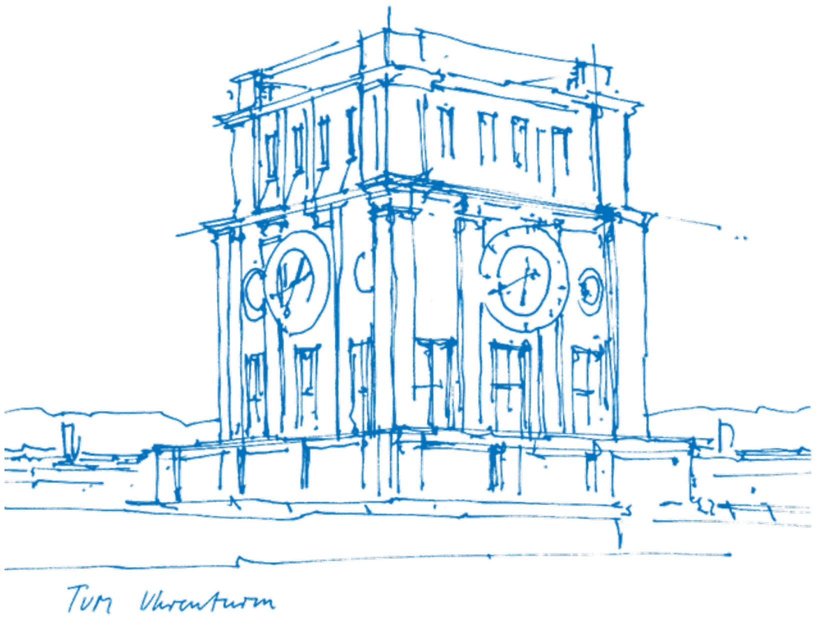
Improving Time Stepping in Partitioned Multi-Physics

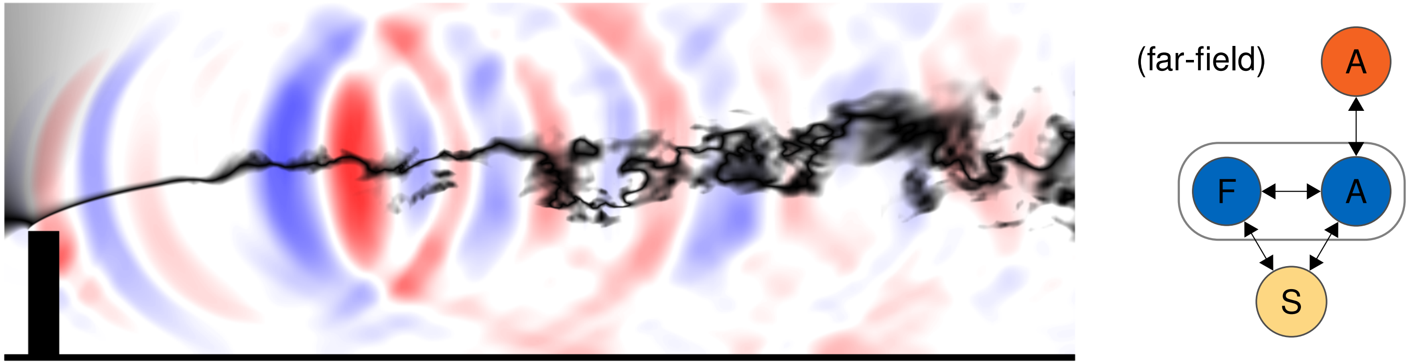
Benjamin Rüth, Benjamin Uekermann

Technical University of Munich
Department of Informatics
Chair of Scientific Computing

