$.*

Thus, the coordination problem makes it hard to predict bidding behavior and the outcome in a FPSB split-award auction. An inefficient WTA or an efficient σ equilibrium are possible as equilibrium outcomes. These problems do not arise in the Dutch or Dutch-FPSB split-award auction, because there is a unique and efficient σ equilibrium.

4.8.2 *Comparison of Purchasing Costs*

For the FPSB split-award auction we can only define the expected costs for the buyer on the condition that all bidders follow the same equilibrium strategy. Hence, we get expected costs for the WTA and for the σ equilibrium in the FPSB split-award auction. When bidders choose a WTA equilibrium, the price for the auctioneer equals the purchasing costs in a single-unit auction. If the bidders choose the σ equilibrium, then bidders in all auction formats, the descending, the Dutch, the FPSB, and the VCG auction, aim for a single share in equilibrium.

With this symmetric σ equilibrium, $n > 2$ and identical 50% shares, the auctions are strategically equivalent to traditional multi-unit auctions with single-unit demand such that we can draw on the well-known revenue equivalence theorem for this environment (Myerson, 1981; Engelbrecht-Wiggans, 1988).

Bidders typically do not have single-unit demand in combinatorial auctions. However, by playing a σ equilibrium and submitting non-competitive bid-to-lose prices for 100% of the business, the sole source award is off-equilibrium and the results are outcome equivalent to an ex-ante split-award auction, in which bidders cannot win more than 50% of the business. This is the reason, why the assumption of single-unit demand can be applied to bidders playing a σ equilibrium and the purchasing costs in the Dutch auction equal the costs in the VCG or descending auction with *dual source efficiency*. This is only true for the FPSB auction provided that bidders are able to coordinate on the split.

Although the split is efficient, purchasing costs in a σ equilibrium are not necessarily lower than in a WTA equilibrium. However, this applies for many settings with *dual source efficiency*.

Corollary 10. *Consider the dual source efficiency split-award auction model including $n > 2$ bidders with cost types Θ_i. In the FPSB split-award auction, the price for the buyer in the σ equilibrium is always lower than in the WTA equilibrium,*

- *if either $C < \frac{\underline{\Theta}}{2\overline{\Theta}}$ applies or*
- *if $\theta_i \sim U\left[\underline{\Theta}, \overline{\Theta}\right]$ applies.*

The expected price for the auctioneer in a σ equilibrium raises with a higher efficiency parameter C, whereas prices in the WTA equilibrium are independent of C. Hence, if the efficiency parameter C is sufficiently low, the costs for the auctioneer are always lower in the σ equilibrium.

Additionally, we show that for all possible $C < \frac{\underline{\Theta}}{\underline{\Theta}+\overline{\Theta}}$ and uniformly distributed cost types, the σ equilibrium yields lower purchasing costs than the

WTA equilibrium. Corollary 9 states that there is always a coordination problem in such a setting and no equilibrium is payoff-dominant. Thus, when costs are assumed to be uniformly distributed over any support $\left[\underline{\Theta}, \overline{\Theta}\right]$, the auctioneer should prefer the Dutch or Dutch-FPSB over the FPSB split-award auction with *dual source efficiency* not only because of its efficiency properties and lower strategical complexity for the bidders but also because of lower expected purchasing costs.

Note that the effect of adding a third bidder on procurement costs is substantial. In a split-award auction, with uniformly distributed cost types $\Theta \in [100, 140]$ and an efficiency parameter $C = 0.3$ procurement costs are reduced by 44.3% in expectation[36], if a third bidder is added. An additional fourth bidder only has and impact of minus 2.6%, a fifth supplier only an impact of minus 1.7%.

4.9 EXPERIMENTAL EVALUATION

We begin with explaining the experimental design. Then, the results for efficiency, procurement costs and bidding behavior in our experiments are discussed.

4.9.1 *Experimental Design*

Our theoretical results for a two-bidder and a three-bidder setting of the three first-price split-award auction mechanisms, i.e., FPSB, Dutch and Dutch-FPSB formats, were tested in experiments in which human subjects interacted in multiple periods. Thus, our treatment variables are the auction format and the number of bidders that result in six different treatments as depicted in Table 1:

Auction format	Number of bidders
FPSB	2
	3
Dutch	2
	3
Dutch-FPSB	2
	3

Table 1: Treatments

Every period in all treatments starts with an information stage for the bidders, in which they are informed about their own costs for supplying 50% or 100% of a fictitious order. The information about the cost draws is private and the participants do not know the costs of their opponents. However, it is common knowledge that the cost parameter Θ is uniformly and independently distributed on $[100.00, 140.00]$ and that the efficiency parameter remains constant at $C = 0.30$ in every period. Hence, the costs of a bidder for the 100%

[36] Procurement prices are expected to drop from a σ equilibrium with the highest pooling price, $2\underline{\Theta}(1 - C)$, to $2E[\Theta_{3:3}]$ in the σ equilibrium with three bidders.

share, Θ, range from 100.00 to 140.00 and his costs for the 50% share, ΘC, from 30.00 to 42.00. Successively, the respective auctions were conducted according to the rules described in Section 4.4.

We implemented upper bounds for the split as well as the sole source price in each auction format. In the FPSB auction, the highest possible bid for the split award is set to 150.00 and the submitted sole source price cannot be higher than 300.00. Both bids must be submitted as multiples of 0.50. The starting price for the split award in the Dutch and in the first phase of the Dutch-FPSB auction is set to 30.00 and increases with a step size of 0.50 every half second until the upper bound of 150.00 is reached. The price for the 100% share is twice the price of the smaller share and rises accordingly, i.e. it cannot exceed the upper bound of 300.00. The same upper bound as for the split price in the FPSB auction applies for all bids submitted in the second phase of the Dutch-FPSB auction.

		Sample size				
		Group 1	Group 2	Group 3	Group 4	Σ
2x2 setting	FPSB	12	12	12	10	46
	Dutch-Dutch	12	12	12	12	48
	Dutch-FPSB	12	12	12	12	48
2x3 setting	FPSB	12	12	12	0	36
	Dutch-Dutch	12	12	12	12	48
	Dutch-FPSB	12	12	12	12	48
						274

Table 2: Matching Group Sample Sizes

Two sessions were conducted for each of the treatment variables. For each session, two matching groups of 12 subjects were defined, who participated in 15 consecutive first-price split-award auctions. Random matching of the subjects to the auctions was applied in each of the 15 periods. Each subject was allowed to participate in one session only. One of the four matching groups for the FPSB auction with 2 bidders comprised only 10 subjects and the experiment could only be conducted for three matching groups for the FPSB format with three bidders. Summarizing, 274 subjects took part in our experiments and Table 2 gives a detailed overview of the sample sizes in the different treatments.

In each period of the treatment with two bidders, the 12 subjects were randomly divided into six auctions consisting of two bidders each. In total, we conducted 360 auctions for the Dutch and Dutch-FPSB formats and 345 auctions for the FPSB format. In the three-bidder treatments the 12 subjects per matching group were randomly distributed to four auctions in every period, which resulted in 240 auctions for the Dutch and Dutch-FPSB auctions and 180 for the FPSB format. In the evaluation of our experiments the matching group average serves as the unit of observation and is employed to calculate the average costs to the auctioneer and efficiencies of the six different treatments. In the two-bidder setting, in each matching group six first-price split-award auctions were carried out in each of 15 consecutive periods. Thus, the average values per matching group are computed based on a sample of 90 auctions with the exception of the FPSB auction in which one matching group contained only 75 auctions. With three bidders, four first-price auctions took place in each of 15 periods and the matching group averages are calculated for a sample of 60 auctions. Finally, note that in the discussion of individual bidding behavior in Section 4.9.4 the unit of observation is the individual decision.

Before the subjects sifted through the instructions on their own, the instructions were read out aloud to all subjects at the start of each session. Additionally, the subjects should answer comprehension questions, which were included at the end of the instructions. All interaction in the experiments was anonymously and computerized using the experimental software z-Tree (Fischbacher, 2007). The subjects were neither allowed to personally interact nor to communicate with each other during the session.

The experiments were carried out at the experimenTUM, the laboratory for experimental economic studies of the Technical University of Munich in 2016. Undergraduate as well as graduate students from the Technical University of Munich from different study programs participated. At the end of the session, each subject was anonymously paid his cumulative earnings from all periods including a show-up fee of 6.00 EUR (6.56 USD). On average subjects earned 20.85 EUR (22.78 USD) and participated between one and a half to two hours in the experiments.

4.9.2 *Theoretical Predictions*

In this section, we determine the equilibrium strategies for the chosen experimental setting from Section 4.9.1 for two and three bidders. Note that the parameters in our experimental setting are chosen such that a σ equilibrium exists in the FPSB, Dutch, and Dutch-FPSB split-award auction.

4.9.2.1 *The FPSB Split-Award Auction*

In our experimental setting the σ equilibrium of the FPSB split-award auction is characterized by a pooling price for the 50% share $p_e^\sigma \in [54.06, 70.00]$. Each of these prices is supported by a bid on the 100% share according to $p_e^s(\Theta_i) \leq G\left(p_e^\sigma, \Theta_i\right)$ for all $\Theta_i \in \left[\underline{\Theta}, \overline{\Theta}\right]$, where $G\left(p_e^\sigma, \Theta_i\right) = p_e^\sigma + \frac{40.00 p_e^\sigma - 42.00\Theta_i + 4200.00}{140.00 - \Theta_i}$. The range of equilibrium pooling prices is restricted by the off-equilibrium non-negative profit condition $\Theta_i \leq G\left(p_e^\sigma, \Theta_i\right)$ for all $\Theta_i \in \left[\underline{\Theta}, \overline{\Theta}\right]$. In the WTA equilibrium, the optimal bid on 100% of the business $p_e^s(\Theta_i) = 0.50\Theta_i + 70.00$ is supported by any bid on 50% share of at least $p_e^\sigma(\Theta_i) \geq p_e^s(\Theta_i) - 30.00$. We predict split prices as high as 70.00.

In the setting with three bidders, there is an inefficient WTA equilibrium with competitive prices for the package of two units, $p_e^s(\Theta_i) = \frac{2.00\Theta_i}{3.00} + \frac{140.00}{3.00}$, and high bid-to-lose prices for the single unit, $p_e^\sigma(\Theta_i) = p_e^s(\Theta_i) - 30.00$. A sole source price as high as the bidder's cost type Θ_i is sufficient to support competitive split prices in the efficient σ equilibrium of

$$p_e^{\sigma}(\Theta_i) = \frac{0.10\left(-(300.00 - 2.00\Theta_i)\,\Theta_i + 280.00\Theta_i - 2800.00\right)}{\Theta_i - 60.00}.$$

Neither the σ nor the WTA equilibrium is payoff-dominant in this setting as shown in Corollary 9 and it is not possible to predict by this criterion, which equilibrium the bidders should select.

4.9.2.2 *The Dutch Split-Award Auction*

In our experimental setting the σ equilibrium of the Dutch auction is defined as a unique pooling price of $p_e^{\sigma 1}\left(\Theta_i, h^0\right) = p_e^{\sigma 2l}\left(\Theta_l, h^1\right) = p_e^{\sigma} = 70.00$ on the 50% share for phases 1 and 2. The winner of phase 1 threatens to accept the remaining 50% of the business at a price of $p_e^{\sigma 2w}\left(\Theta_w, h^1\right) = 0.70\Theta_w$. The conditions for a σ equilibrium are fulfilled and we expect the same split prices of 70.00 as in the FPSB format.

With more than two bidders there is still only a unique and efficient σ equilibrium in the Dutch auction, which always results in split allocation with two bidders winning a single unit sequentially in each phase. The bidder with the lowest cost draw should win the first unit for a price of $p_e^{\sigma 1}\left(\Theta_i, h^0\right) = 0.15\,(\Theta_i + 140.00) + 0.05\,(140.00 - \Theta_i)$ and play a threat of $p_e^{\sigma 2w}\left(\Theta_w, h^1\right) = 0.70\Theta_w$ in phase 2. The second-lowest cost draw is supposed to accept a counteroffer of $p_e^{\sigma 2l}(\Theta_l) = 0.15\,(\Theta_l + 140.00)$ for the remaining share in phase 2.

4.9.2.3 *The Dutch-FPSB Split-Award Auction*

The range of pooling prices for the 50% share in phases 1 and 2 in the σ equilibrium of the Dutch-FPSB auction is analogous to the FPSB format with $p_e^{\sigma 1}\left(\Theta_i, h^0\right) = p_e^{\sigma 2l}\left(\Theta_l, h^1\right) = p_e^{\sigma} \in [54.06, 70.00]$. Each of the equilibrium pooling prices is supported by the winner of 50% of the business from phase 1 threatening to submit a price of $p_e^{\sigma 2w}\left(\Theta_w, h^1\right) = \max\{p_e^{\sigma}, \Theta_w - p_e^{\sigma}\}$ for the remaining 50% of the business in phase 2. The σ equilibrium exists, and based on payoff dominance we predict the Dutch-FPSB auction to produce the same split prices as its Dutch counterpart. As the Dutch-FPSB is strategically equivalent to the Dutch auction with $n > 2$ bidders, the same σ equilibrium as above emerges.

4.9.2.4 *Efficiency and Purchasing Costs*

As the two ascending auctions are characterized solely by efficient σ equilibria we expect the latter two formats to yield the efficient split award more often than the FPSB format in the experiments. The expected procurement costs for the buyer in the Dutch and the Dutch-FPSB split-award auctions are $E\left[p_b^{\sigma}\right] = 140.00$ in the two-bidder environment. This applies as well for FPSB auctions in which all σ equilibria are payoff-dominant over the respective hybrid equilibria. If we consider non-payoff-dominant σ equilibria, predictions about an expected purchasing price cannot be made.

The predictions concerning efficiency are independent of the number of bidders. However, for $n = 3$ bidders the coordination problem in the FPSB auction involves solely the σ and the WTA equilibrium as hybrid equilibria do not exist anymore. The expected VCG price for buying the split award is 78.00 in the three-bidder setting, which equals the costs for the auctioneer in each format, when bidders coordinate on the efficient σ equilibrium. Whereas this can be expected in the ascending formats, also a WTA equilibrium with expected costs for the auctioneer of 126.67 can be supported in the FPSB auction.

4.9.3 *Welfare Results*

First, the results on efficiency and procurement costs are discussed.

4.9.3.1 *Efficiency*

Result 1. *All three auction formats almost always implement the efficient split award with three bidders. With only two bidders, the Dutch-FPSB auction leads to higher proportion of split awards (81.44%) than the Dutch auction (64.47%), for which again the split is more often awarded than in the FPSB split-award auction (44.99%).*

First, we analyze the proportion of auctions that result in the efficient split out of all non-deleted auctions.[37] The allocations of the different treatments are summarized in Table 3 below and standard deviations (sd) are provided in brackets. Contrary to theory, many sole source awards are observed in all three auction formats in the two-bidder setting. The Dutch-FPSB auction results significantly more often in the split allocation than the Dutch format (p-value of 0.03). Moreover, the FPSB auction has lower efficiency than the Dutch-FPSB format with p-value of 0.00 and than the Dutch auction with p-value of 0.02 which is in line with the prediction.[38] As predicted, the split was awarded in nearly all of the auctions independent of the auction design in the three-bidder setting. Only the Dutch auction ended in one profitable sole source award.

Furthermore, we compare the share of efficient allocations, i.e. the proportion of allocations, which resulted in an efficient split award for the two lowest cost types. Again, standard deviations (sd) are added in Table 3. Obviously, the two different metrics for efficiency are identical for the setting with two bidders. The share of efficient allocations rises when increasing the number

[37] The omitted auctions are the result of a small number of participants repeatedly bidding below their costs. In the two-bidder setting two individuals bid below their costs in the FPSB auction, eight do so in the Dutch auction, and nine do so in the Dutch-FPSB format. In the three-bidder setting, seven participants violate individual rationality in the FPSB split-award auction, seven do so in the Dutch format, and six do so in the Dutch-FPSB counterpart.

[38] In this section a Welch test is used for all significance tests between two samples. The Welch test is computed with the matching group average values.

of bidders from two to three in the FPSB auction from 44.99% to 71.82% (p-value of 0.00). It stays constant for the Dutch and Dutch-FPSB formats with p-values of 0.98 and 0.06, respectively. As expected, with three bidders all three auction mechanisms do not differ statistically in the share of efficient allocations with p-value of 0.73 between the FPSB and the Dutch-FPSB auction, p-value of 0.20 between the FPSB and the Dutch format, and p-value of 0.37 for the difference between the Dutch-FPSB and Dutch auction.

				Efficiency		
		Total Auctions	Omitted Auctions	Split Awards	Efficient Allocations	Allocative Efficiency
2 x 2 Setting	FPSB	345	2	44.99% (sd = 6.78%)	44.99% (sd = 6.78%)	75.44% (sd = 2.80%)
	Dutch	360	12	64.47% (sd = 9.55%)	64.47% (sd = 9.55%)	82.74% (sd = 4.47%)
	Dutch - FPSB	360	15	81.44% (sd = 2.15%)	81.44% (sd = 2.15%)	90.37% (sd = 1.01%)
2 x 3 Setting	FPSB	180	10	100.00%	71.82% (sd = 4.06%)	98.80% (sd = 0.55%)
	Dutch	240	7	100.00%	64.33% (sd = 8.89%)	98.31% (sd = 0.84%)
	Dutch-FPSB	240	16	100.00%	70.15% (sd = 7.97%)	99.09% (sd = 0.22%)

Table 3: Efficiency

Often a relative measure of allocative efficiency is used to characterize the result of combinatorial auctions. The last column of Table 3 provides the mean allocative efficiency based on the definition of Kwasnica et al. (2005).[39] Standard deviations (sd) of costs are given in brackets. Mean allocative efficiency of the Dutch-FPSB auction is highest in both settings with two and three bidders, and values of 90.37% and 99.09%, respectively. In the two-bidder setting, just like the other two measures of efficiency, the share is higher than in the Dutch auction (p-value of 0.04) which in turn has higher allocative efficiency than the FPSB format with p-value of 0.04. The metric is close to 100% for all different auction formats with three bidders and there are no significant differences with p-values of at least 0.16.

Moreover, the fact that the share of efficient allocations is statistically below one (p-values of 0.01 and lower) for all three auctions formats in the three-bidder setting might be explained by the high values of allocative efficiency. If the split is not won by the two lowest cost types, the cost draw of the bidder with highest costs is relatively close to the two lowest cost draws. To explain the high number of sole source awards in each split-award auction format in the two-bidder setting and the discrepancy in efficiency between the two ascending auctions, we analyze the bidding behavior in more detail in Section 4.9.4.

4.9.3.2 *Procurement Costs*

Result 2. *With only two bidders the FPSB and Dutch-FPSB auction formats yield substantially lower procurement costs of 130.49 and 130.08, respectively,*

[39] Assume N^{σ}_{winner}, N^{s}_{winner} are the sets of bidders, who won the split, respectively the sole source award, and $N^{\sigma}_{optimal}$ is the set of the two bidders with the lowest cost type per auction. Then, the allocative efficiency of a split-award auction with *dual source efficiency* is defined as

$$\text{Allocative efficiency} = \frac{\Sigma_{i \in N^{\sigma}_{optimal}} C\Theta_i}{\Sigma_{i \in N^{\sigma}_{winner}} C\Theta_i + \Sigma_{i \in N^{s}_{winner}} \Theta_i}.$$

than the Dutch split-award auction with a value of 155.03. Ranging from 75.49 to 79.16, the procurement costs of all auction formats with three bidders are considerably lower than in the two-bidder setting but follow the same ranking.

The average price the auctioneer has to pay in each treatment defines the overall procurement costs. Table 4 below gives an overview of the procurement costs of the different treatments with the standard deviations (sd) given in brackets. With values of 155.03 and 79.16, the Dutch auction results in higher average costs than the other two auction formats in the two- and three-bidder setting, respectively (p-values of 0.01 for all comparisons). As predicted, there is no significant difference in costs between the FPSB and the Dutch-FPSB split-award auctions with p-values of 0.93 and 0.20 in case of two and three bidders.

The procurement costs for the FPSB auction in the two-bidder setting do not differ significantly from the theoretical prediction of 140 (p-value of 0.08) with 95% confidence interval of $[116.86, 144.12]$.[40] However, the average prices of the Dutch-FPSB and the Dutch format are statistically lower and higher than the predicted 140 with 95% confidence intervals of $[126.28, 133.88]$ and $[141.93, 168.13]$, respectively. Nevertheless, note that the costs of the Dutch-FPSB auction are still fully within the predicted range of supportable equilibrium pooling prices of $[108.12, 140.00]$. Furthermore, as predicted, the average prices in all auction formats are significantly below the expected VCG price of 168 with p-values of 0.02 and lower.

Finally, as expected, all procurement costs in the three-bidder setting are substantially lower compared to the respective costs in the two-bidder setting (all p-values of 0.00) but the ranking remains constant. For the FPSB and the Dutch-FPSB auction the average prices are statistically lower than the expected VCG price of 78.00 with 95% confidence intervals of $[74.27, 76.71]$ and $[74.74, 77.99]$ and p-values of 0.01 and 0.05, respectively, whereas they correspond to the prediction for the Dutch format with 95% confidence interval of $[77.17, 81.15]$ and p-value of 0.16. Explanations for the remarkably high prices of the Dutch auction in the two-bidder setting and the unexpectedly low prices of the FPSB and the Dutch-FPSB auctions in the setting with three bidders are given in Section 4.9.4.

		Average Procurement Costs		
		Overall	Split Award	Sole Source Award
2 x 2 Setting	FPSB	130.49 (sd = 8.57)	125.04 (sd = 11.52)	134.73 (sd = 6.30)
	Dutch	155.03 (sd = 8.23)	152.24 (sd = 7.18)	158.80 (sd = 10.38)
	Dutch-FPSB	130.08 (sd = 2.39)	128.32 (sd = 1.84)	137.90 (sd = 8.07)
2 x 3 Setting	FPSB	75.49 (sd = 0.49)	75.49 (sd = 0.49)	-
	Dutch	79.16 (sd = 1.25)	79.02 (sd = 1.1)	115.00
	Dutch-FPSB	76.37 (sd = 1.02)	76.36 (sd = 1.02)	-

Table 4: Procurement Costs

[40] A student t-test is used for all single sample significance tests.

In the two-bidder setting, there is a trade-off between the higher efficiency of the Dutch auction and the lower procurement costs of the FPSB auction. We find higher prices for both the sole source and the split award in the Dutch auction than in the FPSB auction. The Dutch-FPSB auction achieves low prices and high efficiency and thus has advantages for the procurement manager in this respect, even resulting in higher efficiency than the Dutch auction with procurement costs that are not significantly different from those of the FPSB auction. Conversely, in the three-bidder setting all three auction formats are equally efficient. Again, the Dutch auction is the most expensive format tested at a 5% level. However, the differences are much smaller than in the two-bidder case.

4.9.4 *Bidding Behavior*

We next discuss the individual bidder behavior in the two-bidder and three-bidder environments for each of the three different auction formats. We estimated fixed-effects regressions for bids and prices of bidders in the split and sole-source award in all treatments and attached the outcomes in Appendix B.2 Section B.2.1. We also included univariate regressions in which the cost draw is the single independent variable. These regressions allow us to interpret all plots of bids and prices on cost draws in Section B.2.2. These plots and the corresponding univariate regressions provide intuitive insights on the subjects' bidding behavior. Finally, Appendix B.2 Section B.2.3 contains plots of bids/prices in split allocations for all treatments across periods. These plots visualize any adaptation in bidding behavior with repeated interaction of the bidders.

Especially in the two-bidder setting, the derived σ equilibria have a collusive flavour. In order to describe the different forms of tacit collusion we apply the following distinction. We define that *pooling behavior* includes high split prices, which lie (1) above the highest possible cost draw for the split, $\overline{\Theta C}$, and (2) within the range of equilibrium predictions. Furthermore, the regression analysis should show that (3) these pooling prices are not significantly influenced by the own cost draw of the bidder. When this is not the case and only (1) and (2) apply, we talk about *tacit collusion*. Prices, which are even higher than the equilibrium predictions, are defined as *strong pooling* or *strong tacit collusion*.

4.9.4.1 *Two-Bidder FPSB Split-Award Auction*

Result 3. *As theory predicts, split-award winners show pooling behavior as nearly all split prices are above 42.00 ($\overline{\Theta C}$) and the average split price is 62.30. Furthermore, the own cost draw does not significantly influence the split prices in the fixed-effects regression analysis. In the sole-source allocations, bidders tried to exclude the split award with high bids on one unit and submitted competitive bids on two units.*

Remember, there is an inherent coordination problem in the FPSB auction format, because the inefficient WTA equilibrium can be simply implemented by the unilateral use of veto bids. Solely the existence of these veto bids might make the σ equilibrium less attractive in practice as its implementation requires both bidders not to use veto bids. Moreover, it is strategically complex for two bidders to coordinate on a split. To support any single-unit pooling price bidders have to bid at least twice the amount on two units such that the auctioneer chooses the split award. Note, however, that such bids only lead to an equilibrium if both bidders actually pool at the same bid for one unit which is very unlikely.

We observe that bidders who win the split-award bid low on the single unit with an average bid of 62.30 and submit a high average bid of 145.70 on the double-unit package. Such bids facilitate coordination on the split award independent of slight deviations by the opponent, especially regarding the choice of an alternative pooling price. However, this strategy is vulnerable against a combination of high single-unit prices and low double-unit bids. In sole source award allocations that do not involve veto bids (9.04% of all bidders make use of their veto power), winners submit average bids of 68.76 and 135.48, and losers bid on average 78.28 and 157.13 on one and two units, respectively. As a result, the sum of both parties average single-unit bids, 147.04, exceeds the winner's average bid on two units. In other words, the winners do not bid high enough on two units and the losers bid too high on one unit, thus, preventing coordination on the split award. This "off-equilibrium" behavior differs from the bidding behavior in the ascending auction formats as is discussed for the Dutch and Dutch-FPSB auctions in Sections 4.9.4.2 and 4.9.4.3, respectively.

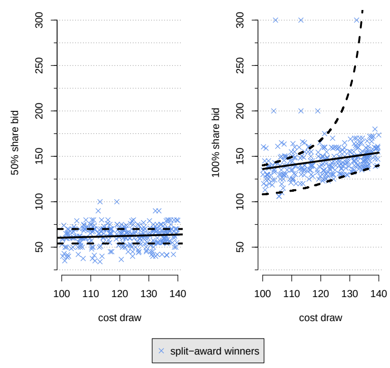

Figure 1: Bids of Split-Award Winners in FPSB (n=2)

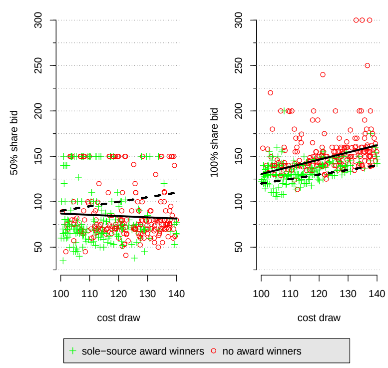

Figure 2: Bids in Sole-Source-Award in FPSB (n=2)

Figure 1 shows the single- and double-unit bids of split award winners plotted against the cost draws. The figure already indicates that winners of the split award pool their single-unit bids and submit high double-unit bids as theory predicts. Also, the bids of split-award winners on the package are increasing with the cost draw. The regression line (solid line) for the single-unit bids lies within the predicted pooling boundaries (dashed lines) and the regression line for the double-unit bids is within the predicted support, too.

The winners of the sole-source award submit bid-to-lose split prices as can be seen in the left plot of Figure 2 in which the univariate regression line (solid line) even has a negative slope. The bid for two units (solid line) is increasing in the cost draw as predicted by theory (dashed line) and illustrated in the right plot of the figure. We observe overbidding which is mainly caused by the very high double-unit bids of the sole-source award losers. We conjecture that many of them aim for the split award.

In Appendix B.2 we summarize the results of a number of fixed-effects regressions, which support the graphical analysis. We also analyze the bidder behavior across the periods in a session. For split-award winners the cost draw is not a significant explanatory variable for the height of the single-unit bid. Manual inspection yields that many bidders stick to their strategy of either bidding on the sole source or the split award over time. We observe a significant decrease in the height of the single-unit bid for ten out of 46 sole-source award bidders across all 15 periods. This might suggest that some sole-source bidders adapt and try to win the split award but do not alter their double-unit bids. The latter bids of split-award winners and the distribution of allocations appears to remain constant over periods. Details can be found in Appendix B.2 in figures 17 and 18, respectively.

4.9.4.2 *Two-Bidder Dutch Split-Award Auction*

Result 4. *Split-award winners achieve above-equilibrium prices with an average of 74.80 in the first phase and an average of 77.23 in the second phase. This can be seen as a form of strong tacit collusion, where bidders agree on higher payoffs.*

In the Dutch auction there is a unique perfect Bayesian σ equilibrium. Therefore, strategic complexity is lower compared to the FPSB format as bidders observe increasing single- and double-unit prices publicly and can constantly compute their respective profits. Furthermore, a payoff-maximizing bidder does not accept two units at a double-unit price below 140.00 (which corresponds to a single-unit price of 70.00) as the split is more profitable. In the experiments, 10.39% of all direct sole-source winners accept a double-unit price below 140.00. With this common knowledge there is a low risk in letting the single-unit price rise to 70.00 as the equilibrium suggests. Even if the opponent accepts at a lower price in the first phase, there is always a chance to win the second unit.

Interestingly, we find average split prices of 74.80 in phase 1 and 77.23 in phase 2, which strictly exceed the equilibrium prediction of 70.00. It appears that bidders implicitly agree upon letting the price rise above the equilibrium prediction to make higher profits in the split allocation. The publicly increasing prices allow both bidders to constantly reinforce this agreement on strong tacit collusion until one bidder accepts, which apparently leads both bidders to exceed the equilibrium price. The first bidder to accept the single-unit price, signals at which price the opponent should accept the second unit. Assume that one unit was sold in phase 1 for a price of 74.00. The loser of phase 1 knows that the price for the remaining share is likely to rise again at least to

74.00, as otherwise the opponent would have accepted both units at a price of 148 in phase 1. In fact, 77.68% of the bidders only accepted the counteroffer for one unit in phase 1, when their payoff for the split award was strictly higher than for the sole source award. Therefore, the loser of the first unit might not want to accept the second unit at a lower price than his opponent in phase 1, because it is not credible for the winner of phase 1 to accept the second unit below the price of the first unit. The price of the first unit is a natural lower bound for the price of the second unit and typically bidders try to go even a bid higher at the risk that the opponent takes both shares.

Of course, if the loser of the first phase lets the price in phase 2 rise too high it becomes more profitable for the winner to accept the second consecutive unit. Similarly, if the price in phase 1 rises too high it might become more profitable for one of the two bidders to directly accept two units. For example, if the first unit is sold at a price of 74.00, a bidder with cost draw 110.00 makes a payoff of 74.00-33.00=41.00. Now, if the second unit reaches a price of 78.00, the winner of the first unit should accept this price as well, as it provides a higher payoff of 152.00-110.00=42.00. Thus, if bidders let the split prices rise too high the sole-source award becomes more profitable at some point. This is also what we see in the data. There is a substantial proportion of sole-source awards in the Dutch auction due to such behavior, although the proportion of efficient split awards has increased significantly compared to the FPSB format.

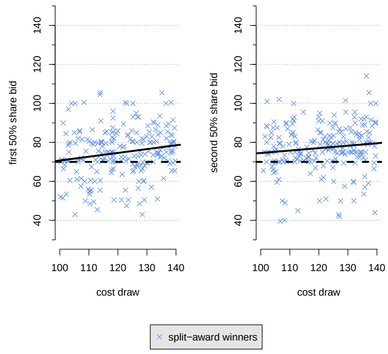

Figure 3: Bids of Split-Award Winners in Dutch-Dutch (n=2)

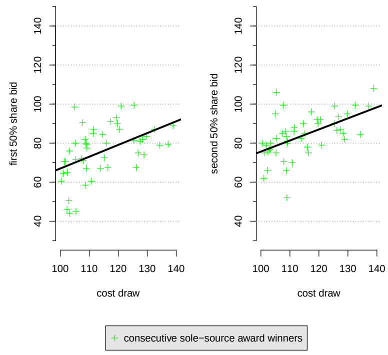

Figure 4: Bids of Consecutive Sole-Source-Award Winners in Dutch-Dutch (n=2)

The first-unit price was taken as a signal for the second-unit price, which was higher in 57.59% of all split allocations. The relation between the two split prices is depicted in Figure 3 in which both univariate regression lines (solid lines) lie entirely above the predicted pooling price of 70.00 (dashed lines). The prices in case of bidders winning two consecutive units are depicted in Figure 4 and are strongly increasing in costs (solid lines), but otherwise show a similar pattern as the split prices. Figure 9 in Appendix B.2 contains the prices at which two units were directly accepted (solid line) which are on average higher than in the split award. Similar to the FPSB auction the distribution of allocations does not change across the periods as is shown in Figure 20 in the appendix. The fixed-effects regressions for split-award winners in Appendix B.2 yield that the cost draws have a significant but small effect on the bid price of the first phase, but not on the price in the second phase. In phase 2, the price of the first phase is a significant covariate.