89th GAMM Annual Meeting
Technical University of Munich
20. March 2018



Fluid-Structure-Acoustics

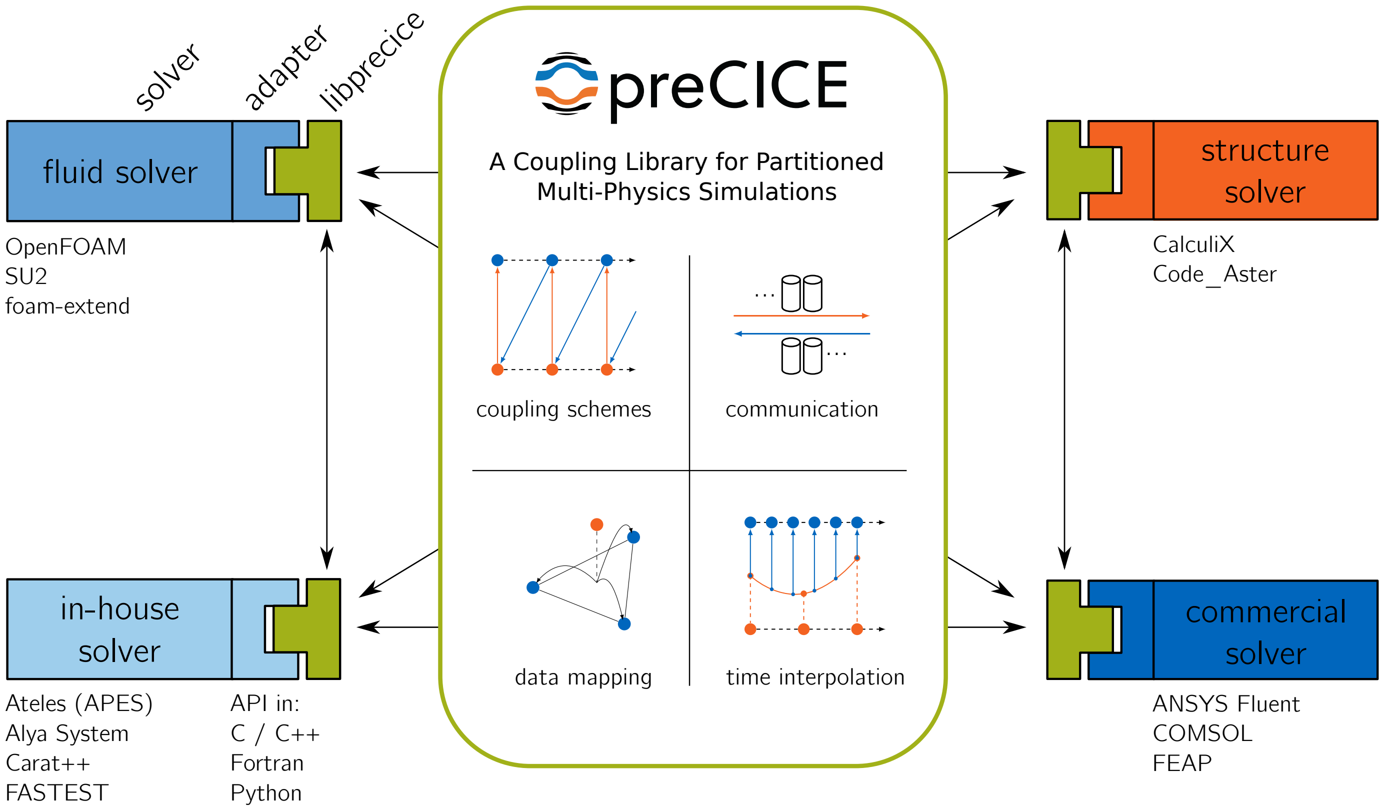


Fluid-Structure-Acoustics simulation and partitioned setup[1].

physics	timescale	solver	scheme	order
(A)	small	Ateles	RK	2 or 4
(A)	small	FASTEST	EE	1
(F)	medium	FASTEST	CN	2
(S)	large	FEAP	N-β	1 or 2

[1] Reimann, T., et al. (2017). Aspects of FSI with aeroacoustics in turbulent flow. In 7th GACM Colloquium on Computational Mechanics.

preCICE[1]



[1] Bungartz, H.-J., et al. (2016). preCICE – A fully parallel library for multi-physics surface coupling. https://doi.org/10.1016/j.compfluid.2016.04.003

preCICE at GAMM

preCICE Coupling Library for Multi-Physics Simulation

Amin Totounferoush, University of Stuttgart S07.01 Coupled Problems
(today in the morning)

Quasi-Newton – A Universal Approach for Coupled Problems and Optimization

Miriam Mehl, University of Stuttgart S07.01 Coupled Problems
(just now)

Multi-physics simulations with OpenFOAM through preCICE

Gerasimos Chourdakis, Technical University of Munich S22.01 Scientific Computing
(Thursday morning)

Improving Time Stepping in Partitioned Multi-Physics

Requirements

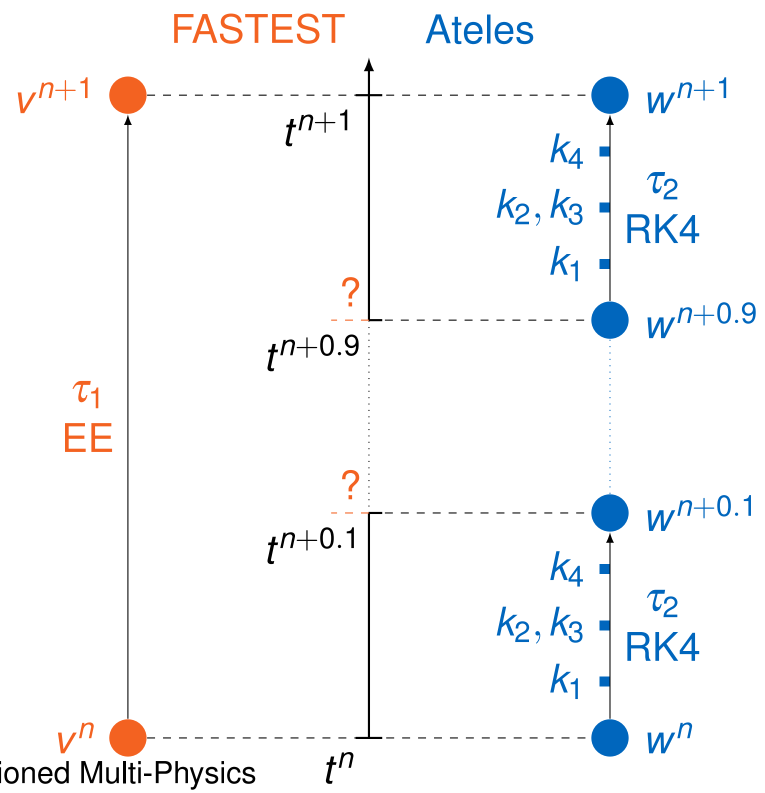
Engineering:

- use different solvers
- use different discretization
- no degradation of solver performance

Informatics:

- black-box approach
- parallel

Multi-Scale Multi-Physics

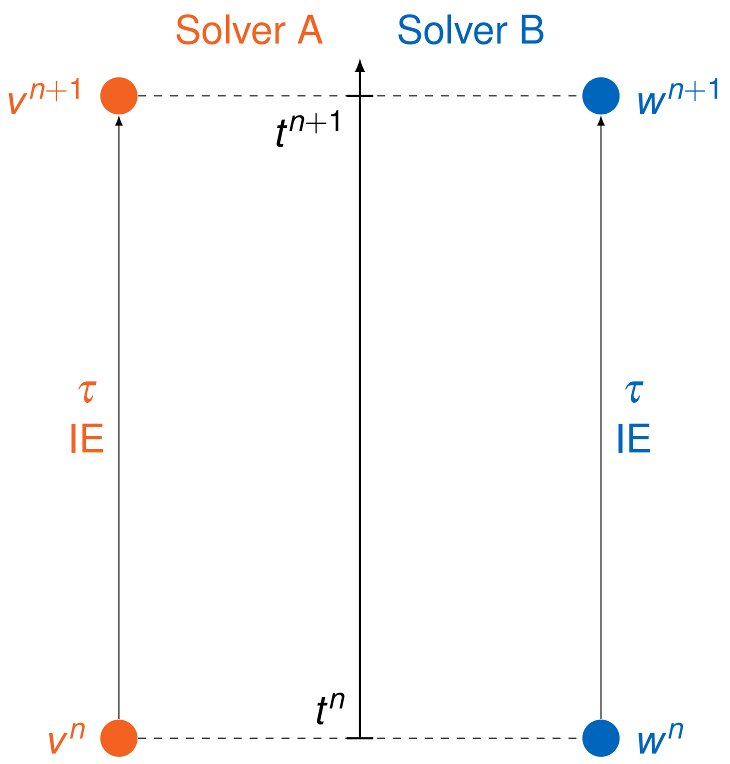


Outline

Partitioned Heat Transport Equation

- introduce the partitioned heat transport equation example
- introduce classical and advanced coupling schemes
- show deficits of classical explicit and implicit coupling schemes
- show advantages of waveform relaxation coupling scheme

Simple setup



Reference Solution: The Monolithic Setup

Heat Transport equation

$$\frac{\partial u(x,t)}{\partial t} = \alpha \frac{\partial^2 u(x,t)}{\partial x^2}, x \in \Omega, t \in \mathbb{R}^+$$