4.9.4.3 *Two-Bidder Dutch-FPSB Split-Award Auction*

Result 5. *Split allocations involve tacit collusion of the bidders with average single-unit prices of 61.80 and 66.57 in phases 1 and 2, respectively. The lower prices avoid sole-source bids as they happen in the Dutch auction. Repeated auctions lead to significantly higher prices in this auction format, as bidders learn to coordinate on higher split prices.*

In the Dutch-FPSB auction we observe average single-unit prices of 61.80 and 66.57 in phases 1 and 2, respectively. Efficient equilibria, in which bidders tacitly collude on split prices below 70.00, are in line with the theoretical predictions for this format. In contrast to the Dutch auction, bidders have a credible threat with low sealed bids on the second unit that would result in a lower overall profit for a bidder, but forces his opponent not to bid too high on the second unit and win. Interestingly, we observe that 61.92% of the winners of the first unit submit bids for the second unit to make as much profit when winning both units as when winning one unit. Such bids are no credible threats in the Dutch-FPSB auction. The strategic option of using threats to implement split prices below 70.00 is rarely used in the experiments, probably because it cannot be directly observed by the opponent.

The credible threat is the reason for the different equilibrium bidding strategies between both ascending auction formats. The fact that this credible threat is not used might suggest that one would see prices similar to the Dutch auction. However, prices are lower in the Dutch-FPSB auction. It appears less certain for a bidder to win the second unit, and bidders tend to accept the first unit already at a lower price in order to secure one unit. Furthermore, we conjecture that the possibility of possible threats in itself leads to higher insecurity and lower bidding already in phase 1. Similar to the Dutch auction, the price of the first unit is a signal for the second unit, and we observe slightly higher prices for the second unit. In summary, bidders are faced with relatively low split prices at which sole-source deviations are less likely to occur. Therefore, we observe even fewer sole-source allocations in the Dutch-FPSB auction than in the Dutch format. Overall, this increases efficiency.

Plots of the bids in the Dutch-FPSB auction can be found in Appendix B.2. In the fixed-effects regressions we find a small but significantly positive effect of the cost draws on both units, and again the price of the first phase was a significant covariate for the price in the second phase (see Table 7 in Appendix B.2). As in the Dutch auction the impact of the cost draws on the bid is very small (e.g., 0.13 for the first bid), which is close to the constant bid price predicted by the theory. In contrast to the other auction formats, we found a significant positive impact of the number of periods on the bid price for the first price. This indicates that the subjects adapted and learned to tacitly collude on higher split prices over time.

4.9.4.4 *Three-Bidder FPSB, Dutch, and Dutch-FPSB Split-Award Auction*

Result 6. *With three bidders almost always the efficient split award is implemented. In the FPSB auction format it is difficult for bidders to realize a sole source allocation, and they need to rely on another bidder. In the two ascending auction formats sole source awards are no equilibrium and they do not happen in the lab.*

Contrary to the ascending auctions, there is an inefficient WTA equilibrium in the FPSB auction. To implement a sole-source allocation in this format at least two bidders have to exclude the split award with high bid-to-lose prices for one unit. Note, however, that in this case only the bidder with the lowest cost type wins and both losers know that they would have won one unit with certainty by coordinating on the split award. This implies that both losers may regret their decision to chose the WTA strategy. In contrast, in the split allocation the only loser could not have won the sole-source award by unilaterally playing a bid-to lose strategy for the split. He could only regret not to bid aggressive enough for the split award after the winning bids are disclosed. The anticipation of this form of loser's regret is a possible explanation for too aggressive bidding in first-price sealed-bid auctions (Filiz-Ozbay and Ozbay, 2007). In our experiments, not only the winning bid(s) but also the winning allocation is disclosed to all bidders after each auction. Hence, we conjecture that anticipated regret of the losers in a WTA equilibrium as described above prevent bidders from trying to win the sole-source allocation in an environment with three bidders, whereas these equilibria can be observed in the experiments with two bidders.

Furthermore, note that a coordination on a WTA equilibrium, in which all bidders submit high bid-to-lose prices for the split award, becomes more difficult with a higher number of bidders in the auction. Indeed, a bidder following a split deviation from such an equilibrium only wins, when there is at least one other bidder placing a competitive bid for the 50% share. However, the chances that there is another deviating bidder of course increases with the number of participants in the auction and the expected payoffs in this case can be very attractive due to the *dual source efficiency* environment. This applies especially for bidders with a high cost draw whose expected payoff in a WTA equilibrium is relatively low.

Finally, a simple and effective strategy in such an environment is to bid competitively for the split as well as for the sole source award. Such a bidding behavior excludes the sole source and is not vulnerable to deviations even from both opponents. Such a strategy is a form of a σ equilibrium with a moderate bid-to-lose price for 2 units.[41] Assume a bidder with costs of 120.00 for two units and 36.00 for one unit submits bids of 124.00 and 38.00, respectively. If all bidders follow a σ strategy with bid-to-lose prices on two units, he has good chances to win the split award. In the case where both opponents try to coordinate on the sole source award with high bid-to-lose prices for one unit

[41] Remember, that in our setting bidding the cost draw for two units is enough to exclude the split award.

and competitive bids for two units, his chances (with regard to his cost draw) are still good to win both units.

We conjecture that the combination of those phenomena leads to the high efficiency and non-appearance of sole source awards in the FPSB auction with three bidders. Note that payoff-dominance or (weakly) dominated strategies cannot be used as an explanation in our environment due to Corollary 9. As predicted, in the ascending auction formats competition and the inherent *dual source efficiency* cost structure lead to low prices for the first and the second unit consecutively, so that the sole-source award is always unprofitable.

As expected in case of competitive bidding for the split award with three bidders in all auction formats the fixed-effects regressions in tables 8 to 10 in Appendix B.2 contain a cost parameter with significant explanatory power that is positively correlated with the corresponding dependent variable. The Wald test is used to test for the correspondence between fixed-effects regression model and the derived equilibrium strategy.

Although the single-unit bids of split-award winners in the three-bidder setting of the FPSB auction are significantly different from the equilibrium strategy (Wald test with $p < 2.2e - 16$), the univariate regression line in the left plot of Figure 13 in Appendix B.2 still indicates correspondence. Similar to the experimental evaluation of bidding in single-unit auctions we observe underbidding in our setting. Moreover, in the right plot the double-unit bids weakly exceed the cost type as predicted by theory.

As can be seen from the regression line in the left plot of Figure 14, the equilibrium strategy is a good predictor for the bidding behavior of split-award winners for the first unit in the Dutch auction (Wald test with $p = 0.34$). Also, the bidding behavior for the second unit corresponds to the theoretical prediction (Wald test with $p = 0.08$). The left plot of Figure 15 in Appendix B.2 shows that split-award winners of the first unit in the Dutch-FPSB auction submit bids weakly below the equilibrium strategy and the fixed-effects regression differs significantly (Wald test with $p < 2.2e - 16$). Although split-award winners of the second unit appear to bid closer to the theoretical prediction in Figure 16, their bidding behavior does not correspond to the equilibrium strategy (Wald test with $p = 0.00$).

Similar to the experimental literature on first-price single-item auctions, we observe underbidding for the split-award winners in the FPSB and the Dutch-FPSB auctions with three bidders. This causes the fixed-effects regression lines to differ significantly from the equilibrium predictions. Moreover, the first- and second-unit bids in the Dutch-FPSB format neither differ from each other (Wald test with $p = 0.99$), nor from the single-unit bid of split-award winners in the FPSB auction (Wald test with $p = 0.23$ and $p = 0.26$ respectively). We conjecture that the strategical differences between both auction formats do not influence average bidding behavior.

Similar to our two-bidder setting, average single-unit prices in the Dutch auction with three bidders are significantly higher than in the other two formats. The bids for the first and second units do not differ significantly from the predictions although they are not statistically different from each other (Wald test with $p = 0.07$). These higher prices in the Dutch format may explain the lower allocative efficiency as well as the lower share of efficient allocations compared to the other two auctions.

4.10 CONCLUSION

Ex-post split-award auctions are a widely used form of combinatorial procurement auctions. In particular, first-price auctions are often chosen for their simplicity. There is often little competition in procurement particularly if there are only a few qualified suppliers for specific products to be procured. Unfortunately, bidding strategies of such auctions are not well understood. However, the analysis of a limited 2×2 market by Anton and Yao (1992) showed that, for the wide-spread FPSB split-award auction with two bidders, there is an efficient split equilibrium and an inefficient WTA equilibrium, leaving the bidders with a veritable coordination problem. The 2-bidder model is specific, because bidders can veto the split award unilaterally, and it is unclear if the results carry over to markets with more than two bidders, and if it is predictive in the lab.

We extend the analysis to $n > 2$ bidders, and also analyze the Dutch split-award auction and the Dutch-FPSB split-award auction, which are wide-spread in procurement practice, but have not yet been studied. For markets with two bidders, we show that the Dutch split-award auction has a unique split equilibrium with a constant pooling price. The Dutch-FPSB also exhibits only efficient split equilibria, but it allows for multiple equilibrium prices. The strategic differences for the bidders arise because of differences in the revealed information in the three auction formats. There is cost equivalence between the Dutch and the Dutch-FPSB auction, while this only applies for the FPSB format when suppliers coordinate in a split equilibrium. In markets with more than two bidders, the FPSB auction still exhibits a WTA equilibrium even though the veto power of bidders ceases. The theoretical results organize important patterns in the experimental results such as pooling prices in the two-bidder auction and the coordination problem.

In our experimental assessments, we found that bidders in the two-bidder FPSB auction indeed selected both types of awards. It is interesting to see that many more split allocations emerged in the Dutch auctions at prices even beyond the equilibrium pooling price. We conjecture that bidders interpret non-acceptance of counteroffers at low prices as an implicit agreement on high prices. These high prices sometimes lead bidders in the first phase to bid on the package of two units or accept also a high price in the second phase, such that there are inefficient sole-source awards. The introduction of a sealed-bid stage in phase 2 of the Dutch-FPSB auction led to lower prices in the first phase. As a consequence, winning the package became less attractive, which led to even higher efficiency. The Dutch-FPSB auction appears as a robust and simple alternative that yields high efficiency and low procurement cost.

Interestingly, with a single additional bidder the inefficiency of the 2-bidder environment is largely gone, even in the FPSB split-award auction, in which there is also an equilibrium selection problem. The veto power of bidders vanishes and instead they want to win the 50% share and coordinate with others. Furthermore, we found that prices drop substantially in all three auction formats as theory predicts. Competition was very effective in the laboratory even though bidders in the FPSB split-award auction could also choose a sole-source award in theory. In summary, first-price combinatorial auctions are highly efficient in our setting. The lower strategic complexity and high efficiency of the Dutch auction formats can be seen as an advantage.

5

CONCLUSION

We first summarize our main findings and subsequently present present some ideas on how to generalize the mayor results. Furthermore, potential promising areas of future research are proposed. The contributions of this thesis are threefold.

First, we derive optimal bidding strategies in various combinatorial (forward) auction formats in a limited 2×2 setting in which two profit-maximizing bidders compete for two units of a homogeneous product (perfect substitutes). We assume a standard IPV model with risk-neural and ex-ante symmetric bidders who possess decreasing marginal values in the number of units obtained. To be precise we concentrate on *dual-winner efficiency* in which it is always social-welfare maximizing to have two winners of one unit each. As demonstrated in Anton and Yao (1992), these assumptions allow us to derive Bayesian Nash equilibrium strategies and guarantee the benchmark allocation of the VCG mechanism to be in the core and there is no threshold problem in the studied combinatorial package auction formats and no exposure problem in comparable non-combinatorial auctions. This allows us to restrict attention solely on strategic differences between the latter two auction mechanisms that are caused by the bidding language. Second, in the same market we examine principal-agent relationships in bidding firms and present different optimal contracts that resolve the agency problem depending on the information asymmetries between both parties. Third, we extend the focus on combinatorial package auctions to predominantly first-price combinatorial ex-post split-award (reverse) auctions with $n \geq 2$ bidders who compete for two 50% shares or 100% of a contract in an IPV procurement setting. Analogue to the forward auctions we assume increasing marginal costs and *dual-source efficiency* such that it is always efficient to have to winners of one 50% share each. Our linear Bayesian Nash equilibrium predictions for this $2 \times n$ procurement market are evaluated with laboratory experiments later on.

In the 2×2 forward auction market we are able to demonstrate a weak form of outcome equivalence between the ascending package auction, the non-combinatorial standard uniform-price multi-unit auction and its ascending-price counterpart which is outcome equivalent to the non-combinatorial SMRA. In all these auction formats bidders reduce demand to obtain one unit each at zero price. The outcome equivalence is only weak because the ascending package auction also possesses an inefficient equilibrium in which each bidder unilaterally vetoes the outcome with two winners. Nevertheless, this equilibrium is strictly dominated in payoff by the efficient equilibrium and the dynamic auction format also allows for coordination advantages on the latter equilibrium. In the procurement setting with reverse auctions and linear equilibria, we are able to demonstrate by the use of the revenue-equivalence theorem that for $n > 2$ bidders all above auctions become in fact strategically and revenue equivalent to all efficient standard multi-unit and sequential auctions with single-unit demand. This includes the generalized VCG mechanism, which additionally, leads to higher (strictly positive) seller revenue in the 2×2 market.

Contrary to the above findings, the FPSB package auction is neither outcome nor strategically equivalent to its non-combinatorial standard multi-unit counterpart in the 2×2 setting. Moreover, general unambiguous revenue rankings between its efficient and inefficient equilibrium are not possible. However, in its efficient pooling equilibrium it strictly dominates its ascending counterpart as well as the VCG mechanism and even the standard discriminatory multi-unit auction in seller revenue. In its inefficient equilibrium it is revenue equivalent to the ascending package auction. Again, for $n > 2$ bidders in the procurement setting, the efficient equilibrium of the FPSB package auction is strategically in line with all other auction formats mentioned above. Nevertheless, the FPSB format possesses an inefficient equilibrium independent of the number of bidders. This is not generally true for other combinatorial (ascending) first-price reverse auctions as shown in the procurement setting. These mechanisms are characterized by uniquely efficient equilibria that are payoff-equivalent to the efficient allocation of their sealed-bid counterpart. In summary, the FPSB package auction is characterized by peculiar bidding behavior that differentiates it from other non-combinatorial auction mechanisms and leads to relatively high seller revenue but also inefficient allocations. This is not necessarily true for the ascending package auction which is weakly outcome equivalent to other efficient non-combinatorial uniform-price auctions but achieves comparably low seller revenue. The generalized VCG mechanism serves as an intermediate that is fully efficient and achieves strictly positive revenue. For $n > 2$ bidders all considered auction formats except the FPSB package auction are strategically equivalent and efficient. It is interesting to observe that in our setting the combinatorial auction formats perform at least as good in terms of revenue and efficiency as all other non-combinatorial mechanisms. In environments with exposure and threshold problems this ranking is likely to persist as the free-rider risk is generally considered less of a threat than the exposure risk (Bykowsky et al., 2000; chakraborty et al., 1995).

In our principal-agent model of bidding firms the profit-maximizing principal uses budget contracts to direct her value-maximizing agent, who bids in the auction, on the efficient equilibrium within the 2×2 forward auction setting. The agency bias causes the agent to inflate his demand on two units in an inefficient equilibrium. We distinguish between an environment of symmetric information about the firm's values between principal and agent in which the principal uses a delegation set of budgets to incentive-align the agent and an asymmetric information setting in which the uninformed principal employs a menu of contingent budgets and transfer payments to incentivize the agent who is informed about the firm's package valuations. In the symmetric information setting of the FPSB package auction, optimal delegation is impossible and the agent cannot be directed on the efficient equilibrium bidding strategy. Moreover, in the asymmetric information setting the agency bias might be so strong that even the use of optimal transfer payments cannot implement the efficient equilibrium. Contrary, in the ascending package auction the efficient equilibrium is straightforward to achieve with an optimal delegation set of budgets in the symmetric and asymmetric information environment. In the special setting in which both units are sold as the single package, the agency problem is easy to overcome in the symmetric information environment but the bias might be so strong that an optimal contract cannot be implemented with asymmetric information. Here, the optimal contract is identical for all auction formats within the respective information environment which is reminiscent of earlier results on optimal delegation in single-object auctions (Burkett, 2015, 2016). Summarizing, in addition to its beneficial efficiency properties, the ascending package auction can also be put forward as an advantageous auction format in case the social-welfare maximizing auctioneer is aware of agency problems within the participating bidding firms.

Finally, in the $2 \times n$ procurement auction setting we consider different combinatorial first-price ex-post split-award auctions that are frequently employed in industry practice. In our theoretical analysis we show that independent of the number of bidders the FPSB auction always possesses at least one inefficient equilibrium whereas the Dutch (ascending firs-price) auctions are characterized by only efficient equilibria. Therefore, general efficiency and revenue rankings are impossible but there is payoff-equivalence if the efficient equilibrium of the FPSB format is played and the open formats appear to be more efficient. In our experimental analysis we show for $n = 2$ bidders that the Dutch auctions are significantly more efficient than their sealed-bid counterpart, whereas the latter achieves higher buyer revenue compared to the Dutch auction but with respect to the Dutch-FPSB format. For $n = 3$ bidders, we find all auction formats to be outcome equivalent. Thus, we can conclude that our theoretical predictions are well reflected in a laboratory setting, which might then indicate that our results offer valid managerial and policy advice. Based on our experimental study the compound Dutch-FPSB auction can be proposed as an efficient and revenue maximizing combinatorial first-price ex-post split-award auction in industrial procurement.

The potential shortcomings of our contributions are obvious. In all applications we never considered package auctions for more than two units or shares, and always focus on *dual-winner* or *dual-source efficiency*, respectively. At first glance it appears unclear whether our insights generalize to markets with more than two auctioned items and different efficiency environments. Let us first consider more complex markets.

In the 2×2 market, the existence of inefficient equilibria in the IPV setting as well as the agency bias in the principal-agent model depend to a large extend on each bidder's power to unilaterally veto the efficient outcome. On the one hand, we have already shown that this veto power generally decreases with an increasing number of bidders. However, any individual bidder can never be entirely certain about his competitors' willingness to coordinate as was shown for the FPSB package auction which still possesses an inefficient equilibrium. In this equilibrium each bidder independently vetoes the efficient allocation such that any one bidder is forced to participate in the inefficient outcome. On the other hand, if the number of possible packages strictly exceeds the number of buyers, which is common in combinatorial auctions in the field, unilateral veto-power against multiple-winner outcomes may still be very strong. Here, the results from the 2×2 market simply extend and the fundamental trade-offs described above are likely to remain valid. Often there are many bidders participating in an auction, but only a few are strong and they decide the outcome. For example, in the German auction in 1999 that was discussed in Section 3.9, there were four bidders and ten units, but in the strategic analysis one can focus on the competition between Mannesmann and T-Mobile. It is reasonable to assume both bidders understood that an outcome with two winners was efficient and payoff-dominant to a single-winner outcome.

The results regarding overcoming the principal-agent problem in the 2×2 market depend on each principal's power to unilaterally veto the efficient outcome. As we have established above, in more complex markets this veto-power can still be strong. The fact that the VCG mechanism is strategy-proof independent of the complexity of the market environment suggests that it makes it easier to solve the agency dilemma more generally in a second-price payment mechanism. Because of the payment function, principals have no incentive to deviate from the welfare-maximizing allocation independent of the opponent's bidding behavior. Contrary, any equilibrium of the FPSB package auction that requires bidders to coordinate involves potentially complex bidding on packages not part of the equilibrium allocation. Each bidder has to submit bids that make a deviation by any of her opponents unprofitable in expectation and thereby prevent him from using her veto-power. Next, we focus on different efficiency settings.

It is worth mentioning that our model also allows to examine other environments than *dual-winner efficiency* in a $2 \times n$ market. Under *single-winner efficiency*, for example, in which it is always efficient to have one buyer winning the package of two units and the lowest possible package-valuation strictly exceeds twice the highest possible single-unit value, no equilibrium with two winners exist. For fully combinatorial auctions there is no equilibrium selection problem and no inefficient allocations arise. The theory from single-object auctions applies (actually, in this case single-package auctions) and there is revenue equivalence between all combinatorial mechanisms considered in this thesis. In our principal-agent model there is no agency bias with respect to the equilibrium allocation in the first place and therefore no agency dilemma that needs to be solved. Principal and agent both aim for the efficient equilibrium with one winner, and the principal-agent problem in the symmetric information environment is then easy to solve as already discussed in Section 3.5.3. The principal can simply provide her agent with a budget for the package that corresponds to her profit-maximizing equilibrium report and with no budget for the single unit. In the asymmetric information setting in which there is an agency bias with respect to the optimal height of the bid, the principal provides her agent with the same optimal contract for the package as is demonstrated in Corollary 1 in Section 3.5.4 and with no budget for the single unit. Nevertheless, as shown before, depending on the extend of the agency-bias it might be ex-ante infeasible for the firm to participate in the mechanism.

At this stage it is important to highlight that the revenue-equivalence result for an environment in which it is always efficient to have a single winner in the $2 \times n$ market does not generally extend to non-combinatorial (sequential) auctions or to auctions that are not fully combinatorial and, for example, contain some element of sequential sales as is demonstrated in Kokott et al. (2018b). Here, the authors analyze three different practically relevant first-price ex-post split-award auctions in a procurement context similar to Anton and Yao (1992) Anton et al. (2010) and Kokott et al. (2018a) but in which it is always efficient to have a single supplier winning the contract (strong economies of scale). Again, the suppliers are assumed to be commonly informed about the efficiency environment whereas the auctioneer is uninformed and thus, cannot ex-ante decide to buy both 50% shares as a single contract in a single-object auction. The authors consider the combinatorial FPSB and Dutch-FPSB package auctions as introduced in Chapter 4, and a non-combinatorial (sequential) Dutch-FPSB auction in which both 50% shares are bought consecutively. Note that the Dutch-FPSB package auction contains a sequential mechanism in case the entire contract is not bought in the first stage and is therefore not expected to be efficient and revenue-equivalent to its sealed-bid counterpart. Remember, that there is hardly any literature on sequential auctions in a setting with bidders demanding multiple units. In particular, we are not aware of the derivation of any bidding strategies for a setting in which it is efficient to have a single winner. Nevertheless, Kokott et al. (2018b) manage to characterize linear bidding equilibria under the assumption that sellers are not allowed to bid below costs or marginal costs at any stage in the auction. This assumption is justified for a large proportion of procurement auctions in which sales representatives participate on behalf of their firms and must not incur losses at any circumstance.

In this setting Kokott et al. (2018b) show that for high economies of scale suppliers bid their marginal cost for one share in both phases of the sequential Dutch-FPSB auction which results in the efficient allocation. With low economies of scale bidders overbid their marginal cost for one share in both phases and therefore also inefficient allocations result in equilibria. Contrary, in the combinatorial FPSB auction only efficient equilibria exist. This is also true for the Dutch-FPSB in which 100% of the contract is sold in the first phase as long as scale economies are sufficiently high. For relatively low economies of scale, however, sequential deviations are profitable. There are no equilibria in which always allocations with two winners result in any of the considered auction formats and unambiguous rankings of buyer revenue cannot generally be computed.

Comparing different combinatorial auction formats across *single-winner efficiency* and *dual-winner efficiency* we find that the FPSB auction possesses an efficient equilibrium in both setting whereas the Dutch-FPSB package format is fully efficient only under *dual-winner efficiency* as are the sequential mechanisms. Bear in mind that there is also an inefficient equilibrium of the FPSB auction under *dual-winner efficiency*. Therefore, we cannot propose a general efficiency ranking of different combinatorial auctions across different efficiency settings. However, the FPSB auction appears to weakly dominate all other formats in terms of auctioneer revenue. In an accompanying experimental study in Kokott et al. (2018b), it is demonstrated that the equilibrium predictions organize the experimental data accurately. The FPSB package auction on average is most efficient and leads to the lowest procurement costs for the buyer. The sequential auction format appears to have no advantages and its frequent use in industrial procurement should be severely questioned. The authors thus argue in favor of an increased employment of combinatorial auction formats in industry practice.

In future research it remains to show that our results generalize to settings with imperfect substitutes and heterogeneous items in the forward auction context and to different share sizes in the procurement setting. In addition, research should strive to analyze combinatorial auction formats in even more general efficiency settings, such as ”mixed winner efficiency” for example, in which it is not pre-specified whether the only the double-winner or single-winner outcome is efficient (Anton et al., 2010). Here, the goal should be to derive general Bayesian Nash equilibrium bidding strategies independent of a certain pre-determined efficiency environment. In this context unambiguous efficiency and revenue rankings among various combinatorial auctions should be achieved and finally an efficient and revenue-maximizing (optimal) combinatorial mechanism proposed.

BIBLIOGRAPHY

Alonso, R., Matouschek, N., 2008. Optimal delegation. The Review of Economic Studies 75, 259–293. doi:doi:10.1111/j.1467-937X.2007.00471.x.

Amador, M., Bagwell, K., 2013. The theory of optimal delegation with an application to tariff caps. Econometrica 81, 1541–1599. doi:doi:10.3982/ECTA9288.

Anton, J.J., Brusco, S., Lopomo, G., 2010. Split-award procurement auctions with uncertain scale economies: Theory and data. Games and Economic Behavior 69, 24–41. doi:doi:10.1016/j.geb.2009.09.001.

Anton, J.J., Yao, D.A., 1992. Coordination in split award auctions. The Quarterly Journal of Economics 107, 681–707. doi:doi:10.2307/2118486.

Armstrong, M., 2000. Optimal multi-object auctions. The Review of Economic Studies 67, 455–481. doi:doi:10.1111/1467-937X.00139.

Ausubel, L., Cramton, P., Milgrom, P., 2006. The Clock-Proxy Auction: A Practical Combinatorial Auction Design. MIT Press, Cambridge, MA. URL: `ftp://www.cramton.umd.edu/papers2000-2004/ausubel-cramton-milgrom-the-clock-proxy-auction.pdf`.

Ausubel, L., Milgrom, P., 2002. Ascending auctions with package bidding. Frontiers of Theoretical Economics 1, 1–42. doi:doi:10.2202/1534-5955.1019.

Ausubel, L.M., Baranov, O.V., 2018. Core-selecting auctions with incomplete information. University of Maryland Working Paper. URL: `http://www.ausubel.com/auction-papers/ausubel-baranov-core-selecting-auctions-with-incomplete-info.pdf`.

Ausubel, L.M., Cramton, P., Pycia, M., Rostek, M., Weretka, M., 2014. Demand reduction and inefficiency in multi-unit auctions. The Review of Economic Studies 81, 1366–1400. doi:doi:10.1093/restud/rdu023.

Baron, D.P., Myerson, R.B., 1982. Regulating a monopolist with unknown costs. Econometrica 50, 911–930. doi:doi:10.2307/1912769.

Bernheim, B.D., Whinston, M.D., 1986. Menu auctions, resource allocation, and economic influence. The Quarterly Journal of Economics 101, 1–31. URL: `10.2307/1884639`.

Bichler, M., Goeree, J.K. (Eds.), 2017. The Combinatorial Clock Auction Designs. Cambridge University Press. pp. 109–371. doi:doi:10.1017/9781316471609.

Bichler, M., Guler, K., Mayer, S., 2015. Split-award procurement auctions - Can Bayesian equilibrium strategies predict human bidding behavior in multi-object auctions? Production and Operations Management 24, 1012–1027. doi:doi:10.1111/poms.12301.

Bichler, M., Paulsen, P., 2018. A principal-agent model of bidding firms in multi-unit auctions. Games and Economic Behavior 111, 20 – 40. doi:doi:10.1016/j.geb.2018.06.002.

Bikhchandani, S., Ostroy, J.M., 2002. The package assignment model. Journal of Economic Theory 107, 377–406. doi:doi:10.1006/jeth.2001.2957.

Bogaschewsky, R., 2015. Elektronische Beschaffung 2015: Stand der Nutzung und Trends. Technical Report. Bundesverband fuer Materialwirtschaft, Einkauf und Logistik. URL: http://www.cfsm.de/fileadmin/Downloads/E-Procurement/Elektronische_Beschaffung_2015.pdf.

Bolle, F., 1997. Necessary Conditions for Efficient Multiple-Bid Auctions. Boston: Kluwer Academic Publishers. pp. 123–143. doi:doi:10.1007/978-94-017-1360-3_9.

Bulow, J., Klemperer, P., 1996. Auctions versus negotiations. American Economic Review 86, 180–94. URL: https://www.jstor.org/stable/2118262.

Bulow, J.I., Levin, J., Milgrom, P., 2009. Winning Play in Spectrum Auctions. Working Paper 14765. NBER. doi:doi:10.3386/w14765.

Burkett, J., 2015. Endogenous budget constraints in auctions. Journal of Economic Theory 158, 1–20. doi:doi:10.1016/j.jet.2015.03.010.

Burkett, J., 2016. Optimally constraining a bidder using a simple budget. Theoretical Economics 11, 133–155. doi:doi:10.3982/TE1745.

Bykowsky, M.M., Cull, R.J., Ledyard, J.O., 2000. Mutually destructive bidding: The fcc auction design problem. Journal of Regulatory Economics 17, 205–228. doi:doi:10.1023/A:1008122015102.

Cantillon, E., Pesendorfer, M., 2006. Auctioning bus routes: The london experience, in: Cramton, P., Shoham, Y., Steinberg, R. (Eds.), Combinatorial Auctions. MIT Press, Cambridge, MA.

Cassady, R.J., 1967. Auctions and Auctioneering. University of California Press, Berkeley and Los Angeles.

chakraborty, B., Sharkey, W.W., Spiegel, Y., Wilkie, S., 1995. Auctioning the airwaves: The contest for broadband pcs spectrum. Journal of Economics & Management Strategy 4, 345–373. doi:doi:10.1111/j.1430-9134.1995.00345.x.

Chakraborty, I., 2006. Characterization of equilibrium in pay-as-bid auctions for multiple units. Economic Theory 29, 197–211. URL: https://doi.org/10.1007/s00199-005-0009-y, doi:doi:10.1007/s00199-005-0009-y.

Chaturvedi, A., Katok, E., Beil, D.R., . Split-award auctions: Insights from theory and experiments. Management Science forthcoming. doi:doi:10.1287/mnsc.2017.2932.

Clarke, E.H., 1971. Multipart pricing of public goods. Public Choice 11, 17–33. doi:doi:10.1007/BF01726210.

Conitzer, V., Sandholm, T., 2006. Failures of the vcg mechanism in combinatorial auctions and exchanges, in: Proceedings of the Fifth International Joint Conference on Autonomous Agents and Multiagent Systems, ACM, New York, NY, USA. pp. 521–528. doi:doi:10.1145/1160633.1160729.

Cox, J.C., Smith, V.L., Walker, J.M., 1983. Tests of a heterogeneous bidder's theory of first price auctions. Economics Letters 12, 207–212. doi:doi:10.1016/0165-1765(83)90039-3.

Cramton, P., 1995. Money out of thin air: The nationwide narrowband pcs auction. Journal of Economics & Management Strategy 4, 267–343. doi:doi:10.1111/j.1430-9134.1995.00267.x.

Cramton, P., 1997. The fcc spectrum auctions: An early assessment. Journal of Economics & Management Strategy 6, 431–495. doi:doi:10.1111/j.1430-9134.1997.00431.x.

Cramton, P., 2013. Spectrum auction design. Review of Industrial Organization 42, 161–190. doi:doi:10.1007/s11151-013-9376-x.

David, L.R., 2000. Auctions on the internet: What's being auctioned, and how? The Journal of Industrial Economics 48, 227–252. doi:doi:10.1111/1467-6451.00122.

Day, R., Milgrom, P., 2008. Core-selecting package auctions. International Journal of Game Theory 36, 393–407. doi:doi:10.1007/s00182-007-0100-7.

Day, R., Raghavan, S., 2007. Fair payments for efficient allocations in public sector combinatorial auctions. Management Science 53, 1389–1406.

Day, R.W., Cramton, P., 2012. Quadratic core-selecting payment rules for combinatorial auctions. Operations Research 60, 588–603. doi:doi:10.1287/opre.1110.1024.

Demange, G., Gale, D., Sotomayor, M., 1986. Multi-item auctions. Journal of Political Economy 94, 863–872.

Dewatripont, P., Bolton, P., 2005. Contract Theory. Contract Theory, MIT Press.

Donohue, K., Katok, E., Leider, S., 2017. The handbook of behavioral operations. Wiley.

Elmaghraby, W., 2007. Auctions within e-sourcing events. Production and Operations Management 16, 409–422.

Engelbrecht-Wiggans, R., 1980. State of the art—auctions and bidding models: A survey. Management Science 26, 119–142. URL: https://doi.org/10.1287/mnsc.26.2.119, doi:doi:10.1287/mnsc.26.2.119, arXiv:https://doi.org/10.1287/mnsc.26.2.119.

Engelbrecht-Wiggans, R., 1987. Optimal constrained bidding. International Journal of Game Theory 16, 115–121. doi:doi:10.1007/BF01780636.

Engelbrecht-Wiggans, R., 1988. Revenue equivalence in multi-object auctions. Economics Letters 26, 15–19. doi:doi:https://doi.org/10.1016/0165-1765(88)90044-4.

Engelbrecht-Wiggans, R., Kahn, C., 1998a. Multi-unit auctions with uniform prices. Economic Theory 12, 227–258. doi:doi:https://doi.org/10.1007/s001990050220.

Engelbrecht-Wiggans, R., Kahn, C., 1998b. Multi-unit pay-your-bid auctions with variable awards. Economics Letters 25-42, 227–258. doi:doi:https://doi.org/10.1006/game.1997.0599.

Engelbrecht-Wiggans, R., Weber, R.J., 1979. An example of a multi-object auction game. Management Science 25, 1272–1277. URL: http://www.jstor.org/stable/2630800, doi:doi:10.2307/2630800.

Epstein, R., Henriquez, L., Catalan, J., Weintraub, G.Y., Martinez, C., 2002. A combinatorial auction improves school meals in chile. Interfaces 32, 593–612.

Fadaei, S., Bichler, M., 2017. Truthfulness with value-maximizing bidders: On the limits of approximation in combinatorial markets. European Journal of Operational Research 260, 767–777. doi:doi:10.1016/j.ejor.2016.12.031.

Filiz-Ozbay, E., Ozbay, E.Y., 2007. Auctions with anticipated regret: Theory and experiment. American Economic Review 97, 1407–1418.

Fischbacher, U., 2007. z-tree: Zurich toolbox for ready-made economic experiments. Experimental Economics 10, 171–178.

Friend, G., 2015. Spectrum auction prices and bias in spectrum values. Report. coleagoconsulting. URL: http://www.coleago.com/spectrum-auction-prices-and-bias-in-spectrum-values/.

Fudenberg, D., Tirole, J., 1991. Game Theory. MIT Press, Cambridge, MA.

Goeree, J., Lien, Y., 2014. An equilibrium analysis of the simultaneous ascending auction. Journal of Economic Theory 153, 506–533. doi:doi:10.1016/j.jet.2014.07.014.

Goeree, J., Lien, Y., 2016. On the impossibility of core-selecting auctions. Theoretical Economics 11, 41–52. doi:doi:10.3982/TE1198.

Green, J., Laffont, J., 1979. Incentives in public decision making. North Holland, Amsterdaml.

Gretschko, V., Rasch, A., Wambach, A., 2014. On the strictly descending multi-unit auction. Journal of Mathematical Economics 50, 79–85.

Grimm, V., Riedel, F., Wolfstetter, E., 2004. The Third Generation (UMTS) Spectrum Auction in Germany. MIT Press. pp. 223–247.

Groves, T., 1973. Incentives in teams. Econometrica 41, 617–631. doi:doi:10.2307/1914085.

Guesnerie, R., Laffont, J.J., 1984. A complete solution to a class of principal-agent problems with an application to the control of a self-managed firm. Journal of Public Economics 25, 329 – 369. URL: http://www.sciencedirect.com/science/article/pii/0047272784900604, doi:doi:https://doi.org/10.1016/0047-2727(84)90060-4.

Gul, F., Stacchetti, E., 1999. Walrasian equilibrium with gross substitutes. Journal of Economic Theory 87, 95–124.

Harris, M., Raviv, A., 1981. Allocation mechanisms and the design of auctions. Econometrica 49, 1477–1499. URL: http://www.jstor.org/stable/1911413, doi:doi:10.2307/1911413.

Harsanyi, J.C., 1967a. Games with incomplete information played by 'bayesian' players, i-iii. part i. the basic model. Management Science 14, 159–182.

Harsanyi, J.C., 1967b. Games with incomplete information played by 'bayesian' players, i-iii. part ii. bayesian equilibrium points. Management Science 14, 320–334.

Harsanyi, J.C., 1967c. Games with incomplete information played by 'bayesian' players, i-iii. part iii. the basic probability distribution of the game. Management Science 14, 486–502.

Hohner, G., Rich, J., Ng, E., Reid, G., Davenport, A., Kalagnanam, J., Lee, H., An, C., 2003. Combinatorial and quantity discount procurement auctions with mutual benefits at mars, incorporated. Interfaces 33, 23–35.

Holmström, B.R., 1977. On Incentives and Control in Organizations. Stanford University.

Holmstrom, B., 1984. On the Theory of Delegation. New York: North-Holland. pp. 115–141.

Jensen, M.C., 1986. Agency costs of free cash flow, corporate finance, and takeovers. The American Economic Review 76, 323–329. URL: http://www.jstor.org/stable/1818789.

Katzman, B., 1999. A two stage sequential auction with multi-unit demands. Journal of Economic Theory 86, 77 – 99. URL: http://www.sciencedirect.com/science/article/pii/S0022053199925103, doi:doi:https://doi.org/10.1006/jeth.1999.2510.

Kirchkamp, O., Reiß, J.P., 2011. Out-of-equilibrium bids in first-price auctions: Wrong expectations or wrong bids. The Economic Journal 121, 1361–1397.

Klemperer, P., 1999. Auction theory: A guide to the literature. Journal of Economic Surveys 13, 227–260.

Kokott, G.M., Bichler, M., P., P., 2018a. Combinatorial first-price auctions: Theory and experiments. Working paper, Technical University of Munich .

Kokott, G.M., Bichler, M., P., P., 2018b. First-price split-award auctions in procurement markets with economies of scale: An experimental study. Production and Operations Management forthcoming.

Krishna, V. (Ed.), 2002. Auction Theory. Elsevier Science.

Krishna, V., 2009. Auction Theory. Academic Press.

Krishna, V., Morgan, J., 2008. Contracting for information under imperfect commitment. The RAND Journal of Economics 39, 905–925. doi:doi:10.1111/j.1756-2171.2008.00043.x.

Krishna, V., Perry, R., 1999. Efficient mechanism design URL: https://ssrn.com/abstract=64934, doi:doi:http://dx.doi.org/10.2139/ssrn.64934.

Krishna, V., Rosenthal, R., 1996. Simultaneous auctions with synergies. Games and Economic Behavior 17, 1–31. doi:doi:10.1006/game.1996.0092.

Kunreuther, H., Kleindorfer, P.R., 1986. A sealed-bid auction mechanism for siting noxious facilities. The American Economic Review 76, 295–299. URL: http://www.jstor.org/stable/1818783.

Kwasnica, T., Ledyard, J.O., Porter, D., DeMartini, C., 2005. A new and improved design for multi-objective iterative auctions. Management Science 51, 419–434.

Lehmann, D., Oćallaghan, L.I., Shoham, Y., 2002. Truth revelation in approximately efficient combinatorial auctions. Journal of the ACM (JACM) 49, 577–602.

Leyton-Brown, K., Shoham, Y., 2008. Essentials of game theory: A concise multidisciplinary introduction. Synthesis Lectures on Artificial Intelligence and Machine Learning 2, 1–88. URL:

https://doi.org/10.2200/S00108ED1V01Y200802AIM003, doi:doi:10.2200/S00108ED1V01Y200802AIM003, arXiv:https://doi.org/10.2200/S00108ED1V01Y200802AIM003.

Lucking-Reiley, D., 2000. Vickrey auctions in practice: From nineteenth-century philately to twenty-first-century e-commerce. Journal of Economic Perspectives 14, 183–192. URL: http://www.aeaweb.org/articles?id=10.1257/jep.14.3.183, doi:doi:10.1257/jep.14.3.183.

Mas-Colell, A., , Whinston, M., Green, J., 1995. Microeconomic Theory. Oxford student edition, Oxford University Press.

Maskin, E., Riley, J., 1984. Monopoly with incomplete information. The RAND Journal of Economics 15, 171–196. URL: http://www.jstor.org/stable/2555674.

Maskin, E., Riley, J., 2000. Asymmetric auctions. The Review of Economic Studies 67, 413–438. URL: http://www.nybooks.com/articles/2017/06/08/a-better-way-to-choose-presidents/, doi:doi:https://doi.org/10.1111/1467-937X.00137.

Maskin, E.S., Riley, J.G., 1989. Optimal multi-unit auctions, in: Hahn, F. (Ed.), The Economics of Missing Markets, Information, and Games. Oxford University Press, Oxford, pp. 312–335.

McAfee, R., McMillan, P.J., 1987. Auctions and bidding. Journal of Economic Literature 25, 699–738.

McMillan, J., 1994. Selling spectrum rights. Journal of Economic Perspectives 8, 145–162. URL: http://www.aeaweb.org/articles?id=10.1257/jep.8.3.145, doi:doi:10.1257/jep.8.3.145.

Milgrom, P., 2000. Putting auction theory to work: The simultaneous ascending auction. Journal of Political Economy 108, 245–272. doi:doi:10.1086/262118.

Milgrom, P., 2007. Package auctions and exchanges. Econometrica 75, 935–965. URL: https://onlinelibrary.wiley.com/doi/abs/10.1111/j.1468-0262.2007.00778.x, doi:doi:10.1111/j.1468-0262.2007.00778.x.

Milgrom, P.R., 1985. The economics of competitive bidding: a selective survey, in: Hurwicz, L., Schmeidler, D., Sonnenschein, H. (Eds.), Social Goal and Social Organization: Essays in Memory of Elisha Pazner. Cambridge University Press.

Milgrom, P.R., 1987. Auction theory, in: Bewley, T. (Ed.), Advances in Economic Theory: Fifth World Congress. Cambridge University Press, Cambridge, UK.

Mirrlees, J.A., 1971. An exploration in the theory of optimum income taxation. The Review of Economic Studies 38, 175–208. doi:doi:10.2307/2296779.

Myerson, R.B., 1981. Optimal auction design. Mathematics of Operations Research 6, 58–73. doi:doi:https://doi.org/10.1287/moor.6.1.58.

von Neumann, J., Morgenstern, O., 1944. Theory of Games and Economic Behavior. Princeton University Press.

Nisan, N., 2000. Bidding and allocations in combinatorial auctions, in: ACM Conference on Electronic Commerce (EC-2000).

Noussair, C., 1995. Equilibria in a multi-object uniform price sealed bid auction with multi-unit demands. Economic Theory 5, 337–51. URL: https://EconPapers.repec.org/RePEc:spr:joecth:v:5:y:1995:i:2:p:337-51, doi:doi:https://doi.org/10.1007/BF01215207.

Ortega-Reichert, A., 1968. Models for Competitive Bidding under Uncertainty. Technical Report. Stanford University.

Palfrey, T.R., 1983. Bundling decisions by a multiproduct monopolist with incomplete information. Econometrica 51, 463–483. URL: http://www.jstor.org/stable/1912001.

Rassenti, S., Smith, V.L., Bulfin, R.L., 1982. A combinatorial auction mechanism for airport time slot allocations. Bell Journal of Economics 13, 402–417. URL: http://www.jstor.org/stable/3003463, doi:doi:10.2307/3003463.

Riley, J.G., Samuleson, J.G., 1981. Optimal auctions. American Economic Review 71, 381–392.

Rothkopf, M.H., 2007. Thirteen reasons why the Vickrey-Clarke-Groves process is not practical. Operations Research 55, 191–197.

Rothkopf, M.H., Pekec, A., Harstad, R.M., 1998. Computationally manageable combinatorial auctions. Management Science 44, 1131–1147.

Rothkopf, M.H., Teisberg, T.J., Kahn, E.P., 1990. Why are vickrey auctions rare? Journal of Political Economy 98, 94–109. URL: https://doi.org/10.1086/261670, doi:doi:10.1086/261670, arXiv:https://doi.org/10.1086/261670.

Sandholm, T., 2002. Algorithm for optimal winner determination in combinatorial auctions. Artif. Intell. 135, 1–54. URL: http://portal.acm.org/citation.cfm?id=506694.506695.

Sano, R., 2011. Incentives in core-selecting auctions with single-minded bidders. Games and Economic Behavior 72, 602–606.

Sano, R., 2012. Non-bidding equilibrium in an ascending core-selecting auction. Games and Economic Behavior 74, 637–650.

Schmidt, K.M., 2004. Efficiency of the British UMTS Auction: A Comment on Borgers and Dustmann. MIT Press. pp. 159–162.

Shapiro, R., Holtz-Eakin, D., Bazelon, C., 2013. The Economic Implications of Restricting Spectrum Purchases in the Incentive Auctions. Working Paper 2541812. SSRN. doi:doi:10.2139/ssrn.2541812.

Varian, H.R., 2009. Online ad auctions. The American Economic Review 99, 430–434. URL: http://www.jstor.org/stable/25592436.

Vickrey, W., 1961. Counterspeculation, auctions, and competitive sealed tenders. The Journal of Finance 16, 8–37. doi:doi:10.1111/j.1540-6261.1961.tb02789.x.

Vickrey, W., 1962. Auction and Bidding Games. Princeton University Press, Princeton. pp. 15–27.

de Vries, S., Vohra, R., 2003. Combinatorial auctions: A survey. INFORMS Journal of Computing 15, 284–309.

Weber, R.J., 1983. Multiple-object auctions, in: Engelbrecht-Wiggans, R., Shubik, M., Stark, J. (Eds.), Auctions, Bidding, and Contracting. New York University Press, New York, NY, USA, pp. 165–191.

Weber, R.J., Milgrom, P.R., 2000. A Theory of Auctions and Competitive Bidding II. Edward Elgar Publishing Ltd.

Williamson, O.E., 1976. Franchise bidding for natural monopolies-in general and with respect to catv. The Bell Journal of Economics 7, 73–104. URL: http://www.jstor.org/stable/3003191.

Wilson, R., 1989. Bidding. Palgrave Macmillan UK, London. pp. 54–63. URL: https://doi.org/10.1007/978-1-349-20215-7_5, doi:doi:10.1007/978-1-349-20215-7_5.

Yokoo, M., Sakurai, Y., Matsubara, S., 2004. The effect of false-name bids in combinatorial auctions: new fraud in internet auctions. Games and Economic Behavior 46, 174 – 188. URL: http://www.sciencedirect.com/science/article/pii/S0899825603000459, doi:doi:https://doi.org/10.1016/S0899-8256(03)00045-9.

A

APPENDIX A

A.1 PROOFS OF CHAPTER 3

A.1.1 *Proof of Proposition 1*

We begin with the reformulation of the principal's optimization problem (SPA-SB) to an information setting in which the principal does not know the package valuation but, different to the asymmetric information setting, its true support $V(2)$ as well as *dual-winner efficiency*.

$$\max_{\left(\beta_2\left(v_i(2)\right),\mu_2\left(v_i(2)\right)\right)} \quad E_{v_i(2)\in V(2)}\left(F_2\left(\beta_2\left(v_i(2)\right)\right)\cdot\left(v_i(2)-\beta_2\left(v_i(2)\right)\right)+ \right.$$

$$\left. + \int_{\underline{v}(2)}^{\beta_2\left(v_i(2)\right)} F_2(x)dx - T\left(\underline{v}(2)\right) - \mu_2\left(v_i(2)\right) \right) \quad \text{(SPA')}$$

$$\text{subject to} \quad v_i(2) \in \underset{\hat{v}_i(2)\in V(2)}{\operatorname{argmax}}\ F_2\left(\beta\left(\hat{v}_i(2)\right)\right)\cdot w\left(v_i(2)\right)+\mu_2\left(\hat{v}_i(2)\right) \quad \text{(SPA-IC')}$$

$$\beta_i(2)\leq v_i(2) \text{ for all } v_i(2)\in V(2). \quad \text{(SPA-A-NL')}$$

In this case, budget and transfer are functions of the report $\hat{v}_i(2)\in V(2)\subset Z(2)$ and the relevant marginal distribution function is $F_2(\cdot)$. Next, we restate the agent's incentive compatibility constraint (SPA-IC'). Following Mirrlees (1971) to rewrite the agent's indirect ex-interim expected utility, his ex-interim expected utility in (SPA-IC') is equated with his ex-interim indirect utility to obtain transfer payment of

$$\mu_2\left(v_i(2)\right) = EU\left(\beta_2\left(\underline{v}(2)\right), v_j(2), \underline{v}(2); F_2\left(\beta_2\left(\underline{v}(2)\right)\right)\right) +$$

$$+\int_{\underline{v}(2)}^{v_i(2)} F_2\left(\beta_2(x)\right)\cdot\frac{\partial w(x)}{\partial x}dx - F_2\left(\beta_2\left(v_i(2)\right)\right)\cdot w\left(v_i(2)\right), \quad \text{(A.1.1)}$$

with $EU\left(\beta_2\left(\underline{v}(2)\right), v_j(2), \underline{v}(2); F_2\left(\beta_2\left(\underline{v}(2)\right)\right)\right) = F_2\left(\beta_2\left(\underline{v}(2)\right)\right)\cdot w\left(\underline{v}(2)\right) + \mu\left(\underline{v}(2)\right)$. The single crossing condition is satisfied as $w(\cdot)$ is strictly increasing.

It also must be the case that $F_2\left(\beta_2\left(v_i(2)\right)\right)$ is a non-decreasing function of $v_i(2)$, which given the assumption on $F_2(\cdot)$ implies that $\beta_2\left(v_i(2)\right)$ must be non-decreasing and that $\mu_2\left(v_i(2)\right)$ is non-increasing.[42] As $\mu_2\left(v_i(2)\right)$ is non-increasing whenever $\beta_2\left(v_i(2)\right)$ is non-decreasing, condition (A.1.1) together with $\beta_2\left(v_i(2)\right)$ being non-decreasing completely characterize the incentive-compatibility constraint on the principal.

Given $\mu_2\left(v_i(2)\right)$ is non-increasing, optimality then requires $\mu_2\left(\overline{v}(2)\right) = 0$ such that $\mu_2\left(v_i(2)\right)$ is weakly positive for all $v_i(2) \in V(2)$. Therefore (A.1.1) can be rewritten as follows,[43]

$$\begin{aligned}\mu_2\left(v_i(2)\right) =& w\left(\overline{v}(2)\right) - \int_{v_i(2)}^{\overline{v}(2)} F_2\left(\beta_2(x)\right) \cdot \frac{\partial w(x)}{\partial x} dx - \\ &- F_2\left(\beta_2\left(v_i(2)\right)\right) \cdot w\left(v_i(2)\right). \end{aligned} \tag{A.1.2}$$

Plugging (A.1.2) into SPA' to rewrite the principal's optimization problem and integrating $\int_{\underline{v}(2)}^{\overline{v}(2)} \int_{v_i(2)}^{\overline{v}(2)} F_2\left(\beta_2(x)\right) \cdot \frac{\partial w(x)}{\partial y} dx dF_2\left(v_i(2)\right)$ by parts to obtain $\int_{\underline{v}(2)}^{\overline{v}(2)} F_2\left(\beta_2\left(v_i(2)\right)\right) \cdot \frac{\partial w\left(v_i(2)\right)}{\partial v_i(2)} \cdot F_2\left(v_i(2)\right) dv_i(2)$, the principal's optimization problem corresponds to,

$$\begin{aligned}\max_{\beta_2\left(v_i(2)\right)} \quad & \int_{\underline{v}(2)}^{\overline{v}(2)} \left(F_2\left(\beta_2\left(v_i(2)\right)\right) \cdot \left(v_i(2) - \beta_2\left(v_i(2)\right) + w\left(v_i(2)\right) + \right.\right. \\ & \left.\left. + \frac{\partial w\left(v_i(2)\right)}{\partial v_i(2)} \cdot \frac{F_2\left(v_i(2)\right)}{f_2\left(v_i(2)\right)} \right) + \int_{\underline{v}(2)}^{\beta_2\left(v_i(2)\right)} F_2(x) dx \right) \cdot \\ & \cdot f_2\left(v_i(2)\right) dv_i(2) - \left(T\left(\underline{v}(2)\right) + w\left(\overline{v}(2)\right)\right) \end{aligned} \tag{A.1.3}$$

$$\text{subject to} \quad \frac{\partial \beta_2\left(v_i(2)\right)}{\partial v_i(2)} \geq 0 \text{ for all } v_i(2) \in V(2); \tag{A.1.4}$$

$$\text{and } \beta_i(2) \leq v_i(2) \text{ for all } v_i(2) \in V(2). \tag{A.1.5}$$

Condition (A.1.4) now reflects the agent's incentive-compatibility constraint on the principal. The solution to the first-order condition of the unconstrained program in (A.1.3) is

$$\beta_2\left(v_i(2)\right) = v_i(2) + w\left(v_i(2)\right) + \frac{\partial w\left(v_i(2)\right)}{\partial v_i(2)} \cdot \frac{F_2\left(v_i(2)\right)}{f_2\left(v_i(2)\right)}. \tag{A.1.6}$$

[42] Integrating (A.1.1) by parts results in $\mu_2\left(v_i(2)\right) = \mu_2\left(\underline{v}(2)\right) - \int_0^{v_i(2)} \frac{\partial F_2\left(\beta_2(x)\right)}{\partial \beta_2} \cdot \frac{\partial \beta_2(x)}{\partial x} \cdot w(x) dx$. Thus, if $F_2(\cdot)$ and $\beta_2(\cdot)$ are non-decreasing functions, $\mu_2\left(v_i(2)\right)$ is a non-increasing function.

[43] Note that $\mu_2\left(\overline{v}(2)\right) = 0$ implies $\mu_2\left(\underline{v}(2)\right) = w\left(\overline{v}(2)\right) - \int_{\underline{v}(2)}^{\overline{v}(2)} F_2\left(\beta_2(x)\right) \cdot \frac{\partial w(x)}{\partial x} dx$. Plugging $\mu_2\left(\underline{v}(2)\right)$ in (A.1.1) results in (A.1.2).

For the report in A.1.6 it is straightforward to very that the second order condition of the principal's unconstrained optimization problem in (A.1.3) is satisfied. As $\beta_2\left(v_i(2)\right) > v_i(2)$ for all $v_i(2) \in V(2)$, given the no-loss condition (A.1.5), the optimal report of the constrained optimization problem corresponds to $\beta_2\left(v_i(2)\right) = v_i(2)$, which also satisfies condition (A.1.4). Substituting $\beta_2\left(v_i(2)\right) = v_i(2)$ into (A.1.2) and into the principal's ex-ante expected profit in (A.1.3) results in the optimal transfer scheme $\mu_2\left(v_i(2)\right)$ in (SPA-SB-T) and the ex-ante participation constraint in (SPA-SB-PC) of Proposition 1, respectively. Finally, substituting the marginal distribution function $G_2\left(v_i(2)\right)$ for $F_2\left(v_i(2)\right)$ for each $v_i(2) \in Z(2)$ results in Proposition 1. □

A.1.2 *Proof of Proposition 2*

For the following argument suppose opponent j follows the equilibrium strategy $b_j = \left(\beta_1, \beta_2\left(v_j(2)\right)\right)$. Regarding condition 1), suppose any bidder i's equilibrium bidding function for one unit $b_i(1)$ varies with $v_i(1)$ in the *dual-winner equilibrium*, so that $\beta_1\left(v_i(1)\right) < \beta_1\left(\hat{v}_i(1)\right)$ with $v_i(1) \neq \hat{v}_i(1)$. Then bidder i with value $\hat{v}_i(1)$ for one unit has an incentive to bid $b_i(1) = \beta_1\left(v_i(1)\right)$ to raise her profit: $\hat{v}_i(1) - \beta_1\left(v_i(1)\right) > \hat{v}_i(1) - \beta_1\left(\hat{v}_i(1)\right)$. Thus, any bidder i with single-unit values of $v_i(1)$ or $\hat{v}_i(1)$ bids $b_i(1) = \beta_1\left(v_i(1)\right)$ independent of her valuation. This reasoning is true for any bidder with any value and results in the equilibrium bidding function for one unit of $\beta_1\left(v_i(1)\right) = \beta_1\left(\hat{v}_i(1)\right) = \beta_1$ for all $v_i(1), \hat{v}_i(1) \in V(1)$, the pooling bid.

The upper bound in condition 2) ensures that any bidder with the lowest single-unit value $\underline{v}(1)$ will not make a negative profit in the *dual-winner equilibrium*: $\underline{v}(1) \geq \beta_1$. Note further that any bidder i with vector of valuations $v_i = \left(\underline{v}(1), \overline{v}(2)\right)$ has the highest incentive to deviate from the dual-winner outcome. The lower bound in condition 2) makes sure this bidder does not deviate at any pooling price $\beta_1 \geq \overline{v}(2) - \underline{v}(1)$. The respective bidder i could marginally overbid twice the pooling bid with her bid on the package of two units to obtain the profit of the single-winner outcome with certainty: $\pi_i(2) = \overline{v}(2) - 2 \cdot \beta_1 - \varepsilon$ for $\varepsilon \to 0$.

Note, that depending on her type, opponent j might in fact bid twice the pooling bid on the package of two units. It is the highest bid on two units that still supports the dual-winner outcome as stated in condition 3) below. For this deviation not to be profitable, the pooling bid has to be of the form $\beta_1 \geq \overline{v}(2) - \underline{v}(1)$. Note that bidder i's profit in the *dual-winner equilibrium* is given by $\pi_i(1) = \underline{v}(1) - \beta_1$. If this bidder with vector of valuations $v_i = \left(\underline{v}(1), \overline{v}(2)\right)$ has no incentive to deviate from the *dual-winner equilibrium*, then no other bidder j with valuations $v_j(1) \geq \underline{v}(1)$ and $v_j(2) \leq \overline{v}(2)$ deviates either. Note that, if bidder i did not bid β_1, but zero for example, then it would not be in equilibrium for bidder j to bid β_1.

For the dual-winner outcome to be chosen by the revenue-maximizing auctioneer for all possible package bids, it has to be true that $2 \cdot \beta_1 \geq$

$sup_{v_i(2)}\left\{\beta_2\left(v_i(2)\right)\right\}$ as stated in condition 3). The supremum is defined as the smallest upper bound or the greatest element in the set. Suppose $2\cdot\beta_1 > sup_{v_i(2)}\left\{\beta_2\left(v_i(2)\right)\right\}$, then for any bidder i, it is a best response to deviate from her equilibrium strategy by underbidding the pooling bid slightly with her single-unit bid (and thus raising her profit in the *dual-winner equilibrium*). This cannot be optimal and therefore we obtain the equilibrium requirement of $2\cdot\beta_1 = sup_{v_i(2)}\left\{\beta_2\left(v_i(2)\right)\right\}$. □

A.1.3 *Proof of Proposition 3*

Assume opponent j sticks to her equilibrium strategy $b_j = \left(\beta_1, \beta_2\left(v_j(2)\right)\right)$ for all possible package values $v_j(1) \in V(1)$ and $v_j(2) \in V(2)$. Then we demonstrate that under conditions 1) to 4), $b_i = \left(\beta_1, \beta_2\left(v_i(2)\right)\right)$ is an equilibrium supporting strategy in relation to any other deviating bidding strategy $\hat{b}_i = \left(\hat{b}_i(1), \hat{b}_i(2)\right)$. Note that the profit in a *dual-winner equilibrium* for any bidder i is given by $\pi_i(1) = v_i(1) - b_i(1)$ for all $v_i(1) \in V(1)$. Let us refer to this as equilibrium profit.

Now we consider three different cases that can result from bidder i playing any deviating strategy $\hat{b}_i = \left(\hat{b}_i(1), \hat{b}_i(2)\right)$ instead of the equilibrium strategy $b_i = \left(\beta_1, \beta_2\left(v_i(2)\right)\right)$. Any deviating strategy might involve changing only the bid for one unit, the bid for two units or both bids. Nevertheless, any of the three mentioned deviations will always end up in one of the following three outcomes given opponent j sticks to the proposed equilibrium strategy $b_j = \left(\beta_1, \beta_2\left(v_j(2)\right)\right)$: no single-winner outcome for principal i, $\beta_1 + \hat{b}_i(1) > \hat{b}_i(2)$, a single-winner outcome, $\beta_1 + \hat{b}_i(1) < \hat{b}_i(2)$, and an indifference condition for bidder i, $\beta_1 + \hat{b}_i(1) = \hat{b}_i(2)$. Let us consider each case in turn:

A) $\beta_1 + \hat{b}_i(1) > \hat{b}_i(2)$

Bidder i deviates to a different dual-winner outcome. However, bidder i would never receive a dual-winner award if $\beta_1 + \hat{b}_i(1) < \beta_2\left(v_j(2)\right)$ for all $v_j(2) \in V(2)$, i.e., $\beta_1 + \hat{b}_i(1) < \beta_2\left(\underline{v}(2)\right)$. Thus, it is a necessary condition for $\hat{b}_i(1)$ to exceed $\beta_2\left(\underline{v}(2)\right) - \beta_1$ to satisfy case A). Due to condition 3), raising $\hat{b}_i(1)$ above β_1 does not increase the probability of winning, which is already equal to one in the *dual-winner equilibrium*, but strictly lowers profits. Therefore, the rationalizable range for the deviating bid is defined by $\hat{b}_i(1) \in \left(\beta_2\left(\underline{v}(2)\right) - \beta_1, \beta_1\right)$. The ex-interim expected profit, $\hat{\Pi}_i(1)$, of submitting a $\hat{b}_i(1)$ from this range for any single-unit value $v_i(1) \in V(1)$ is given by equation (A.1.7):

$$\hat{\Pi}_i(1) = \left(v_i(1) - \hat{b}_i(1)\right)\cdot P\left(\beta_2\left(v_j(2)\right) \leq \beta_1 + \hat{b}_i(1)\right) \qquad \text{(A.1.7)}$$

Now, (A.1.7) can be simplified as follows: As the function $\beta_2(\cdot)$ is continuous and strictly increasing, for any $\hat{b}_i(1) \in \left(\beta_2\left(\underline{v}(2)\right) - \beta_1, \beta_1\right)$, a unique proxy valuation for the package of two units $\hat{v}_i(2) \in V(2)$ can be defined, so that $\hat{b}_i(1) = \beta_2\left(\hat{v}_i(2)\right) - \beta_1$. Using this expression for $\hat{b}_i(1)$ to rewrite equation (A.1.7) we obtain equation (A.1.8):

$$\hat{\Pi}_i(1) = \left(v_i(1) - \beta_2\left(\hat{v}_i(2)\right) + \beta_1\right) \cdot F_2\left(\hat{v}_i(2)\right) \tag{A.1.8}$$

Deviating single-unit bids of the form $\hat{b}_i(1) \in \left(\beta_2\left(\underline{v}(2)\right) - \beta_1, \beta_1\right)$ imply a focus on a deviation weakly below the pooling equilibrium price of β_1. In addition, any bidder i with single-unit value of $v_i(1) = \underline{v}(1)$ earns least in a dual-winner outcome and therefore has the highest incentive to deviate to a lower bid on one unit in equilibrium. To cancel this incentive, her equilibrium profit of $\pi_i(1) = \underline{v}(1) - \beta_1$ has to exceed her ex-interim expected profit from deviating, $\hat{\Pi}_i(1)$, which is ensured in inequality (A.1.9):

$$\underline{v}(1) - \beta_1 \geq \left(\underline{v}(1) - \beta_2\left(\hat{v}_i(2)\right) + \beta_1\right) \cdot F_2\left(\hat{v}_i(2)\right) \tag{A.1.9}$$

Rearranging we obtain condition 4) for all $v_i(1) \in V(1)$ and $\hat{v}_i(2) \in V(2)$,

$$\beta_2\left(\hat{v}_i(2)\right) \geq \beta_1 + \frac{\beta_1 - \underline{v}(1) \cdot (1 - F_2\left(\hat{v}_i(2)\right)}{F_2\left(\hat{v}_i(2)\right)} \equiv G\left(\hat{v}_i(2), \beta_1\right) \tag{A.1.10}$$

If bidder i with the lowest value for one unit has no incentive to deviate, then in fact no player with a higher value can have an incentive to deviate independent of the valuation for two units. The proposed *dual-winner equilibrium* is preferred to a deviation from case A) by any bidder i as long as all deviating bids for two units are bounded from below by $G\left(v_i(2), \beta_1\right)$. This is true for all valuations $v_i \in V$.