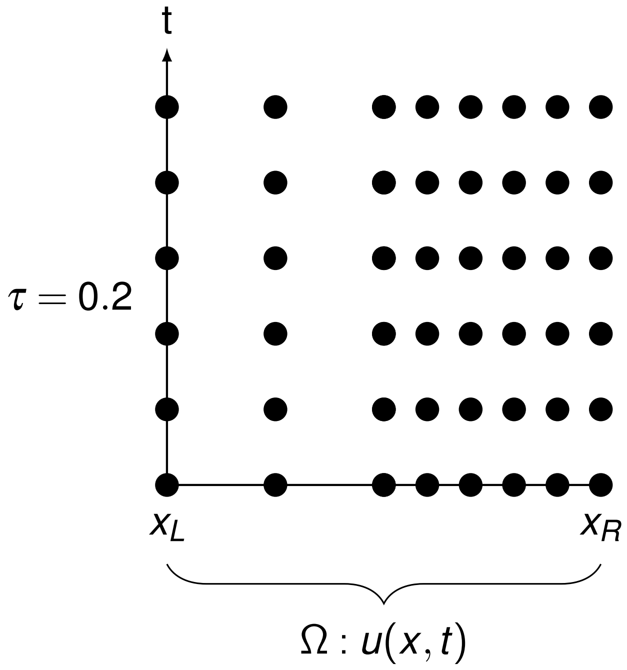
Dirichlet boundary conditions

$$u(x = x_L, t) = u_L^D, u(x = x_R, t) = u_R^D$$

Initial condition

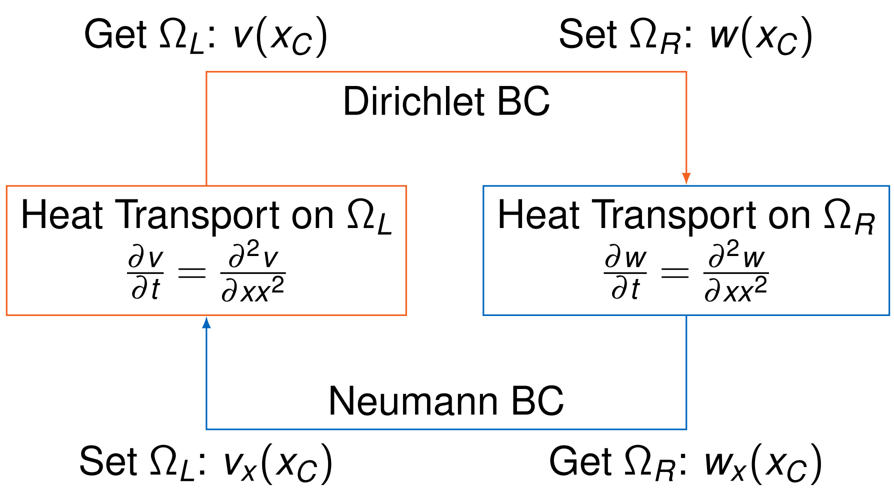
$$u(x, t = 0) = u_0(x)$$

Discretization

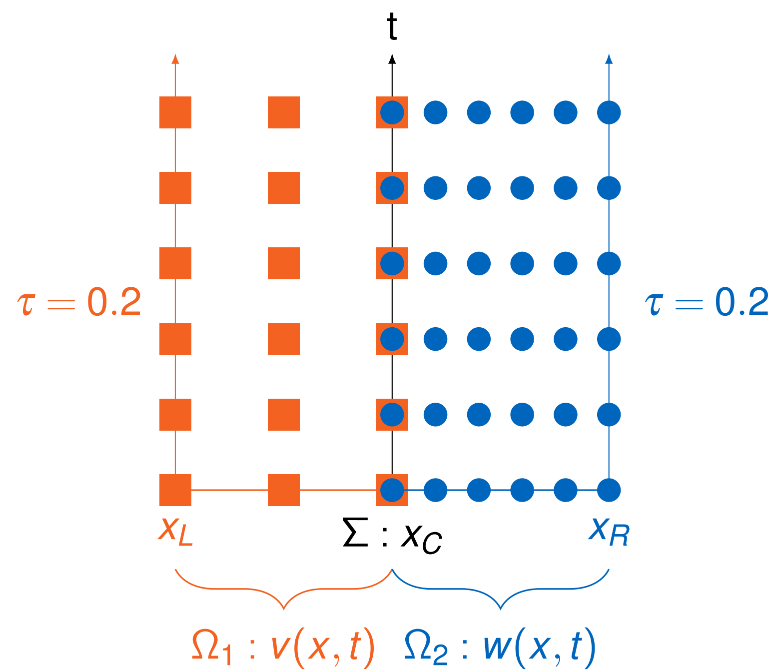


The Partitioned Setup

Dirichlet-Neumann coupling

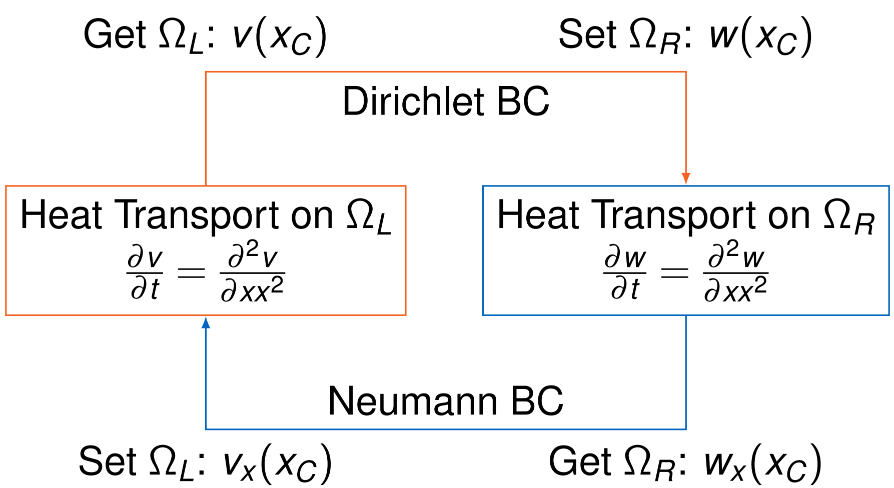


Partitioning

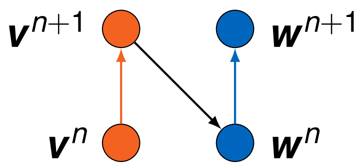


Classical Coupling Schemes

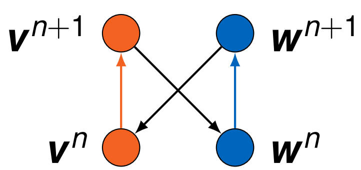
Dirichlet-Neumann coupling



Explicit coupling

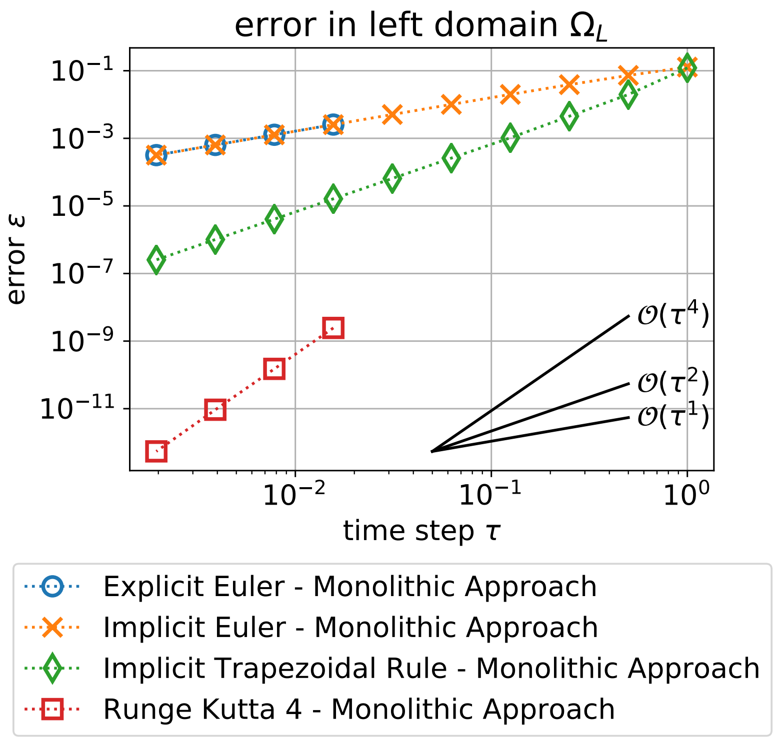


Implicit coupling



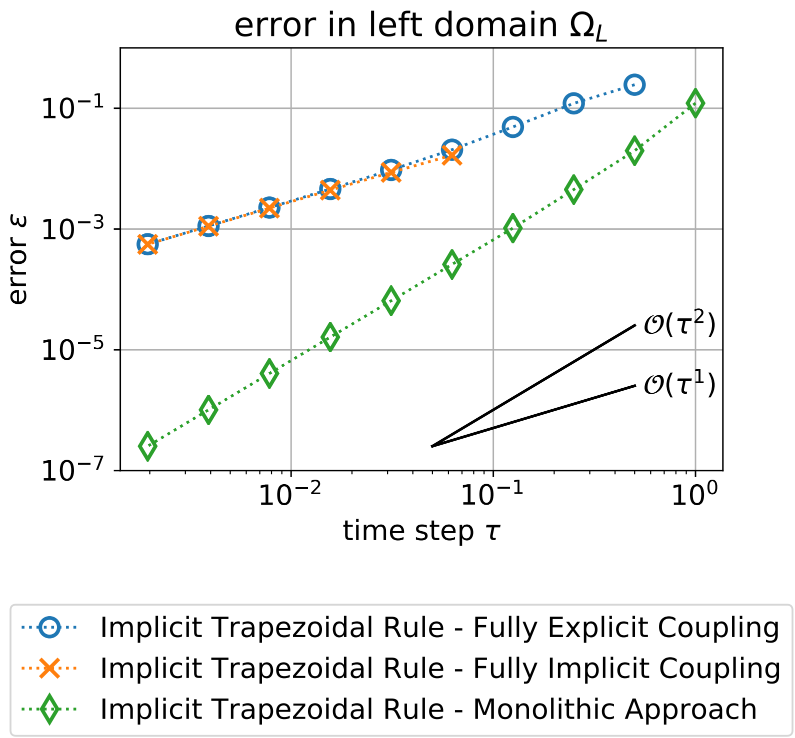
Convergence order in time

- use constant spatial meshwidth h
- refine temporal meshwidth τ
- compare to monolithic reference solution $\boldsymbol{u}^n$ with fine τ



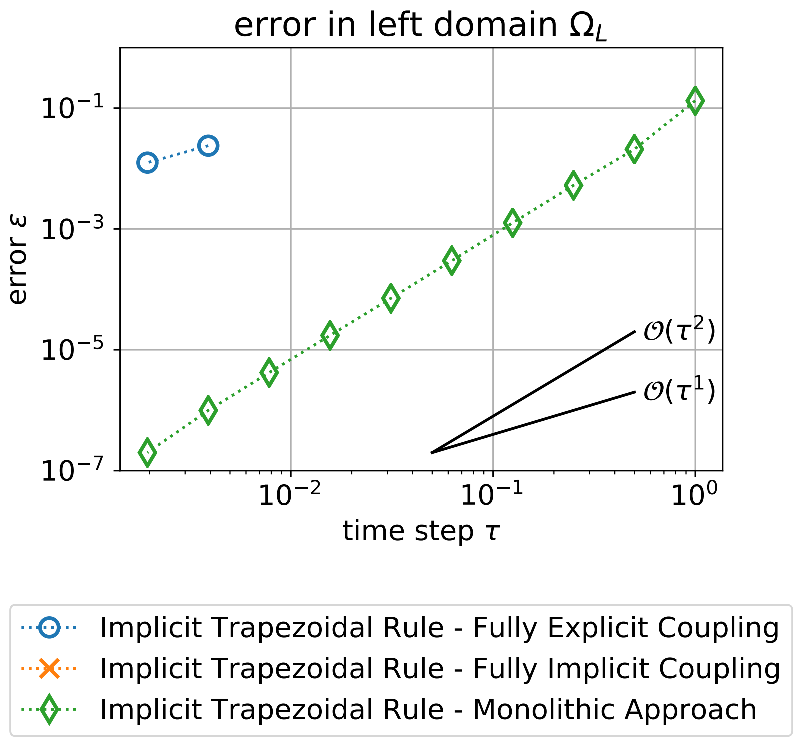
Order Degradation: Trapezoidal rule

- order reduced to $\mathcal{O}(\tau)$
- $h = 0.2$
- stability problems for Fully implicit coupling



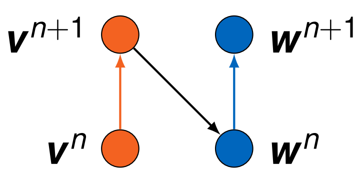
Order Degradation: Trapezoidal rule

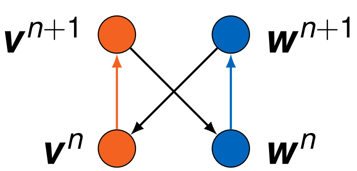
- order reduced to $\mathcal{O}(\tau)$
- $h = 0.01$
- stability problems for Fully implicit coupling
- stability problems for Fully explicit coupling



Semi Implicit-Explicit Coupling

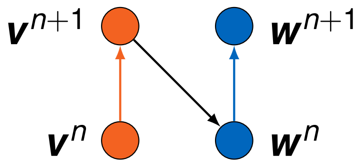
	update scheme	stability	order
fully explicit	$\mathbf{v}^{n+1} = \mathbf{v}^n + \frac{\tau}{2}\left[f_v(\mathbf{v}^n, t_n) + f_v(\mathbf{v}^{n+1}, t_{n+1})\right]$ $\mathbf{w}^{n+1} = \mathbf{w}^n + \frac{\tau}{2}\left[f_w(\mathbf{w}^n, t_n) + f_w(\mathbf{w}^{n+1}, t_{n+1})\right]$	depends on τ	$\mathcal{O}(\tau)$
fully implicit	$\mathbf{v}^{n+1} = \mathbf{v}^n + \frac{\tau}{2}\left[f_v(\mathbf{v}^n, t_n) + f_v(\mathbf{v}^{n+1}, t_{n+1})\right]$ $\mathbf{w}^{n+1} = \mathbf{w}^n + \frac{\tau}{2}\left[f_w(\mathbf{w}^n, t_n) + f_w(\mathbf{w}^{n+1}, t_{n+1})\right]$	depends on τ	$\mathcal{O}(\tau)$

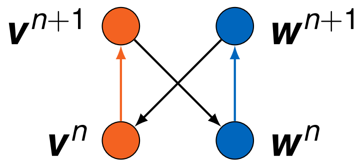




Semi Implicit-Explicit Coupling

	update scheme	stability	order
fully explicit	$\mathbf{v}^{n+1} = \mathbf{v}^n + \frac{\tau}{2}\left[f_v(\mathbf{v}^n, t_n, c_n) + f_v(\mathbf{v}^{n+1}, t_{n+1}, c_n)\right]$ $\mathbf{w}^{n+1} = \mathbf{w}^n + \frac{\tau}{2}\left[f_w(\mathbf{w}^n, t_n, c_{n+1}) + f_w(\mathbf{w}^{n+1}, t_{n+1}, c_{n+1})\right]$	depends on τ	$\mathcal{O}(\tau)$
fully implicit	$\mathbf{v}^{n+1} = \mathbf{v}^n + \frac{\tau}{2}\left[f_v(\mathbf{v}^n, t_n, c_{n+1}) + f_v(\mathbf{v}^{n+1}, t_{n+1}, c_{n+1})\right]$ $\mathbf{w}^{n+1} = \mathbf{w}^n + \frac{\tau}{2}\left[f_w(\mathbf{w}^n, t_n, c_{n+1}) + f_w(\mathbf{w}^{n+1}, t_{n+1}, c_{n+1})\right]$	depends on τ	$\mathcal{O}(\tau)$