B) $\beta_1 + \hat{b}_i(1) < \hat{b}_i(2)$

Bidder i deviates in a way that the auctioneer never selects any dual-winner outcome. She definitely does not win both units either if $\hat{b}_i(2) < \beta_2\left(\underline{v}(2)\right)$, which then defines the lower bound of her deviating bid for the package. If $\hat{b}_i(2) > \beta_2\left(\overline{v}(2)\right)$, the bidder wins both units, but lowering the respective bid until $\hat{b}_i(2) = \beta_2\left(\overline{v}(2)\right)$ strictly dominates in profit without changing the probability of winning. Thus, we obtain a rationalizable range for bidder i's deviating package bid of $\hat{b}_i(2) \in \left(\beta_2\left(\underline{v}(2)\right), \beta_2\left(\overline{v}(2)\right)\right)$ with an ex-interim expected deviating profit of $\hat{\Pi}_i(2) = \left(v_i(2) - \hat{b}_i(2)\right) \cdot P\left(\beta_2\left(v_j(2)\right) \leq \hat{b}_i(2)\right)$ for all $v_i(2) \in V(2)$. Let us again use the proxy notation $\hat{b}_i(2) = \beta_2\left(\hat{v}_i(2)\right)$ to rewrite above profit as $\hat{\Pi}_i(2) = \left(v_i(2) - \hat{\beta}_i(2)\right) \cdot F_2\left(\hat{v}_i(2)\right)$. Bidder i prefers the equilibrium profit $\pi_i(1)$ to the deviating profit of $\hat{\Pi}_i(2)$ if the following inequality (A.1.11) holds:

$$0 \geq F_2\left(\hat{v}_i(2)\right) \cdot \left(\frac{\beta_1 - v_i(1)}{F_2\left(\hat{v}_i(2)\right)} + v_i(2) - \hat{b}_i(2)\right) \tag{A.1.11}$$

The above inequality is true for all valuations $v_i \in V$ and $\hat{v}_i(2) \in V(2)$ if the term in squared brackets is weakly negative. We already know the sufficient condition to satisfy case A) is $\beta_2\left(\hat{v}_i(2)\right) \geq G\left(\hat{v}_i(2), \beta_1\right)$. It ensures opponent j's equilibrium-supporting bid on two units to be sufficiently high. Then, principal i's deviation to a different dual-winner award is less likely to succeed and therefore does not offer high enough expected profit for the deviation to be worthwhile. Note, that a similar reasoning applies in case B): Again, principal j's equilibrium-supporting bid on two units must be sufficiently high. In this case, principal i's deviation to a single-winner award is less likely to succeed and therefore not profitable enough in expectation. As a result the sufficient condition from case A) can be used in case B) as well. Thus, the term in squared brackets in condition (A.1.11) is in fact weakly negative for all $v_i \in V$ and $\hat{v}(2) \in V(2)$ because the following inequality (A.1.12) is true:

$$\frac{\beta_1 - v_i(1)}{F_2\left(\hat{v}_i(2)\right)} + v_i(2) \leq G\left(\hat{v}_i(2), \beta_1\right) \tag{A.1.12}$$

Using the definition of $G\left(\hat{v}_i(2), \beta_1\right)$ and rearranging, we obtain (A.1.13):

$$\underline{v}(1) - v_i(1) \leq F_2\left(\hat{v}_i(2)\right) \cdot \left(\beta_1 + \underline{v}(1) - v_i(2)\right) \tag{A.1.13}$$

The LHS of the above inequality is weakly negative. Now we have to distinguish two cases regarding the RHS of inequality (A.1.13): If $\beta_1 + \underline{v}(1) - v_i(2) \geq 0$, the inequality always holds. If $\beta_1 + \underline{v}(1) - v_i(2) < 0$, we have to show that $\beta_1 + \underline{v}(1) - v_i(2) \geq \underline{v}(1) - v_i(1)$. This is true for all $v_i \in V$ given the lower bound of condition 1). As inequality (A.1.13) holds, inequality (A.1.12) must also be true. Remember from A) that $\beta_2\left(\hat{v}_i(2)\right) \geq G\left(\hat{v}_i(2), \beta_1\right)$ must be given, which implies that inequality (A.1.11) is satisfied. Therefore, any deviation considered in case B) is not profitable.

C) $\beta_1 + \hat{b}_i(1) = \hat{b}_i(2)$

In this case, bidder i deviates as if she were indifferent between the dual-winner and single-winner outcome. Remember from condition 1) that any bidder i with valuations of $v_i = \left(\underline{v}(1), \overline{v}(2)\right)$ is indifferent between the *dual-winner equilibrium* and any single-winner outcome at the pooling price. Hence, the deviating behavior in case C) does in fact define her equilibrium strategy. Rewrite as the deviation in case C) to $\beta_1 = \hat{b}_i(2) - \hat{b}_i(1)$ and bear in mind the lower bound from condition

1): $\beta_1 \geq \overline{v}(2) - \underline{v}(1)$. Combining these two equations by substituting for β_1 and rearranging, we obtain inequality (A.1.14):

$$\underline{v}(1) - \hat{b}_i(1) \geq \overline{v}(2) - \hat{b}_i(2) \tag{A.1.14}$$

Now, consider player i with values of $v_i = \big(v_i(1), v_i(2)\big)$ in which $v_i(1) > \underline{v}(1)$ and $v_i(2) \leq \overline{v}(2)$. For any such bidder condition (A.1.14) holds with strict inequality and she strictly prefers any deviating dual-winner award (LHS) to any single-winner award, which contradicts case C). Note that for bidder i with valuations of $v_i = \big(v_i(1), v_i(2)\big)$ in which $v_i(1) \geq \underline{v}(1)$ and $v_i(2) < \overline{v}(2)$, the same reasoning holds.

Note in particular that by strictly decreasing the deviating bid on two units from $\hat{b}_i(2)$ to $\hat{b}_i(2)'$, so that the deviation from case C) becomes $\beta_1 + \hat{b}_i(1) > \hat{b}_i(2)'$, the bidder changes from a case C)-deviation to a case A)-deviation. As the latter always leads to some dual-winner outcome, it dominates case C) for all $v_i = \big(v_i(1), v_i(2)\big)$ with $v_i(1) > \underline{v}(1)$ and $v_i(2) \leq \overline{v}(2)$, and for all $v_i = \big(v_i(1), v_i(2)\big)$ in which $v_i(1) \geq \underline{v}(1)$ and $v_i(2) < \overline{v}(2)$. Finally, as a case A)-deviation is not beneficial, a case C)-deviation cannot possibly be either.

□

A.1.4 *Proof of Proposition 4*

In a *single-winner equilibrium*, any bidder i solely aims for the package of two units for all package valuations of $v_i \in V$. This scenario is strategically equivalent to the well-known first-price sealed-bid auction for a single package, in which two units are sold as the only bundle. In this standard auction format, the equilibrium strategy of any bidder i takes the form of $b_i(2) = \beta_2\big(v_i(2)\big)$ from condition (1)).

Note that the *single-winner equilibrium* requires any bidder i to possess ultimate "veto" power on the dual-winner outcome to make it unprofitable for her opponent to deviate from equilibrium. Suppose opponent j follows the proposed equilibrium strategy and submits a very low "veto" bid on one unit, such as $b_j(1) = 0$, for example. Then bidder i would have to submit a deviating single-unit bid, $\hat{b}_i(1)$, to retain the chance of winning the dual-winner outcome that satisfies the next inequality (A.1.15):

$$\hat{b}_i(1) > \beta_2\big(\underline{v}(2)\big) = \underline{v}(2) \tag{A.1.15}$$

In inequality (A.1.15), $\beta_2\big(\underline{v}(2)\big)$ is the optimal bid on the package of bidder i with lowest valuation for two units. Add the valuation for one unit $v_i(1)$ on both sides of inequality (A.1.15) and rearrange to obtain inequality (A.1.16):

$$v_i(1) - \underline{v}(2) > v_i(1) - \hat{b}_i(1) \tag{A.1.16}$$

The LHS of inequality (A.1.16) is strictly negative if $\underline{v}(2) > v_i(1)$ for all single-unit valuations $v_i(1) \in V(1)$, i.e., if $\underline{v}(2) > \overline{v}(1)$ is true. The last inequality holds by assumption. As the LHS of (A.1.16) is strictly negative, the RHS of inequality (A.1.16) must be strictly negative. Note that the RHS corresponds to bidder i's profit in the forced deviating dual-winner outcome. Thus, if opponent j submits a "veto" bid in form of condition (2)), any deviating single-unit bid $\hat{b}_i(1)$ of bidder i to enforce the dual-winner outcome results in strictly negative profit. As she receives weakly positive expected profit in the *single-winner equilibrium*, a deviating bid of $\hat{b}_i(1)$ is strictly dominated by any single-unit bid that supports the *single-winner equilibrium*. By symmetry, only a bid of the form $b_i(1) < \underline{v}(2) - \overline{v}(1)$ supports the *single-winner equilibrium* for all $v_i \in V$ with certainty. □

A.1.5 *Proof of Proposition 5*

Remember from Proposition 3 that any principal i submits the payoff-maximizing pooling bid of $\beta_1 = \overline{v}(2) - \underline{v}(1)$ in the *dual-winner equilibrium* and obtains respective equilibrium profit of $v_i(1) - \overline{v}(2) + \underline{v}(1)$ with certainty. The principal's ex-interim expected equilibrium profit in the *single-winner equilibrium* is $\int_{\underline{v}(2)}^{v_i(2)} F_2(x)\,dx$, as in the standard FPSB auction, in which two units are sold as the sole package to two bidders. For principal i let us define the difference between expected profits in the *dual-winner* and *single-winner equilibrium* as a function $\Delta\big(v_i(1), v_i(2)\big) : V \to \mathbb{R}$ on the compact set $V \subset \mathbb{R}^2$, with

$$\Delta\big(v_i(1), v_i(2)\big) = v_i(1) - \overline{v}(2) + \underline{v}(1) - \int_{\underline{v}(2)}^{v_i(2)} F_2(x)\,dx \qquad \text{(A.1.17)}$$

The above function is continuous, due to the differentiability of its constituents. It follows that it possesses a global maximum and a global minimum on V. Moreover, $\Delta\big(v_i(1), v_i(2)\big)$ is strictly increasing in its first argument and strictly decreasing in its second argument. Consequently, the function does not have a critical point in the interior of its domain, but on the boundary. Its maximum occurs at $\big(\overline{v}(1), \underline{v}(2)\big)$ and the minimum at $\big(\underline{v}(1), \overline{v}(2)\big)$, with values of $\Delta\big(\overline{v}(1), \underline{v}(2)\big) = \overline{v}(1) - \overline{v}(2) + \underline{v}(1)$ and $\Delta\big(\underline{v}(1), \overline{v}(2)\big) = \underline{v}(1) - \overline{v}(2) + \underline{v}(1) - \int_{\underline{v}(2)}^{\overline{v}(2)} F_2(x)\,dx$, respectively. Remember that *dual-winner efficiency* is defined by $\overline{v}(2) < 2 \cdot \underline{v}(1)$. This implies the maximum $\Delta\big(\overline{v}(1), \underline{v}(2)\big)$ is always strictly positive and the minimum $\Delta\big(\underline{v}(1), \overline{v}(2)\big)$ is strictly positive for all package valuations $v_i \in V$ if inequality (A.1.18) holds:

$$\underline{v}(1) - \overline{v}(2) + \underline{v}(1) - \int_{\underline{v}(2)}^{\overline{v}(2)} F_2(x)\,dx > 0 \qquad \text{(A.1.18)}$$

Using integration by parts, inequality (A.1.18) can be rewritten to (A.1.19):

$$\int_{\underline{v}(2)}^{\overline{v}(2)} f_2(x) \cdot x dx > 2 \cdot \overline{v}(2) - 2 \cdot \underline{v}(1) \tag{A.1.19}$$

Thus, $\Delta\left(v_i(1), v_i(2)\right)$ is strictly positive for all package valuations $v_i \in V$ if $\int_{\underline{v}(2)}^{\overline{v}(2)} f_2(x) \cdot x dx > 2 \cdot d$ with $\beta_1 = d$. □

A.1.6 *Proof of Lemma 1*

We prove the lemma by eliminating weakly dominated strategies. Suppose agent i follows the set of strategies $\hat{b}_i = \left(b_i(1), \hat{b}_i(2)\right)$ with fixed $b_i(1) \in \left[0, a_i(1)\right]$ and $\hat{b}_i(2) < a_i(2)$. Let us first focus on the setting in which opponent j submits bids of $b_i(1) + b_j(1) < b_j(2)$. Then,

a) if $b_j(2) < \hat{b}_i(2)$ agent i wins both units. However, he also wins the package with strategy $b_i = \left(b_i(1), a_i(2)\right)$, and the higher bid on two units does not impact his utility.

b) If $b_j(2) = \hat{b}_i(2)$ agent i might win two units through randomization by the auctioneer. With strategy b_i agent i would win the package, which provides a strictly higher utility.

c) If $b_j(2) > \hat{b}_i(2)$ agent i wins nothing, but might have won the package with strategy b_i in case $a_i(2) > b_j(2)$, such that $\hat{b}_i$ was strictly dominated. Otherwise he is indifferent between strategies $\hat{b}_i$ and b_i.

Let us now consider the setting in which opponent j submits bids of $b_i(1) + b_j(1) \geq b_j(2)$. Here,

d) if $b_i(1) + b_j(1) < \hat{b}_i(2)$ agent i wins the package in any case and is indifferent between $\hat{b}_i$ and b_i.

e) If $b_i(1) + b_j(1) \geq \hat{b}_i(2)$ agent i wins one unit, but could win the package with strategy b_i as long as $b_i(1) + b_j(1) < a_i(2)$, which would strictly dominate $\hat{b}_i$. For $b_i(1) + b_j(1) \geq a_i(2)$ agent i is indifferent between strategies $\hat{b}_i$ and b_i.

Thus, any strategy set $\hat{b}_i = \left(b_i(1), \hat{b}_i(2)\right)$ with $\hat{b}_i(2) < a_i(2)$ is weakly dominated by the set of strategies $b_i = \left(b_i(1), a_i(2)\right)$ for all $b_i(1) \in \left[0, a_i(1)\right]$. □

A.1.7 *Proof of Lemma 2*

We prove this lemma by eliminating weakly dominated strategies. From Lemma 1, we know that any strategy $b_i(2) < a_i(2)$ is weakly dominated. Now, we can concentrate on strategies $b_i' = \left(0, a_i(2)\right)$, $b_i'' = \left(a_i(1), a_i(2)\right)$, and the strategy set $\hat{b}_i = \left(\hat{b}_i(1), a_i(2)\right)$ with $\hat{b}_i(1) \in \left(0, a_i(1)\right)$. In what follows, we show that $\hat{b}_i$ is weakly dominated. Let us first analyze agent j submitting bids of $\hat{b}_i(1) + b_j(1) < a_j(2)$. Then,

a) if $a_j(2) < a_i(2)$ agent i wins both units just as with strategy $b_i' = \big(0, a_i(2)\big)$. A strategy $b_i'' = \big(a_i(1), a_i(2)\big)$ might still lead to winning the package, but it could also lead to $a_i(1) + b_j(1) > a_i(2)$, such that the agent wins only one unit. In this case, strategy b_i'' is strictly dominated. $\hat{b}_i$ is weakly dominated in each of the cases, because a payment is not considered in utility u_i.

b) If $a_j(2) = a_i(2)$ he might win the package or nothing due to the randomization of the auctioneer as with strategy b_i'. Strategy b_i'' does either not change the outcome or it leads to winning one unit with certainty in case of $a_i(1) + b_j(1) > a_i(2)$. In the latter case, b_i'' dominates the other strategies if $u_i(1) > 0.5 \cdot u_i(2)$. If $u_i(1) < 0.5 \cdot u_i(2)$, then the agent is indifferent between $\hat{b}_i$ and b_i'. Strategy $\hat{b}_i$ is again weakly dominated in each of the cases.

c) If $a_j(2) > a_i(2)$ he wins nothing independent of strategy $\hat{b}_i$ or b_i'. With strategy b_i'' agent i will not win anything if $a_i(1) + b_j(1) < a_j(2)$, or he will win one unit if $a_i(1) + b_j(1) > a_j(2)$. In the earlier case the bid on a single unit is irrelevant, in the latter case, b_i'' strictly dominates the other strategies b_i' and weakly dominates $\hat{b}_i$.

Let us now consider opponent j submitting bids of $\hat{b}_i(1) + b_j(1) \geq a_j(2)$. Then,

d) if $\hat{b}_i(1) + b_j(1) < a_i(2)$, the outcomes of this case correspond to a) for agent i.

e) If $\hat{b}_i(1) + b_j(1) = a_i(2)$ he wins one unit just as with strategy b_i''. If $a_i(2) > a_j(2)$, then b_i' strictly dominates $\hat{b}_i$. Also, if $a_i(2) = a_j(2)$ and $u_i(1) < 0.5 \cdot u_i(2)$, then b_i' strictly dominates $\hat{b}_i$. If $a_i(2) = a_j(2)$ and $u_i(1) > 0.5 \cdot u_i(2)$, then b_i'' strictly dominates b_i'. Strategy $\hat{b}_i$ is weakly dominated in all cases.

f) If $\hat{b}_i(1) + b_j(1) > a_i(2)$ he wins one unit just as with strategy b_i''. With strategy b_i' we have to distinguish different cases. Case $a_j(2) > a_i(2)$ corresponds to the outcome in c) and b_i' is strictly dominated by b_i'', where agent i would win one unit. If in case $a_j(2) < a_i(2)$, then b_i' strictly dominates the other strategies. Finally, case $a_j(2) = a_i(2)$ is reflected in b) and b_i' is strictly dominated by b_i'' if $u_i(1) > 0.5 \cdot u_i(2)$. With $u_i(1) < 0.5 \cdot u_i(2)$ then strategy b_i' would strictly dominate b_i'. Also here strategy $\hat{b}_i$ is weakly dominated.

In summary, the strategy set $\hat{b}_i = \big(\hat{b}_i(1), a_i(2)\big)$ with $\hat{b}_i(1) \in \big(0, a_i(1)\big)$ is always weakly dominated by the set of strategies $b_i = \big(b_i(1), a_i(2)\big)$ with $b_i(1) \in \big\{0, a_i(1)\big\}$. □

A.1.8 *Proof of Proposition 6*

Suppose opponent j follows the proposed equilibrium strategy and bids $b_j = \big(0, a_j(2)\big)$. In this case bidder i cannot win one unit independent of which strategy he chooses from the set of weakly dominant strategies defined in Lemma 2. Remember, he always bids his full budget on two units which exceeds his single unit budget by assumption. With $b_i = \big(a_i(1), a_i(2)\big)$ he either wins the package or nothing because $a_i(1) \leq a_i(2)$. Therefore, agent i is indifferent between both strategies and $b_i = \big(0, a_i(2)\big)$ is an equilibrium. In this equilibrium no agent i would want to deviate even if he knew the opponent's type. Again, assume opponent j to follow the proposed equilibrium strategy. For package budgets of the form $a_i(2) > a_j(2)$, bidder i wins two units following the equilibrium strategy and has no incentive to adjust. In case of $a_i(2) < a_j(2)$, agent i does not win anything independent of his strategy and therefore is indifferent to deviating. □

A.1.9 *Proof of Proposition 7*

We prove the theorem by contradiction. Assume there is a set of budget combinations $S \subset A$ for which bidders submit bids $b_i = \big(a_i(1), a_i(2)\big)$. For all draws of package budget combinations not in S bidders bid on the large package only, i.e., $b_i = \big(0, a_i(2)\big)$. Let us focus on a bidder i with any draws of package budgets $a_i \in A$ and suppose his opponent j possesses budget draws of $a_j \in S$. Based on Lemma 2 opponent j employs the strategy $b_j = \big(a_j(1), a_j(2)\big)$. Bidder i's ex-interim expected payoff of bidding on the large package only is $EU_i^p = u_i(2) \cdot P\big(a_i(2) \geq a_j(2)\big)$, whereas, his ex-interim expected payoff of using $b_i = \big(a_i(1), a_i(2)\big)$, is

$$\begin{aligned} EU^s p_i =& u_i(2) \cdot P\Big(a_i(2) \geq a_j(2) \cap a_i(2) \geq a_i(1) + a_j(1)\Big) + \\ &+ u_i(1) \cdot P\Big(a_i(1) + a_j(1) \geq a_i(2) \cap a_i(1) + a_j(1) \geq a_j(2)\Big) \end{aligned} \tag{A.1.20}$$

in which $P(X)$ denotes the probability of event X to occur. To keep the proof traceable we will not express the relevant probabilities with distribution function $Q(a_j)|_{a_j(1) \leq a_j(2)}$. Define the difference between bidder i's ex-interim expected payoffs, ΔEU_i, as $\Delta EU_i = EU^s p_i - EU_i^p$. We now demonstrate that $\Delta EU_i \leq 0$ for all $a_i \in A$ which corresponds to showing that set S is empty, i.e., there cannot exist a Bayesian Nash equilibrium strategy in which any agent i bids $b_i = \big(a_i(1), a_i(2)\big)$.

First, observe that bidder i's package draw for two-units of $a_i(2) \in \big(\hat{a}_i(2), \overline{a}(2)\big)$, in which $\hat{a}_i(2)$ is some budget close to $\overline{a}(2)$, cannot belong to set S independent of the budget for one unit. For the highest possible budget draw for two units, $a_i(2) = \overline{a}(2)$, the difference in bidder i's expected payoffs is weakly negative for all possible single-unit budgets, i.e., $\Delta EU_i \leq 0 \forall a_i(1) \in A(1)$, as

$$u_i(2) \cdot P\Big(\overline{a}(2) \geq a_i(1) + a_j(1)\Big) + u_i(1) \cdot P\Big(a_i(1) + a_j(1) \geq \overline{a}(2)\Big) \leq u_i(2). \tag{A.1.21}$$

Let us distinguish two different cases. If $\overline{a}(2) < 2 \cdot \overline{a}(1)$ then $P\Big(\overline{a}(2) \geq a_i(1) + a_j(1)\Big) < 1$ and $P\Big(a_i(1) + a_j(1) \geq \overline{a}(2)\Big) > 0$. As $u_i(1)$is strictly smaller than $u_i(2)$ the LHS is strictly smaller than the RHS and (A.1.21) holds strictly for bidder i with highest package budget and any single-unit budget draw. If $\overline{a}(2) \geq 2 \cdot \overline{a}(1)$ then $P\Big(\overline{a}(2) \geq a_i(1) + a_j(1)\Big) = 1$ and $P\Big(a_i(1) + a_j(1) \geq \overline{a}(2)\Big) = 0$, so (A.1.21) holds with equality. Bidder i is indifferent between bidding on the package only and bidding on one and two units for all possible single-unit budget draws because he wins the large package anyway. WLOG in this case we can assume bidder i with package budget draws $a_i(2) \geq \check{a}_i(2)$ to bid on the large package only, independent of the single-unit budget draw, with $\check{a}_i(2)$ being defined as the lowest package budget such that $P\Big(\check{a}_i(2) \geq a_i(1) + a_j(1)\Big) = 1$. Note that for both cases (A.1.21) also holds strictly for slightly lower two-unit budgets than $\overline{a}(2)$, $a_i(2) \in \Big(\hat{a}_i(2), \overline{a}(2)\Big)$, independent of the budget draw for one unit.

Second, define the set of budget combinations with the highest package budget draw in S as $H \subseteq S$. Let us from now on focus on bidder i with budget draws of $a_i \in H$. By definition, if his package budget draw, $a_i(2)$, is marginally increased he does not belong to set S anymore. Bidder i's expected payoff from bidding on the large package only, $b_i = \Big(0, a_i(2)\Big)$, remains unaltered whereas his expected payoff from bidding on the single unit and the package corresponds to

$$\begin{aligned} EU_i^{sp} = & u_i(2) \cdot P\Big(a_i(2) \geq a_j(2) \cap a_i(2) \geq a_i(1) + a_j(1)\Big) + \\ & + u_i(1) \cdot P\Big(a_i(2) \geq a_j(2) \cap a_i(1) + a_j(1) \geq a_i(2)\Big). \end{aligned} \tag{A.1.22}$$

The bidder's probability of winning the single unit must include the opponent not having a higher package budget draw than himself, $a_j(2) \leq a_i(2)$. Otherwise, by symmetry, opponent j would not bid on the single unit independent of his corresponding budget and bidder i could not win one unit anyway. Note at this stage, there might be budget combinations with lower budget draws for two units for which opponent j bids on the large package only. However, if we can show that bidder i prefers to bid on the large package only, if we assume all bidders with lower package budget draws to bid on both packages, he will not change this behavior if some bidders with lower package budgets bid on the large package only.

By definition for bidder i with a budget draw from set S the difference in his expected payoff must be positive for all his budget combinations, i.e. $\Delta EU_i > 0$ for all $a_i \in H \subseteq S$, which corresponds to

$$\begin{aligned} &u_i(2) \cdot P\Big(a_i(2) \geq a_j(2) \cap a_i(2) \geq a_i(1) + a_j(1)\Big) + \\ &+ u_i(1) \cdot P\Big(a_i(2) \geq a_j(2) \cap a_i(1) + a_j(1) \geq a_i(2)\Big) > \\ &> u_i(2) \cdot P\Big(a_i(2) \geq a_j(2)\Big). \end{aligned} \tag{A.1.23}$$

We use conditional probability to rewrite the LHS of A.1.23 and cancel out $P\Big(a_i(2) \geq a_j(2)\Big)$ to obtain

$$\begin{aligned} &u_i(2) \cdot P\Big(a_i(2) \geq a_i(1) + a_j(1) | a_i(2) \geq a_j(2)\Big) + \\ &+ u_i(1) \cdot P\Big(a_i(2) \leq a_i(1) + a_j(1) | a_i(2) \geq a_j(2)\Big) > a_i(2). \end{aligned} \tag{A.1.24}$$

As $u_i(1)$is strictly smaller than $u_i(2)$ the LHS is strictly smaller than the RHS in and in fact $\Delta EU_i < 0$ for all $a_i \in H \subseteq S$. Hence, bidder i has an incentive to deviate and bid for the large package only. Thus, set H cannot belong to S. Finally, as it is always possible to define a subset H in S in which bidder i has the highest budget for two units in the set S, there cannot be a set $S \subset V$ as defined above, and the argument unravels for all types. Therefore, the proposed Bayesian Nash equilibrium strategy is in fact unique. □

A.1.10 *Proof of Lemma 3*

Any agent i can be coordinated on winning one unit together with his opponent with certainty. For this, the sum of both single-unit bids must exceed each agent's package bid. Remember from Lemma 1 that any agent always spends his entire package budget constraint and from Lemma 2 that if he submits a non-zero bid on one unit, he bids his entire single-unit budget constraint. Therefore, both principals have to implement a budget constraint scheme so that the sum of both single-unit budget constraints exceeds each agent's package budget constraint in the dual-winner outcome.

In this allocation, principal i chooses the vector of budget constraints a_i such that condition $a_i(1) + \underline{a}(1) \geq a_i(2)$ is satisfied. She does not know firm j's budget for one unit. Thus, she has to make sure her two-unit budget constraint is below the sum of both single-unit budget constraints. This has to be true for all possible single-unit budget constraints of her opponent, especially the smallest budget constraint of $\underline{a}(1)$. Thus, we get $a_i(1) + \underline{a}(1) \geq a_i(2)$.

For agent i to in fact submit a positive single-unit bid, his certain utility from bidding on this package must exceed his ex-interim expected utility from winning two units. This is ensured in condition $u_i(1) \geq u_i(2) \cdot P\Big(a_i(2) \geq a_j(2) \cap a_i(2) \geq a_j(1)\Big)$. On the RHS, agent i does not bid on one unit, but can win two units instead. Here,
$P\Big(a_i(2) \geq a_j(2) \cap a_i(2) \geq a_j(1)\Big)$ is the probability with which his package budget constraint exceeds opponent j's single- and package budget constraints. The budget constraint $a_i(2)$ is chosen so that

$P\left(a_i(2) \geq a_j(2) \cap a_i(2) \geq a_j(1)\right)$ is low enough for the RHS to be lower than the LHS. Thus, agent i prefers to bid on one unit and win with certainty. $\square$

A.1.11 *Proof of Proposition 8*

For the following line of argument suppose the opposing principal j manages to implement the equilibrium strategy from Proposition 3. In Proposition 3, every principal chooses the same pooling price of $\beta_1 = \overline{v}(2) - \underline{v}(1)$ in the *dual-winner equilibrium*. Thus, according to Lemma 3, any principal i has to provide her agent with the same single-unit budget constraint which must correspond to the pooling bid, i.e., $a_i(1) = \overline{v}(2) - \underline{v}(1)$ for all $i \in I$. Moreover, any principal i selects a package budget constraint that matches her equilibrium bid on two units, $a_i(2) = \beta_2\left(v_i(2)\right)$. We will now demonstrate that the principal cannot implement her *dual-winner equilibrium* strategies from Proposition 3 given conditions (CFPA-D-IC) and (CFPA-D-NL) for all $v_i \in V$.

Let us focus on the following conditions for a principal with vector of valuations $v_i = \left(v_i(1) = \overline{v}(1), v_i(2) = \overline{v}(2)\right)$ and the focal point pooling bid of $\beta_1 = \overline{v}(2) - \underline{v}(1)$:

$$\overline{a}(2) < \overline{a}(1) \tag{A.1.25}$$

$$\overline{a}(1) \leq \overline{v}(1) \tag{A.1.26}$$

$$\beta_2\left(\overline{v}(2)\right) = 2 \cdot \left(\overline{v}(2) - \underline{v}(1)\right) \tag{A.1.27}$$

For principal i the budget for two units is determined by $\overline{a}(2) = \beta\left(\overline{v}(2)\right)$ according to Proposition 3. Condition (A.1.25) corresponds to (1)) from Lemma 3 and resembles the incentive compatibility constraint (CFPA-D-IC) for principal i. Condition (A.1.26) is the no-loss condition (CFPA-D-NL) for one unit and (A.1.27) is the equilibrium bid on two units from Proposition 3 with single-unit bid of $\beta_1 = \overline{v}(2) - \underline{v}(1)$. Let us summarize conditions (A.1.25) and (CFPA-D-NL), and use $\overline{a}(2) = \beta_2\left(\overline{v}(2)\right)$ to combine with condition (A.1.27) to obtain

$$2 \cdot \left(\overline{v}(2) - \underline{v}(1)\right) < \overline{v}(1) \tag{A.1.28}$$

Condition (A.1.28) cannot be satisfied whenever the condition in Proposition 8 holds. In this case, any firm i with package value of $v_i(2) = \overline{v}(2)$ cannot implement budget constraints that satisfy two restrictions: They correspond to its principal's equilibrium strategy and at the same time direct its agent to bid truthfully on both packages. Hence, the *dual-winner equilibrium* cannot be supported as a solution to the *principal-agent* 2×2 *first-price sealed-bid package auction model*. $\square$

A.1.12 *Proof of Proposition 9*

Assume opponent j submits the equilibrium bids from (CFPA-SB-bids) to the auction. Now, suppose agent i chooses a deviating report of $\hat{d} > d$ such that $\beta_1\left(\hat{d}\right) > \beta_1(d)$ for one unit, which by construction implies a deviating package bid of $\beta_2\left(\hat{d}\right) = 2 \cdot \beta_1\left(\hat{d}\right)$. This deviation results in the single-winner outcome for agent i with certainty as $2 \cdot \beta_1\left(\hat{d}\right) > \beta_1\left(\hat{d}\right) + \beta_1(d)$ and $2 \cdot \beta_1\left(\hat{d}\right) > \beta_2(d)$. For this deviation to be unprofitable, incentive-compatibility constraint (CFPA-SB-IC) corresponds to

$$u_i(1) + m_i(1) \geq u_i(2) + m_i(2). \tag{A.1.29}$$

Any profit-maximizing principal chooses $m_i(2) = 0$ and $m_i(1)$ as small as possible. Therefore, A.1.29 can be rewritten as

$$m_i(1) = u_i(2) - u_i(1). \tag{A.1.30}$$

The agent type with highest incentive to deviate has package valuations of $v_i = \left(\overline{v}(2), \underline{v}(1)\right)$. He has to receive a transfer of $m_i(1) = w\left(\overline{v}(2)\right) - w\left(\underline{v}(1)\right)$ in order to report the true d to the contract. As principal i does not know the package valuation $v_i(l)$ for all $l \in L$, each agent has to receive the same transfer and the principal could now offer a menu of payments $m(1) = w\left(\overline{v}(2)\right) - w\left(\underline{v}(1)\right)$ with corresponding reports of $\beta_1(d) = d + m(1)$ and $\beta_2(d) = 2 \cdot \left(d + m(1)\right)$ to the auction.