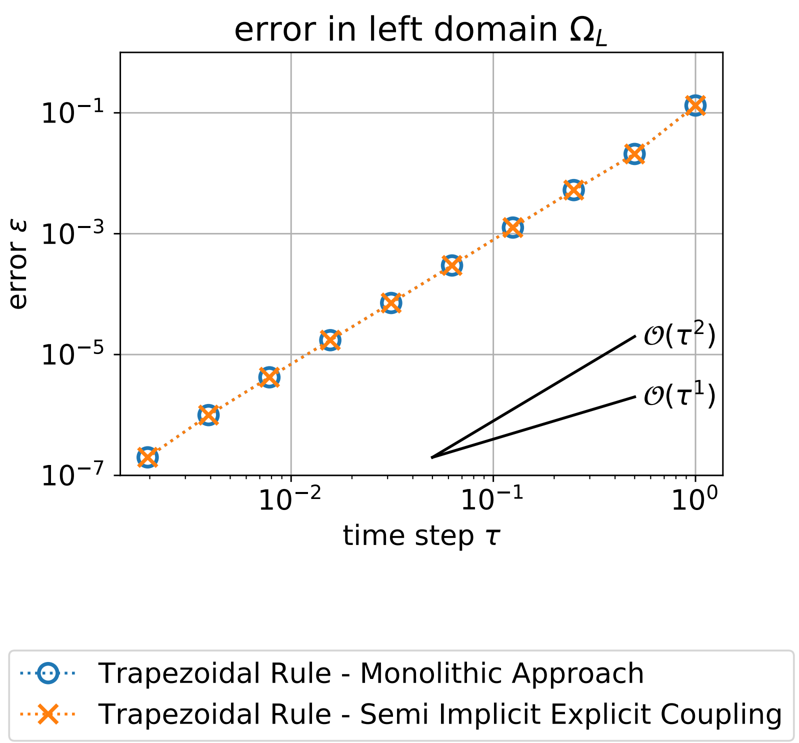


Semi Implicit-Explicit Coupling

	update scheme	stability	order
fully explicit	$\mathbf{v}^{n+1} = \mathbf{v}^n + \frac{\tau}{2}\left[f_v(\mathbf{v}^n, t_n, c_n) + f_v(\mathbf{v}^{n+1}, t_{n+1}, c_n)\right]$ $\mathbf{w}^{n+1} = \mathbf{w}^n + \frac{\tau}{2}\left[f_w(\mathbf{w}^n, t_n, c_{n+1}) + f_w(\mathbf{w}^{n+1}, t_{n+1}, c_{n+1})\right]$	depends on τ	$\mathcal{O}(\tau)$
fully implicit	$\mathbf{v}^{n+1} = \mathbf{v}^n + \frac{\tau}{2}\left[f_v(\mathbf{v}^n, t_n, c_{n+1}) + f_v(\mathbf{v}^{n+1}, t_{n+1}, c_{n+1})\right]$ $\mathbf{w}^{n+1} = \mathbf{w}^n + \frac{\tau}{2}\left[f_w(\mathbf{w}^n, t_n, c_{n+1}) + f_w(\mathbf{w}^{n+1}, t_{n+1}, c_{n+1})\right]$	depends on τ	$\mathcal{O}(\tau)$
semi explicit-implicit	$\mathbf{v}^{n+1} = \mathbf{v}^n + \frac{\tau}{2}\left[f_v(\mathbf{v}^n, t_n, c_n) + f_v(\mathbf{v}^{n+1}, t_{n+1}, c_{n+1})\right]$ $\mathbf{w}^{n+1} = \mathbf{w}^n + \frac{\tau}{2}\left[f_w(\mathbf{w}^n, t_n, c_n) + f_w(\mathbf{w}^{n+1}, t_{n+1}, c_{n+1})\right]$	???	???

Higher Order: Semi Implicit-Explicit Coupling

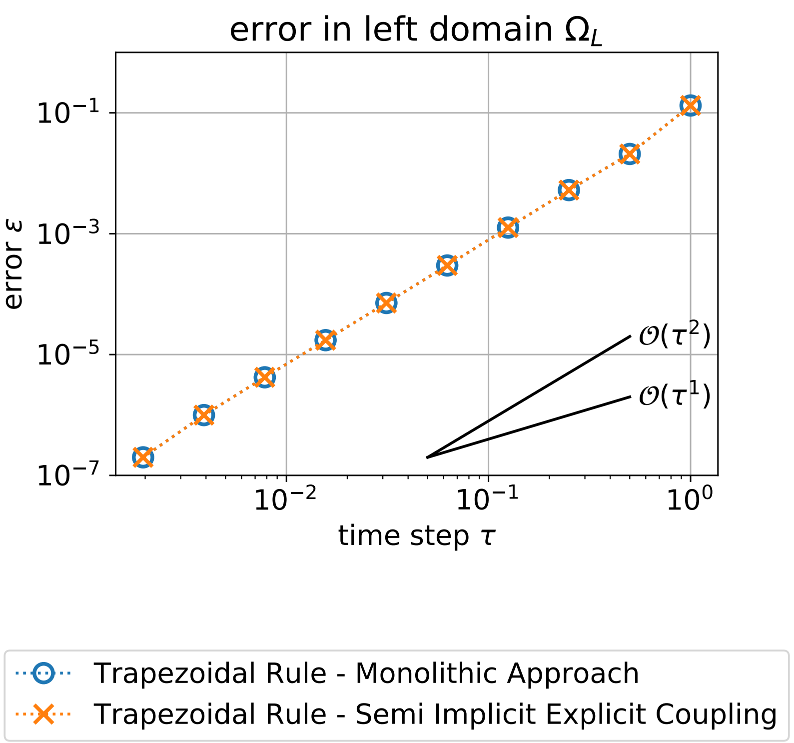
- order $\mathcal{O}\left(\tau^2\right)$ maintained for semi explicit-implicit coupling
- no stability problems for semi explicit-implicit coupling



Higher Order: Semi Implicit-Explicit Coupling

- order $\mathcal{O}\left(\tau^2\right)$ maintained for semi explicit-implicit coupling
- no stability problems for semi explicit-implicit coupling