However, as any value of d could be the result of various $\overline{v}(2) \in \overline{V}(2)$ and $\underline{v}(1) \in \underline{V}(1)$ pairings, any agent i has an incentive to choose the pairing that offers the highest payment and still satisfies $d = \overline{v}(2) - \underline{v}(1)$. Hence, as the principal also does not know the exact valuation support of $V(l)$ for all $l \in L$, the transfer must be constant over all possible values of $d \in D$ and high enough such that agent i with package valuations of $v_i = \left(\overline{\overline{v}(2)}, \underline{\underline{v}(1)}\right)$ will not deviate. Therefore, the optimal payment scheme consists of constant transfers in height of $\mu_1 = w\left(\overline{\overline{v}(2)}\right) - w\left(\underline{\underline{v}(1)}\right)$ and $\mu_2 = 0$.

Finally, suppose agent i chooses a deviating report of $\hat{d} < d$ that results in a bid for one unit of $\beta_1\left(\hat{d}\right) < \beta_1(d)$ and implies a deviating bid on the package of $\beta_2\left(\hat{d}\right) = 2 \cdot \beta_1\left(\hat{d}\right)$. This deviation results in the single-winner award for firm j, as $2 \cdot \beta_1\left(\hat{d}\right) < \beta_1\left(\hat{d}\right) + \beta_1(d)$ and $2 \cdot \beta_1\left(\hat{d}\right) < \beta_2(d)$. According to the incentive compatibility constraint A.1.30, this deviation cannot be optimal for any agent i. This leaves the reports of $\beta_1(d) = d + m(1)$ and $\beta_2(d) = 2 \cdot \left(d + m(1)\right)$ with transfers of $\mu_1 = w\left(\overline{\overline{v}(2)}\right) - w\left(\underline{\underline{v}(1)}\right)$ and $\mu_2 = 0$ as the only solution to the principal's optimization problem. □

A.1.13 *Proof of Proposition 10*

In this proof we first demonstrate that the proposed *dual-winner equilibrium* constitutes an ex-post equilibrium and consecutively show its uniqueness as *dual-winner equilibrium*. Let both bidders follow the described equilibrium strategy and become active on the single unit first. This situation corresponds to case (A)) of the possible outcomes of the 2×2 *ascending package auction* in Appendix A.2. Here, the auction immediately stops at a price of zero for one unit. Any bidder i receives equilibrium profit of $v_i(1)$ with certainty. We assume opponent j follows the proposed equilibrium strategy to show its existence.

Assume principal i tries to win two units by excluding the dual-winner outcome with a sufficiently low bid on one unit. In this case the highest obtainable profit in the singe-winner outcome is $v_i(2) - v_j(2)$. As soon as i becomes active on the package of two units, bidder j also starts bidding, which corresponds to case (E)) of Appendix A.2. Given *dual-winner efficiency*, the profit is always smaller than the proposed equilibrium profit and such a deviation cannot be viable independent of the height of the bids. Next, suppose principal i tries to win one unit by implementing the dual-winner outcome with a sufficiently high single-unit bid. Here, any immediate accompanying positive bid on two units prevents the termination of the auction at zero dual-winner outcome prices (and bidder j then also becomes active on two units as described in case of Appendix A.2). This reduces equilibrium profit and therefore does not constitute a viable deviation. In fact, any strategy that involves becoming active on two units simultaneously to start bidding on one unit is strictly dominated in profit. However, any strategy that dictates to start bidding solely on one unit is a best response against principal j's equilibrium strategy, independent of what it prescribes to bid on two units (and even how high to bid on one unit). As in case of Appendix A.2, the dual-winner outcome results at zero prices with certainty and the bidding behavior on two units simply constitutes a threat. Finally, note that the proposed *dual-winner equilibrium* strategy would not change for any bidder i by knowing the other bidder j's package valuations and, therefore, constitutes an ex-post equilibrium.

Now, we demonstrate that the proposed *dual-winner equilibrium* strategy weakly dominates any other strategy aimed at enforcing the dual-winner outcome.

Assume bidder j does not follow the proposed equilibrium strategy anymore. Note, we have already shown that if principal j bids such as to implement the dual-winner outcome, any strategy by i that enforces the dual-winner outcome at zero prices as in case case of Appendix A.2 is a best response independent of the height of the bids. However, suppose j tries to enforce the single-winner outcome by bidding sufficiently low on one unit and simultaneously becoming active on two units. In this case principal i cannot enforce the dual-winner outcome and her "‘threat"’-bidding on two units becomes active. This corresponds to case of Appendix A.2. Staying active on two units until the opponent drops out or until her respective valuation is reached, $b_i(2) = v_i(2)$, maximizes her chances of obtaining profit of $v_i(2) - v_j(2)$ instead of zero and does not influence the price paid for two units. Any bid of $b_i(2) < v_i(2)$ reduces the respective chances and a bid of $b_i(2) > v_i(2)$ on two units might result in losses. Finally suppose principal j simply becomes active on both packages until his respective values are reached. Here, if i follows the proposed *dual-winner equilibrium* strategy, the setting corresponds to case of Appendix A.2 and the dual-winner outcome will result with certainty, given *dual-winner* *efficiency*, but at positive price. Corresponding profit is $v_i(1) - v_j(2)/2$. Remember, in this case the height of i's bid on two units is irrelevant for determining the allocation. However, if principal i lowers her bid on one unit marginally to $b_i(1) < v_i(1)$ it simply reduces the chances of winning one unit but does not affect the dual-winner price and is therefore dominated. If the corresponding bid is lowered significantly, the setting matches case , the single-winner outcome results and we are not looking at a dual-winner outcome enforcing deviation anymore. Any bid $b_i(1) > v_i(1)$ is dominated as it might result in losses.

Given the described bidding behavior by principal j, it might be profitable for i to deviate from a dual-winner outcome enforcing strategy if $v_i(2) - v_j(2) > v_i(1) - v_j(2)/2$. This is why a single-winner enforcing strategy as described in Proposition 11 is not weakly dominated. Moreover, j's strategy might in fact be part of such a deviation. □

A.1.14 *Proof of Proposition 11*

We have already shown in the proof of Proposition 11 that the proposed *single-winner equilibrium* strategy is not weakly dominated. Next, we demonstrate that this strategy in fact constitutes an ex-post equilibrium.

Suppose both bidders do not begin to bid on one unit, but start bidding on two units and continue to be active until the lowest respective value is reached, i.e., $p(2) = \min\left\{v_i(2), v_j(2)\right\}$. With this strategy any bidder i obtains an ex-interim expected equilibrium profit of $F_2\left(v_i(2)\right) \cdot \left(v_i(2) - v_j(2)\right)$. Bidder i has the highest package value with probability of $F_2\left(v_i(2)\right)$. In this case, she wins two units at a price of the second highest value, $v_j(2)$, and receives a profit of $v_i(2) - v_j(2)$. From now on, assume opponent j follows the proposed equilibrium strategy. Bidder i has no chance to enforce the dual-winner outcome because at a single-unit price that satisfies $2p(1) > \underline{v}(2)$ opponent j has already dropped out and "'vetoed"' the dual-winner outcome. From this point on bidder i competes for two units if remaining active as is demonstrated in case (E)) of the possible outcomes of 2×2 *ascending package auction* in Appendix A.2. Bidder i is indifferent between submitting any single-unit bid of $b_i(1) \in \left\{0, v_i(1)\right\}$ as long as she continues to be active on the large bundle until the price reaches her corresponding value of $b_i(2) = v_i(2)$. She obtains an ex-interim expected profit of $F_2\left(v_i(2)\right) \cdot \left(v_i(2) - v_j(2)\right)$. However, if bidder i decides to drop out on two units before the price reaches her value of $v_i(2)$, she strictly lowers her probability of winning. This strictly decreases her ex-interim expected profit and cannot be optimal. In the *single-winner equilibrium*, by symmetry, both bidders quit bidding on one unit before its price reaches $\underline{v}(2)/2$ and remain active on the package of two units until its price reaches their respective valuations. With *dual-winner efficiency* being common knowledge, the proposed equilibrium strategies are independent of the bidders' actual package valuations and therefore are rationalizable ex-post. □

A.1.15 *Proof of Corollary 1*

In the *dual-winner equilibrium* from Proposition 10, the bidder with the lowest value for one unit obtains the lowest profit of $\underline{v}(1)$ with certainty. According to Proposition 11, the highest possible profit achievable in the *single-winner equilibrium* is $\overline{v}(2) - \underline{v}(2)$. Using the definition of *dual-winner efficiency* we obtain

$$\overline{v}(2) - \underline{v}(2) < 2\underline{v}(1) - \underline{v}(2) < 2\underline{v}(1) - \underline{v}(1). \qquad \text{(A.1.31)}$$

The last inequality stems from the fact that $\underline{v}(2) > \underline{v}(1)$. Therefore, the profit in the *dual-winner equilibrium* is strictly greater than in the *single-winner equilibrium* for all possible bidder's valuations $v_i \in V$. □

A.1.16 *Proof of Proposition 12*

Definition 1 implies that both agents start bidding on the package. Each of them remains active until his budget for two units is reached. Any bidder i, who is outbid on the package with $a_i(2) < a_j(2)$, can then start bidding on one unit. However, as $a_i(1) < a_i(2)$, bidder i cannot become winning unilaterally with his bid on one unit. Both bidders have a higher utility for the package, and therefore agent j, who is the standing winner on the package, would not bid on a single unit. Note that the low bidder i on the package can also not win by starting to bid on the single unit only, because he would be overbid by opponent j as well, who submits his equilibrium bid on the package only. In summary, even knowing the opponent's type, no agent can benefit by deviating from his equilibrium strategy. Thus, straightforward bidding constitutes an ex-post equilibrium in the second stage of the *principal-agent 2 × 2 ascending package auction model*. □

A.1.17 *Proof of Proposition 13*

In Proposition 11, any principal i does not bid on the package of two units. She remains active on the package of one unit at most until the price reaches her corresponding valuation of $b_i(1) = v_i(1)$. To implement the principal's equilibrium strategy for her agent, the principal provides a zero budget constraint on the large package. This eliminates the agent's possibility to win the package and the principal can simply provide her agent with a budget constraint in the amount of her valuation for one unit. From the beginning there is no over-demand and the auction terminates immediately. □

A.1.18 *Proof of Proposition 14*

Suppose opponent j follows the proposed equilibrium strategy. At the beginning of the auction both agents are active on both units and the uniform unit price starts to increase. Let us now distinguish two cases:

a) Whenever agent i faces budget constraints of the form $a_i(1) \geq a_i(2)/2$ straightforward bidding is clearly optimal. If the opposing agent j drops out before the unit price rises up two $a_i(2)/2$, i.e. $a_i(2)/2 > a_j(1)$, the auction terminates and agent $i$wins two units. If the unit price passes $a_i(2)/2$, because $a_i(2)/2 \leq a_j(1)$, agent i can no longer demand two units as this would exceed his double-unit budget constraint of $a_i(2)$. He reduces demand to one unit and remains active until the price reaches his corresponding budget of $a_i(1)$ such that he can still win the small award if $a_i(1) \geq min\left\{a_j(2)/2, a_j(1)\right\}$.

   In case agent i does not engage in straightforward bidding but begins demanding one unit only until his respective budget of $a_i(1)$ is reached, the budget constraint for one unit prohibits him from demanding two units later on. Such a strategy prevents him from winning two units and as agent i strictly prefers two to one unit such behavior cannot be optimal. The same is true for not demanding two units from the start as he might miss out winning the package.

b) If agent i is provided with budgets of the form $a_i(1) < a_i(2)/2$, straightforward bidding is optimal, too. Suppose the uniform unit price reaches $a_i(1)$. If opponent j also reduces demand at exactly this price, $a_i(1) = a_j(1)$, when facing budgets of $a_j(1) < a_j(2)/2$, both agents win one unit each. If agent j has already reduced demand to one unit, $a_i(1) > a_j(2)/2$, with budgets of $a_j(1) \geq a_j(2)/2$, agent i wins one unit. Whereas if opponent j has already reduced demand before $a_i(1) > a_j(1)$, agent i wins two units. Otherwise agent i is overbid and wins nothing. Using the same argument as in a) it follows that any bidding behavior other than straightforward bidding cannot improve agent i.

□

A.1.19 *Proof of Proposition 15*

It will be convenient to focus on the principal's and agent's true lowest pooling prices of $\beta_1^p(d) = d - \eta\left(\overline{v}(2)\right) + \eta\left(\underline{v}(1)\right)$ and $\beta_1^a(d) = d$ with $d = \overline{v}(2) - \underline{v}(1) \in D$, respectively, as they maximize each party's profit. The principal can implement higher pooling prices by the same logic as in the following proof, but this is not optimal as the profit-maximizing lowest pooling price serves as a natural focal point.

Define principal i's lowest possible pooling price as $\beta_1^p\underline{(d)} = \underline{d} - \eta\left(\underline{\overline{v}(2)}\right) + \eta\left(\overline{\underline{v}(1)}\right)$ and her highest possible pooling price as $\beta_1^p\overline{(d)} = \overline{d} - \eta\left(\overline{\overline{v}(2)}\right) + \eta\left(\underline{\underline{v}(1)}\right)$ in the asymmetric information environment. The agent's respective pooling prices are $\beta_1^a\underline{(1)} = \underline{d}$ and $\beta_1^a\overline{(1)} = \overline{d}$. Note that the principal's true lowest pooling price, $\beta_1^p(d)$, is strictly smaller than the agent's true lowest pooling price, $\beta_1^a(d)$, given $\eta\left(\overline{v}(2)\right) > \eta\left(\underline{v}(1)\right)$, and it is also true that

$\beta_1^p(\underline{d}) < \beta_1^a(\underline{d})$ and $\beta_1^p(\overline{d}) < \beta_1^a(\overline{d})$. Therefore, it is generally impossible for the uninformed principal to implement a report in height of her desired true lowest pooling price of $\beta_1(d) = \beta_1^p(d)$ by the use of a menu of contingent budget constraints only and the contract results in an inherent welfare loss for the principal as the agent always over-reports $\beta_1(d) = \beta_1^a(d)$.

The only exception is the implementation of the upper bound on the menu of budget constraints $\beta_1(d) \leq \beta_1^p(\overline{d})$ for all $d \in D$. The principal knows the supports D and Z and can define the upper bound $\beta_1^p(\overline{d})$. It cannot be optimal for any principal to allow bids on one unit that exceed this highest possible pooling price. Any agent i with true $d > \beta_1^p(\overline{d})$ automatically chooses $\beta_1(d) = \beta_1^p(\overline{d})$ as it is the closest possible budget to his optimal budget constraint for one unit of $\beta_1^a(d) = d$ and as choosing a lower report would result in the single-winner award for the opponent. The principal cannot implement a lower upper bound than $\beta_1^p(\overline{d})$ as this is her true lowest pooling price in case of $d = \overline{d}$. Given it is true that $d = \overline{d}$, suppose firm j reports $\beta_1(d) < \beta_1^p(\overline{d})$ to the auction. Now it is profitable for principal i to deviate to a report that exceeds $\beta_1(d)$ and is still smaller than $\beta_1^p(\overline{d})$ which results in the single-winner award for him as illustrated in the proof of Proposition 3.

Note that the implemented reports from Proposition 15 satisfy the agent's and the principal's highest lower bound for the bid on the package as specified in Proposition 3 and Corollary 2, respectively, with $G^p\left(\bar{v}_i(2), d\right) = 2 \cdot d$ and $G^a\left(\bar{v}_i(2), d\right) = 2 \cdot d$. As the principal generally cannot enforce the submission of her true lowest pooling price, she must adhere to the agent's report of the latter's true lowest pooling price. Therefore, a necessary condition for a budget constraint contract to exist is that the agent's true lowest pooling price $\beta_1^a(d) = \bar{v}(2) - \underline{v}(1)$ is smaller than the principal's true largest pooling price of $\underline{v}(1) - \eta\left(\underline{v}(1)\right)$ which is guaranteed as long as the following necessary condition is satisfied:

$$2 \cdot \underline{v}(1) \geq \bar{v}(2) + \eta\left(\underline{v}(1)\right). \tag{A.1.32}$$

Note that if the above condition is satisfied, *dual-winner efficiency* for the agent's utility function is given such that they can coordinate on their *dual-winner equilibrium* as specified in Proposition 3. As the vector of reports b_i satisfies both parties' *dual-winner equilibrium* it simultaneously fulfills the no-loss condition (B-CFPA-SB-NL) in the principal's optimization problem in (B-CFPA-SB).

Next, we have to verify that the above necessary condition is also sufficient for the vector of reports in Proposition 15 to be optimal. To do so we demonstrate that the principal cannot improve by trying to implement her true lowest pooling price of $\beta_1^p(d) = d - \eta\left(\bar{v}(2)\right) + \eta\left(\underline{v}(1)\right)$ together with transfer payments to the agent. In this case a principal i would have to design a contract $\left(\beta(d), \mu(d)\right)$ to direct her agent to submit the equilibrium strategy of $\beta_1(d) = \beta_1^p(d)$ and $\beta_2(d) = 2 \cdot \beta_1^p(d)$.

Suppose the opposing principal j manages to implement these equilibrium reports to the auction. Then agent i with high enough valuation for the package and low enough value for the single unit has an incentive to deviate to report $\hat{d} = d + \varepsilon$ with $\varepsilon \to 0$ to the contract such that $\beta_1\left(\hat{d}\right) > \beta_1^p(d)$ and $\beta_2\left(\hat{d}\right) > \beta_1\left(\hat{d}\right) + \beta_1^p(d)$. In this case she obtains the single-winner award at a price of $2 \cdot \beta_1^p(d) + \varepsilon$ with $\varepsilon \to 0$. For this deviation to be unprofitable, the following incentive-compatibility constraint then needs to be satisfied:

$$v_i(1) - \beta_1^p(d) + m_i(1) \geq v_i(2) - 2 \cdot \beta_1^p(d) + m_i(2) \tag{A.1.33}$$

Profit maximization by the principal requires $m_i(2) = 0$ and, using the definitions of $\beta_1^p(d)$ and $\beta_1^a(d)$, A.1.33 then corresponds to

$$m_i(1) \geq v_i(2) - v_i(1) - \overline{v}(2) + \eta\left(\overline{v}(2)\right) + \underline{v}(1) - \eta\left(\underline{v}(1)\right) \tag{A.1.34}$$

The agent with highest incentive to deviate has valuations of $v_i = \left(\overline{v}(2), \underline{v}(1)\right)$ and has to receive a transfer of $m_i(1) = \eta\left(\overline{v}(2)\right) - \eta\left(\underline{v}(1)\right)$ to truthfully report d to the contract such that $\beta_1(d) = \beta_1^p(d)$ and $\beta_2(d) = 2 \cdot \beta_1^p(d)$ is reported to the auction. As principal i is not informed about package valuations $v_i(l)$ for all $l \in L$, she has to implement the constant transfer payments of $m(1) = \eta\left(\overline{v}(2)\right) - \eta\left(\underline{v}(1)\right)$ and $m(2) = 0$ with corresponding budget menu of $\beta_1(d) = \beta_1^p(d)$ and $\beta_2(d) = 2 \cdot \beta_1^p(d)$.

However, as $\beta_1(d)$ could result as a combination of different $\overline{v}(2) \in \overline{V}(2)$ and $\underline{v}(1) \in \underline{V}(1)$ pairings, agent i will choose to report the pairing that offers the highest transfer payment to the contract. Agent i with package valuations of $v_i = \left(\overline{\overline{v}(2)}, \underline{\underline{v}(1)}\right)$ has the highest incentive to deviate and the optimal payment scheme must involve constant transfers in height of $m(1) = \eta\left(\overline{\overline{v}(2)}\right) - \eta\left(\underline{\underline{v}(1)}\right)$ because the principal does not know the true valuation support, $V(l)$, for all $l \in L$.

The principal's profit in the *dual-winner equilibrium* under the pure budget constraints contract is

$$\pi_i^{BC}(1) = \begin{cases} v_i(1) - \eta\left(v_i(1)\right) - d & \text{if } d \leq \overline{\beta_1^p}(d) \\ v_i(1) - \eta\left(v_i(1)\right) - \overline{\beta_1^p}(d) & \text{if } d > \overline{\beta_1^p}(d) \end{cases} \tag{A.1.35}$$

and under the contract with transfers it corresponds to

$$\pi_i^T(1) = v_i(1) - \eta\left(v_i(1)\right) - \beta^p(1) - \eta\left(\overline{\overline{v}(2)}\right) + \eta\left(\underline{\underline{v}(1)}\right) \tag{A.1.36}$$

with $\beta^p(1) = \overline{v}(2) - \eta\left(\overline{v}(2)\right) - \underline{v}(1) + \eta\left(\underline{v}(1)\right)$. Principal i prefers the budget constraint contract to the contract with transfers if $\pi_i^{BC}(1) \geq \pi_i^T(1)$ which is always satisfied for $d \leq \overline{\beta_1^p}(d)$ as $\eta\left(\overline{v}(2)\right) - \eta\left(\underline{v}(1)\right) - \eta\left(\overline{\overline{v}(2)}\right) + \eta\left(\underline{\underline{v}(1)}\right) \leq 0$. For $d > \overline{\beta_1^p}(d)$, $\pi_i^{BC}(1)$ becomes even larger relative to $\pi_i^T(1)$ by definition.

As the optimal transfer payment $m(1) = \eta\left(\overline{\overline{v}(2)}\right) - \eta\left(\underline{\underline{v}(1)}\right)$ is a function of the agent's utility only and is independent of the implemented pooling price, it is always optimal for the principal to condition on her lowest possible pooling price, $\beta_1^p(d) = d - \eta\left(\overline{v}(2)\right) + \eta\left(\underline{v}(1)\right)$, in the contract with transfer payments as higher prices cannot be achieved with lower transfer costs. □

A.2 ASCENDING AUCTION FORMATS FROM CHAPTER 3

This appendix includes detailed descriptions of the possible outcomes of the 2×2 *ascending package auction* as well as the 2×2 *ascending uniform-price auction*. In what follows, we begin with the 2×2 *ascending package auction*:

A) Both bidders are only active on one unit with current price of $p_c(1) = p_c(2)/2$. Then the dual-winner outcome is winning and there is no demand for the single-winner outcome. Thus, the auction stops with both bidders winning one unit each at a price of $p(1) = p_c(2)/2$. If both bidders only bid on one unit directly at the beginning of the auction, the bidding process stops immediately at price of $p(1) = 0$.

B) Bidder i bids on the package while bidder j only bids on a single unit. In this case one of the two allocations must be non-winning and its price rise. If $p_c(1) > p_c(2)$, the price for the package rises and in case of $p_c(1) < p_c(2)$, the price for the single unit increases. If bidder i reduces demand to one unit, we are in A). In case bidder i releases the button and does not switch to one unit, then bidder j wins the package at price $p(2) = p_c(1)$, because the auctioneer always allocates all objects and bidder j does not suffer as $v_j(1) < v_j(2)$. However, if bidder j releases the button on one unit and does not switch to the package, then the single-winner outcome is winning and not over-demanded. Thus, the package is assigned to bidder i at a price of $p(2) = p_c(1)$. If bidder j switches to the package, we are in C).

C) Both bidders are only active on the package and its price rises at $p_c(2)$, given excess demand, until one bidder stops bidding on the package. There is no current demand for the dual-winner outcome. The bidder, who dropped out on two units might still be allowed to bid on one unit, such that we are in B). If a bidder i drops out of the auction, the auction stops, as there is no overdemand for the single-winner outcome, and j wins the package at a price of $p(2) = p_c(2)$. If both bidders reduce demand to one unit simultaneously, we are in A). If both bidders drop out from the auction at the same time, the auctioneer randomly assigns the package to one of the bidders at the current price.

D) One bidder i bids on the package and the single unit, the other bidder j only on a single unit. In this case one of the two allocations must be non-winning and its price rise. If $2p_c(1) > p_c(2)$, the price for the package rises and in case $2p_c(1) < p_c(2)$, the single-unit price increases. In case bidder i stops bidding on the package, we are in A) and if bidder i stops bidding on the one unit, we are in B). In case bidder i releases both buttons, bidder j wins the package at price $p(2) = p_c(2)$. If bidder

j stops bidding on one unit, there is no demand for the dual-winner outcome and bidder i wins the package at price $p(2) = 2p_c(1)$.

E) One bidder i bids on the package and the single unit, the other bidder j only on the package. Here, there is excess demand on the package and its price rises. Moreover, if the dual-winner outcome is non-winning the single-unit price increases, too. If bidder i stops bidding on the single unit, we are in C). In case bidder i stops bidding on the package, we are in B). If bidder j stops bidding on the package, there is no overdemand and i wins both units at price $p(2) = p_c(2)$.

F) Both bidders are active on the package and the single unit such that prices rise on all clocks. If one of them stops bidding on one unit, we are in E). If one of them stops bidding on the package, we are in D). If both release the button for the package, the setting corresponds to A). If both simultaneously release the button for the single unit, we are in C). In case one bidder stops bidding on the package and her opponent stops bidding on one unit, we are in B). If both bidders simultaneously release both buttons, the auctioneer assigns each bidder i one object at price of $p(1) = p_c(2)/2$.

In contrast, the following outcomes are possible in the 2×2 *ascending uniform-price auction*:

A) Both bidders only bid on one unit with current price of $p_c(1)$. As there is no excess demand the auction stops and each bidder wins one unit at unit price of $p(1)$. If both bidders only demand one unit at the beginning of the auction, the process terminates immediately at unit price of $p(1) = 0$.

B) Bidder i demands two units, while bidder j only bids for one unit. Thus, there is excess demand and the current unit price $p_c(1)$ increases continuously. If bidder i stops bidding on the second unit, we are in A). In case bidder j stops demanding one unit, there is no excess demand and bidder i wins both units at twice the current unit price of $p(1)$. If bidder i reduces demand to zero, then the auction stops and the auctioneer awards both units to bidder j at the current unit price of $p(1)$.

C) Both bidders bid on two units. If one bidder drops out of the auction, there is no overdemand and the other bidder gets the package of both units at twice the current unit price of $p(1)$. In case one bidder reduces demand to one unit, we are in B). If both bidders simultaneously stop bidding on the second unit, we are in A). Finally, in case both bidders reduce demand to zero at the same time, the auctioneer assigns each bidder i one object at unit price of $p(1)$.

B

APPENDIX B

B.1 PROOFS OF CHAPTER 4

B.1.1 *Proof of Corollary 4*

Using the same logic as in proposition 1 of Anton and Yao (1992), it is possible to show that split prices $p_e^{\sigma 1}\left(\Theta_i, h^0\right)$ and $p_e^{\sigma 2l}\left(\Theta_l, h^1\right)$ in a σ equilibrium of the Dutch split-award auction have to be constant. Otherwise, the bidder with the lower split price always has an incentive to deviate from the equilibrium strategy. We will show that the only possible split price in a σ equilibrium is $p_e^{\sigma} = \underline{\Theta}\,(1 - C)$. First, we assume that a σ equilibrium with a constant split price $p_e^{\sigma'}\left(\Theta_i, h^0\right) > p_e^{\sigma} = \underline{\Theta}\,(1 - C)$ exists. In this case, there is always a $\varepsilon > 0$ and a profitable sole source deviation for a bidder $\hat{\Theta} = \underline{\Theta}$ by accepting the counteroffer for 100% of the business at a price of $\hat{p}^{s1}\left(\hat{\Theta}\right) = 2p_e^{\sigma'}$, as $E\left[\hat{\Pi}^s\left(\underline{\Theta}\right)\right] > E\left[\Pi_e^{\sigma'}\left(\underline{\Theta}\right)\right]$ is true for $p_e^{\sigma'} > (1 - C)\,\underline{\Theta}$.

Second, we assume a σ equilibrium such that $p_e^{\sigma'}\left(\Theta_i, h^0\right) < p_e^{\sigma} = \underline{\Theta}\,(1 - C)$ exists. Supplier A with $\Theta_A \neq \overline{\Theta}$ sticks to such an equilibrium strategy and accepts the 50% share in the first phase; then, he makes a profit of $p_e^{\sigma'} - \Theta_A C > 0$ in equilibrium. However, in this case there is a split deviation for the loser of phase 1, which generates a higher payoff than in equilibrium. This player knows that the additional costs for providing 100% of the business are $\Theta_A\,(1 - C)$ for the winner of phase 1. Hence, the deviating bidder can accept the remaining share at a price of $\underline{\Theta}\,(1 - C)$ knowing that the other bidder cannot accept any previous offer without reducing his already achieved payoff ($\Theta_A = \underline{\Theta}$ is a null set) . A threat of the winner to accept the 50% share for a lower price than $\underline{\Theta}\,(1 - C)$ is not credible because both bidders are assumed to be payoff-maximizing. Following such a strategy, a deviating bidder wins 50% of the business with probability 1 and achieves a higher payoff than by playing a σ equilibrium with a split price $p_e^{\sigma'}\left(\Theta_i, h^0\right) < (1 - C)\,\underline{\Theta}$. □

B.1.2 *Proof of Proposition 18*

The Dutch split-award auction is modeled as a two-stage game, where the action of phase 1 is observed by both players before phase 2 starts. Because the σ equilibrium includes pooling prices, which are independent on the respective cost type, the loser of phase 1 does not get any information about the cost type of his opponent. Hence, his beliefs μ are not updated after phase 1 and remain the same as ex-ante.

Furthermore, it has to be proven that the strategy profile $\left(S_e^{\text{PBE2}}, \mu\right)$ is sequentially rational given the system of beliefs. Therefore, we show to prove that there is neither a sole source nor a split deviation for any bidder $i \in \{A, B\}$ that yields a higher expected payoff than the equilibrium payoff of $\Pi_e^{\sigma}(\Theta_i) = \underline{\Theta}(1-C) - \Theta_i C$.

SOLE SOURCE DEVIATIONS: In each round r, the counteroffers c_r^{σ} and c_r^{s} are presented according to the pricing rule, $2c_r^{\sigma} = c_r^{s}$, which satisfies the buyer's indifference condition. Thus, only sole source deviations with prices $\hat{p}^{s1}\left(\hat{\Theta}\right) > 2\hat{\Theta}(1-C)$ have to be considered. For all other sole source deviations, a bidder makes a higher profit by accepting the split in the same round. However, sole source deviations with prices greater than $2\hat{\Theta}(1-C)$ can never be realized, as the opponent who sticks to the equilibrium strategy accepts the split at a price of $\underline{\Theta}(1-C)$ in equilibrium, and counteroffers for 100% of the business greater than $2\underline{\Theta}(1-C)$ are not presented. This excludes sole source deviations in phase 1. As the remaining 50% share is offered in phase 2, only the winner of phase 1 can follow a sole source deviation in this stage. Again, if the other bidder sticks to the equilibrium strategy, the sum of the split price in phase 1 and a counteroffer for the remaining share cannot be greater than $2\underline{\Theta}(1-C)$, which excludes profitable sole source deviations in phase 2 as well.

SPLIT DEVIATIONS: The expected payoff of all possible split deviations with split prices lower than $\underline{\Theta}(1-C)$ is obviously less than the equilibrium payoff, as the probability of winning 50% does not increase with a lower split price.

In phase 1, a split deviation with a split price higher than $\underline{\Theta}(1-C)$ is not possible, when the other bidder follows the equilibrium strategy. However, this does not apply in phase 2 because it would not be a credible threat for bidder A, the winner of phase 1 with a cost of type $\Theta_A \in \left(\underline{\Theta}, \overline{\Theta}\right]$, to accept the remaining 50% share in phase 2 for a price $\underline{\Theta}(1-C)$. Bidder A can only accept a counteroffer for the remaining 50% share when his additional costs $\Theta_A(1-C)$ in the case of winning the remaining 50% share are covered and he makes at least the same payoff as in phase 1. Although such split deviations with split prices $\hat{p}^{\sigma 2l}\left(\hat{\Theta}, h^1\right) > \underline{\Theta}(1-C)$ are possible, we show in the following that they yield lower expected payoff than the equilibrium strategy.

Assume a bidder tries to deviate with a split price $\underline{\Theta}(1-C) < \hat{p}^{\sigma 2l}\left(\hat{\Theta}, h^1\right) \leq \overline{\Theta}(1-C)$ in phase 2. The upper boundary arises because the winner of phase 1 accepts the counteroffer for the remaining 50% share latest at a price of $(1-C)\overline{\Theta}$. Such a strategy can be expressed by $\hat{p}^{\sigma 2l}\left(x, h^1\right) = x(1-C)$ with the variable $x \in \left(\underline{\Theta}, \overline{\Theta}\right]$. A deviating bidder B risks losing the whole business because he knows that supplier A will fulfill his threat at $p_e^{\sigma 2w}\left(\Theta_A, h^1\right) = \Theta_A(1-C)$. Therefore, bidder B faces a trade-off because he does not know the cost type of the other bidder; recall that the split price in phase 1 is constant and independent of the cost type Θ_A, which is why the loser does not get any information about the cost type of the winner in phase 1.

Split deviations with $\hat{p}^{\sigma 2l}\left(x, h^1\right)$ can only be excluded, when for all possible cost types $\Theta_i \in \left[\underline{\Theta}, \overline{\Theta}\right]$ and for all $x \in \left(\underline{\Theta}, \overline{\Theta}\right]$, the expected payoff of such a deviation is less than the σ equilibrium payoff, i.e.,

$$\begin{aligned}\Delta^{\Pi}\left(x, \Theta_i\right) = & \left(\hat{p}^{\sigma 2l}\left(x, h^1\right) - C\Theta_i\right) P\left(\hat{p}^{\sigma 2l}\left(x, h^1\right) \leq \Theta_{1:n-1}\left(1-C\right)\right) - \\ & - \left(\underline{\Theta}\left(1-C\right) - C\Theta_i\right) < 0 \\ & \left(x\left(1-C\right) - C\Theta_i\right)\left(1 - F\left(x\right)\right) - \left(\underline{\Theta}\left(1-C\right) - C\Theta_i\right) < 0\end{aligned}$$

for all $\Theta_i \in \left[\underline{\Theta}, \overline{\Theta}\right]$ and for all $x \in \left(\underline{\Theta}, \overline{\Theta}\right]$. As $\Delta^{\Pi}\left(x, \Theta_i\right)$ is strictly increasing in Θ_i, it suffices to show that $\Delta^{\Pi}\left(x, \overline{\Theta}\right) < 0$ applies for all x. This is fulfilled by the necessary condition (4.5.2). The strategy in phase 2 of the winner of the 50% share in phase 1 is credible, as the payoff of this threat equals the equilibrium payoff when it must be carried out. □

B.1.3 *Proof of Proposition 19*

First, we will show that there is no WTA equilibrium in a Dutch split-award auction. Subsequently, hybrid equilibria are excluded. If there is a WTA equilibrium strategy, such a strategy would have to be payoff-dominant over all possible split and sole source deviations.

SOLE SOURCE DEVIATIONS: Similar to the WTA equilibrium from proposition 17, a bidder must accept a counteroffer for the sole source award, c_r^s, in round r for a price of $p_e^{s1}\left(\Theta_i, h^0\right) = \Theta_i + \frac{\int_{\Theta_i}^{\overline{\Theta}}\left(1-F(t)\right)dt}{\left(1-F(\Theta_i)\right)}$ in order to ensure that there is no sole source deviation that yields a higher payoff than the equilibrium strategy in a (potential) WTA equilibrium (S_e^{PBE1}, μ). Trying to win the sole source award sequentially cannot be an equilibrium strategy. Assume such a equilibrium (S_e^{PBE2}, μ) exists. Then, the price in phase 2 must be higher than $\Theta_w\left(1-C\right)$ in order to make at least as much payoff than already achieved. This cannot be an equilibrium strategy with *dual source efficiency*, as there is a $\varepsilon > 0$ such that even an opponent with cost type $\overline{\Theta}$ makes a strictly positive payoff by accepting a counteroffer at $c_r^{\sigma} = \Theta_w\left(1-C\right) - \varepsilon$.

SPLIT DEVIATIONS: Next, we show that if all bidders follow such a strategy, $p_e^{s1}\left(\Theta_i, h^0\right)$, there is at least one bidder who has an incentive to deviate. Consider a bidder with the highest cost type $\overline{\Theta}$ who makes a payoff of zero in equilibrium by accepting the counteroffer $c_r^s = p_e^{s1}\left(\overline{\Theta}\right)$ in round r. With *dual source efficiency*, there is always a $\varepsilon > 0$ and a round q preceding round r with a counteroffer $c_q^s = 2\overline{\Theta}C + \varepsilon < c_r^s$, in which this bidder makes a higher payoff than in the potential WTA equilibrium. Hence, there is no WTA equilibrium in a Dutch split-award auction with *dual source efficiency*.

In the *dual source efficiency* split-award auction model, each potential hybrid equilibrium with a strategy set S_e^{PBE2} or S_e^{PBE3} (in pure strategies) can be described by disjunct intervals $I_1^s, I_2^s, ..., I_t^s$ and $I_1^\sigma, I_2^\sigma, ..., I_u^\sigma$. If both bidders have the same cost type $\Theta_i \in I_1^s, I_2^s, ..., I_t^s$, the sole source award emerges. The same is true for the split award when both suppliers have costs $\Theta_i \in I_1^\sigma, I_2^\sigma, ..., I_u^\sigma$. Furthermore, $t, u \in \mathbb{N}$ and $I_1^s \dot\cup I_2^s \dot\cup ... \dot\cup I_t^s \dot\cup I_1^\sigma \dot\cup I_2^\sigma \dot\cup ... \dot\cup I_u^\sigma = \left[\underline{\Theta}, \overline{\Theta}\right]$ applies. We divide the different hybrid equilibria in two types: hybrid equilibria with $I_t^s = \left(\tau, \overline{\Theta}\right]$ and hybrid equilibria with $I_u^\sigma = \left[\tau, \overline{\Theta}\right]$. The strategic parameter τ indicates the cost type for which bidders change their strategy in a hybrid equilibrium.

Using the same reasoning as above for the WTA equilibrium, the bidder with the highest cost type has an incentive for a split deviation in the hybrid equilibrium with $I_t^s = \left(\tau, \overline{\Theta}\right]$. Hence, such hybrid equilibria do not exist.

Next, assume a hybrid equilibrium with an interval $I_u^\sigma = \left[\tau, \overline{\Theta}\right]$. As in the analysis of Anton et al. (2010), a bidder with cost type τ must be indifferent between winning the split for $p_e^{\sigma 1}\left(\tau, h^0\right)$ or the sole source award for $p_e^{s1}\left(\tau, h^0\right) = 2p_e^{\sigma 1}\left(\tau, h^0\right)$. Otherwise, this bidder would not change his strategy, i.e. $p_e^{\sigma 1}\left(\tau, h^0\right) = \tau\left(1 - C\right)$ follows from $E\left[\Pi_e^{s1}\left(\tau, h^0\right)\right] = E\left[\Pi_e^{\sigma 1}\left(\tau, h^0\right)\right]$. This directly implies that $p_e^{\sigma 1}\left(\tau, h^0\right) = \tau\left(1 - C\right)$ is the equilibrium split price for bidder τ and all other bidders with $\Theta_i \in I_1^\sigma \dot\cup I_2^\sigma \dot\cup ... \dot\cup I_u^\sigma$ as well, as split prices must be constant based on corollary 4. However, all cost types for which the strategy changes in equilibrium must be indifferent between both awards, such as τ for $I_u^\sigma = \left[\tau, \overline{\Theta}\right]$. As this is never true for multiple disjunct intervals, a hybrid equilibrium with $u > 1$ can be excluded. Hence, it suffices to show that no hybrid equilibrium with $I_1^s = \left[\underline{\Theta}, \tau\right)$ and $I_1^\sigma = \left[\tau, \overline{\Theta}\right]$ exists.

Assume there is such an equilibrium with an arbitrary parameter $\tau \in \left(\underline{\Theta}, \overline{\Theta}\right)$. The sole source price $p_e^{s1}\left(\Theta_i, h^0\right)$ for $\Theta_i \in \left[\underline{\Theta}, \tau\right)$ must assure that there is no sole source deviation in equilibrium, i.e., that $E\left[\Pi_e^{s1}\left(x, h^0\right)\right] = \left(p_e^{s1}\left(x, h^0\right) - \Theta_i\right)\left(1 - F\left(x\right)\right)$ is maximized for $x = \Theta_i$. Similar to Anton et al. (2010), this applies with the following sole source prices

$$p_e^{s1}\left(\Theta_i, h^0\right) = \Theta_i + \tau\left(1 - 2C\right)\frac{\left(1 - F\left(\tau\right)\right)}{\left(1 - F\left(\Theta_i\right)\right)} + \frac{\int_{\Theta_i}^{\tau} 1 - F\left(t\right) dt}{\left(1 - F\left(\Theta_i\right)\right)}$$

for types $\Theta_i \in \left[\underline{\Theta}, \tau\right)$.

Furthermore, for a type τ there must be no incentive to deviate from equilibrium by accepting the offer for 50% for a price $p_e^{\sigma 1}\left(y, h^0\right) = 0.5p_e^{s1}\left(y, h^0\right) <$

$0.5p_e^{s1}\left(\tau, h^0\right)$ with $y \in \left[\underline{\Theta}, \tau\right)$. Such a deviation would yield the expected payoff $E\left[\hat{\Pi}^\sigma\left(y, h^0\right)\right] = \left(0.5p_e^{s1}\left(y, h^0\right) - \tau C\right)\left(1 - F\left(y\right)\right)$ dependent on variable y. Solving the first-order condition yields:

$$\frac{d}{dy}E\left[\hat{\Pi}^\sigma\left(y, h^0\right)\right] = 0$$
$$0.5\left(-f\left(y\right)\left(y - 2C\tau\right) + \left(1 - F\left(y\right)\right) - \left(1 - F\left(y\right)\right)\right) = 0$$
$$-f\left(y\right)\left(y - 2C\tau\right) = 0.$$

The solution of the first-order condition is $y = 2C\tau < \tau$, and it can be shown that it is the unique maximum of the expected payoff function, as f is positive and the derivative of $E\left[\hat{\Pi}^\sigma\left(y, h^0\right)\right]$ is positive (negative) for all values for y that are lower (higher) than $2C\tau$. Hence, there is always a profitable split deviation with $y < \tau$ for a bidder with cost type τ in a hybrid equilibrium, as $E\left[\hat{\Pi}^\sigma\left(y, h^0\right)\right] > E\left[\hat{\Pi}^\sigma\left(\tau, h^0\right)\right] = E\left[\Pi_e^\sigma\left(\tau, h^0\right)\right]$.

Hence, we proved that no such hybrid equilibrium can emerge and that, when a σ equilibrium exists, it is unique. □

B.1.4 *Proof of Corollary 5*

The results for (i) and (ii) of proposition 1 in Anton and Yao (1992) can be easily transferred to the Dutch-FPSB split-award auction case. In the Dutch split-award auction, the bidding strategy of a bidder w, the winner of phase 1, has to assure that it yields at least the same payoff as in phase 1, as the payoff is realized by carrying out the threat. However, when phase 2 is a FPSB mechanism, it suffices that the offer for the remaining share impedes split deviations. As long as bidder w bids at least p_e^σ for the remaining share, the probability that the threat must be carried out and the payoff of bidder w decreases is changed to zero, as the opponent has no incentive to deviate from equilibrium. This makes it easier for the winner to exclude split deviations in phase 2 and various split prices can emerge in equilibrium. □

B.1.5 *Proof of Proposition 20*

As in the proof of proposition 18, the pooling price in phase 1 does not allow for any updating of the beliefs about the opponent's cost type. In what follows, we show that deviations from the pooling equilibrium are unprofitable for all different cost types in every stage of the game. Hence, the sequential rationality assumption for perfect Bayesian equilibria applies.

Assume a σ equilibrium in which both bidders win 50% of the business for a constant split price $p_e^{\sigma 1}\left(\Theta_i, h^0\right) = p_e^{\sigma 2l}\left(\Theta_l, h^1\right) = p_e^\sigma \in \left[\overline{\Theta}C, \underline{\Theta}\left(1 - C\right)\right]$ exists.

SOLE SOURCE DEVIATIONS: With the same logic as in the proof of proposition 18 no bidder has an incentive to deviate for the sole source award in equilibrium.

SPLIT DEVIATIONS: Split deviations in phase 1 can be easily excluded, as the expected payoff is either strictly lower than the equilibrium payoff (for deviations with a lower split price than p_e^σ) or zero (for deviations with higher split prices). Bidder w, the winner of the first 50% share, submits a quote of $p_e^{\sigma 2w}\left(\Theta_w, h^1\right) = \max\{p_e^\sigma, \Theta_w - p_e^\sigma\}$ in phase 2 in order to implement the σ equilibrium strategy. This threat is credible as his equilibrium payoff of phase 1 does not change in expectation. The probability that his opponent deviates is zero, as the expected payoff of such a split deviation is lower than the equilibrium payoff. This is assured by condition (4.5.3). We skip this line of reasoning because it is based on the proof of proposition 1 (Case 1 deviation) in Anton and Yao (1992) for the FPSB split-award auction. □

B.1.6 *Proof of Proposition 22*

Similar to a setting with two bidders, the existence of such an inefficient equilibrium can be shown in the FPSB auction with $n > 2$ bidders:

SOLE SOURCE DEVIATIONS: In order to avoid any sole source deviation, the sole source price $p_e^s(\Theta_i)$ has to maximize the expected payoff of winning 100% of the business

$$E\left[\Pi_e^{WTA}(\Theta_i)\right] = \left(p_e^s(\Theta_i) - \Theta_i\right) P\left(p_e^s(\Theta_i) \le p_e^s(\Theta_{1:n-1})\right)$$
$$\text{for every } \Theta_i \in \left[\underline{\Theta}, \overline{\Theta}\right].$$

SPLIT DEVIATIONS: Additionally, split deviations can be excluded by sufficiently high equilibrium prices for the split, which satisfy

$$p_e^\sigma(\Theta_i) > \overline{\Theta} - \underline{\Theta}C.$$

These prices assure that the auctioneer never chooses the split, as the sum of any possible split deviation $\hat{p}^\sigma\left(\hat{\Theta}\right) \ge \underline{\Theta}C$ and the lowest split price of another supplier is strictly greater than $\overline{\Theta}$, the highest possible price the auctioneer has to pay for the sole source award in equilibrium. Hence, the payoff of such a deviating strategy is always zero. □

B.1.7 *Proof of Proposition 23*

We will show that in equilibrium neither sole source nor split nor hybrid deviations are attractive for bidders.

SOLE SOURCE DEVIATIONS: With a sole source deviation $\left(\hat{p}^s\left(\hat{\Theta}\right), \hat{p}^\sigma\left(\hat{\Theta}\right)\right)$, a bidder aims to win the sole source award and excludes the split (for himself). By differentiating the following cases, it is shown that there is no sole source deviation, which yields a higher expected payoff than a σ equilibrium.

i) $2\overline{\Theta}C \leq \underline{\Theta}$

All possible deviating sole source prices $\hat{p}^s\left(\hat{\Theta}\right)$ have to be greater than or equal to $\underline{\Theta}$, which is a necessary condition such that the assumption of individual rationality is fulfilled. As all other bidders play the σ equilibrium, the highest possible price for the auctioneer is $2\overline{\Theta}C$ by awarding the split. Therefore, the auctioneer never allocates the sole source award to any bidder with $\hat{p}^s\left(\hat{\Theta}\right) \geq \underline{\Theta} > 2\overline{\Theta}C$ and the expected payoff of such deviations is zero.

ii) $\frac{\underline{\Theta}}{2\overline{\Theta}} < C < \frac{\underline{\Theta}}{\overline{\Theta}+\underline{\Theta}}$:

We show that there is no sole source deviation with prices $\hat{p}^s\left(\hat{\Theta}\right)$. A sole source deviation with price $\hat{p}^s\left(\hat{\Theta}\right) = 2p_e^\sigma\left(\underline{\Theta}\right)$ is payoff-dominant over all possible deviations with lower sole source prices, because the probability to win the sole source award is 1 for all deviations of this type. Hence, it suffices to show that deviations with prices greater than or equal to $2p_e^\sigma\left(\underline{\Theta}\right)$ are not attractive for the bidders. The upper bound for deviating sole source prices is $2\overline{\Theta}C$, because the probability that a sole source award is chosen is zero, when a bidder submits a higher price for the sole source award.

Let us first derive the optimal deviating sole source price $\hat{p}^s\left(\hat{\Theta}\right) \in \left[2p_e^\sigma\left(\underline{\Theta}\right), 2\overline{\Theta}C\right]$ for all possible cost types $\hat{\Theta} \in \left[\underline{\Theta}, 2\overline{\Theta}C\right]$. As $p_e^\sigma\left(\Theta_i\right)$ is continuous, we can express all possible $\hat{p}^s\left(\hat{\Theta}\right)$ by $2p_e^\sigma\left(\Theta\right)$ with variable $\Theta \in \left[\underline{\Theta}, \overline{\Theta}\right]$ and the expected profit of a sole source deviation as

$$\begin{aligned} E\left[\hat{\Pi}^s\left(\hat{\Theta}\right)\right] &= \left(\hat{p}^s\left(\hat{\Theta}\right) - \hat{\Theta}\right) P\left(\hat{p}^s\left(\hat{\Theta}\right) \leq p_e^\sigma\left(\Theta_{2:n-1}\right) + p_e^\sigma\left(\Theta_{1:n-1}\right)\right) \\ &= \left(2p_e^\sigma\left(\Theta\right) - \hat{\Theta}\right) P\left(2p_e^\sigma\left(\Theta\right) \leq p_e^\sigma\left(\Theta_{2:n-1}\right) + p_e^\sigma\left(\Theta_{1:n-1}\right)\right). \end{aligned}$$

Obviously, the probability that the deviating sole source price is lower than the split prices of the second lowest and lowest order statistic is always less than the probability that it is lower than two times the split price of the second lowest order statistic:

$$\begin{aligned} E\left[\hat{\Pi}^s\left(\hat{\Theta}\right)\right] &< \left(2p_e^\sigma\left(\Theta\right) - \hat{\Theta}\right) P\left(2p_e^\sigma\left(\Theta\right) \leq 2p_e^\sigma\left(\Theta_{2:n-1}\right)\right) \\ &= \left(2p_e^\sigma\left(\Theta\right) - \hat{\Theta}\right) P\left(p_e^\sigma\left(\Theta\right) \leq p_e^\sigma\left(\Theta_{2:n-1}\right)\right) \\ &= \left(2p_e^\sigma\left(\Theta\right) - \hat{\Theta}\right) \left(\left(1 - F\left(\Theta\right)\right)^{n-1} + (n-1) F\left(\Theta\right)\left(1 - F\left(\Theta\right)\right)^{n-2}\right) \\ &< \left(p_e^\sigma\left(\Theta\right) - C\hat{\Theta}\right) \left(\left(1 - F\left(\Theta\right)\right)^{n-1} + (n-1) F\left(\Theta\right)\left(1 - F\left(\Theta\right)\right)^{n-2}\right) \\ &\leq E\left[\Pi_e^\sigma\left(\hat{\Theta}\right)\right]. \end{aligned}$$

This assessment is true, as $\Theta = \hat{\Theta}$ maximizes the payoff function and

$$\begin{aligned}
C &< \frac{\underline{\Theta}}{\underline{\Theta} + \overline{\Theta}} \\
\overline{\Theta} C &< \underline{\Theta}\left(1 - C\right) \\
p_e^{\sigma}\left(\Theta\right) &< \hat{\Theta}\left(1 - C\right) \text{ because } p_e^{\sigma}\left(\Theta\right) \leq \overline{\Theta} C \text{ and } \underline{\Theta} \leq \hat{\Theta} \quad | + p_e^{\sigma}\left(\Theta\right) \\
2 p_e^{\sigma}\left(\Theta\right) - \hat{\Theta} &< p_e^{\sigma}\left(\Theta\right) - C\hat{\Theta}.
\end{aligned}$$

Therefore, all possible sole source deviations can be excluded.

SPLIT DEVIATIONS: Split prices $p_e^{\sigma}\left(\Theta_i\right)$ have to maximize the expected payoff of winning 50% of the business for a bidder with cost type Θ_i.

$$E\left[\Pi_e^{\sigma}\left(\Theta_i\right)\right] = \left(p_e^{\sigma}\left(\Theta_i\right) - \Theta_i C\right) P\left(p_e^{\sigma}\left(\Theta_i\right) \leq p_e^{\sigma}\left(\Theta_{2:n-1}\right)\right) \quad \text{for every } \Theta_i \in \left[\underline{\Theta}, \overline{\Theta}\right]$$

in equilibrium. The first-order condition can be simplified to:

$$\frac{d}{d\Theta_i}\left\{\left(\left(1 - F\left(\Theta_i\right)\right)^{n-1} + \left(n-1\right) F\left(\Theta_i\right)\left(1 - F\left(\Theta_i\right)\right)^{n-2}\right) p_e^{\sigma}\left(\Theta_i\right)\right\} = \Theta_i C\left(\left(n-1\right)\left(n-2\right)\left(1 - F\left(\Theta_i\right)\right)^{n-3} F\left(\Theta_i\right)\left(-f\left(\Theta_i\right)\right)\right).$$

By applying the boundary condition $p_e^{\sigma}\left(\overline{\Theta}\right) = \overline{\Theta} C$ and integration on both sides, we get

$$\left(\left(1 - F\left(\Theta_i\right)\right)^{n-1} + \left(n-1\right) F\left(\Theta_i\right)\left(1 - F\left(\Theta_i\right)\right)^{n-2}\right) p_e^{\sigma}\left(\Theta_i\right) \Big|_{\Theta_i}^{\overline{\Theta}} = \int_{\Theta_i}^{\overline{\Theta}} xC\left(\left(n-1\right)\left(n-2\right)\left(1 - F\left(x\right)\right)^{n-3} F\left(x\right)\left(-f\left(x\right)\right)\right) dx.$$

Solving for $p_e^{\sigma}\left(\Theta\right)$ results in the equilibrium split price for a bidder with cost type Θ_i:

$$p_e^{\sigma}\left(\Theta_i\right) = C\Theta_i + C\frac{\int_{\Theta_i}^{\overline{\Theta}}\left(1 - F\left(x\right)\right)^{n-1} + \left(n-1\right) F\left(x\right)\left(1 - F\left(x\right)\right)^{n-2} dx}{\left(1 - F\left(\Theta_i\right)\right)^{n-1} + \left(n-1\right) F\left(\Theta_i\right)\left(1 - F\left(\Theta_i\right)\right)^{n-2}}.$$

HYBRID DEVIATIONS: Least, one have to exclude hybrid deviations, i.e. deviating strategies, for which the sole source as well as the split award can emerge with strictly positive probability.

i) $2\overline{\Theta} C \leq \underline{\Theta}$

When this condition applies, hybrid deviations are excluded due to the individual rationality assumption following the same logic as discussed above for sole source deviations.

ii) $\frac{\underline{\Theta}}{2\overline{\Theta}} < C < \frac{\underline{\Theta}}{\overline{\Theta}+\underline{\Theta}}$:
In contrast to the other two types of deviations, the supplier's offers are influencing each other in a hybrid deviation, as a low bid for the sole source award can lower the probability of winning the split award and vice versa. Because all other bidders follow the equilibrium strategy, only bidders with cost types $\hat{\Theta} \leq \overline{\Theta}C + \hat{p}^{\sigma}\left(\hat{\Theta}\right)$ have positive probability of winning the sole source award. The lower bound $2p_e^{\sigma}\left(\underline{\Theta}\right) \leq \hat{p}^{s}\left(\hat{\Theta}\right)$ emerges, as all lower deviating sole source prices are dominated. The same logic applies for deviating split prices, which are bounded below by $p_e^{\sigma}\left(\underline{\Theta}\right)$ and above by $\overline{\Theta}C$. The expected payoff of such deviations $\left(\hat{p}^{s}\left(\hat{\Theta}\right), \hat{p}^{\sigma}\left(\hat{\Theta}\right)\right)$ is

$$
\begin{aligned}
E\left[\hat{\Pi}^{hybrid}\right] = \left(\hat{p}^{s}\left(\hat{\Theta}\right) - \hat{\Theta}\right) P\Bigg(\hat{p}^{s}\left(\hat{\Theta}\right) < \min\Big\{ & p_e^{\sigma}\left(\Theta_{1:n-1}\right) + \\
& + \min\left\{p_e^{\sigma}\left(\Theta_{2:n-1}\right), \hat{p}^{\sigma}\left(\hat{\Theta}\right)\right\}, \\
& \max\left\{\Theta_{1:n-1}, p_e^{\sigma}\left(\Theta_{1:n-1}\right) + \overline{\Theta}C\right\}\Big\}\Bigg) \\
& + \left(\hat{p}^{\sigma}\left(\hat{\Theta}\right) - \hat{\Theta}C\right) P\Big(\hat{p}^{\sigma}\left(\hat{\Theta}\right) < p_e^{\sigma}\left(\Theta_{2:n-1}\right) \\
& \wedge \hat{p}^{s}\left(\hat{\Theta}\right) \geq \hat{p}^{\sigma}\left(\hat{\Theta}\right) + p_e^{\sigma}\left(\Theta_{1:n-1}\right)\Big).
\end{aligned}
$$

We define $\hat{p}^{s}\left(\hat{\Theta}\right) = p_e^{\sigma}\left(x_1\right) + p_e^{\sigma}\left(x_2\right)$ and $\hat{p}^{\sigma}\left(\hat{\Theta}\right) = p_e^{\sigma}\left(x_2\right)$ with $x_1 \in \left[\underline{\Theta}, \overline{\Theta}\right]$, $x_2 \in \left[\hat{\Theta}, \overline{\Theta}\right]$ and $p_e^{\sigma}\left(x_1\right) + p_e^{\sigma}\left(x_2\right) > 0$. It is known that

$$
\begin{aligned}
C &< \frac{\underline{\Theta}}{\underline{\Theta}+\overline{\Theta}} \\
\overline{\Theta}C &< \underline{\Theta}\left(1 - C\right) \\
p_e^{\sigma}\left(x_1\right) &< \Theta_i\left(1 - C\right) \text{ as } p_e^{\sigma}\left(x_1\right) \leq \overline{\Theta}C \text{ and } \underline{\Theta} \leq \Theta_i \\
p_e^{\sigma}\left(x_2\right) + p_e^{\sigma}\left(x_1\right) - \Theta_i &< p_e^{\sigma}\left(x_2\right) - \Theta_i
\end{aligned}
$$

with $\Theta_i \in \left[\underline{\Theta}, \overline{\Theta}\right]$. We want to find x_1 and x_2 such that the following expected payoff function is maximized:

$$
\begin{aligned}
E\left[\hat{\Pi}^{hybrid}\left(\hat{\Theta}\right)\right] = &\left(p_e^\sigma(x_1) + p_e^\sigma(x_2) - \hat{\Theta}\right) P\Bigg(p_e^\sigma(x_1) + p_e^\sigma(x_2) < \\
&< \min\left\{p_e^\sigma(\Theta_{1:n-1}) + \min\left\{p_e^\sigma(\Theta_{2:n-1}), p_e^\sigma(x_2)\right\},\right. \\
&\left.\max\left\{\Theta_{1:n-1}, p_e^\sigma(\Theta_{1:n-1}) + \overline{\Theta}C\right\}\right\}\Bigg) + \\
&+ \left(p_e^\sigma(x_2) - \hat{\Theta}C\right) \cdot P\Bigg(p_e^\sigma(x_2) < p_e^\sigma(\Theta_{2:n-1}) \wedge \\
&\wedge\, p_e^\sigma(x_1) + p_e^\sigma(x_2) \geq p_e^\sigma(x_2) + p_e^\sigma(\Theta_{1:n-1})\Bigg).
\end{aligned}
$$

For deviations with $x_1 \geq x_2$ we can show that

$$
\begin{aligned}
E\left[\hat{\Pi}^{hybrid}\left(\hat{\Theta}\right)\right] < &\left(p_e^\sigma(x_2) - C\hat{\Theta}\right) P\Bigg(p_e^\sigma(x_2) \leq p_e^\sigma(\Theta_{2:n-1}) \\
&\wedge\, p_e^\sigma(x_1) < p_e^\sigma(\Theta_{1:n-1})\Bigg) \\
&+ \left(p_e^\sigma(x_2) - \hat{\Theta}C\right) P\Bigg(p_e^\sigma(x_2) \leq p_e^\sigma(\Theta_{2:n-1}) \wedge \\
\leq &\wedge\, p_e^\sigma(x_1) \geq p_e^\sigma(\Theta_{1:n-1})\Bigg) \\
&\left(p_e^\sigma(x_2) - \hat{\Theta}C\right) P\left(p_e^\sigma(x_2) \leq p_e^\sigma(\Theta_{2:n-1})\right) \\
\leq &E\left[\Pi_e^\sigma\left(\hat{\Theta}\right)\right].
\end{aligned}
$$

If condition (4.6.6) applies, deviations with $x_1 < x_2$ can be excluded. □

Remark: If one wants to test condition (4.6.6) for a specific setting, it is more convenient to use the stricter condition

$$
\begin{aligned}
E\left[\hat{\Pi}\left(\hat{\Theta}\right)\right] < &\left(p_e^\sigma(x_1) + p_e^\sigma(x_2) - \hat{\Theta}\right) P\Bigg(p_e^\sigma(x_2) \leq p_e^\sigma(\Theta_{2:n-1}) \wedge \\
&\wedge\, p_e^\sigma(x_1) < p_e^\sigma(\Theta_{1:n-1})\Bigg) \\
&+ \left(p_e^\sigma(x_2) - C\hat{\Theta}\right) P\Bigg(p_e^\sigma(x_2) \leq p_e^\sigma(\Theta_{2:n-1}) \wedge \\
&\wedge\, p_e^\sigma(x_1) \geq p_e^\sigma(\Theta_{1:n-1})\Bigg) \\
< &\left(p_e^\sigma(x_1) + p_e^\sigma(x_2) - \hat{\Theta}\right) P\left(p_e^\sigma(x_1) < p_e^\sigma(\Theta_{1:n-1})\right) \\
&+ \left(p_e^\sigma(x_2) - C\hat{\Theta}\right) P\Bigg(p_e^\sigma(x_2) \leq p_e^\sigma(\Theta_{2:n-1}) \wedge \\
&\wedge\, p_e^\sigma(x_2) \geq p_e^\sigma(\Theta_{1:n-1})\Bigg) \mid \text{ as } x_1 < x_2
\end{aligned}
$$

and check, whether it is lower than the equilibrium payoff for all possible cost types and all possible combinations of x_1 and x_2.

B.1.8 *Proof of Proposition 24*

Assume there is a hybrid equilibrium with cost intervals $I_1^s, I_2^s, ..., I_t^s$ and $I_1^\sigma, I_2^\sigma, ..., I_u^\sigma$, for which the sole source award, respectively the split award, is the equilibrium outcome when the interval includes the two lowest cost draws of the n competitors. Furthermore, $t, u \in \mathbb{N}$ and

$$I_1^s \cup I_2^s \cup ... \cup I_t^s \cup I_1^\sigma \cup I_2^\sigma \cup ... \cup I_u^\sigma = \left[\underline{\Theta}, \overline{\Theta}\right]$$

applies. The functions $p_i^s(\Theta)$ for $i \in 1, ..., t$ and $p_j^\sigma(\Theta)$ for $j \in 1, ..., u$ are the relevant equilibrium prices for the sole source and split awards in the intervals I_i^s and I_i^σ, respectively. Then, every possible hybrid equilibrium must include a strategic parameter $\tau_1 \in \left(\underline{\Theta}, \overline{\Theta}\right)$, for which the equilibrium results in the same award, when all bidders have cost draws higher than τ. Without loss of generalization, we assume for the proof that $I_1^s = \left[\underline{\Theta}, \tau_1\right]$ and $I_1^\sigma = \left(\tau_1, \overline{\Theta}\right]$. Then, the following conditions have to apply for a hybrid equilibrium:

1. $p_1^s(\tau_1) = 2p_1^\sigma(\tau_1)$
2. $p_1^s(\tau_1) - \tau_1 = p_1^\sigma(\tau_1) - C\tau_1$
3. $p_1^\sigma(\tau_1) \leq \overline{\Theta}C$.