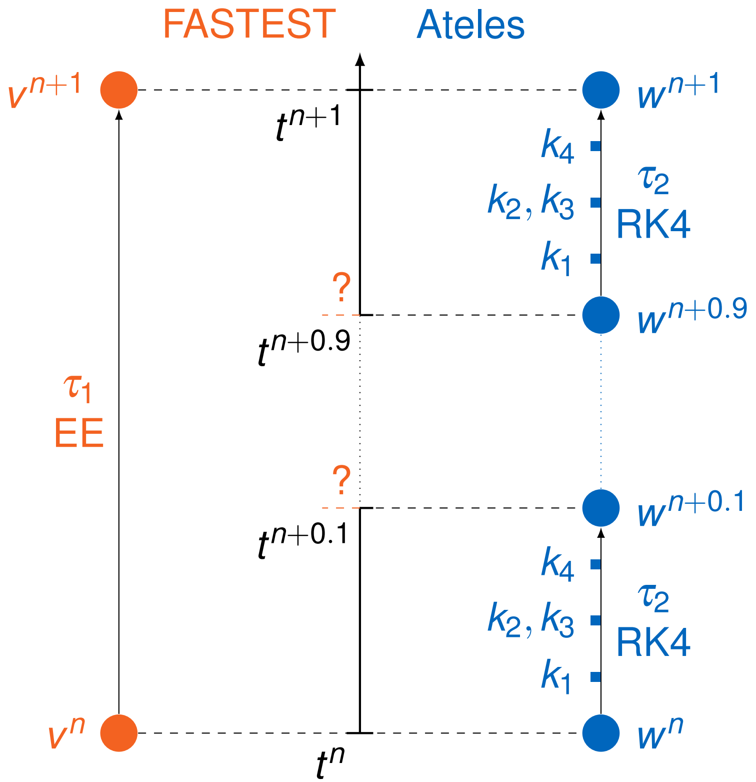
	stability	order
fully explicit	depends on τ	$\mathcal{O}\left(\tau\right)$
fully implicit	depends on τ	$\mathcal{O}\left(\tau\right)$
semi explicit-implicit	unconditionally	$\mathcal{O}\left(\tau^2\right)$



Intermediate Summary

Semi-Explicit-Implicit	Goal
identical timesteps	subcycling
simple schemes	substepping
identical solvers	inhomogeneous setup
$\mathcal{O}(\tau^2)$	Higher order
taylored schemes	general solution strategy

Multi-Scale Multi-Physics



What is Waveform Relaxation?

Background Information

Algorithm[1]

We want to solve the coupled problem

$$F_v(v, c) = 0, \; F_w(w, c) = 0.$$

with v, w, c known for $t < t_n$ on the window $T_n = [t_n, t_{n+1}]$.

1. set $k = 0$ and extrapolate $c^0(t) = c_n$ for $t \in T$
2. solve decoupled F_v, F_w using c^k to obtain v^{k+1}, w^{k+1} for $t \in T$
3. use v^{k+1}, w^{k+1} to obtain c^{k+1}
4. if not converged:
   a. set $k = k + 1$ and go to step 2,
   b. otherwise proceed to next window T_{n+1}

[1] Adapted from *Maciejewski, M., et al. (2017). Application of the Waveform Relaxation Technique to the Co-Simulation of Power Converter Controller and Electrical Circuit Models. https://doi.org/10.1109/MMAR.2017.8046937*

Waveform Relaxation (WR) Coupling Scheme

WR with our example

- Semi-Explicit-Implicit coupling equals WR with linear interpolation of

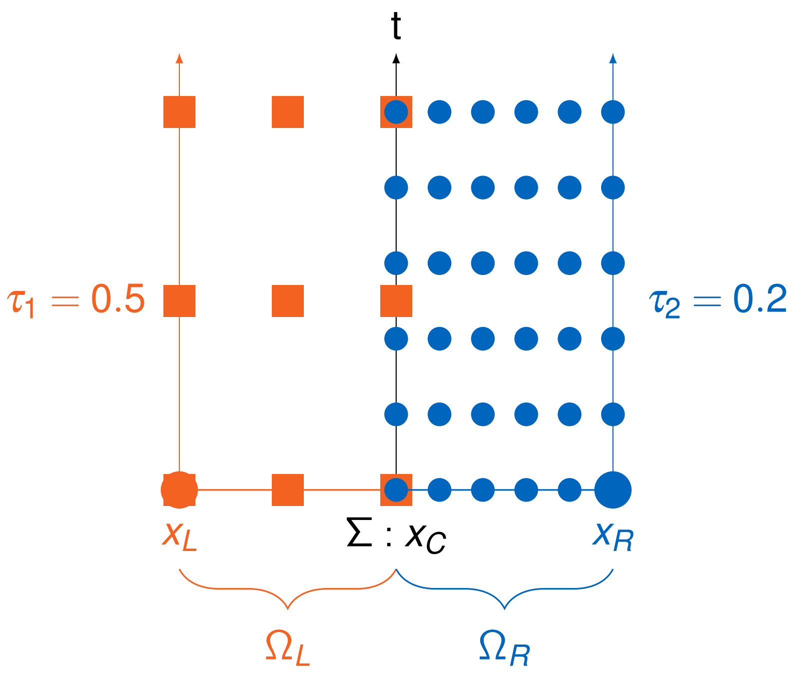
$$c^k(t) = \frac{c_n(t_{n+1} - t)}{\tau} + \frac{c^k_{n+1}(t - t_n)}{\tau}.$$

- Semi-Explicit-Implicit coupling:

$$\boldsymbol{v}^{n+1} = \boldsymbol{v}^n + \frac{\tau}{2}\left[f_v(c^k(t_n)) + f_v(c^k(t_{n+1}))\right]$$
$$\boldsymbol{w}^{n+1} = \boldsymbol{w}^n + \frac{\tau}{2}\left[f_w(c^k(t_n)) + f_w(c^k(t_{n+1}))\right]$$

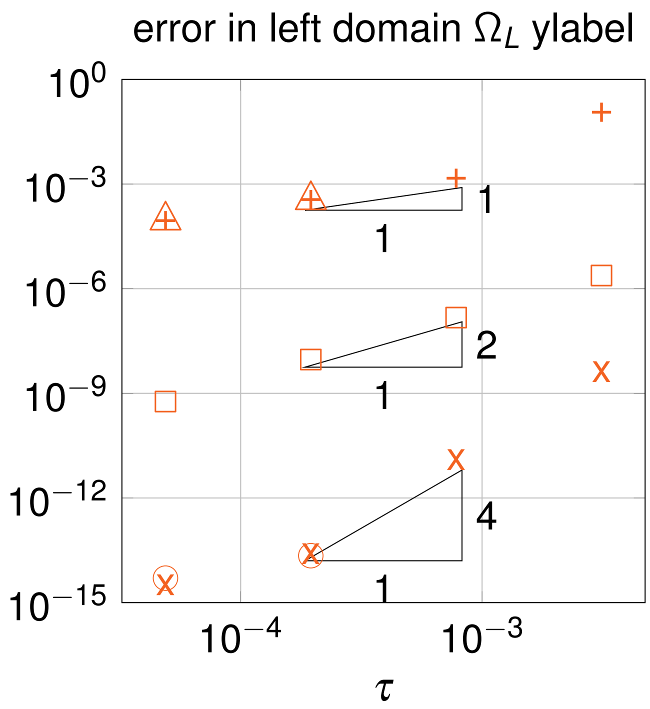
- Other interpolation methods are spline or dense output interpolation

Multi-Scale Setup



High Order and Subcycling

	scheme-solvers	time step	order	stable
○	M-RK4	$\tau_1 = \tau_2$	$\mathcal{O}(\tau^4)$	small τ_2
+	Im-TR/TR	$\tau_1 = \tau_2$	$\mathcal{O}(\tau)$	small τ_2
△	Im-RK4/RK4	$\tau_1 = \tau_2$	$\mathcal{O}(\tau)$	small τ_2
x	WR-RK4/RK4	$\tau_1 > \tau_2$	$\mathcal{O}(\tau^4)$	small τ_2
□	WR-RK4/TR	$\tau_1 = \tau_2$	$\mathcal{O}(\tau^2)$	$\forall \tau_2$

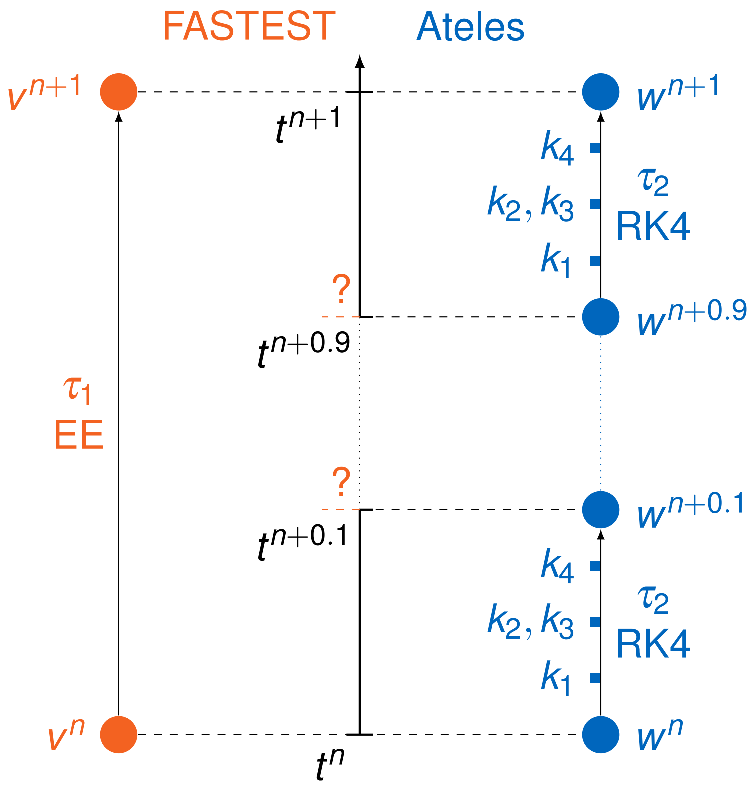


Conclusion

Partitioned Heat Transport

- ✓ introduce the partitioned heat transport equation example
- ✓ introduce classical and advanced coupling schemes
- ✓ deficits of classical explicit and implicit coupling schemes
  - order and stability degradation
- ✓ advantages of waveform relaxation coupling scheme
  - order and stability maintained
  - inhomogeneous setup
  - subcycling

Multi-Scale



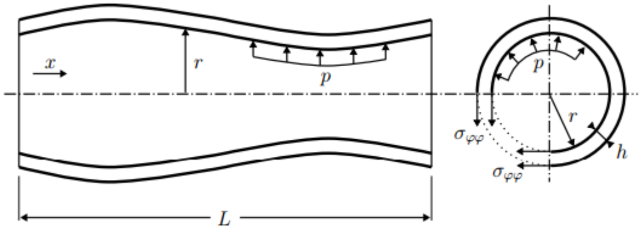
Outlook

Implementation

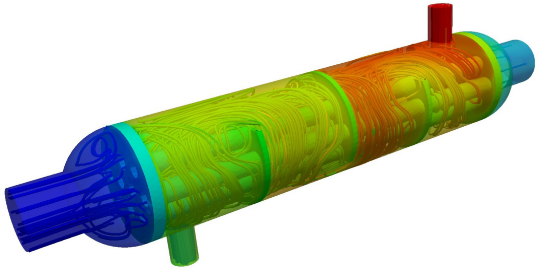
- Interpolation methods?
- Convergence of acceleration schemes
- Parallel performance

Further Tests

1D Tube[1]:



preCICE examples[2]:



[1] figure from Degroote, J., et al. (2008). Stability of a coupling technique for partitioned solvers in FSI applications. https://doi.org/10.1016/j.compstruc.2008.05.005

[2] figure from Cheung Yau, L. (2016). Conjugate Heat Transfer with the Multiphysics Coupling Library preCICE. TUM.