The first two conditions for hybrid equilibria have been established by Anton et al. (2010) for the FPSB auction and two bidders. When all cost types belong to I_1^s, the auctioneer chooses the sole source award in equilibrium; the split is selected, when all cost types are in I_1^σ. Then, the auctioneer must be indifferent between both awards in the case that all bidders have the same cost type τ_1, as the price functions $p_e^s(\Theta_i)$ and $p_e^\sigma(\Theta_i)$ are increasing and continuous. Furthermore, a bidder must be indifferent between winning the sole source award or the split award, when his cost parameter is τ_1. Otherwise, he would not change his strategy for this cost type in equilibrium. The third condition is a standard requirement for split prices in equilibrium with more than two bidders.[44]

In what follows, we show that these three assumptions can never be met simultaneously with *dual source efficiency*:

Combining (i) and (ii) results in $p_1^\sigma(\tau_1) = \tau_1(1 - C)$. Hence, with (iii)

$$p_1^\sigma(\tau_1) \leq \overline{\Theta}C$$
$$C \geq \frac{\tau_1}{\tau_1 + \overline{\Theta}}$$

[44] This is different to the case with two bidders, in which split prices are constant and can be greater than or equal to $\overline{\Theta}C$. In such a setting, hybrid equilibria cannot be excluded.

must apply. This is never true for $\tau_1 \in \left(\underline{\Theta}, \overline{\Theta}\right)$ with *dual source efficiency*. □

B.1.9 *Proof of Proposition 25*

In order to prove sequential rationality, it has to be shown that there are no payoff-dominant sole source or split deviations in phase 1 as well as in phase 2. In contrast to the setting with two bidders, the equilibrium strategies for both phases are increasing. Hence, the losers of phase 1 have full information about the cost type of the winner of phase 1 (reflected by history h^1), while the cost types of the losers remain private. This updating process has to be considered for the derivation of the equilibrium strategies.

SPLIT DEVIATIONS: The equilibrium bidding strategy in phase 1 and 2 is similar to the equilibrium strategy in an ex-ante split-award auction, in which two times 50% are auctioned off sequentially in two first-price auctions. The winning bid of phase 1 is revealed to all bidders and the winner of the first auction cannot participate in the auction for the remaining 50% share,[45] which is the reason why only deviations for the split award are possible. Hence, a strategy which maximizes the expected payoff in a sequential ex-ante split-award auction, also excludes all split deviations in the Dutch split-award auction. In proposition 15.1 of Krishna (2009), an equilibrium in a sequential multi-unit forward auction is characterized, which can be easily transferred to our setting. Both environments are comparable, as the winner of the first 50% share cannot win the remaining 50% share due to *dual source efficiency* and the assumption of individual rationality. Hence, the following equilibrium emerges:

Bidders accept the counteroffer for the split award in phase 1 at a price of

$$p_e^{\sigma 1}\left(\Theta_i, h^0\right) = \frac{\int_{\Theta_i}^{\overline{\Theta}} p_e^{\sigma 2l}\left(t, h^1\right)(n-1)\left(1-F(t)\right)^{n-2} f(t)\, dt}{\left(1-F(\Theta_i)\right)^{n-1}}.$$

For phase 2, bidders are asymmetric, because $n-1$ losers and one winner of phase 1 compete for the remaining 50% share. The losers of phase 1 approve the counteroffer for the 50% share at a price of

$$p_e^{\sigma 2l}\left(\Theta_l, h^1\right) = C\Theta_l + C\frac{\int_{\Theta_l}^{\overline{\Theta}}\left(1-F(t)\right)^{n-2} dt}{\left(1-F(\Theta_l)\right)^{n-2}}$$

in equilibrium.

Additionally, we have to define a strategy for the winner of phase 1, which is off-equilibrium. When supplier w wins 50% of the business in phase 1 and is also the winner of the remaining 50% share in phase 2, the auctioneer pays him the sum of both split prices. Due to individual rationality this sum must

[45] Otherwise it would not be an ex-ante split-award auction.

be at least as high as the costs for 100% of the business, Θ_w. Furthermore, the overall payoff of bidder w has to be at least as high as the payoff in phase 1, as bidders are assumed to be payoff-maximizing and otherwise the strategy would not be a credible threat. Therefore, bidder w can only accept counteroffers in phase 2, which are at least as high than his additional costs $\Theta_w (1 - C)$,

$$p_e^{\sigma 2w}\left(\Theta_w, h^1\right) \geq \Theta_w (1 - C) .$$

A loser of phase 1 with cost type Θ_l faces the following maximization problem in phase 2:

$$\begin{aligned}\max_z E\left[\Pi^{\sigma 2l}\left(z, h^1\right)\right] &= \left(p_e^{\sigma 2l}\left(z, h^1\right) - \Theta_l C\right) \cdot \\ &\cdot P\Big(p_e^{\sigma 2l}\left(z, h^1\right) < \min\left\{p_e^{\sigma 2l}\left(\Theta_{1:n-2}, h^1\right), p_e^{\sigma 2w}\left(\Theta_w, h^1\right)\right\} \\ &\mid \Theta_{1:n-2} \geq \Theta_w\Big) \\ &= \left(p_e^{\sigma 2l}\left(z, h^1\right) - \Theta_l C\right) \frac{\left(1 - F(z)\right)^{n-2}}{\left(1 - F(\Theta_w)\right)^{n-2}}.\end{aligned}$$

As $\Theta_w (1 - C) > p_e^{\sigma 2l}\left(\Theta_l, h^1\right)$ applies for all $\Theta_w, \Theta_l \in \left[\underline{\Theta}, \overline{\Theta}\right]$ with *dual source efficiency*, the $n - 1$ losers know that Θ_w never wins the remaining share in equilibrium. Otherwise it would not be a a σ equilibrium and this is also the reason, why $\overline{\Theta} C$ and not $p_e^{\sigma 2w}\left(\Theta_w, h^1\right)$ is the upper limit for $p_e^{\sigma 2l}\left(\Theta_l, h^1\right)$. Nevertheless, the beliefs about the cost types of the $n - 1$ losers are updated after phase 1, as every supplier knows that the costs of every loser of phase 1 cannot be lower than Θ_w, which is identical to the ex-ante format discussed in Krishna (2009). Inserting the equilibrium strategy $p_e^{\sigma 2l}\left(\Theta_l, h^1\right)$ with $z = \Theta_l$ from above maximizes the expected payoff.

The expected equilibrium payoffs in phase 2 has to be considered for the derivation of the strategy in phase 1. As the winner of phase 1 never wins the remaining share in phase 2, the same logic as in the proof of ex-ante split-award auctions yields $p_e^{\sigma 1}\left(\Theta_i, h^0\right)$.

However, there could be profitable sole source deviations, which are only possible in the ex-post format. In what follows, we show that such deviations do not exist with *dual source efficiency*.

SOLE SOURCE DEVIATIONS: In phase 1, a bidder can deviate from equilibrium by accepting the 100% share before the first 50% is awarded. Such a sole source deviation is only possible for types $\hat{\Theta} \in \left[\underline{\Theta}, 2\overline{\Theta} C\right]$ and has to satisfy

$$2 p_e^{\sigma 1}\left(\underline{\Theta}, h^0\right) \leq \hat{p}^{s1}\left(\hat{\Theta}, h^0\right) \leq 2\overline{\Theta} C.$$

The probability of winning the auction by accepting a counteroffer for 100% at a price of $2 p_e^{\sigma 1}\left(\underline{\Theta}, h^0\right)$ is 1. Therefore, no price which is lower than this bound can yield a higher expected payoff and such deviations can be neglected. There is an upper bound for sole source deviations, because the split is accepted by a bidder at the latest in round r with counteroffers $c_r^s = 2\overline{\Theta}C$ and $c_r^\sigma = \overline{\Theta}C$.

Because all price functions are continuous, the deviating sole source price $\hat{p}^{s1}\left(\hat{\Theta}, h^0\right)$ can be expressed by $\hat{p}^{s1}\left(\hat{\Theta}, h^0\right) = 2p_e^{\sigma 1}\left(\Theta, h^0\right)$ with $\Theta \in \left[\underline{\Theta}, \overline{\Theta}\right]$. In what follows we show that the expected payoff for a sole source deviation is strictly lower than the expected payoff in a σ equilibrium:

$$
\begin{aligned}
E\left[\hat{\Pi}^s\left(\hat{\Theta}, h^0\right)\right] &= \left(\hat{p}^s\left(\hat{\Theta}, h^0\right) - \hat{\Theta}\right) P\left(\hat{p}^s\left(\hat{\Theta}, h^0\right) < 2p_e^{\sigma 1}\left(\Theta_{1:n-1}, h^0\right)\right) \\
&= \left(2p_e^{\sigma 1}\left(\Theta, h^0\right) - \hat{\Theta}\right) P\left(2p_e^{\sigma 1}\left(\Theta, h^0\right) < 2p_e^{\sigma 1}\left(\Theta_{1:n-1}, h^0\right)\right) \\
&= \left(2p_e^{\sigma 1}\left(\Theta, h^0\right) - \hat{\Theta}\right) P\left(p_e^{\sigma 1}\left(\Theta, h^0\right) < p_e^{\sigma 1}\left(\Theta_{1:n-1}, h^0\right)\right) \\
&= \left(p_e^{\sigma 1}\left(\Theta, h^0\right) - \hat{\Theta}C + p_e^{\sigma 1}\left(\Theta, h^0\right) - (1-C)\,\hat{\Theta}\right)\left(1 - F(\Theta)\right)^{n-1} \\
&< \left(p_e^{\sigma 1}\left(\Theta, h^0\right) - \hat{\Theta}C\right)\left(1 - F(\Theta)\right)^{n-1} + \\
&\quad \underbrace{\left(\overline{\Theta}C - (1-C)\,\hat{\Theta}\right)}_{<0,\text{ as } C<\frac{\hat{\Theta}}{\hat{\Theta}+\overline{\Theta}}}\left(1 - F(\Theta)\right)^{n-1} \\
&< \left(p_e^{\sigma 1}\left(\Theta, h^0\right) - \hat{\Theta}C\right)\left(1 - F(\Theta)\right)^{n-1} \\
&\leq \max_{\Theta}\left(p_e^{\sigma 1}\left(\Theta, h^0\right) - \hat{\Theta}C\right)\left(1 - F(\Theta)\right)^{n-1} \\
&= E\left[\Pi_e^\sigma\left(\hat{\Theta}, h^0\right)\right].
\end{aligned}
$$

The winner w of phase 1 has the chance to deviate for the sole source award in phase 2. We know that in equilibrium the highest possible price for the auctioneer is $\overline{\Theta}C$, which determines the upper bound for any sole source deviation. However, accepting the split for this price is unprofitable regardless of the cost type of bidder w, because the additional costs for producing 100% of the business are not covered at this price:

$$
\begin{aligned}
\overline{\Theta}C &< \Theta_w\,(1-C) \\
C &< \frac{\Theta_w}{\Theta_w + \overline{\Theta}}.
\end{aligned}
$$

This applies for all Θ_w with *dual source efficiency*. □

B.1.10 *Proof of Corollary 6*

The expected payoff in a σ equilibrium, $\underline{\Theta}\,(1-C) - C\Theta_i$, must be for all cost types $\Theta_i \in \left[\underline{\Theta}, \overline{\Theta}\right]$ greater than or equal to the expected payoff of any other possible equilibrium. Condition (i) is necessary in order to achieve payoff dominance over all σ equilibria. The derivation of the expected payoff function used in condition (ii), which assures payoff dominance of the split over hybrid equilibria with parameter τ, can be found in Anton et al. (2010). Condition (iii) is adapted from proposition 5 of Anton and Yao (1992). □

B.1.11 *Proof of Corollary 8*

It is known that truthful bidding, i.e. $C\Theta_i$ for 50% as well as Θ_i for 100% of the business is a weakly dominant strategy for bidders $i \in \{A, B\}$ in a VCG mechanism. Hence, the price of the auctioneer can be calculated as

$$\begin{aligned} p_b^{VCG}(\Theta_A, \Theta_B) =& \Theta_A C + \left(\Theta_B - (\Theta_A C + \Theta_B C)\right) + \Theta_B C + \\ & \left(\Theta_A - (\Theta_A C + \Theta_B C)\right) \\ =& (\Theta_A + \Theta_B)(1 - C) \\ \geq& 2\underline{\Theta}(1 - C) \\ =& p_b^{\sigma}(\Theta_A, \Theta_B) \text{ for all } \Theta_A, \Theta_B \in \left[\underline{\Theta}, \overline{\Theta}\right]. \end{aligned}$$

For every possible combination of cost types, the VCG costs are higher than or equal to the purchasing costs in a payoff-dominant σ equilibrium in one of the three first-price split-award auctions analyzed above. □

B.1.12 *Proof of Proposition 26*

For the following proofs, we need the expected payoffs for a cost type Θ_i

$$\begin{aligned} E\left[\Pi_e^s(\Theta_i)\right] =& \int_{\Theta_i}^{\overline{\Theta}} \left(1 - F(x)\right)^{n-1} dx \text{ in a WTA equilibrium and} \\ E\left[\Pi_e^{\sigma}(\Theta_i)\right] =& C \int_{\Theta_i}^{\overline{\Theta}} \left(1 - F(x)\right)^{n-1} + (n-1) F(x) \left(1 - F(x)\right)^{n-2} dx \text{ in a} \\ & \sigma \text{ equilibrium.} \end{aligned}$$

We show that for all possible settings with *dual source efficiency*, regarding the parameters C, F, $\underline{\Theta}$, $\overline{\Theta}$ and n, there is always an interval $\left(\Theta^*, \overline{\Theta}\right]$ for which the σ equilibrium yields a higher payoff than the WTA equilibrium, i.e., for which

$$\begin{aligned} E\left[\Pi_e^s(\Theta_i)\right] < E\left[\Pi_e^{\sigma}(\Theta_i)\right] \text{ for all } \Theta_i \in \left(\Theta^*, \overline{\Theta}\right] \\ E\left[\Pi_e^s(\Theta_i)\right] - E\left[\Pi_e^{\sigma}(\Theta_i)\right] < 0 \text{ for all } \Theta_i \in \left(\Theta^*, \overline{\Theta}\right]. \end{aligned}$$

We show that this is true for high cost types:

$$\lim_{\Theta_i \to \overline{\Theta}} E\left[\Pi_e^s(\Theta_i)\right] - E\left[\Pi_e^\sigma(\Theta_i)\right] < 0$$

$$\lim_{\Theta_i \to \overline{\Theta}} \int_{\Theta_i}^{\overline{\Theta}} \left(1 - F(x)\right)^{n-1} dx -$$

$$C \int_{\Theta_i}^{\overline{\Theta}} \left(1 - F(x)\right)^{n-1} + (n-1) F(x) \left(1 - F(x)\right)^{n-2} dx < 0$$

$$\lim_{\Theta_i \to \overline{\Theta}} \frac{\int_{\Theta_i}^{\overline{\Theta}} \left(1 - F(x)\right)^{n-1} dx}{\int_{\Theta_i}^{\overline{\Theta}} \left(1 - F(x)\right)^{n-1} + (n-1) F(x) \left(1 - F(x)\right)^{n-2} dx} < C$$

$$\| \frac{0}{0} \text{L'Hospital}$$

$$\lim_{\Theta_i \to \overline{\Theta}} \frac{-\left(1 - F(\Theta_i)\right)^{n-1}}{-\left(1 - F(\Theta_i)\right)^{n-1} - (n-1) F(\Theta_i) \left(1 - F(\Theta_i)\right)^{n-2}} < C$$

$$\lim_{\Theta_i \to \overline{\Theta}} \frac{1}{1 + (n-1) \frac{F(\Theta_i)}{1 - F(\Theta_i)}} < C$$

$$0 < C.$$

As this inequality applies for all possible C, the σ equilibrium always yields more payoff than the WTA equilibrium for cost types in an interval $\left(\Theta^*, \overline{\Theta}\right]$. Therefore, a WTA equilibrium cannot be payoff-dominant for all cost types with *dual source efficiency*. □

B.1.13 *Proof of Corollary 9*

In order to prove that the σ equilibrium is payoff-dominant for an arbitrary setting with *dual source efficiency*, more than two bidders and uniformly distributed cost parameters with support $\left[\underline{\Theta}, \overline{\Theta}\right]$, we show that

$$E\left[\Pi_e^s(\Theta_i)\right] - E\left[\Pi_e^\sigma(\Theta_i)\right] < 0 \ \forall \Theta_i \in \left[\underline{\Theta}, \overline{\Theta}\right].$$

We know from proposition 26 that the σ equilibrium yields higher expected payoff than the WTA equilibrium for bidders with high cost types. Therefore, it suffices to show that for a bidder with cost type $\Theta_i = \underline{\Theta}$, the WTA equilibrium yields a higher expected profit than the σ equilibrium:

$$E\left[\Pi_e^s\left(\underline{\Theta}\right)\right] - E\left[\Pi_e^\sigma\left(\underline{\Theta}\right)\right] > 0$$

$$\int_{\underline{\Theta}}^{\overline{\Theta}} \left(1 - F\left(x\right)\right)^{n-1} dx -$$

$$C \int_{\underline{\Theta}}^{\overline{\Theta}} \left(1 - F\left(x\right)\right)^{n-1} + \left(n-1\right) F\left(x\right)\left(1 - F\left(x\right)\right)^{n-2} dx > 0$$

$$\left(1 - C\right) \int_{\underline{\Theta}}^{\overline{\Theta}} \frac{\left(\overline{\Theta} - x\right)^{n-1}}{\left(\overline{\Theta} - \underline{\Theta}\right)^{n-1}} dx - C \int_{\underline{\Theta}}^{\overline{\Theta}} \left(n-1\right) \frac{\left(x - \underline{\Theta}\right)\left(\overline{\Theta} - x\right)^{n-2}}{\left(\overline{\Theta} - \underline{\Theta}\right)^{n-1}} dx > 0$$

$$\left(1 - C\right) \left[-\frac{\left(\overline{\Theta} - x\right)^n}{n} \right]_{\underline{\Theta}}^{\overline{\Theta}} - C\left(n-1\right) \left[-\frac{\left(\overline{\Theta} - x\right)^n}{n\left(n-1\right)} \right]_{\underline{\Theta}}^{\overline{\Theta}} > 0$$

$$\underbrace{\left(1 - 2C\right)}_{>0} \underbrace{\frac{\left(\overline{\Theta} - \underline{\Theta}\right)^n}{n}}_{>0} > 0.$$

Hence, for small cost types, a WTA equilibrium yields higher payoff than a σ equilibrium, which is sufficient to prove the corollary. □

Remark regarding integration by parts:

$$\int_{\underline{\Theta}}^{\overline{\Theta}} \left(x - \underline{\Theta}\right)\left(\overline{\Theta} - x\right)^{n-2} dx = \left[-\frac{\left(\overline{\Theta} - x\right)^{n-1}}{n-1} \left(x - \underline{\Theta}\right) \right]_{\underline{\Theta}}^{\overline{\Theta}} - \int_{\underline{\Theta}}^{\overline{\Theta}} -\frac{\left(\overline{\Theta} - x\right)^{n-1}}{n-1} dx$$

$$= 0 + \left[-\frac{\left(\overline{\Theta} - x\right)^n}{n\left(n-1\right)} \right]_{\underline{\Theta}}^{\overline{\Theta}}.$$

B.1.14 *Proof of Corollary 10*

We show that the expected price for the buyer in the σ equilibrium is lower than in the WTA equilibrium, when either C is sufficiently low or the cost types are uniformly distributed with *dual source efficiency*.

First, assume $C < \frac{\underline{\Theta}}{2\overline{\Theta}}$. Then,

$$E\left[p_b^\sigma\left(\cdot,\cdot\right)\right] = 2CE\left[\Theta_{3:n}\right] < E\left[\Theta_{2:n}\right] = E\left[p_b^{WTA}\left(\cdot,\cdot\right)\right]$$

$$C < \frac{E\left[\Theta_{3:n}\right]}{2E\left[\Theta_{2:n}\right]}.$$

The function $f(x,y) = \frac{x}{2y}$ is decreasing in y and increasing in x. As $E[\Theta_{2:n}] < \overline{\Theta}$ and $E[\Theta_{3:n}] > \underline{\Theta}$, the inequality applies.

Second, with a uniform distribution the expectation value of the k-th lowest order statistic $\Theta_{k:n}$ can be expressed by

$$E[\Theta_{k:n}] = \underline{\Theta} + \frac{k}{n+1}\left(\overline{\Theta} - \underline{\Theta}\right).$$

Therefore, we can show that

$$\begin{aligned} E\left[p_b^{\sigma}(\cdot,\cdot)\right] = 2CE[\Theta_{3:n}] &< E[\Theta_{2:n}] = E\left[p_b^{WTA}(\cdot,\cdot)\right] \\ 2C\left(\underline{\Theta} + \frac{3}{n+1}\left(\overline{\Theta} - \underline{\Theta}\right)\right) &< \underline{\Theta} + \frac{2}{n+1}\left(\overline{\Theta} - \underline{\Theta}\right) \\ C &< \frac{\underline{\Theta} + \frac{2}{n+1}\left(\overline{\Theta} - \underline{\Theta}\right)}{2\underline{\Theta} + \frac{6}{n+1}\left(\overline{\Theta} - \underline{\Theta}\right)}. \end{aligned}$$

As the right hand side approaches 0.5 with $n \to \inf$, it suffices to show that the condition applies for $n = 3$ and $C = \frac{\underline{\Theta}}{\underline{\Theta}+\overline{\Theta}}$. This is true, as

$$\begin{aligned} \frac{\underline{\Theta}}{\underline{\Theta}+\overline{\Theta}} &< \frac{\underline{\Theta} + \frac{1}{2}\left(\overline{\Theta} - \underline{\Theta}\right)}{2\underline{\Theta} + \frac{3}{2}\left(\overline{\Theta} - \underline{\Theta}\right)} \\ \frac{\underline{\Theta}}{\underline{\Theta}+\overline{\Theta}} &< \frac{\underline{\Theta}+\overline{\Theta}}{\underline{\Theta}+3\overline{\Theta}} \\ 3\overline{\Theta}\underline{\Theta} + \underline{\Theta}^2 &< \underline{\Theta}^2 + 2\overline{\Theta}\underline{\Theta} + \overline{\Theta}^2 \\ 0 &< \overline{\Theta}\left(\overline{\Theta} - \underline{\Theta}\right). \end{aligned}$$

□

B.2 STATISTICS OF LABORATORY EXPERIMENTS

This appendix contains fixed-effects regressions of bids and prices for bidders in split and sole-source awards for all treatments in subsection B.2.1. Moreover, fixed-effects regressions of the allocation (split vs. sole-source award) are included. We also added the univariate regressions of bids and prices on the cost draws and plotted them in Section B.2.2. Finally, plots of split-award winner bids and prices over periods of all treatments are attached to provide insight into the development of bidding behavior with repeated interaction in auctions. We also added plots to visualize the distribution of allocations over periods for the two-bidder treatments.

B.2.1 *Regression Tables*

In Tables 5 to 10 the dependent variable is depicted in the left column and the intercept as well as all independent variables in the columns to the right. For each dependent variable we provide estimates for the coefficients of all explanatory variables for the univariate regression on cost draws _X and the fixed-effects regression _X(P). The corresponding p-values are included in brackets below the coefficients. The columns ”subjects” and ”subjects per period” contain the number of subjects with significant (at least at the 5% level) fixed-effects and fixed-effects over the periods of the experiment, respectively. The column ”period” is the fixed-effect over periods for the reference subject and included for completeness. NA-values for the latter column occur if a subject ends up in the corresponding allocation only once. The right outermost column contains the R^2 as a measurement for the explanatory power of the linear regression model. Note that we used a logistic regression for the fixed-effects model of the binary outcome allocation with ajusted measurement of explanatory power McFadden R^2.

allocation	**intercept**	**#subjects**	**period**	**#subjects per period**	**McFadden R^2**		
FPSB_2_A	0.82151	2 (all negative)/47	-0.15915	3 (all positive)/48	0.15533		
	(0.4737)	*	(0.2383)	*			
Split Bidders							
single-unit bid	**intercept**	**teta**	**double-unit bid**	**#subjects**	**period**	**#subjects per period**	**adjusted R^2**
FPSB_2_S_S	51.88973	0.08566					
	(1.04e-13)***	(0.117)					
FPSB_2_S_S(P)	34.49385	0.0571	0.07806	8 (all positive)/46	0.35459	2 (one positive and negative)/46	0.5286
	(8.66e-05)***	(0.242545)	(0.012520)*	*	(0.633163)	*	
double-unit bid	**intercept**	**teta**	**single-unit bid**	**#subjects**	**period**	**#subjects per period**	**adjusted R^2**
FPSB_2_S_D	91.11576	0.44934					
	(7.21e-13)***	(9.22e-06)***					
FPSB_2_S_D(P)	62.887305	0.481243	0.368754	3 (all positive)/46	0.094282	2 (all negative)/46	0.3685
	(0.001058)**	(3.50e-06)***	(0.012520)*	*	(0.953452)	*	
Sole-Source Bidders							
single-unit bid	**intercept**	**teta**	**double-unit bid**	**#subjects**	**period**	**#subjects per period**	**adjusted R^2**
FPSB_2_SS_S	100.8732	-0.1387					
	(8.7e-10)***	(0.303)					
FPSB_2_SS_S(P)	81.1726	-0.3662	0.444	5 (4 positive, 1 negative)/46	3.4386	11 (1 positive, 10 negative)/46	
	(6.98e-05)***	(0.001212)**	(2.85e-12)***	*	(0.033868)*	*	
double-unit bid	**intercept**	**teta**	**single-unit bid**	**#subjects**	**period**	**#subjects per period**	**adjusted R^2**
FPSB_2_SS_D	51.50754	0.7907					
	(1.18e-05)***	(6.31e-15)***					
FPSB_2_SS_D(P)	1.70465	0.83653	0.35606	7 (all positive)/46	-1.25169	1 (positive)/46	
	(0.926719)	(<2e-16)***	(2.85e-12)***	*	(0.390034)	*	

Table 5: FPSB (n=2) Regressions

In the fixed-effects regression FPSB_2_S_S(P) in Table 5 the cost draw is not a statistically significant explanatory variable for the single-unit bid of split award winners whereas each bidder’s double-unit bid possesses relevant explanatory power. In regression FPSB_2_S_D(P) the double-unit bid strictly increases in costs and the single-unit bid has explanatory power. In the fixed-effects regressions FPSB_2_SS_S(P) and FPSB_2_SS_D(P) the single- and double unit bids of sole-source award bidders are statistically significant increasing in the bid for the alternative number of units. The latter possesses more explanatory power than for the split award winners which corresponds to our conjectures about the different relations of single- and double-unit bids for split award winners and sole-source bidders. The negative influence of the cost parameter in FPSB_2_SS_S(P) might be explained by a large number of sole-source bidders, who submit high bids on one unit to exclude the split award.

allocation	intercept	#subjects	period	#subjects per period	McFadden R^2		
DU_2_A	1.90367	0	-0.013362	0/49	0.1415133		
	(0.2575)	*	(0.9398)	*			
Split Bidders							
first single-unit bid	**intercept**	**teta**		**#subjects**	**period**	**#subjects per period**	**adjusted R^2**
DU_2_S_1	51.20217	0.19497					
	(3.22e-08)***	(0.00854)**					
DU_2_S_1(P)	52.2339	0.14209		4 (1 positive, 3 negative)/47	4.51003	2 (1 positive, 1 negative, 2 NA)/47	0.6171
	(1.93e-05)***	(0.01678)*		*	0.10722	*	
second single-unit bid	**intercept**	**teta**	**single-unit bid**	**#subjects**	**period**	**#subjects per period**	**adjusted R^2**
DU_2_S_2	62.10713	0.12452					
	(5.98e-12)***	(0.0767)					
DU_2_S_2(P)	46.61149	0.06771	0.50969	25 (24 positive, 1 negative)/47	-0.71126	8 (6 positive and 2 negative, 3 NA)/47	0.8001
	(9.74e-09)***	(0.088325)	(<2e-16)***	*	(0.142994)	*	
Sole-source Winners							
direct double-unit bid	**intercept**	**teta**		**#subjects**	**period**	**#subjects per period**	**adjusted R^2**
DU_2_SS_D	95.6622	0.5888					
	(0.00108)**	(0.02207)*					
DU_2_SS_D(P)	122.4552	0.09057		6 (all positive)/33	1.03293	1 (positive, 14 NA)/33	0.6203
	(0.00176)**	(0.76205)		*	(0.46129)	*	
first single-unit bid	**intercept**	**teta**		**#subjects**	**period**	**#subjects per period**	**adjusted R^2**
DU_2_SS_1	6.4632	0.6052					
	(0.733072)	(0.000589)***					
DU_2_SS_1(P)	-6.74E+00	5.44E-01		0/30	2.22E+00	0 (16 NA)/30	0.7367
	(0.913)	(0.149)		*	(0.535)	*	
second single-unit bid	**intercept**	**teta**	**single-unit bid**	**#subjects**	**period**	**#subjects per period**	**adjusted R^2**
DU_2_SS_2	18.9572	0.5672					
	(0.212)	(7.56e-05)***					
DU_2_SS_2(P)	54.87064	-0.09605	1.06658	0/30	-3.48774	0 (16 NA)/30	0.7518
	(0.3252)	(0.7897)	(0.0773).	*	(0.3008)	*	

Table 6: Dutch-Dutch (n=2) Regressions

In regression DU_2_S_1(P) in Table 6 the price of the first unit is increasing in the cost parameter. Considering that the average price is above the equilibrium prediction. This suggests that high cost bidders let the price rise higher above 70 as compared to their low cost counterparts. Interestingly, the price for the second unit in the regression model DU_2_S_2(P) does not significantly depend on the cost draw but on the height of the price for the first unit. Especially, the last observation is in line with our explanation of the bidding behavior in the split awards of the Dutch auction with two bidders. Regarding the second unit, bidders appear to be influenced much more by the signal of their counterpart rather than by their own cost type. Eight subjects even increased the price for the second unit statistically across the periods to make even more profit (see figure 19).

In the fixed-effects regressions there are almost no independent variables with explanatory power. This can be explained by our conjecture that sole-source awards occur when bidders let the price of the first and/or the second unit rise too high. Similar to the split regression, the model DU_2_SS_2(P) describes sole-source winners who obtain two single units consecutively. The price for the second unit is influenced by the price of the first unit. If split-award winners hesitate to accept a second-unit price below the first-unit price, sole-source winners are even more likely to do so.

allocation	intercept	#subjects	period	#subjects per period	McFadden R^2		
DUSB_2_A	-6.82E-03	0/49	2.59E-01	1 (negative)/49	0.1907833		
	(0.9961)	*	(0.2414)	*			
Split Bidders							
first single-unit bid	**intercept**	**teta**		**#subjects**	**period**	**#subjects per period**	**adjusted R^2**
DUSB_2_S_W_1	51.01613	0.09005					
	(3.53e-15)***	(0.0776)					
DUSB_2_S_W_1(P)	51.05587	0.13715		34 (almost all negative)/49	0.26856	26 (almost all positive, 2 NA)/49	0.8192
	(<2e-16)***	(1.83e-07)***		*	(0.512747)	*	
second single-unit bid	**intercept**	**teta**	**single-unit bid**	**#subjects**	**period**	**#subjects per period**	**adjusted R^2**
winner of first unit	49.61836	0.32714					
DUSB_2_S_W_2	(7.18e-07)***	(7.33e-05)***					
DUSB_2_S_W_2(P)	19.79858	0.446	0.124	4 (all positive)	0.07968	6 (4 negative, 2 positive, 2 NA)/49	0.4933
	(0.254066)	(1.11e-08)***	(0.537509)	*	(0.943477)	*	
winner of second	48.96511	0.14544					
DUSB_2_S_L_2	(2.61e-11)***	(0.0127)*					
DUSB_2_S_L_2(P)	4.83E+01	1.20E-01	1.81E-01	23 (almost all positive)/49	-2.73E-01	15 (mix, 1 NA)/49	0.7046
	(1.88e-09)***	(0.002013)**	(0.014269)*	*	(0.628269)	*	
Sole-source Winners							
direct double-unit bid	**intercept**	**teta**		**#subjects**	**period**	**#subjects per period**	**adjusted R^2**
DUSB_2_SS_D	83.8373	0.4807					
	(0.00769)**	(0.07491)					
DUSB_2_SS_D(P)	-10.7642	1.3082		0/13	-0.5096	0 (8 NA)/13	0.415
	(0.8837)	(0.1137)		*	(0.8631)	*	
first single-unit bid	**intercept**	**teta**		**#subjects**	**period**	**#subjects per period**	**adjusted R^2**
DUSB_2_SS_1	22.2237	0.3737					
	(0.2648)	(0.0381)*					
DUSB_2_SS_1(P)	6.783	0.4602		0/25	1.517	0 (16 NA)/25	0.8669
	(0.9588)	(0.6678)		*	(0.2148)	*	
second single-unit bid	**intercept**	**teta**	**single-unit bid**	**#subjects**	**period**	**#subjects per period**	**adjusted R^2**
DUSB_2_SS_2	0.5297	0.6516					
	(0.974)	(4.21e-05)***					
DUSB_2_SS_2(P)	216.8512	-2.3889	2.4789	0/25	-2.8574	0 (16 NA)/25	0.9321
	(0.224)	(0.174)	(0.123)	*	(0.201)	*	

Table 7: Dutch-FPSB (n=2) Regressions

In the fixed-effects regressions for the Dutch-FPSB auction in table 7 we observe almost identical relationships between prices and explanatory variables than in the Dutch format. However, in the split-award regression models DUSB_2_S_W_1(P), DUSB_2_S_W_2(P) and DUSB_2_S_L_2(P) the cost draw has statistically significant positive influence on the dependent variable. This is not surprising as with lower prices, that are not supported by tacit collusion to the same extent as in the Dutch auction, bidders are inclined to take into account their single-unit costs. Moreover, the price for the first unit is not a reliable predictor for the bids on the second unit by winners of the first unit. As depicted in the right plot of figure 10 they bid much higher to not risk winning the respective unit and sustain the split. In the left and right plot of this figure the univariate regression lines lie within the predicted boundaries. The prices and bids of sole-source winners are illustrated in figures 11 and 12.

Note, however, that contrary to the Dutch format, 26 winners of the first unit in split-award allocations significantly increase the price for the respective unit over the number of periods as is shown in figure 21. Regarding the second unit 15 winners substantially adapt the price at which they accept although not all of them let the price rise higher. The distribution of allocations does not change over the periods as illustrated in plot 22. These observations might indicate that subjects manage to overcome the initial uncertainty with respect to the second period and eventually end up in a similar strong tacit collusion as described for the Dutch auction and even support prices above equilibrium.

single-unit bid	intercept	teta	1/(teta-60)	double-unit bid	#subjects	period	#subjects over period	adjusted R^2
FPSB_3_S	2.09512	0.32766						
	(0.753)	(5.53e-09)***						
FPSB_3_S(P)	-7.92E+01	5.75E-01	1.38E+03	2.08E-01	0/34	-8.71E-02	3 (2 positive, 1 negative)/34	0.3942
	(0.117803)	(0.044607)*	(0.142947)	(8.70e-10)***	*	(0.903771)	*	
double-unit bid	**intercept**	**teta**		**single-unit bid**	**#subjects**	**period**	**#subjects over period**	**adjusted R^2**
FPSB_3_D	17.3928	0.9504						
	(0.243)	(8.1e-14)***						
FPSB_3_D(P)	17.35231	0.7494		0.39953	2 (all positive)/34	0.05135	3 (all positive)/34	0.7784
	(0.1358)	(<2e-16)***		(7.05e-10)***	*	(0.95894)	*	

Table 8: FPSB (n=3) Regressions

first single-unit bid	intercept	teta	single-unit bid	#subjects	period	#subjects per period	adjusted R^2
DU_3_1	26.9637	0.10899					
	(<2e-16)***	(5.24e-14)***					
DU_3_1(P)	26.12313	0.092455		23 (all positive)/47	0.229195	12 (all negative, 6 NA)/47	0.6826
	(<2e-16)***	(1.09e-14)***		*	(0.028266)*	*	
second single-unit bid	**intercept**	**teta**	**single-unit bid**	**#subjects**	**period**	**#subjects per period**	**adjusted R^2**
DU_3_2	25.18748	0.12198					
	(<2e-16)***	(<2e-16)***					
DU_3_2(P)	15.03993	0.120899	0.253872	3 (3 positive)/48	0.023405	2 (all negative, 3 NA)/48	0.6084
	(9.50e-08)***	(<2e-16)***	(2.09e-05)***	*	(0.84849)	*	

Table 9: Dutch-Dutch (n=3) Regressions

first single-unit bid	intercept	teta		#subjects	period	#subjects over period	adjusted R^2
DUSB_3_W_1	18.61513	0.16846					
	(<2e-16)***	(<2e-16)***					
DUSB_3_W_1(P)	17.888482	0.185402		10 (all positive)/49	-0.20185	8 (all positive, 2 NA)/49	0.7758
	(<2e-16)***	(<2e-16)***		*	(0.03235)*	*	
second single-unit bid	**intercept**	**teta**	**single-unit bid**	**#subjects**	**period**	**#subjects over period**	**adjusted R^2**
winner of first unit	45.426	0.4004					
DUSB_3_W_2	(0.0126)*	(0.0123)*					
DUSB_3_W_2(P)	8.6789	0.32287	1.08283	9 (7 positive, 2 negative)/49	-0.39624	7 (4 positive, 3 negative, 2 NA)/49	0.8237
	(0.651303)	(0.052737).	(0.148866)	*	(0.623098)	*	
winner and loser of second	7.2542	0.28409					
DUSB_3_L_2	(0.238)	(2.28e-08)***					
DUSB_3_L_2(P)	-5.794843	0.26957	0.282631	7 (all positive)/46	-0.042096	6 (1 positive, 5 negative)/46	0.5344
	(0.584099)	(6.51e-10)***	(0.161048)	*	(0.956202)	*	

Table 10: Dutch-FPSB (n=3) Regressions

For the split award with three bidders the fixed-effects regressions in tables 8 to 10 contain a cost parameter with significant explanatory power that is positively correlated with the corresponding dependent variable. Also, in regressions FPSB_3_S(P) and FPSB_3_D(P) the height of the single- and double-unit bid significantly depends on the height of the bid for two and one units, respectively. The lower a bidder's competitive bid for one unit the more willing he is to submit a low bid on two units and vice versa as would be expected in a competitive outcome.

Interestingly, in panel regression DU_3_2(P) for the second unit in the Dutch auction the price of the first unit still contains high explanatory power whereas this is not the case for the respective regression DUSB_3_L_2(P) in the Dutch-FPSB format. To explain this observation note that in the Dutch auction 12 subjects significantly decrease the price at which they accept the first unit over the periods. It appears that if some bidders accepted the first unit at a higher price in the earlier periods then the price of the second unit was also accepted at higher prices. This connection then vanishes with more periods. In contrast, in the Dutch-FPSB format eight bidders let the price at which they accept the first unit rise significantly higher with more periods. These bidders have accepted the first unit at rather low prices in early periods which did not constitute a relevant signal for the price of the second unit. Generally, bidder behavior is rather constant over periods as is depicted in plots 23 to 25.

B.2.2 *Figures*

In the plots of bids and prices against cost draws in Figures 9 to 25 we include the solid univariate regression lines and the dashed equilibrium strategies. If there are no significant fixed and period effects in the linear regressions of bids and prices the univariate regression on the cost draws helps to visualize the subjects' underlying bidding behavior.

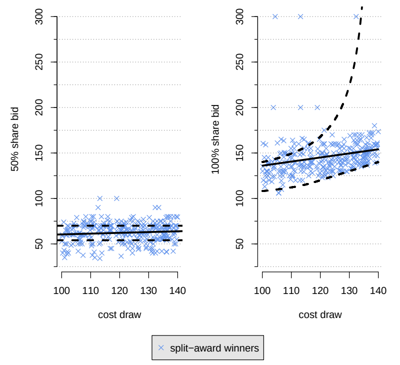

Figure 5: Bids of Split-Award Winners in FPSB (n=2)

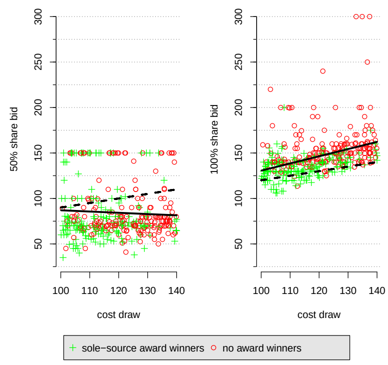

Figure 6: Bids of Sole-Source-Award Winners in FPSB (n=2)

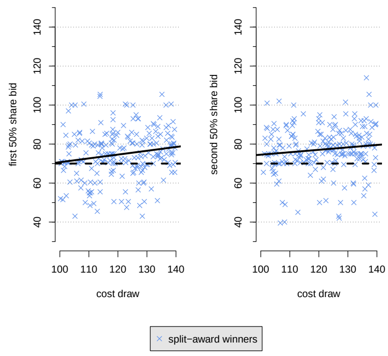

Figure 7: Bids of Split-Award Winners in Dutch-Dutch (n=2)

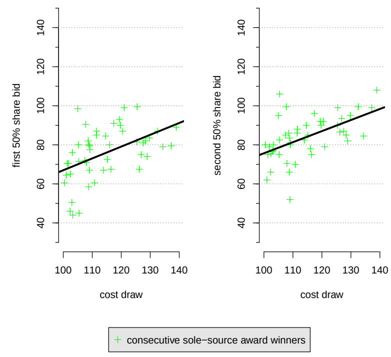

Figure 8: Bids of Consecutive Sole-Source-Award Winners in Dutch-Dutch (n=2)

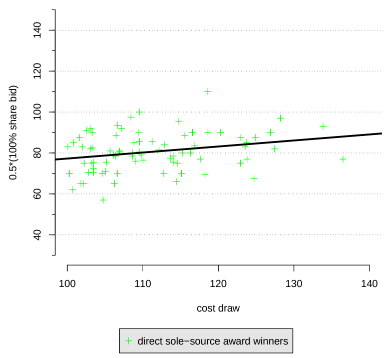

Figure 9: Bids of Direct Sole-Source-Award Winners in Dutch-Dutch (n=2)

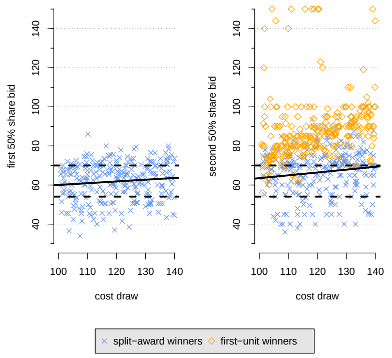

Figure 10: Bids of Split-Award Winners in Dutch-FPSB (n=2)

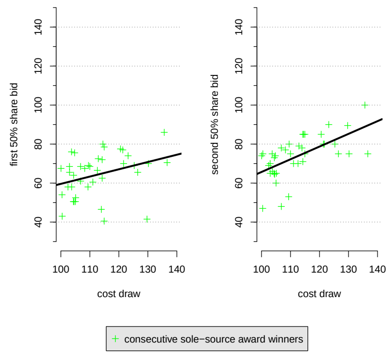

Figure 11: Bids of Consecutive Sole-Source-Award Winners in Dutch-FPSB (n=2)

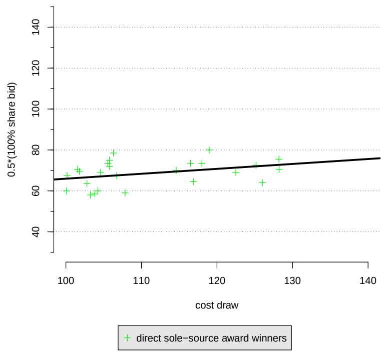

Figure 12: Bids of Direct Sole-Source-Award Winners in Dutch-FPSB (n=2)

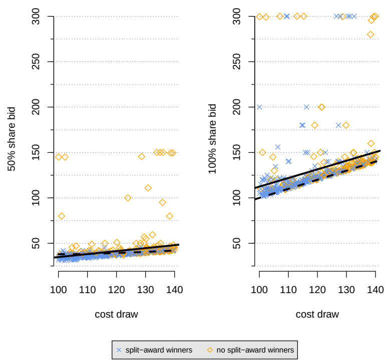

Figure 13: Bids of Split-Award Bidders in FPSB (n=3)

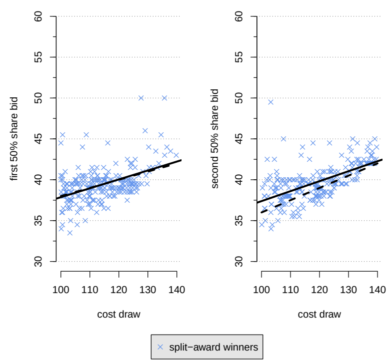

Figure 14: Bids of Split-Award Bidders in Dutch-Dutch (n=3)

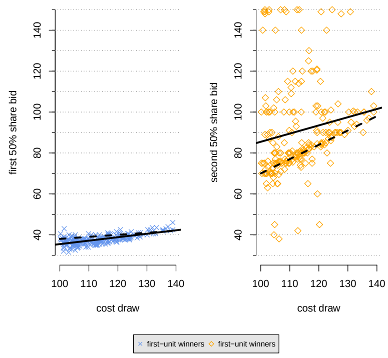

Figure 15: Bids of First-Unit Split-Award Winners in Dutch-FPSB (n=3)

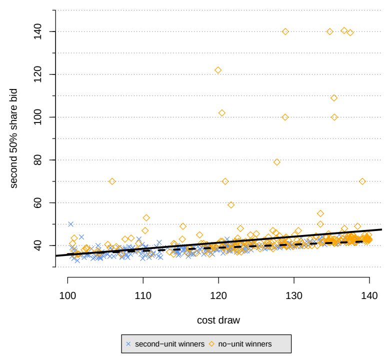

Figure 16: Bids of Second-Unit Split-Award Winners in Dutch-FPSB (n=3)

B.2.3 *Period Plots*

In the following plots of bids and prices against periods the solid black line represents the average bid/price in period 15 and the solid grey line depicts the average bid/price over all periods. For the two-bidder treatments we included dashed lines for the range of constant pooling prices. Furthermore, for the treatments with two bidders we also added plots of the distribution of allocations against periods.

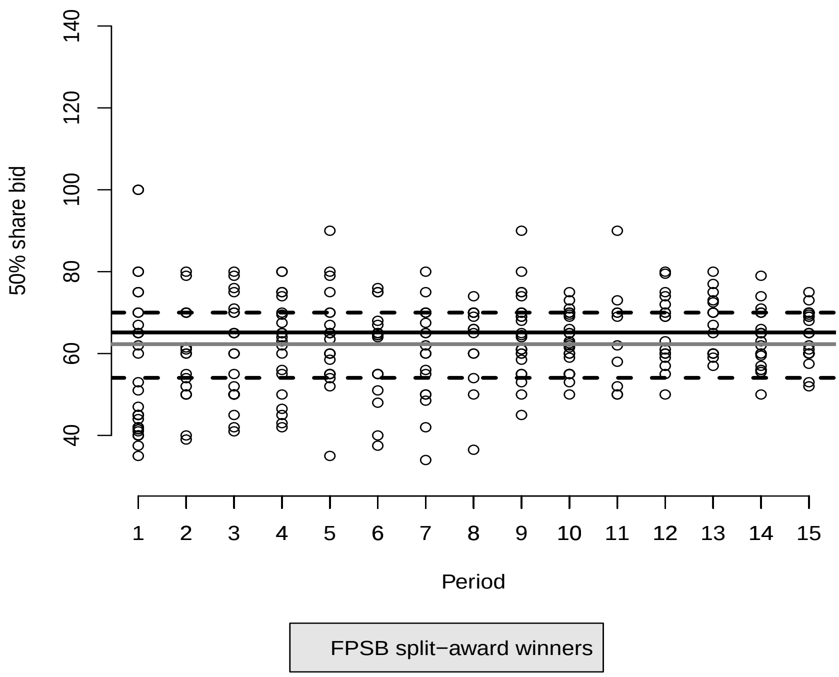

Figure 17: Bids of Split-Award Winners in FPSB (n=2)

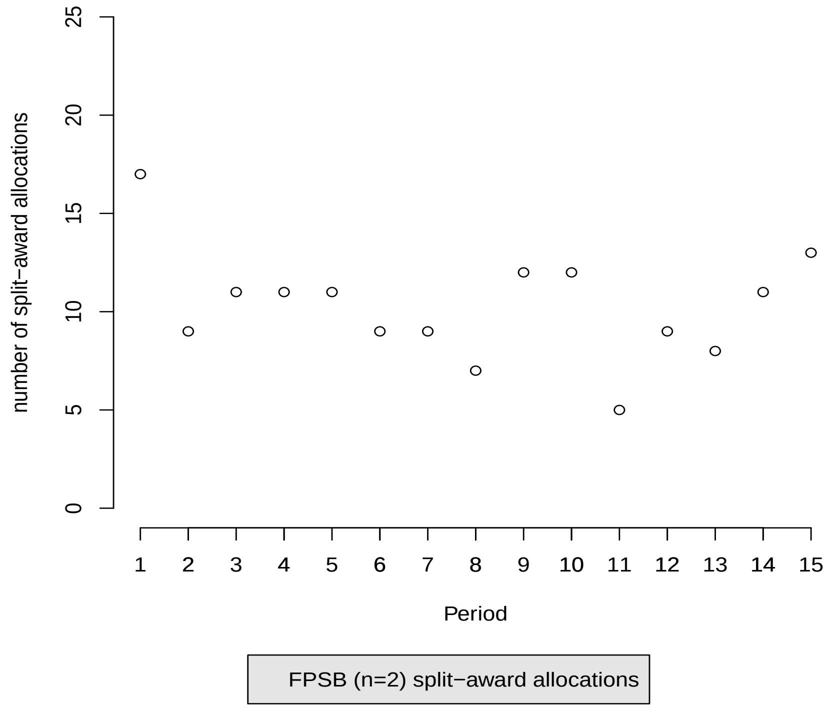

Figure 18: Distribution of allocations in FPSB (n=2)

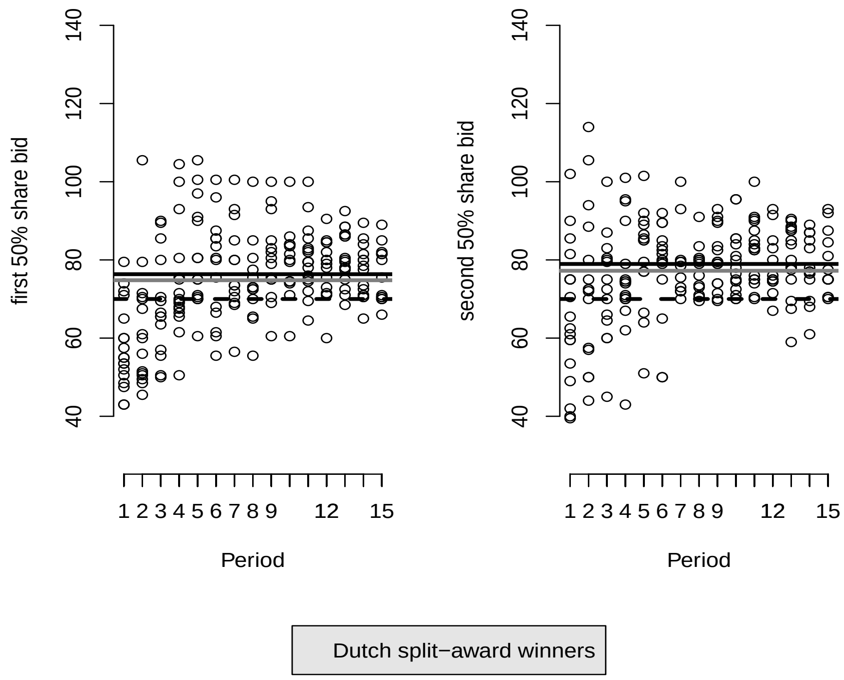

Figure 19: Prices of Split-Award Winners in Dutch-Dutch (n=2)

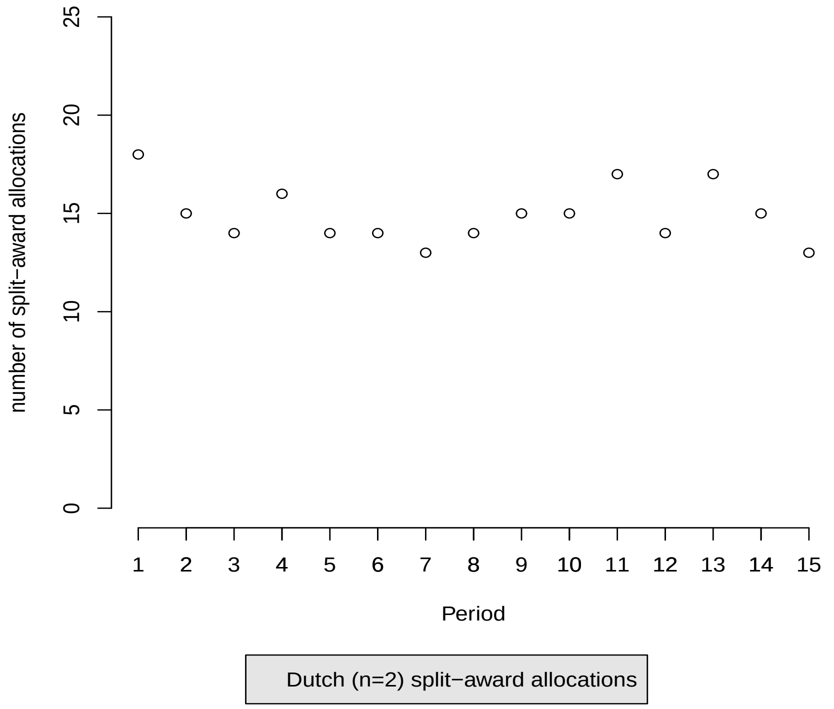

Figure 20: Distribution of allocations in Dutch (n=2)

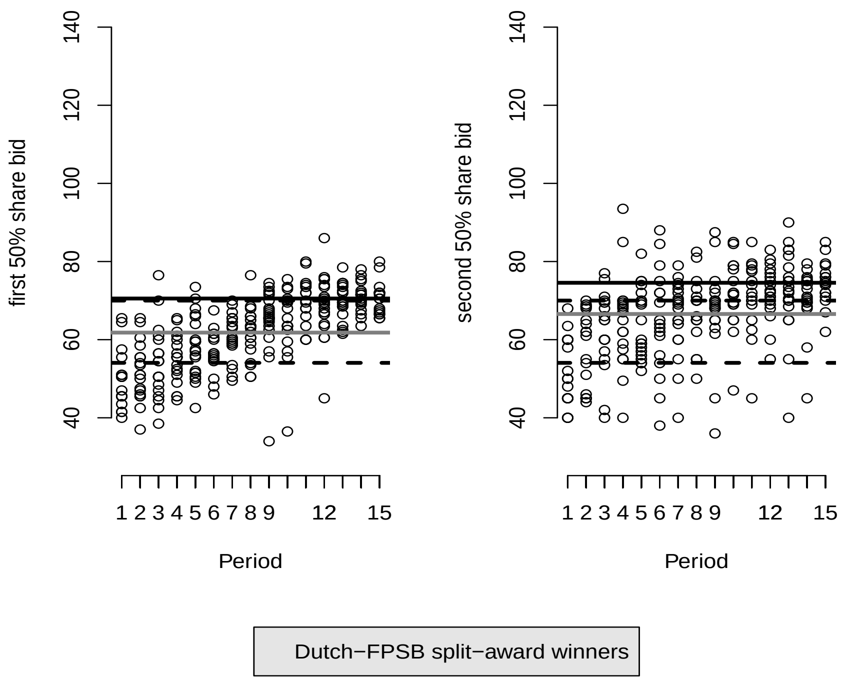

Figure 21: Prices of Split-Award Winners in Dutch-FPSB (n=2)

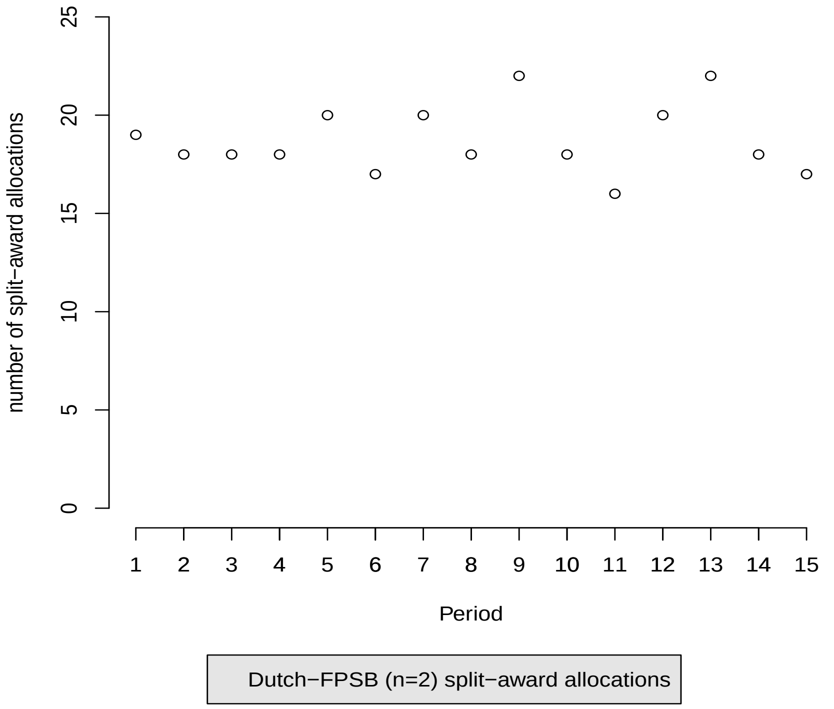

Figure 22: Distribution of allocations in Dutch-FPSB (n=2)

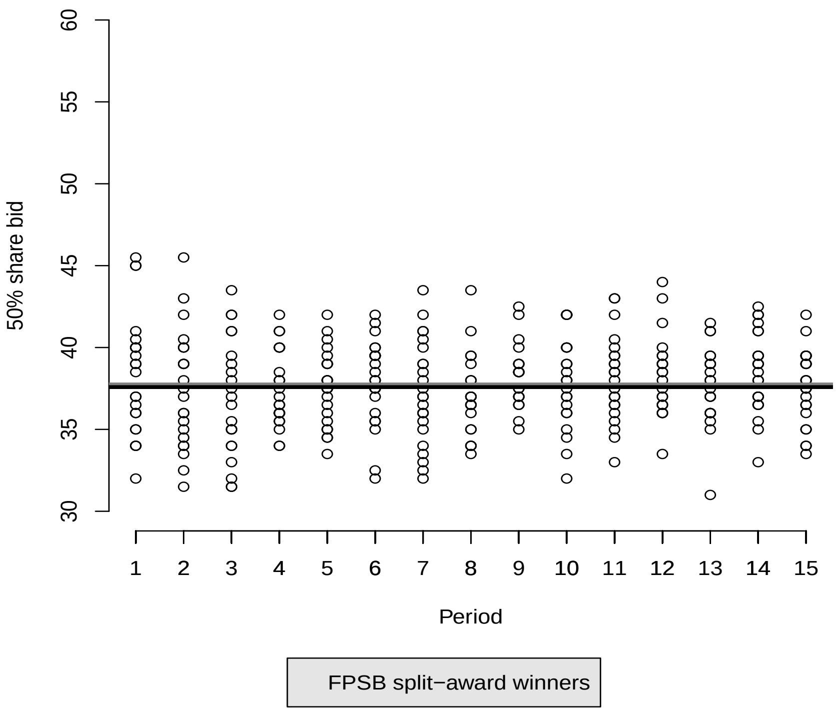

Figure 23: Bids of Split-Award Winners in FPSB (n=3)

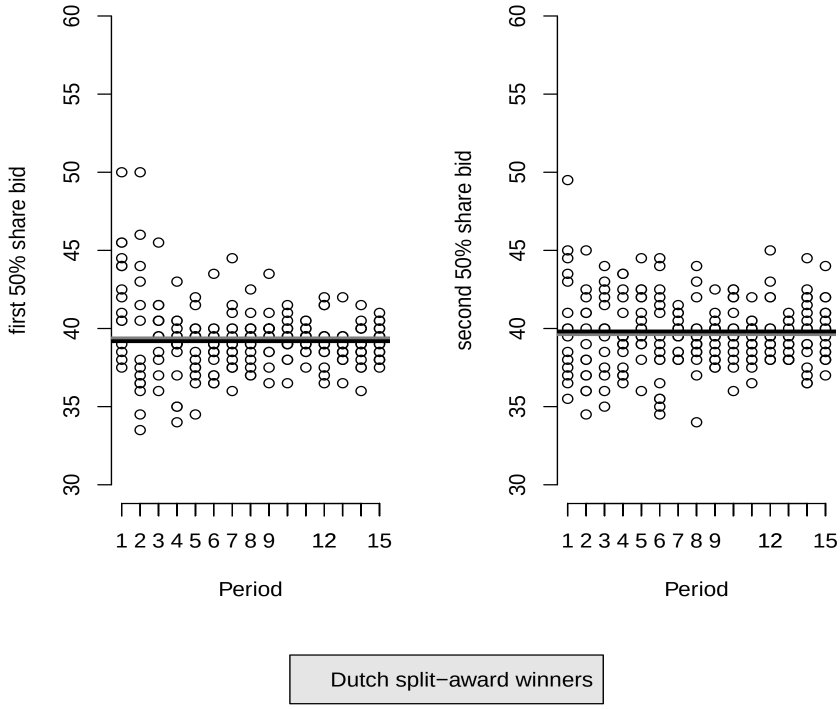

Figure 24: Prices of Split-Award Winners in Dutch-Dutch (n=3)

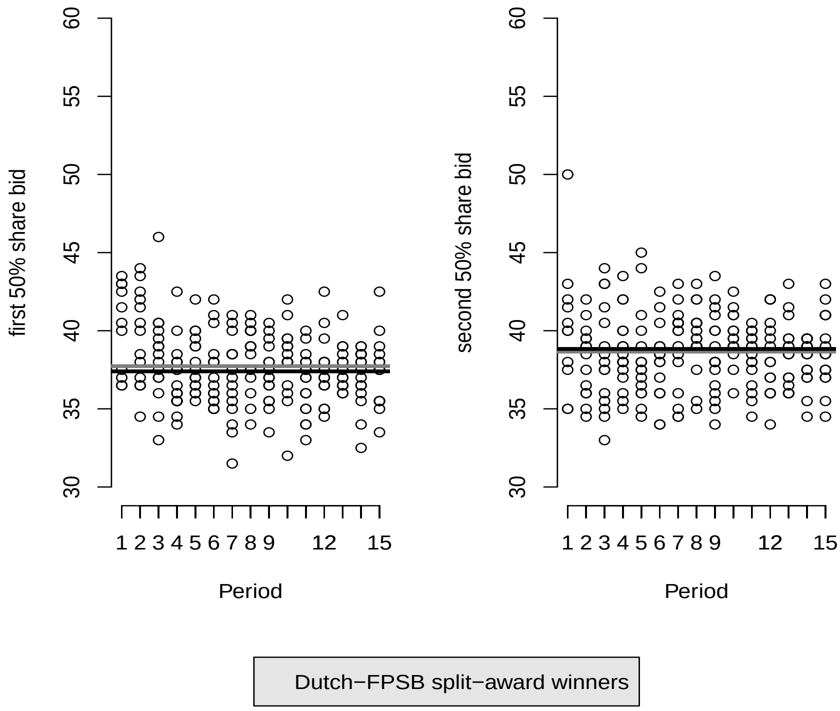

Figure 25: Prices of Split-Award Winners in Dutch-FPSB (n=3)

APPENDIX C

The contributions of the authors Martin Bichler and Per Paulsen to the publication Bichler and Paulsen (2018), and of Martin Bichler, Gian-Marco Kokott and Per Paulsen to the working paper Kokott et al. (2018a) are explained in the two following sections.

C.1 CONTRIBUTIONS TO BICHLER AND PAULSEN (2018) IN CHAPTER 3

The main contribution of Martin Bichler to Bichler and Paulsen (2018) was the motivation of the topic in Section 3.1, placing it into the existing literature in Section 3.2 and validating the obtained insights with examples from practice in Section 3.9. Per Paulsen focused on the formulation of the principal-agent problem in Section 3.3 as well as its solution for different multi-unit auction formats in Section A.1 in close correspondence with Martin Bichler. Most of the publication was written in an iterative cooperative process by both authors.

Martin Bichler

Per Paulsen

Per Paulsen

C.2 CONTRIBUTIONS TO KOKOTT ET AL. (2018A) IN CHAPTER 4

Martin Bichler's primary contribution to Kokott et al. (2018a) was the motivation of the topic in Section 4.1 as well as the interpretation of the obtained results within the context of the existing literature in Sections 4.2 and 4.10. Gian-Marco Kokott's principal contribution was the definition of the ascending auction formats in Section 4.4 and the derivation of equilibrium strategies in Section B.1 as well as the welfare analyses of Sections 4.7 and 4.8 in close consultation with Martin Bichler and Per Paulsen. The latter author implemented and evaluated the laboratory experiments in Section 4.9 in close correspondence with Martin Bichler and Gian-Marco Kokott. The experiments were jointly conducted by Per Paulsen and Gian-Marco Kokott at the experimenTUM, the laboratory for experimental economic studies of the Technical University of Munich. The working paper was written in an alternating concerted procedure by all three authors.

Martin Bichler

Gian-Marco Kokott

Per Paulsen